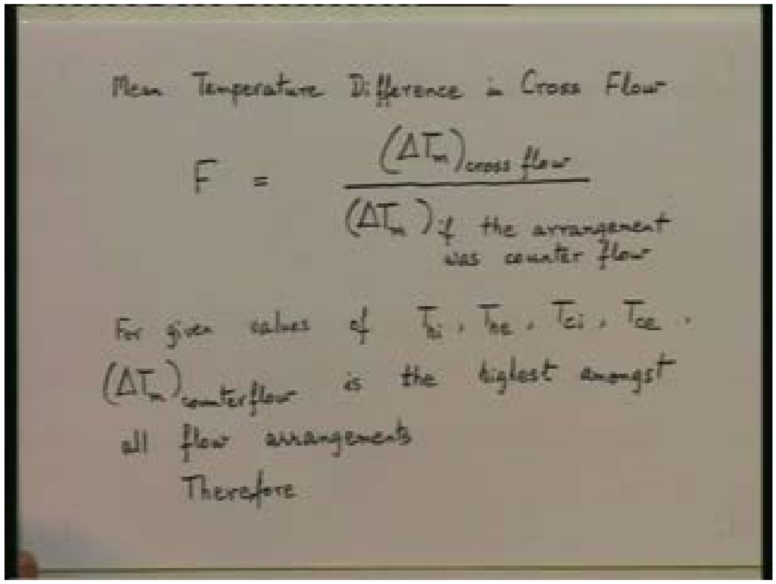
Heat and Mass Transfer
Prof. S.P. Sukhatme
Department of Mechanical Engineering
Indian Institute of Technology, Bombay
Lecture No. 27
Heat Exchangers – 3

We defined the term called mean temperature difference and we derived expressions for the mean temperature difference in parallel flow and in counter flow. You would recall that the logarithm of delta T_i delta T_e occurs in the denominator of the expressions, both the expressions. For this reason the mean temperature difference in a heat exchanger is also referred to as the logarithmic mean temperature difference and the symbol capital LMTD - all letters in capital - LMTD is used instead of delta T_m by many authors and many writers.

Then, we moved on to discussing how to obtain the mean temperature difference in cross flow. Now, you will recall that the, what emerged after discussing the temperature profiles on the hot side of the hot side fluid and the cold side was that in cross flow - unlike the parallel flow or counter flow - the temperature is not a function of one variable, the temperature is a function of 2 variables x and y. If both fluids are unmixed - temperature T_h is a function of x and y, temperature T_c is function of x and y. If both fluids are mixed, one fluid is a temperature, one fluid's temperature is a function of x, the other fluid's temperature is a function of y. And if one is mixed and the other is unmixed, the unmixed is a function of x and y whereas the mixed one is a function only of one variable. But regardless of whichever case it is, the fact is the temperature on the two sides are not function of one variable x as is the case in counter flow or cross flow but are a function of x and y. And therefore, in order to obtain the mean temperature difference it becomes necessary to perform numerical integrations to obtain expressions for the mean temperature difference. And you will also recall that towards the end, I said the mean temperature difference is given, the values are given in terms of a correction factor F and F is defined as the mean temperature difference in cross flow divided by the mean temperature difference if the arrangement had been counter flow; this is where we stopped. So, let us take off from that again.

(Refer Slide Time: 03:30)

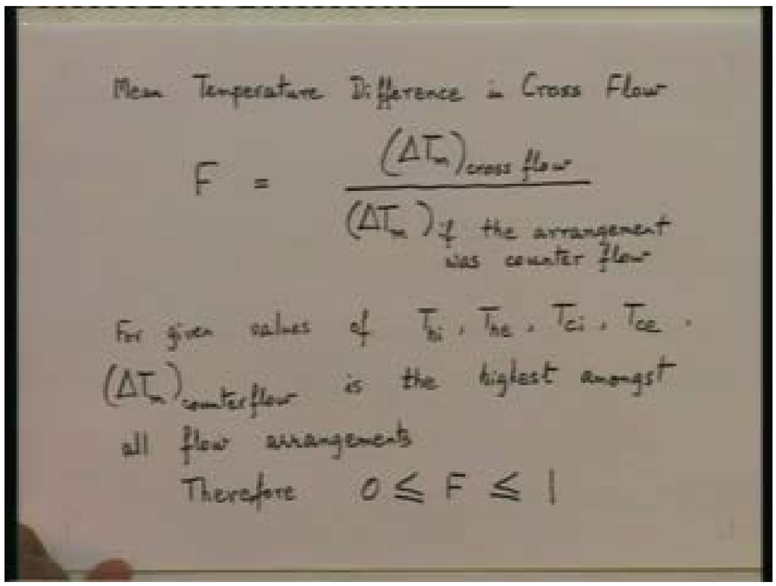


We were deriving getting results for the mean temperature difference in cross flow; we were deriving the results for this case and, towards the end, we said F - we are going to express the results in the form of correction factor F. And F is equal to the delta T_m which we were looking for - The delta T_m in cross flow, this is the quantity we are looking for, divided by the delta T_m. If the arrangement had been counter flow, delta T_m if the arrangement was counter flow and we know how to calculate the denominator and in case the fluid is unmixed on one side, you have to take the mean temperature of the fluid. The bulk mean temperature of the fluid leaving the heat exchanger and substitute that in the expression for delta T_m for obtaining the counter flow delta T_m.

Now the results for given, before I go to the actual presentation of the values of F, how they are presented, let me just make a statement. For given values of T_{hi}, T_{he}, T_{ci} and T_{ce} that is given values of the inlet and outlet temperatures on the hot side and cold side, I am making a statement - for given values of T_{hi}, T_{he}, T_{ci} and T_{ce} for the given values, the delta T_m in counter flow is the highest. That is, for given values of this 4 quantities, if I calculate delta T_m in parallel flow, if I calculate delta T_m in counter flow, in cross flow for any of the 3 cases, what I will find is the delta T_m in counter flow is the highest of all flow arrangements, amongst all flow arrangements; this is what we will find. Therefore, it follows therefore since delta T_m in counter flow is always going to be the highest, it follows that F - the correction factor F - which we have defined above here must be always in the range 0 to 1.

It must be a fair number which will range between 0 and 1; 0 less than equal to F less than or equal to 1 that is follows. And with this definition of F, if we are to calculate the q for a cross flow arrangement

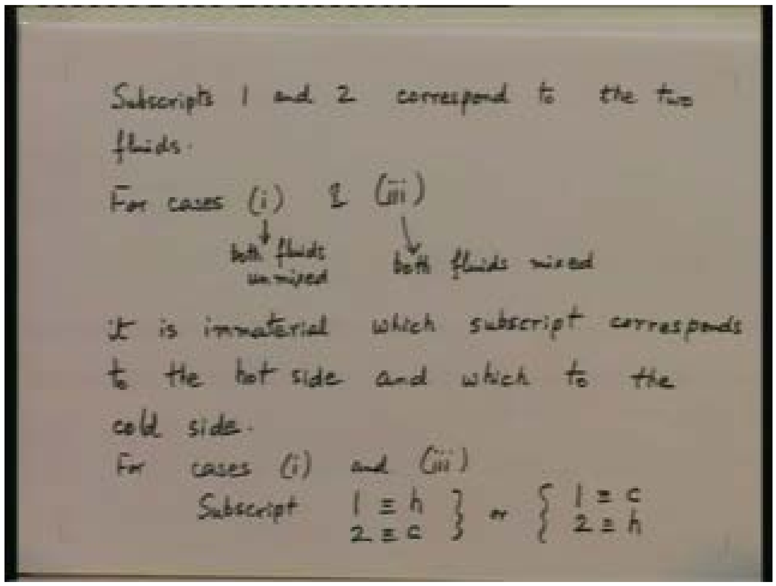
(Refer Slide Time: 07:30)



Therefore, if we are to calculate q for a cross flow arrangement, we will get q for a cross flow arrangement is equal to U A delta T_m cross flow which is with the definition of F equal to UA F delta T_m counter flow. So, when we have a cross flow problem, a situation of cross flow, we will calculate the delta T_m in counter flow; get F which has been obtained by numerical integration. Then which, I will tell you shortly how we get it from the various charts or graphs available and substitute into this as our basic performance equation.

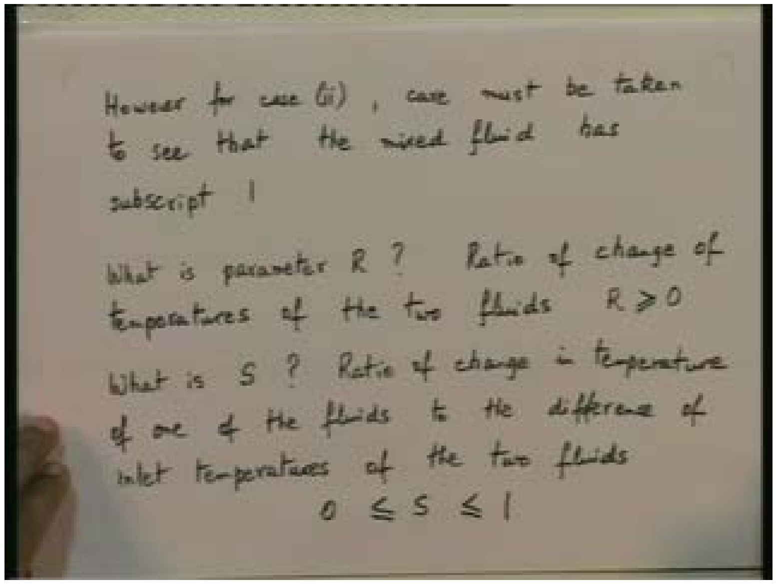
Now the quantity F - how do we get it? F based on the numerical integrations, F is plotted; the results which have been obtained by numerical integration F is plotted as a function of two parameters R and S, dimensionless parameters R and S. R is defined as, let me put down the definitions R is equal to T_{1i} minus T_{1e} divided by T_{2e} minus T_{2i} and I will tell you in a moment what is 1; what are the subscripts 1 and 2? And S is equal to T_{2e} minus T_{2i} divided by T_{1i} minus T_{2i}. Now, the subscripts 1 and 2 correspond to the 2 fluids, the subscripts 1 and 2 correspond to the 2 fluids correspond to the 2 fluids.

(Refer Slide Time: 09:54)



Subscripts 1 and 2 correspond to the two fluids. For cases 1 and 3, what are cases 1 and 3? Cases 1 and 3 is - case 1 is both fluids unmixed and case 3 is both fluids mixed. For cases 1 and 3 of cross flow, these 2 cases of cross flow it is immaterial which subscript corresponds to the hot side and which to the cold side. It doesn't matter you can take, either way the results are valid. That means for this situation, cases 1 and 3, these situation for cases 1 and 3 subscript 1 can be equal to h and subscript 2 can be equal to c. Or we may have the reverse - subscript 1 can be equal to c and subscript 2 can be equal to h. It doesn't matter which way you take it; both are equally acceptable. The results - numerical results - come out right. It is only for case 2 that you have to be a little careful; for case 2, let me put down what you do.

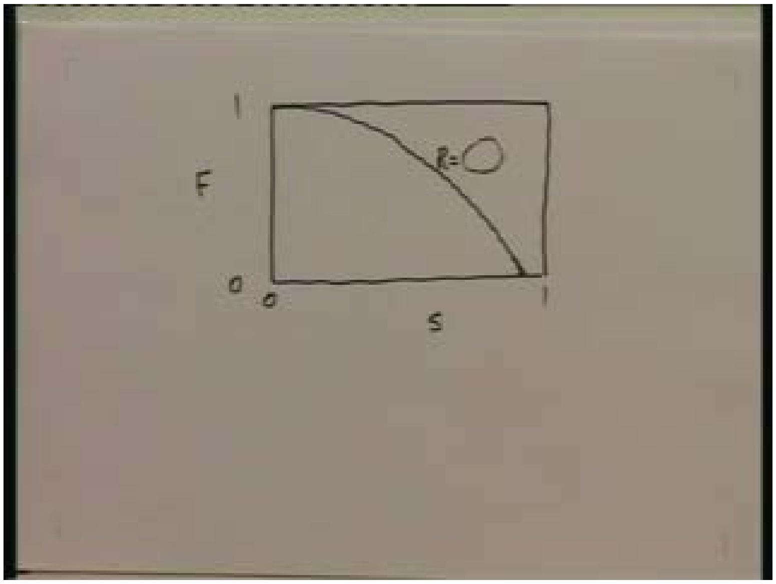
(Refer Slide Time: 12:34)



For case 2 that is one fluid mixed and the other unmixed, care has to be taken; care must be taken to see that the mixed fluid has subscript 1. The mixed fluid maybe the hot fluid or the cold fluid, it doesn’t matter which one but take care to see that the mixed fluid is given subscript 1 and the unmixed fluid automatically becomes subscript 2. That is the only precaution we have to take. So, these are 2 parameters in terms of which the values of F are found out. Now what are these parameters? What is the dimensionless parameter R? If you look at it, it is nothing but the ratio of the change of temperatures of the 2 fluids, that is what is R. And R by definition therefore will be a positive quantity but unbounded greater than or equal to 0 and the limit 0.

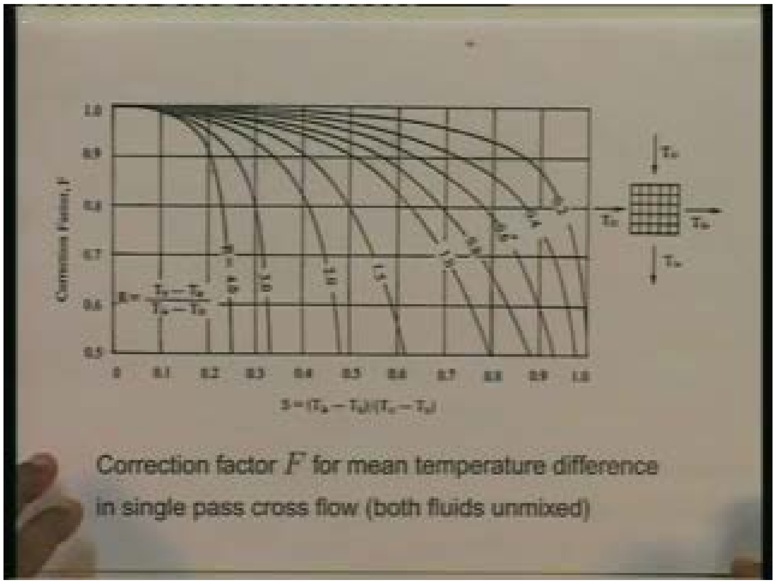
What is S? The parameter S the second dimensionless quantity - it is the ratio of change in temperature of one of the fluids, ratio of change of temperature of one of the fluids to the difference of inlet temperatures of the 2 fluids - T_{hi} minus T_{ci}, that is what it is. So, automatically by definition you can see the parameter S is going to be a number which is going to be between 0 and 1. The particular results that we get for R, the result that we get for R - if you were to plot them. First just let me show the variation; it looks something like this.

(Refer Slide Time: 15:44)



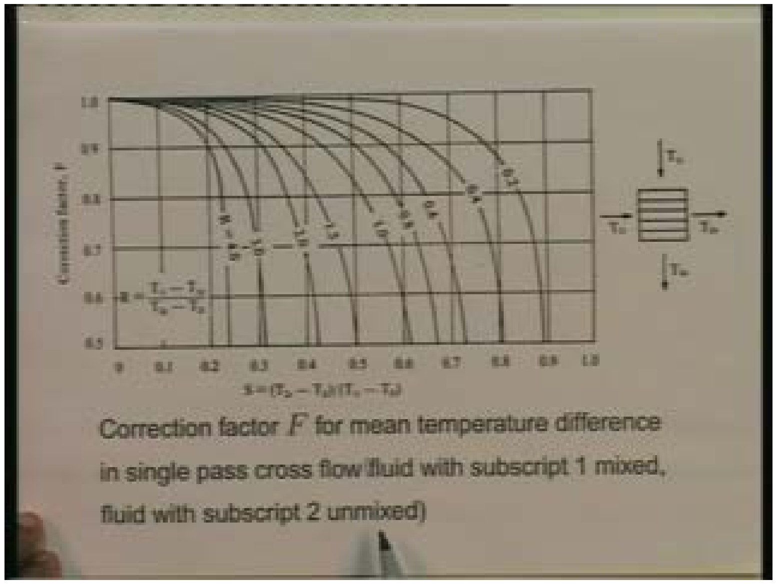
Suppose this is the results that are obtained based on the numerical integration for any of the cases 1 to 3. If I plot F against S, S can range from 0 to 1, F can also range from 0 to 1. Then, for a particular value of R a typical variation of S maybe something like this. This is some particular value of R, some specified value of R. We will get, this is the kind of variation which we get for F and these has been obtained based on the numerical integration which I mentioned to you earlier. Now, let us look at actual results which have been obtained. I am going to show you 3 charts for the 3 cases of cross flow.

(Refer Slide Time: 16:47)



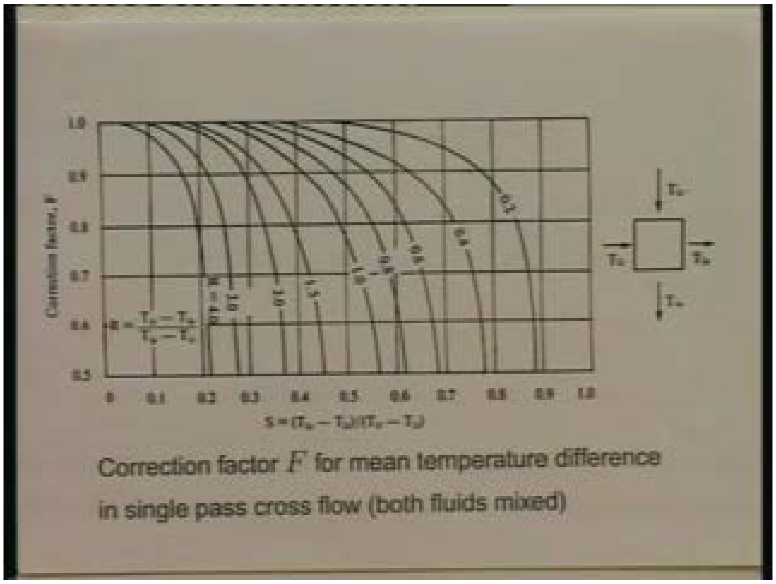
This is the chart which I am showing; you know the correction factor F for the mean temperature difference in single pass cross flow both fluids unmixed - case 1, what we have called. These are results based on the numerical integration to obtain the mean temperature difference in cross and to get the correction factor F. So, on the y axis you have the correction factor, on the x axis you have the parameter S and the different values for the different graphs are the values of R ranging here from .2 to 4. The value of R equal to 0 will be the horizontal line and the vertical line, this will correspond to R equal to 0. So, this is these are the results for the correction factor F for the case of both fluids unmixed.

(Refer Slide Time: 17:44)



Now, let us look at the next results. I am just showing these results which are to be used for actual calculations. The next figure shows the correct, let me read out, shows the correction factor F for mean temperature difference in single pass cross flow fluid with subscript 1 - mixed, fluid with subscript 2 – unmixed. The nature of the curves is of course the same. But the values are very different for the different cases. Again it is F plotted against S with R as the parameter varying from 0 or to 4. 0 is this case the vertical line that is 0 then .2, .4, .6, .8 all the way up to 4. These are typically values encountered in practice; R greater than 4 is probably not encountered very often so the values are not plotted there at all.

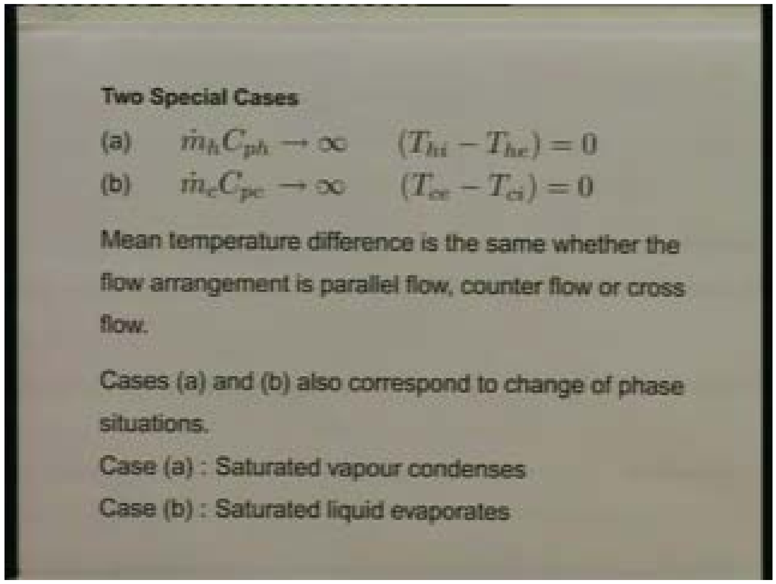
(Refer Slide Time: 18:43)



And then finally, here is the correction factor F - results for correction factor F plotted for mean temperature difference in single pass cross flow for the case of both fluids mixed; similar results but of course numerical values are different. So, depending upon which cross flow case you have, you will go to the appropriate figure and read off the values of S that is the point I want to mean.

We will do a numerical example so that you will be to use, learn how to use these graphs. Now, let us just talk briefly about some special cases before we do some numerical examples, going to do a couple of numerical examples to illustrate all these ideas. First, let us just look at two special cases; these two special cases are the following.

(Refer Slide Time: 19:30)

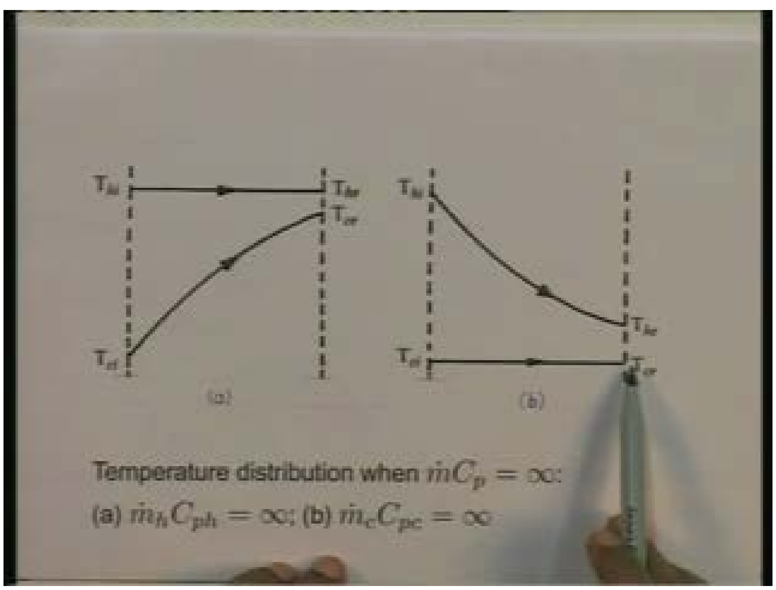


a m dot h C_{ph} tending to infinity - this is one special case. The value of the product of the flow rate and the specific heat on the hot side is very high tending to infinity or automatically it follows that q is finite and m dot h C_{ph} tends to infinity. Then, the change of temperature on the hot side of the hot side fluid must be tending to 0. b is the reverse case; the other case that is m dot c C_{pc} tends to infinity. The product of the flow rate and the heat capacity on the cold side is tending to infinity and therefore since it is tending to infinity T_{ce} minus T_{ci} is tending to 0. So, these are 2 special cases.

Now, if you were to calculate the mean temperature difference for these 2 cases that is either a or b, if you were to calculate the mean temperature difference for these 2 cases, what you will find is the value of the mean temperature difference will be the same whether the arrangement is parallel flow, counter flow or cross flow, any of the cases of cross flow. You can do that yourself and convince yourself that that is what will happen. The fluid temperature on one side doesn't change and as a result, the values of mean temperature difference come out to be the same whatever be the flow arrangement. I would like you to check it on your own and in your mind you should be able to explain why this is also happening.

Cases a and b - incidentally one way of looking at them is to say that m dot h C_{ph} is tending to infinity. The other way of looking at case a is to say it is a case in which a saturated vapor is condensing. Suppose I have a saturated vapor at some temperature T_s and it is condensing at the same temperature. Obviously, since it is saturated vapor, the temperature will not change as it changes from vapor to liquid so automatically the temperature T_s will not change on the hot side. So, case a also corresponds to a saturated vapor condensing and vice versa. Case b corresponds to a saturated liquid evaporating. So, remember case a and b can be interpreted in two ways; one interpretation is the flow rates are very high on one or the other side, the other interpretation is that case a corresponds to the case of saturated vapor condensing and case b to a saturated liquid evaporating. So, these are 2 special cases and as I said it is worth noting that the mean temperature difference is the same in these cases regardless of the flow arrangement. Now, the same idea, again I have shown in a sketch here so I will be sort of repeating things here.

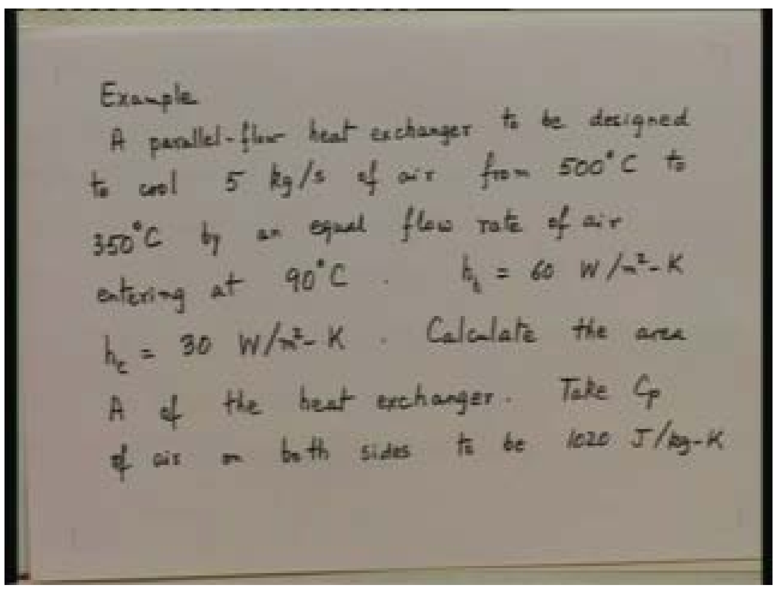
(Refer Slide Time: 22:22)



This is the figure a corresponding to m dot h C_{ph} tending to infinity; the temperature on the hot side doesn't change or this could be a vapor condensing. The temperature on the cold side is increasing. The b is the case of m dot c C_{pc} tending to infinity so the temperature on the cold side doesn't change; this could be a saturated liquid evaporating and the hot side fluid is giving up heat slowly like this. The way I have shown it, it is parallel flow but remember - even if it is counter flow all that will happen is I will get a mirror image kind of variation of T_c in the other direction. And therefore you can see why the main temperature difference is the same whether it is counter flow, cross flow or parallel flow. So, these are 2 special cases.

Now, let us do a couple of problems to illustrate all these ideas of mean temperature difference for different flow arrangements. Let us take one example - a numerical example.

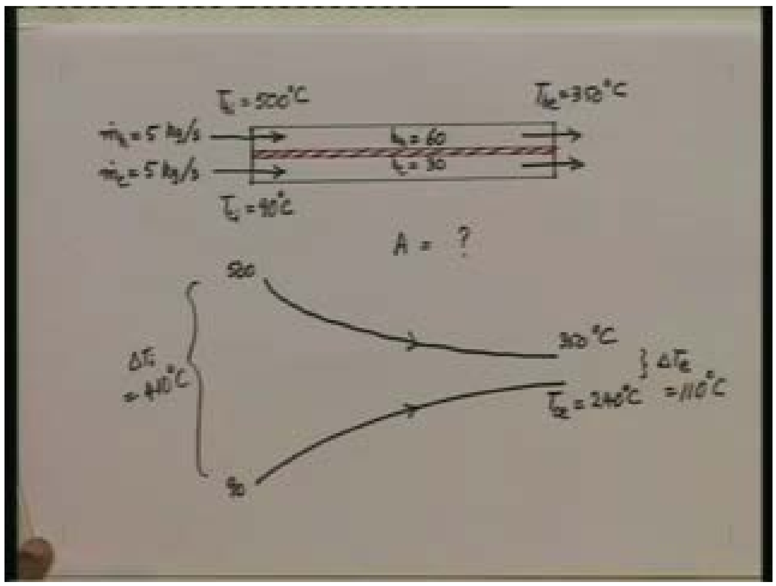
(Refer Slide Time: 23:34)



We have a parallel flow heat exchanger to be designed, a parallel flow heat exchanger to be designed to cool 5 kilograms per second of air from 500 centigrade to 350 centigrade by an equal flow rate of air entering at 90 degrees centigrade. 5 kg per second of air to be cooled from 500 to 350 on the cooling side, the same flow rate of air entering T_{ci} being 90 degree centigrade. We are given that the heat transfer coefficient on the hot side h_h is equal to 60 and on the cold side h_c is equal to 30 - these are the values of h. Calculate the area A of the heat exchanger, that is the example to be solved.

Take C_p; we need the value of C_p; take C_p of air on both sides to be 1020 joules per Kelvin per kilogram Kelvin, take this value of C_p. So, design a parallel flow heat exchanger; that means find the area A of a parallel flow heat exchanger for given conditions. Let us just draw a sketch of what we have, what we are asked to do. This is, let us say - let me draw the sketch of the heat exchanger.

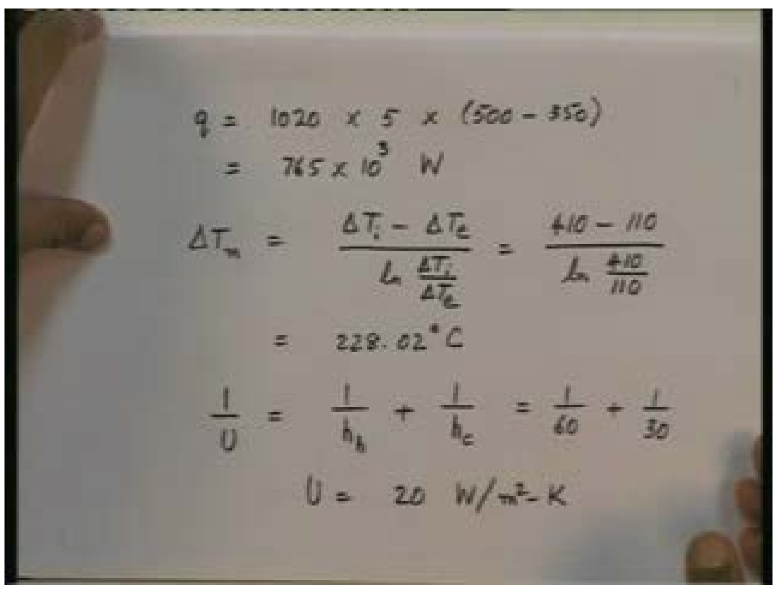
(Refer Slide Time: 26:34)



Let us say this is our heat exchanger, this is the heat transfer surface, the fluid on the hot side, it is a parallel flow heat exchanger. m dot h equal to 5 kilograms per second, m dot c is given to be the same - 5 kilograms per second. T_{hi}, you are told the inlet temperature on the hot side equal to 500 centigrade; T_{ci} equal to 90 degrees centigrade, T_{he} given to be 350 centigrade. The value of h_h heat transfer coefficient on the hot side is equal to 60 in the usual units, Watts per meter squared Kelvin, and the value of h_c on the cold side equal to 30 Watts per meter squared Kelvin. So, find the area, these are the exit points; find the area A of the heat exchanger, the configuration is parallel flow.

We can sketch the temperature profiles - on the hot side going to be something like this, on the cold side it is going to be something like this. This is 500, this is 90, this is 350. Now, the flow rates on the hot side and the cold side are equal and the C_ps are given to be equal. Therefore, it follows that the change of temperature on the hot side and the change temperature on the cold side must be equal. So, the change of temperature on the hot side is 150 so it follows that the change of temperature on the cold side must be also 150. So, T_{ce} must be equal to 240, that follows I don't have to even do any real calculation for that. So, delta T_i is equal to 410 and delta T_e is equal to 110 degree centigrade. These are temperature profiles on the 2 sides; these are the values of delta T_i and delta T_e. It is relatively a very simple problem. We have to remember we have to substitute into the expression q is equal to U A delta T_m. So, we need to get q, we need to get U, we need to get delta T_m then we will get the value of A so let us get them one by one. Let us get the value of, first, the value of q.

(Refer Slide Time: 30:44)

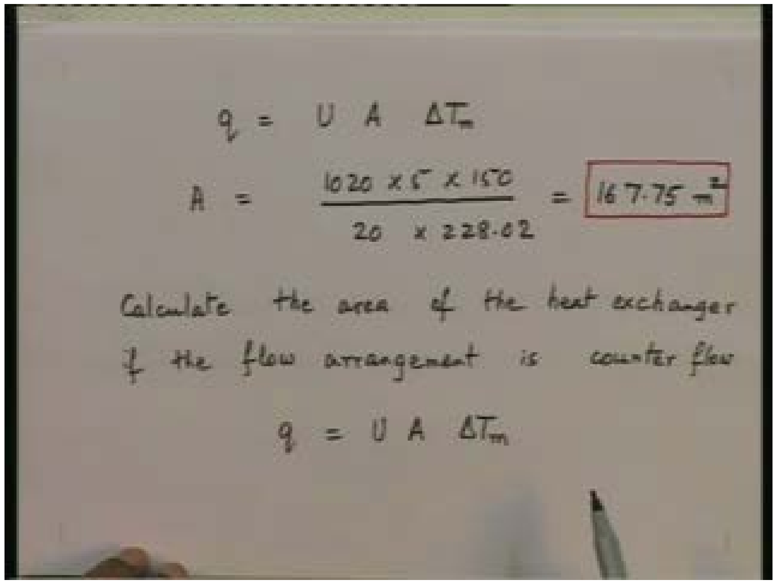


q is equal to 1020 that is the specific heat into the flow rate 5 into the change of temperature on the hot side or cold side - it doesn't matter which side you take - 500 minus 350. And that comes out to be 765 into 10 to the power of 3 Watts. This is the heat transfer rate in the heat exchanger. The value of delta T_m in parallel flow is equal to delta T_i minus delta T_e divided by log to the base e delta T_i by delta T_e. So, it is 410 minus 110 divided by logarithm to the base e 410 divided by 110 and that comes out to be 2208.02 degrees centigrade.

Now, we are given the values of h_h and h_c; we aren't told anything about fouling so we will have to make an assumption. First of all about fouling and secondly about the thermal resistance B by k - we are not told anything about that. But, notice the values of h_h and h_c are low; they are low because we have got air flowing as the heat transfer medium. So, values of h_h and h_c are bound to be of the order 50, 60, 100 something like that, not going to be in thousands with air flows. Automatically, it follows that the thermal resistance of any metal wall you can take, any typical wall, a millimeter thick, 2 millimeters thick, take a conductivity of a metal of 10, 20. You will immediately see its thermal resistance will be insignificant compared to 1 upon h_h or 1 upon h_c.

Similarly, if you look at any of the fouling factors that I have given you, you will remember those fouling factors are for a situation involving liquids. With gases fouling factors which are in the region of .30s, 130s, 230s, 4 in the usual units will be there, effect will be negligible compared to the thermal resistance offered because of the heat transfer coefficient. So, with gas flows the values of a neglecting fouling is a good assumption. Neglecting the thermal resistance of the metal wall if that data is not available is also a good assumption. And we can say in this situation 1 upon U is equal to 1 upon h_h plus 1 upon h_c. We just leave out B by k saying it is negligible and we will also not consider fouling because it is bound to be negligible compared to the values of h that we are seeing for gas flows. So, all that we get is this is equal to 1 upon 60 plus 1 upon 30 and therefore the value of U comes out to be equal to 20 Watts per meter squared Kelvin. Now substitute into our basic performance equation to get the value of A.

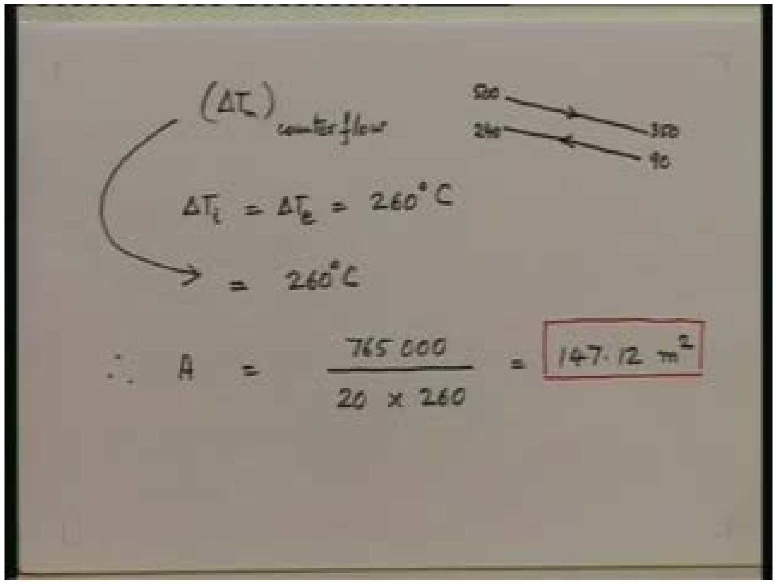
(Refer Slide Time: 34:24)



So, our basic performance equation is q is equal to U A delta T_m; in this case delta T_m in parallel flow. So, we will get A is equal to q which is 10, q which is equal to 1020 into 5 into 150 divided by A UA. U is 20 sorry U delta T_m so U is 20 and delta T_m is 2208.02. So, we get the area A of the heat exchanger to be 167.75 square meters, that is the answer to the problem - 167.75 square meters for the given data that we have got.

Now, let us say instead of a parallel flow heat exchanger, the arrangement instead of being parallel flow is counter flow. I ask you to calculate the area A of the heat exchanger if the flow arrangement is counter flow; let us do a further calculation. Calculate the area of the heat exchanger if the flow arrangement is counter flow. In this case, notice we again have to substitute into the same expression q is equal to U A delta T_m. q is not going to change here is my basic expression, q is equal to U A delta T_m. q is not going to change, U is not going to change, what is going to change is the delta T_m. So, get a new value of delta T_m for counter flow situation and again substitute into our basic equation. What is delta T_m in counter flow? Now let us calculate that.

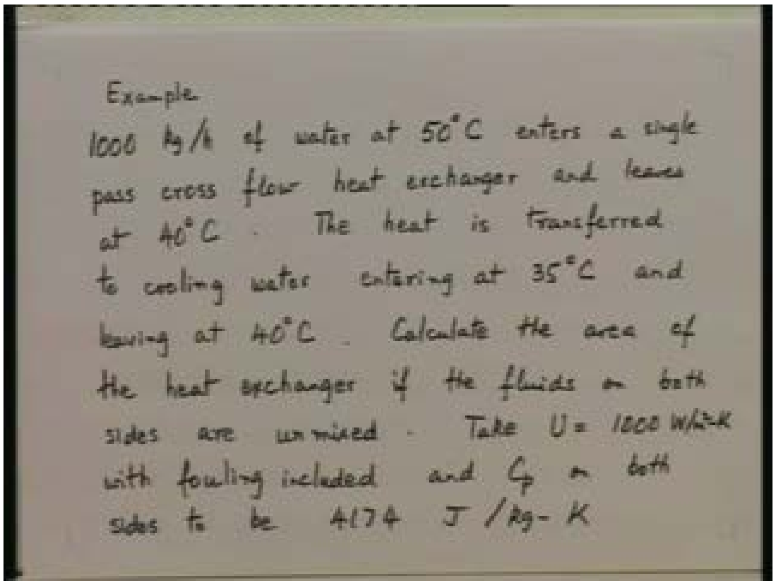
(Refer Slide Time: 36:59)



delta T_m in counter flow would be, notice this is a case of counter flow with m dot h C_{ph} equal to m dot c C_{pc}, notice that. Therefore, T in this case with m dot h C_{ph} equal to m dot c C_{pc}, the temperature profile on the 2 sides are going to be parallel lines like this. So, we are going to have 500 here, hot side fluid entering temperature, 350 leaving. Entering T_{ci} 90, leaving T_{ce} 240 and we will get the value of delta T_i and delta T_e - both delta T_i and delta T_e. Both in this case will be 260 degree centigrade. So, delta T_m counter flow will be equal to the same, delta T_m counter flow will also be equal to 260 degree centigrade. This is the case of the 2 temperature profiles being parallel to each other; this is that special case which we talked of earlier. As I said, q and U don't change therefore A will be equal to the value of q which is 765000 Watts divided by the value of U and the value of delta T_m - 260. So, we get 147.12 square meters; so this is the value for the area A, the arrangement is counter flow.

I will make a statement here without proving it that if it is a cross flow arrangement, any kind of cross flow arrangement, then you are going to get since this is the counter flow arrangement is 147.1 2 and in parallel flow you have got 167.75. A cross flow arrangement, any cross flow arrangement, would give an area between these 2 extremes; parallel flow gives the highest, count flow gives the least. You will get a value for cross flow in between these two; I am just making that as a statement. So, this is a good numerical example to illustrate how we substitute into our basic performance equation - q is equal to U A delta T_m - and calculate the area A of a heat exchanger for a given application. Now, let us do one more problem, another numerical example.

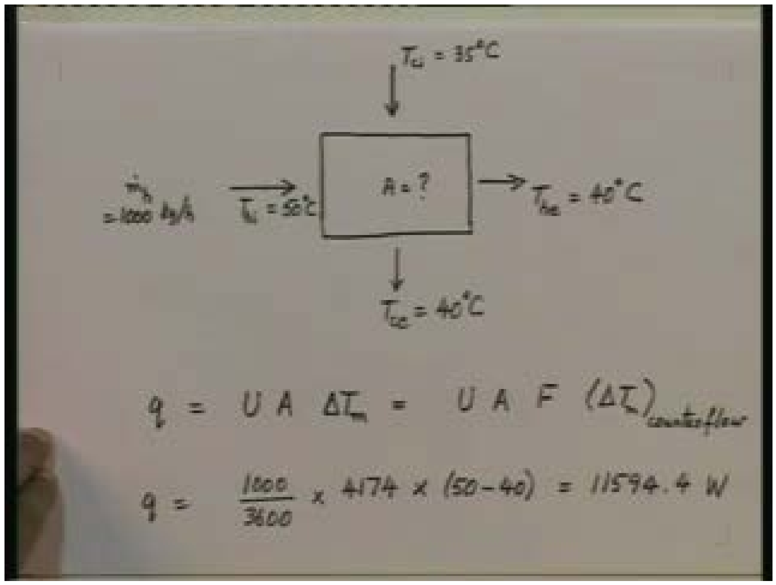
(Refer Slide Time: 39:55)



The problem we are going to do now is the following - we say 1000 kilograms per hour of water at 50 degrees centigrade enters a single pass cross flow heat exchanger and leaves at 40 degree centigrade. A hot water stream, 1000 kilogram per hour of water, enters at 50, leaves at 40. The heat from this hot water, the heat is transferred to cooling water entering at 35 centigrade and leaving at 40 degree centigrade. The cooling water which is cooling this hot water enters at 35 and leaves at 40 degree centigrade. Calculate the area of the heat exchanger if the fluids on both sides are unmixed.

Take U to be 1000 Watts per meter squared Kelvin with fouling included; that means the effects of fouling are included in this value of U. Take the value of U to be 1000 Watts per meter square Kelvin and the value of C_p on both sides, the C_p for water on both sides to be 4174 Joules per kilogram Kelvin; take the value of C_p to be the following. It is a straight forward example of calculating area, very similar to the earlier one excepting that now the flow configuration is cross flow with both fluids unmixed. Now, let us just draw a sketch.

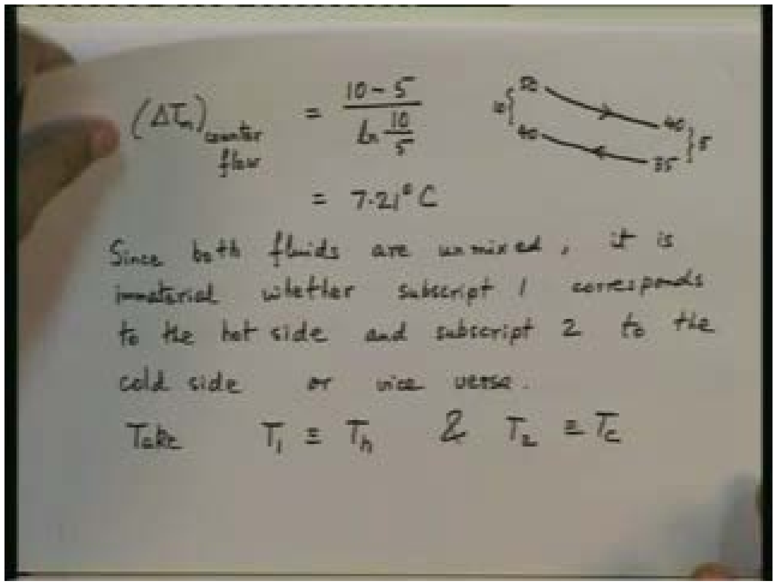
(Refer Slide Time: 43:57)



Here is let us say a schematic diagram. This is the area, the cross flow situation; let us say this is the heat transfer area A which we have to find out. This is the hot fluid entering here, m dot h equal to 1000 kg per hour, T_{hi} entering at a temperature of 50 degree centigrade, leaving at a temperature of - hot fluid leaving at a temperature of 40 degrees centigrade, cold fluid T_{ci} entering with the temperature of 35 centigrade and leaving with a temperature of 40 degrees centigrade. Calculate the area A.

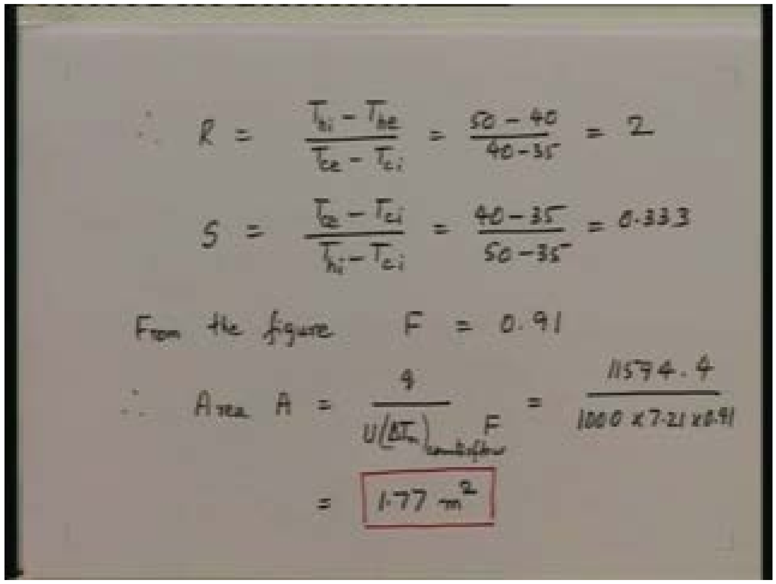
Now again our basic performance equation is q is equal to U A delta T_m. In this case, we are going to get to the correction factor F so we will write this as U A F which will come from that graphs, multiplied by delta T_m if the arrangement had been counter flow. So, if I want to get the value of A, I should get q, I should get U, I should get F; so let us get each of the quantities. First of all – q; what is q? q is nothing but the flow rate on the hot side 1000 divided by 3600 so many kilograms per second into 4174, is the value of C_p, into the change of temperature on the hot side 50 minus 40 and that is equal to 11594.4 so many Watts; so that is the value of q? What is delta T_m in counter flow? Let us get the value of delta T_m in counter flow.

(Refer Slide Time: 45:57)



delta T_m will be equal to, this is the, let us draw a sketch for. If it is a counter flow situation starting with 50 going out at 40 on the hot side, cold side entering at 35 and leaving at 40, so we have delta T_i is equal to 10 and delta T_e equal to 5. So, we have 10 minus 5 divided by logarithm 10 divided by 5 which comes out to be 7.21 degrees centigrade. Now, let us go the charts since both fluids are unmixed; since both fluids are unmixed, it is immaterial whether subscript 1 corresponds to the hot side and subscript 2 to the cold side or vice versa. It doesn't matter which you take; so we say take T_1 equal to T_h and T_2 equal to T_c. Let us take that; it doesn't matter which you take, you will get the same answers.

(Refer Slide Time: 48:15)

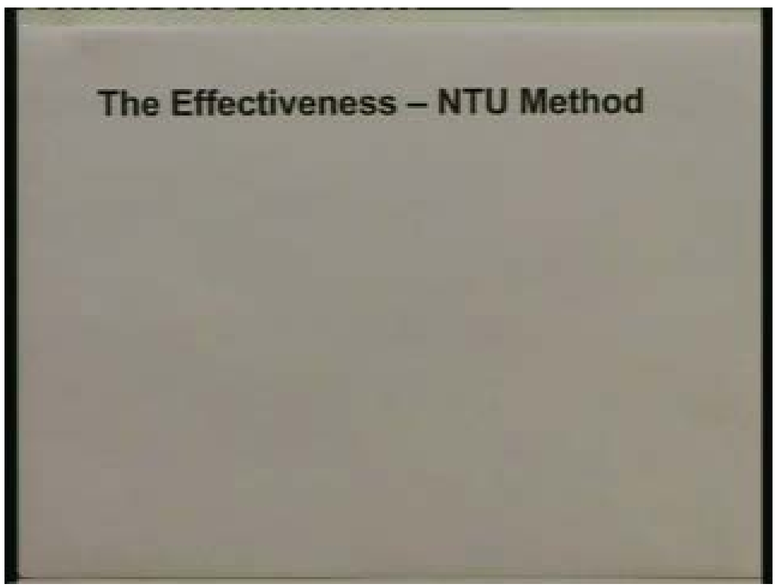


Therefore R is equal to T_{hi} minus T_{he} divided by T_{ce} minus T_{ci} which is equal to 50 minus 40 upon 40 minus 35 which is equal to 2. And S is equal to T_{ce} minus T_{ci} upon T_{hi} minus T_{ci} and that will be equal to for40 minus 35 divided by - T_{hi} minus T_{ci} - 50 minus 35 which is .333.

Now, we go to figure which I had shown you earlier for the case of the results obtained for single pass cross flow; recall I had given you some situations there. And if you recall, let us just show that figure again. Both fluids unmixed here; we have this is the situation correction, factor F for both fluids unmixed. Now, in this case R is equal to 2 and S is equal to .333 so this is the graph of R is equal to 2; here this is the graph of R is equal to 2. Take S equal to .333, somewhere here, go up here to R is equal to 2 which means somewhere out here you will go at this point, then go horizontally and read of the value of F. So, here is the value of R equal to 2; get S equal to about .3, it will come somewhere here. Go horizontally and you will get F is equal to .91 from the figure, F is equal to .91. Therefore, now last step, area A is equal to q divided by U delta T_m counter flow into F so it is equal to 11594.4 divided by the value of U is 1000. Value of delta T_m in counter flow is 7.21 and the value of F is .91 so we get area A equal to 1.77 square meters and that is the answer for this particular problem; that is the answer for this particular problem.

So now, for different flow configurations through the help of these 2 examples, we have illustrated how to obtain the area A for a given heat exchanger. Now this is one technique, the q, using the equation q is equal to U A delta T_m - this is one technique for finding out. The other technique which is also used which is the other method which is also used for finding out the effect the area A of the heat exchanger or the performance of heat exchanger is what is called as the effectiveness NTU method.

(Refer Slide Time: 51:59)



And the effectiveness NTU method is, there are certain reasons why this particular method has been developed. So, next time we will look at the effectiveness NTU method; we will discuss it, derive some relations for it and show how it is also applied for doing calculations with heat exchanges.