EXCELing with Mathematical Modeling
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Week – 02
Lecture – 07 (Basics of Excel - III)

Hello, welcome to the course EXCELing with Mathematical Modeling.

In my last two lectures, we have learned about the basics of Microsoft Excel.

Today we will be continuing that and we focus on how to solve a differential equation with the help of Microsoft Excel.

Now, when we try to solve a differential equation with Microsoft Excel, the solution will be of numerical type.

So, I will give you a numerical formula which we will be using in solving the differential equation with the help of this Microsoft Excel.

So, what I am going to derive is Euler's method of solving a first order differential equation.

So, you consider a differential equation of the form

$$\frac{dy}{dx} = f(x, y), \qquad y(x_0) = y_0.$$

Now, either you know a method of solving that or otherwise you would not be able to solving that when you have to take the help of numerical methods, if it cannot solve it analytically.

So to solve this by Euler's method let me draw and explain, say, you have curve like this.

This is your initial condition, $y(x_0) = y_0$.

So, when I say solve this differential equation, you have an initial point (x_0, y_0).

You need to find what is (x_1, y_1), (x_2, y_2), (x_3, y_3) and so on and ultimately join those points to get a graph.

So that is how your numerical methods work.

So, if this is your actual point (x_1, y_1) in the curve, then, this is your x_1 and this is your y_1.

Now, what we do is we draw a tangent from here, where it intersects this point.

So, this is our approximate y_1, your x_1 remains same, but you get an approximate y_1.

So, you calculate the slope at this point. So, if I write this as (x_1, y_1), this is the true y_1 and this is the approximate y_1.

So, if I calculate the slope, this value of the slope is

$$\frac{y_1 - y_0}{x_1 - x_0} = \frac{dy}{dx}|_{(x_0,y_0)} = f(x_0, y_0).$$

So, I can write $y_1 - y_0 = (x_1 - x_0) f(x_0, y_0)$.

If I take this distance to be h, then $(x_1 - x_0) = h$ and we get

$$y_1 = y_0 + hf(x_0, y_0).$$

So, I get an approximate formula, which will give me y_1, provided I know what is my x_0 and y_0 and this h, is called the increment.

So, if this is your x_0 then if I add $x_0 + h$ to this x_0, I get what is called x_1.

So, likewise if I want my x_2 , I have to add $x_1 + h = x_2$.

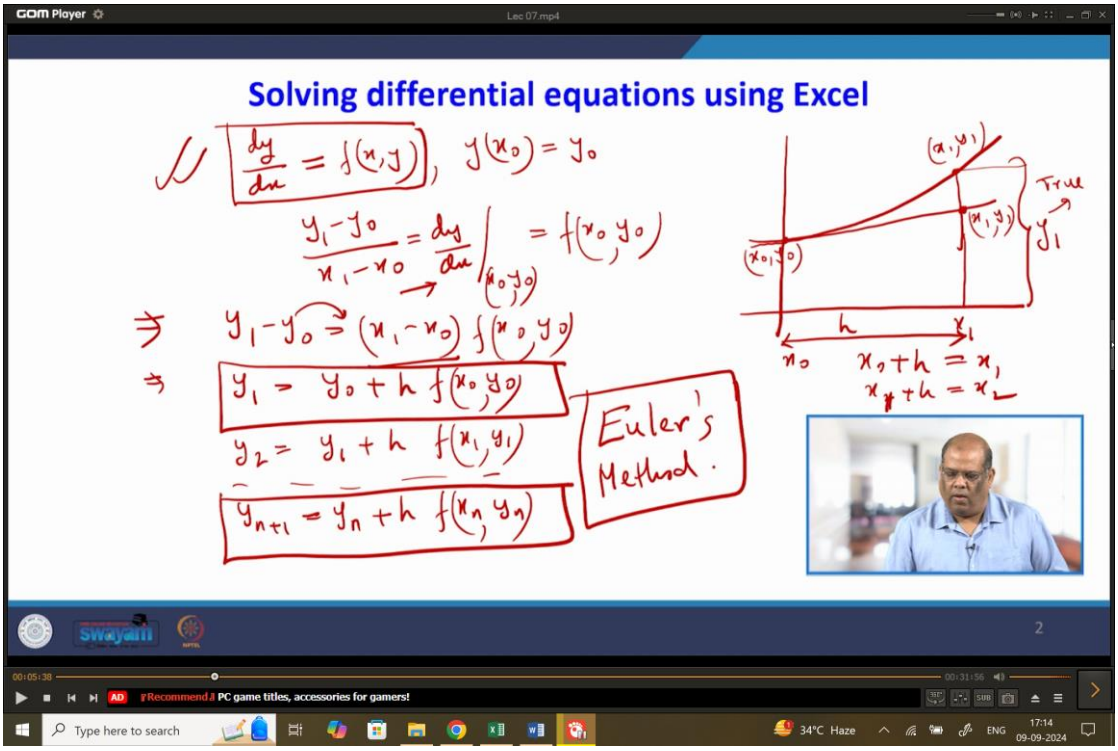
So, in the next step, the y_2, you can see from here, it will be

$$y_2 = y_1 + hf(x_1, y_1)$$

and if I generalize this

$$y_{n+1} = y_n + hf(x_n, y_n).$$

So, this formula or this method is called Euler's method of solving a differential equation of first order of this form.



Now, let us do this in Microsoft Excel, you have to just remember this formula.

Before I continue, let me rewrite the formula here for convenience.

So, if my differential equation is

$$\frac{dy}{dx} = f(x, y), \qquad y(x_0) = y_0.$$

Then the formula is, for getting y_1,

$$y_1 = y_0 + hf(x_0, y_0).$$

The example which we will take is

$$\frac{dy}{dx} = x^2 - y, \qquad y(0) = 0.5$$

and this increment $h = 0.1$

Now, let us solve this with the help of Microsoft Excel.

So, just like before you open a Microsoft Excel sheet and we will be using this formula.

So, here I put the value of $h = 0.1$.

So the equation which we will be solving is

$$\frac{dy}{dx} = x^2 - y,$$

our initial condition is at the point 0, the value of y is 0.5.

So this is say the x value, this is the y value, make them a bit large.

So here the value is 0, and here the value is 0.5.

So, this is given and this is the value of h. So, now I have to calculate these values.

Now, I give an equal increment of $h = 0.1$.

So, what I will write is this is equal to this plus this and I drag them up to this much.

As you can see that all of them become 0.1 which is not supposed to be.

So, this is 0.1, this is supposed to be 0.2, this is supposed to be 0.3 and so on.

So, where is the mistake?

The mistake is that when I write this formula this is C5 plus K3 where K3 is this value.

This is supposed to be a constant value and to make this value to be constant what you have to do is in this K3 you have to put a $ here and you have to put a $ here.

So, if you write a cell number like dollar K$3, then this will be taken as to be constant.

Otherwise what will happen, that the according to the inbuilt system here this will tend to take all these values in the next steps, and these values are zeros, basically there is no entry.

So, that is why you have to be careful when you want to take this as a constant value here you have to put a $ in between.

So now if I drag from here to here

You see now the values changes, there is an increment of 0.1.

Now I have to calculate this value, this is equal to, so the formula is

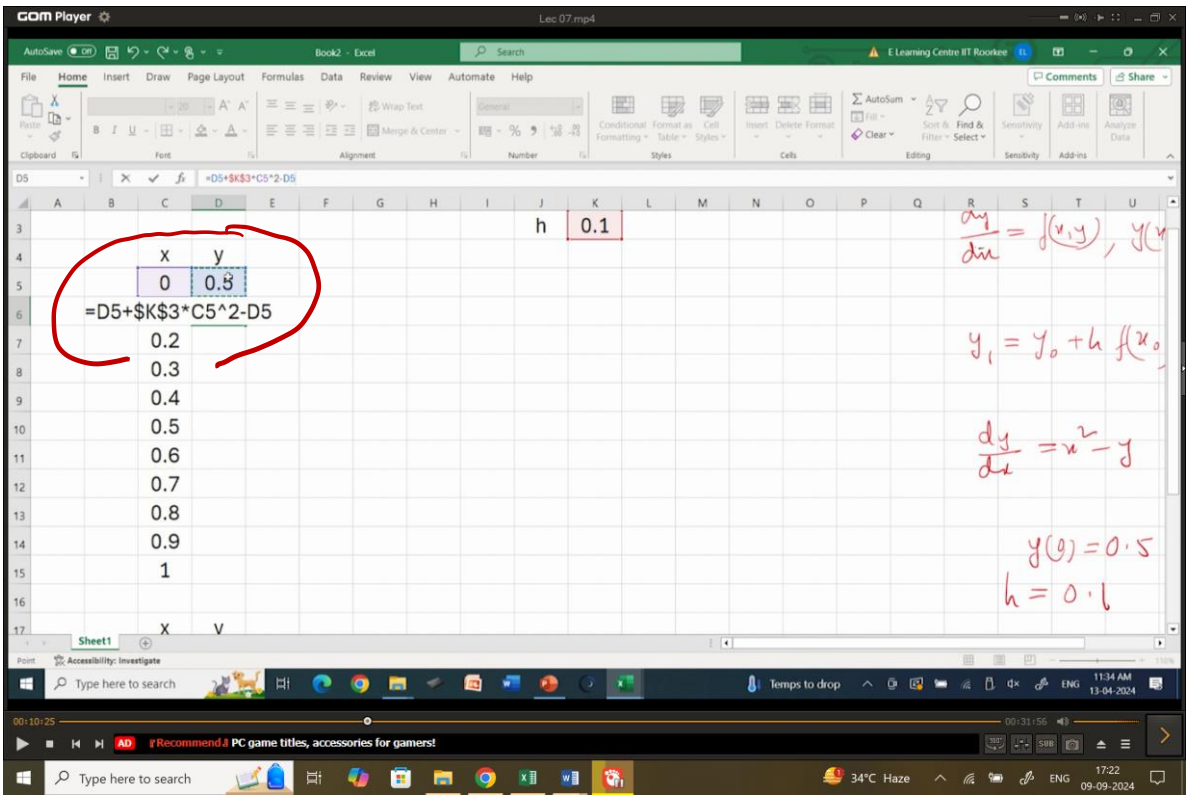
$$y_1 = y_0 + hf(x_0, y_0).$$

So, this is your $f(x, y)$. as you can see from here.

If I compare this $f(x, y) = x^2 - y$. So, this becomes $f(x_0, y_0) = {x_0}^2 - y_0$, and my formula is complete, that is,

$$y_1 = y_0 + hf(x_0, y_0),$$

here , $y_0 = 0.5$ and $h = 0.1$.



So, what is the whatever may be the functional form in this case it is $f(x_0, y_0) = {x_0}^2 - y_0$, you just put the initial values x_0 and y_0 here.

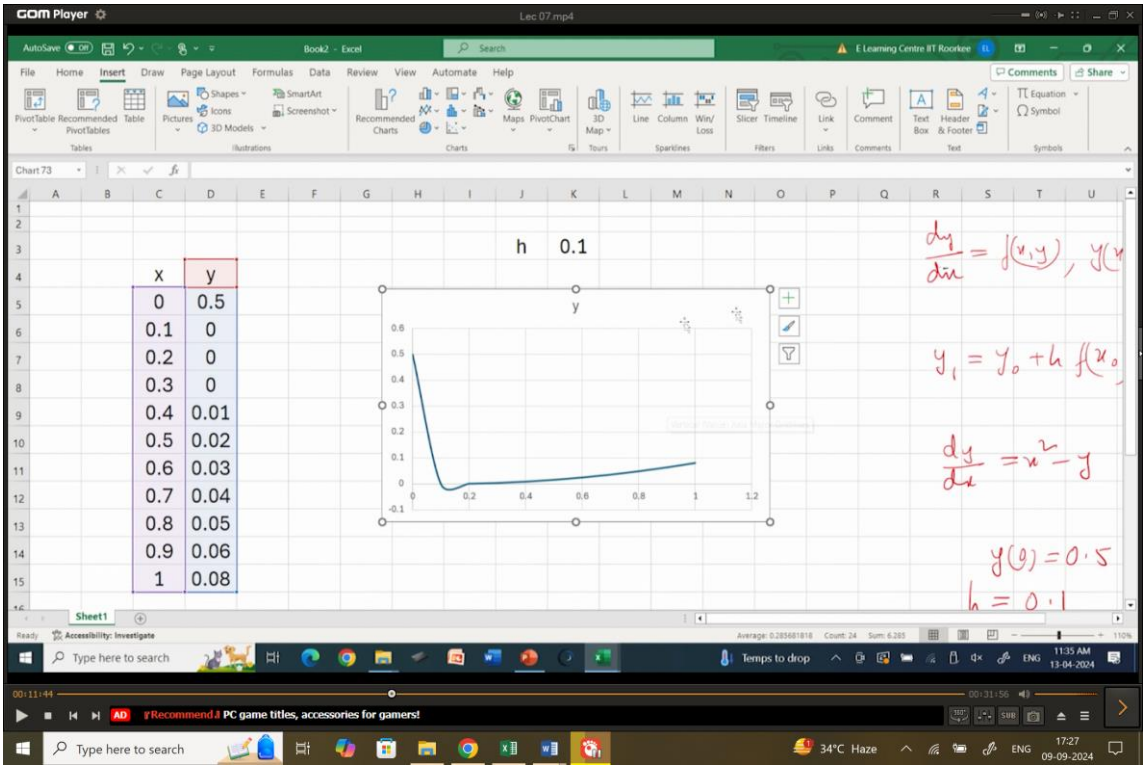
So, in this case $x_0 = 0$. So, 0^2 which is lying in this cell and (– y) which lies here.

So, please note and you have already observed that you have to put the formula in one cell only and then the rest of the cell follows the same formula when you drag.

So, if I press enter this becomes 0 and I drag say up to this. So, you get values.

Now, I have to plot them. So, you take the x value, y value, select up to this much, go to insert, go to the plots and I plot this value.

Either you can choose along with the points or if you want you can just make it as a smooth, it is totally up to you.



Now, I will level them a bit, so here is the plus sign, the moment is plus sign is here, I do not like the grid lines, so I will remove the grid lines, if you want the grid lines you can keep.

The chart title is there, the axis title I want, and the legends I want, what does this graph mean actually.

So, the axis title I just type it here. I put, this is x value, here I put y value, to keep the same format and here I can change, something like Growth, some title.

Now, if I want to change this legend, this one.

So, what I have to do is and this is important, you select this curve as you have selected.

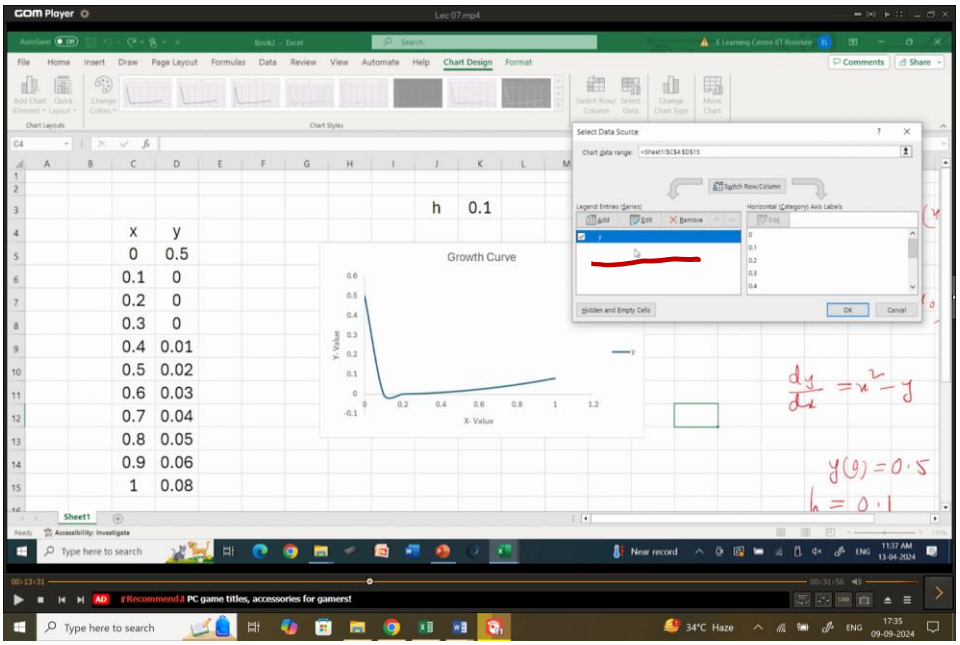
If I select outside, this is gone. If I want to select and you can see that this has been selected.

Go to chart design, and then select data.

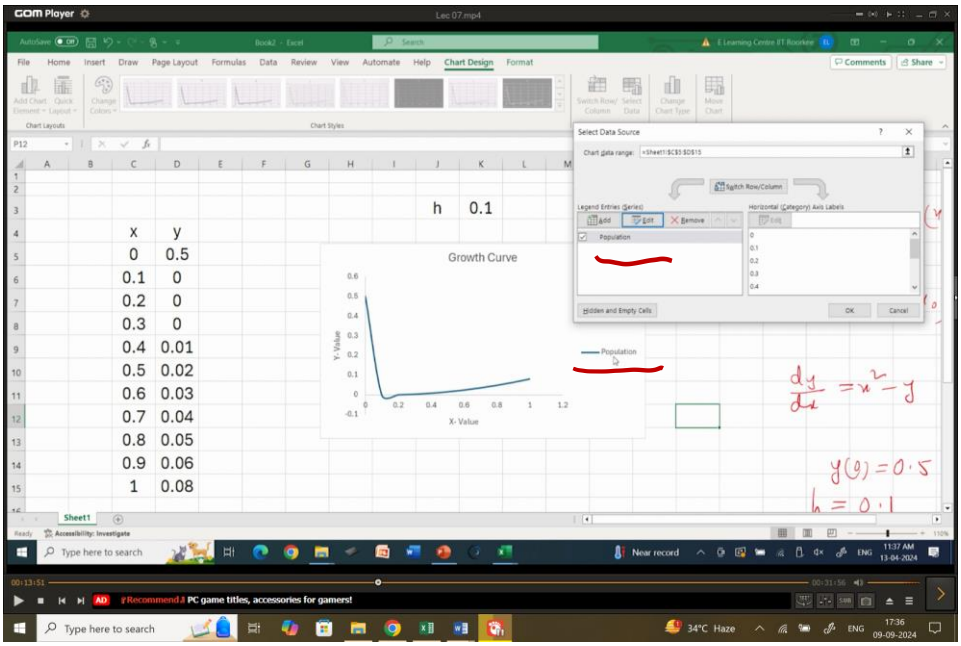
The moment you select data you will see here this y has come here.

So this is the legend.

You have to click this y and go to this edit. So here something is there.



You remove this and you type here say whatever you want to type say let me type the word population. And, if I click okay, you see that this y changes now to population.



So, one more time, if you want to change this legend, you have to go to you select this whole curve, you go to select data, you go to this population and click edit.

The moment you click edit, this has come.

So, you replace this by the name, say I put, species and I click okay. So, see this changes to species.

So, that is how you know, decorate your curve, you put the essential information, that is, the axis, the title of the chart and whatever is shown in the graph.

So, this is solving a first order differential equation using Microsoft Excel.

So let me write another example here.

Solve

$$\frac{dy}{dx} = 1 + 3\cos\left(5\sqrt{x} - 9\right), \qquad y(0) = 3000.$$

So I put my $h = 0.01$, and say, the time, I put a restriction, say, $0 \le t \le 10$.

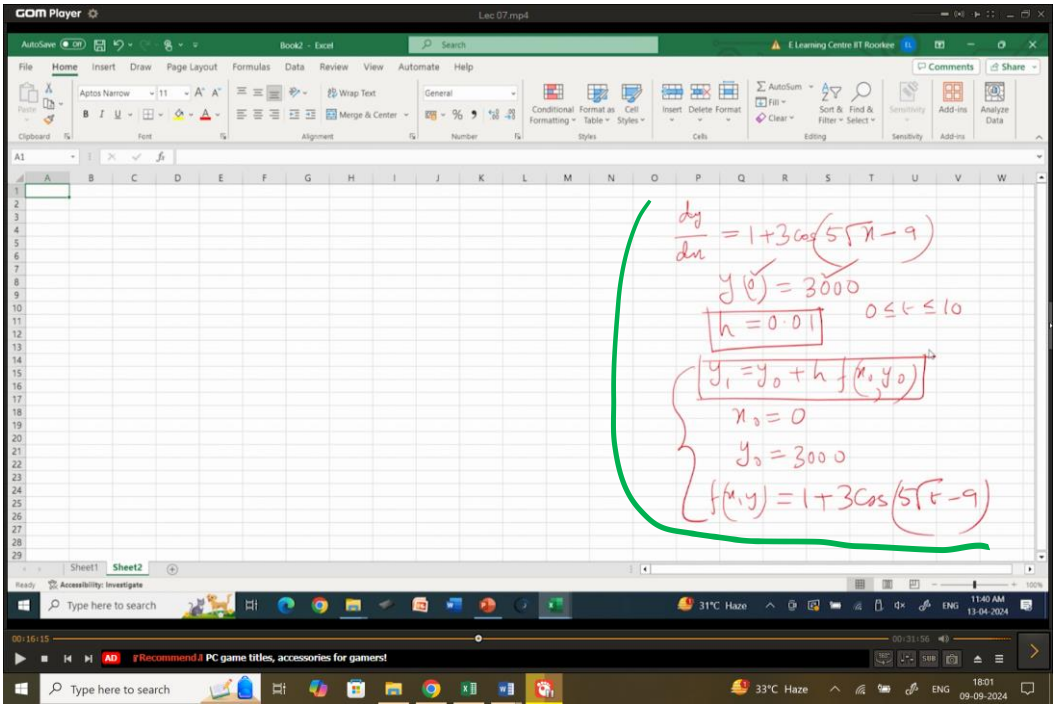
So for the benefit, I write the formula also, that your

$$y_1 = y_0 + hf(x_0, y_0),$$

So, the h=0.01 value is already there, your $x_0 = 0$ and your $y_0 = 3000$ and your

$$f(x, y) = 1 + 3\cos\left(5\sqrt{x} - 9\right)$$

So, everything is there and you just use the formula.



So, for practice let us solve this again, with the help of Euler's method using Microsoft Excel.

So, I put $h = 0.01$, that is your step 1.

The next step is the x values and the y values.

So, this value $x_0 = 0$, this value $y_0 = 3000$.

If I want to calculate this I put " = ", that is, $x_0 + h$, which is this, and as explained before, I have to make this a constant value. So, I put a $ here and I put a $ here.

Enter and I get this. So, I drag them. So, a little more.

So I can go from 0 to 10, so I choose some number.

Now to calculate the value of y, I have to use this formula, that is,

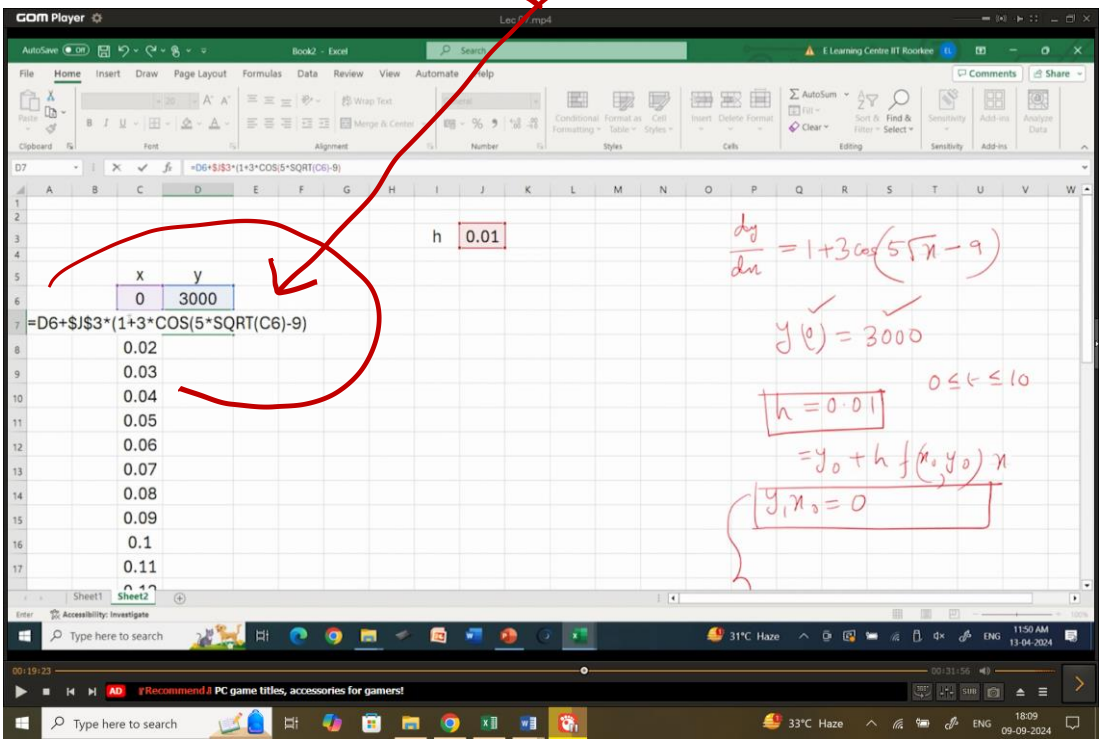
$$y_1 = y_0 + hf(x_0, y_0),$$

Now this one which will be equal to $y_0 + hf(x_0, y_0)$, I have to make this h, a constant. So, I put a $ here and I put a $ here.

So, this is $y_0 + \$h\$ * f(x_0, y_0)$.

So, this is my function $f(x, y) = 1 + 3\cos(5\sqrt{x} - 9)$. So, I have to put x_0 and y_0 the values here to calculate.

The function is $f(x_0, y_0) = 1 + 3\cos(5\sqrt{x_0} - 9)$



Let us see this is the first bracket, this is the cosine bracket, and ultimately the final bracket, and it give me a number.

So, then as you drag this, say, up to this much, and it gives some values.

So, now I have to plot them, I go to insert, I go to this curve and I plot. So you get a curve here.

If you want to change this, say you want to change the steps because you feel that these values are quite close.

So, if I change this number h= 0.1 and just click it outside, you see everything changes. So these all values have been calculated.

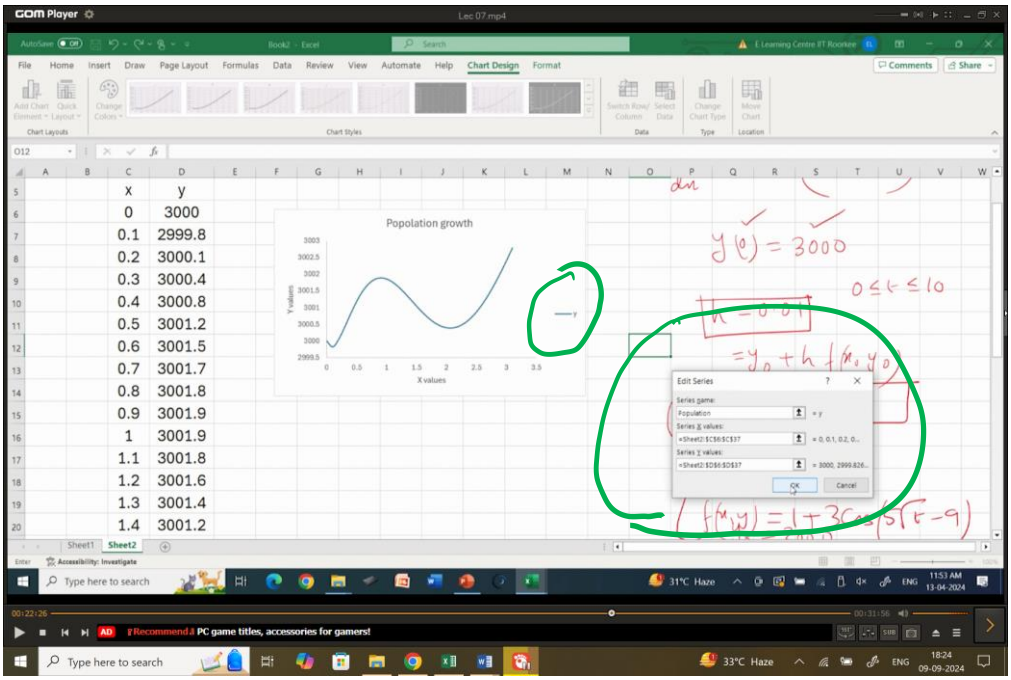
So when you want to change any data, this will be automatically be applied to all the cells and to the curve.

So, if I take it is to be h=0.1, then I get a lot of more dynamics. But, if it is h=0.01, then all I have to do is I have to calculate more values. So you get this curve.

And as usual, if you want to label it, so I do not like the grid lines, I remove it.

I like the legend. I have the chart title. I want the axis title.

So, as usual, you can type your axis title here, say, x value, here it is y value, sorry, as usual you can type the chart title population growth and if you want to change this again you have to select this curve go to chart design, select this data, select this y, go to edit and edit this and I write, say, population.



x	y
0	3000
0.1	2999.8
0.2	3000.1
0.3	3000.4
0.4	3000.8
0.5	3001.2
0.6	3001.5
0.7	3001.7
0.8	3001.8
0.9	3001.9
1	3001.9
1.1	3001.8
1.2	3001.6
1.3	3001.4
1.4	3001.2

The moment you click okay; you will see that this has changed to population.

So, that is how we solve our first order differential equation with the help of Microsoft Excel.

Our next step is to solve a system of differential equation first order.

So, for which I will explain the theory quickly.

It is just the extension of the formula.

So, here we now solve a system of differential equation, system of first order differential equation of the form

$$\frac{dx}{dt} = f_1(x,y), \frac{dy}{dt} = f_2(x,y), \quad x(t_0) = x_0, \;\; y(t_0) = y_0 \, .$$

If we use the same Euler's formula, we just extend it and it will be of the form

$$x_1 = x_0 + hf_1(x_0, y_0),$$
$$y_1 = y_0 + hf_2(x_0, y_0).$$

So, with this, as the formula for a system of differential equation, in this case two, we now solve the example

$$\frac{dx}{dt} = (10 - x - y)\, x,$$
$$\frac{dy}{dt} = (15 - x - 3y)\, y \quad x(0) = 4, \;\; y(0) = 4 \, .$$

and we take the increment h =0.1.

This is the range for $0 \leq t \leq 7$, you can keep your t values in between.

So, let us solve this using Microsoft Excel.

Let me rewrite the problem here. So, you have to solve

$$\frac{dx}{dt} = (10 - x - y)x,$$
$$\frac{dy}{dt} = (15 - x - 3y)y \quad x(0) = 4, \;\; y(0) = 4 \, .$$

and your $0 \leq t \leq 7$ and h = 0.1.

Let us solve this.

So, as usual, I put the value of h = 0.1.

This is t, this is x, this is y. Make them a bit large, center them.

So, this value is t = 0, this value is x = 4 and this value is y = 4.

So, I give the increment, this is equal to, this value plus the increment in h.

I have to make this a constant, you put a $, enter, you get the increment, you drag up to few cells and you get the increment.

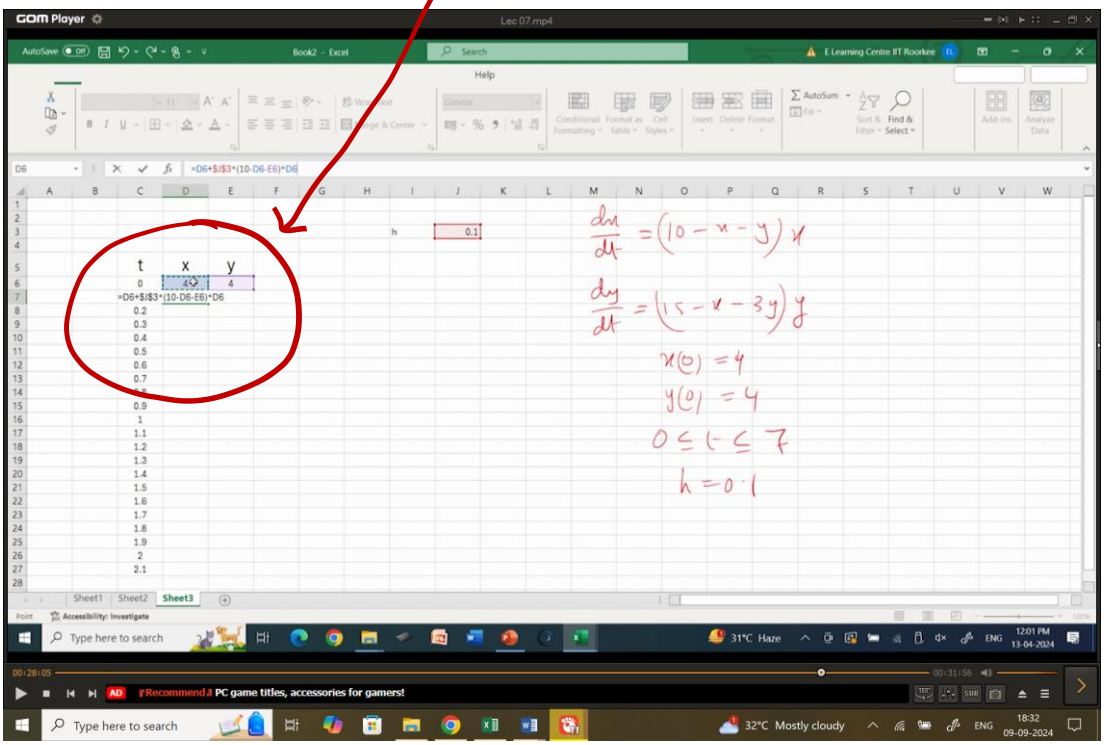
Though I am supposed to go up to 7, but you get the point that I can drag this up to 7 also.

Now to calculate this x, this is equal to

$$x_1 = x_0 + hf_1(x_0, y_0),$$

$$y_1 = y_0 + hf_2(x_0, y_0),$$

So, this is your $x_1 = x_0 + \$h\$ * (10 - x_0 - y_0)x_0$



and enter.

So, you get this value, you drag it up to here.

Again here, this is, $y_1 = y_0 + \$h\$ * (15 - x_0 - 3y_0)y_0$

Let us just quickly check this, yeah this is fine.

So, now let us plot this, go to insert, click this and this choice.

So, now you get this curve, as usual, if you want to level it, you have this axis title, you have the legend, you have the axis, you have the chart title.

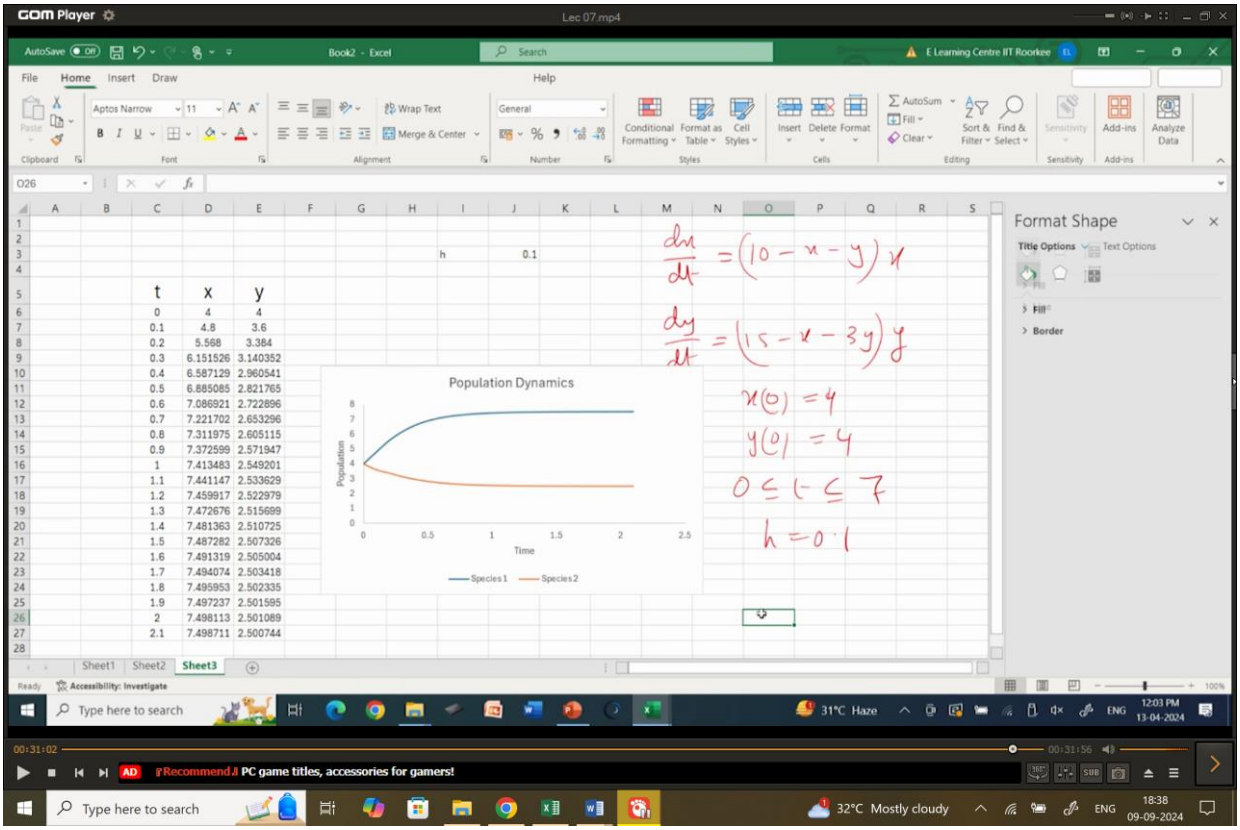
So, again if you want to change this, you just click this, go to chart design, select data, series 1, edit and you type as species 1. And, this becomes species 1.

You click this, go to edit and type species 2. This becomes species 2.

This you type as time.

This you type as population and you can change the title to population dynamics and you get this curve.

So, both of them starts with the same initial condition and reach to certain values.



So, now you have an understanding that how you will be able to solve system of equation or a single equation with the help of Microsoft Excel and we will be using this to solve our models which we are going to study in our future classes.

Till then bye-bye.