EXCELing with Mathematical Modeling
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Lecture – 43 (Economic Model)

Hello, welcome to the course EXCELing with Mathematical Modelling.

Today, we will be talking about an economic model namely Harrod model.

So, Roy F. Harrod, he introduced the concept of this warranted growth, the natural growth and the actual growth.

So, the warranted growth rate is the growth rate at which all savings is absorbed into investments.

So, he has three variables defined.

The first one let us denote by G_n which is called gross domestic product GDP or national income.

So, we use the word say national income.

Then you have savings (S_n) and you have investment (I_n).

Savings of the people, investment of the people.

Now, the first assumption.

Assumption 1:

So, the first assumption is in a country people's savings depends directly on national income. So, the savings depends directly on national income which will give you

$$S_n \propto G_n$$

$$\Rightarrow S_n = a\, G_n \; (a > 0)$$

Assumption 2:

Investment made by the people depends on the difference between the national income of the current year and the previous year, again depends directly.

So, your investment that will depend directly on the difference of the national income of the current year and the previous year.

So, this will give you

$$I_n \propto \; (G_n - G_{n-1})$$

$$\Rightarrow I_n = b\,(G_n - G_{n-1}), \qquad b > a$$

Assumption 3:

All savings made by the people are invested which means

$$S_n = I_n$$

So, we have these three assumptions of the Harrod model.

Now, let us see what it tells about the national income.

So, you have

$$S_n = a\, G_n$$

$$I_n = b\,(G_n - G_{n-1})$$

$$S_n = I_n$$

So let us start say equation 1, this is equation 2 and this is equation 3.

So from 2, I get

$$I_n = b\,(G_n - G_{n-1})$$

Now since

$$I_n = S_n$$

This is replaced by S_n using 3.

$$S_n = b\,(G_n - G_{n-1})$$

And we have

$$S_n = a\, G_n$$

So, it is

$$\Rightarrow a\, G_n = b\,(G_n - G_{n-1}) \Rightarrow (b-a)\, G_n = b\, G_{n-1}$$

$$\Rightarrow\; G_n = \frac{b}{b-a}\, G_{n-1}$$

So, we get a relation between the national income which depends on the previous year's income, previous year's national income.

And you can simplify this a bit.

So,

$$G_n = \frac{b}{b-a} G_{n-1}$$

$$G_{n-1} = \frac{b}{b-a} G_{n-2}$$

$$------$$

$$G_n = \frac{b}{b-a} G_{n-1}$$

$$= \left(\frac{b}{b-a}\right)\left(\frac{b}{b-a}\right) G_{n-2}$$

$$= \left(\frac{b}{b-a}\right)\left(\frac{b}{b-a}\right)\left(\frac{b}{b-a}\right) G_{n-3}$$

$$= \left(\frac{b}{b-a}\right)^3 G_{n-3}$$

so if I replace or if I go till the n terms this will be n and this is

$$G_n = \left(\frac{b}{b-a}\right)^n G_0$$

So If I replace this by some k,

$$G_n = k^n G_0, \qquad \text{where } k = \frac{b}{b-a}$$

Let us now see the dynamics of this national income for this various values of k.

And we will do it numerically for which we will be using Microsoft Excel.

So, I already have it opened.

So, here g n plus 1 is equal to g 0 b by b minus a whole to the power n. I replace this by k and I get this is g 0 k to the power n.

So I put the initial values say 10 to each of them we have 4 cases to discuss which is the values of k 1.5, 0.5, minus 0.5 and minus 1.5.

So the very first case we take where your k is greater than 1 and I take the value to be 1.5.

and this is equal to so if I put n equal to 0 I get the next one g1 which is g0 which is 10 here multiplied by k which is a constant and whole to the power n equal to 0.

So, this value is a constant, this value is also a constant and we calculate till 10 of the values.

If I want to plot them, I will get something like this and let me name the chart tile So, this is the case where k is greater than 1.

So, I will explain this curve later on let me first generate them.

Now, let us k lies between 0 and 1, which is 0.5.

So, this is equal to again the initial value multiplied by k to the power n. These two values are constants and I drag them.

So, now if I want to solve them first I highlight this then I press control and click this then I press shift and click the down cursor.

So, these two columns are highlighted go to insert go to the chart and draw this.

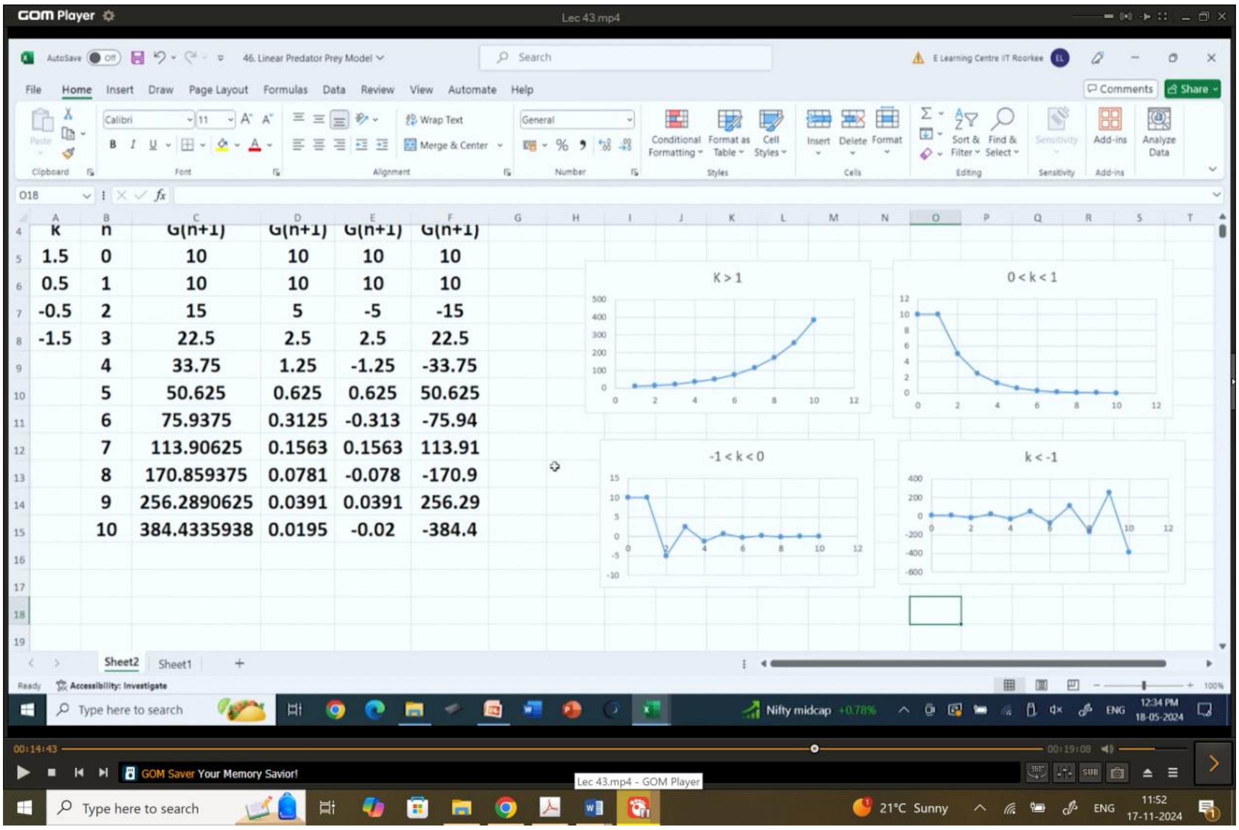
So, this is the case where 0 less than k less than 1.

Third one is say minus 1 less than k less than 0.

So, again the initial value multiplied by k is constant to the power 0.

And again if I plot this I highlight them press control go here press shift and down.

So, insert go to the chart and this some sort of oscillatory behavior.



So, here it is minus 1 less than k less than 0 and the last one when k is less than minus 1.

So, again this is equal to the initial value multiplied by k to the power n this is also a constant and this is also a constant.

So, I again choose this 10 values, press control, press shift and then down cursor value, these two columns are selected, go to insert, go to charts, so oscillatory behavior but some sort of increasing oscillation.

So, this is the case where your key is less than minus 1.

So, let us discuss them in terms of the Harrod model.

So, when k is greater than 1, then what happens?

So, as you can see when k is greater than 1, your g n it increases without limit.

So, it is just increasing without limit.

and it becomes an ever increasing amount as your time passes.

So that is the case when your k is greater than 1 and your g n was b by b minus a whole to the power n into g 0 which we have replaced by k g 0 where k is equal to b by b minus a. When your k lies between 0 and 1,
then you see that your g n will decrease over time and ever increasing it is decreasing as the time passes.

When your k lies between minus 1 and 0, you get that your g n is oscillating around this baseline 0.
And as time passes, this oscillators becomes a damp one and approaches the value 0.

And finally, when your k is less than minus 1, then what you get is an explosive kind of oscillation around the 0 baseline with increasing oscillations.

Now, what happens when your k is equal to 1?

So, if your k is equal to 1, then b by b minus a equal to 1 which implies b equal to b minus a. So, your a equal to 0.

So, if your a equal to 0, your g n becomes b by b whole to the power n 0 and this gives you only g 0.

So, your g n will be equal to g 0 for all future time and when your k is equal to minus 1, then your b by b minus a equal to minus 1, this will give b equal to minus b plus a or 2b is equal to a. So, if you substitute it here, you will get your g n is equal to, so a is 2b, so b by b minus 2b whole to the power n into g 0.

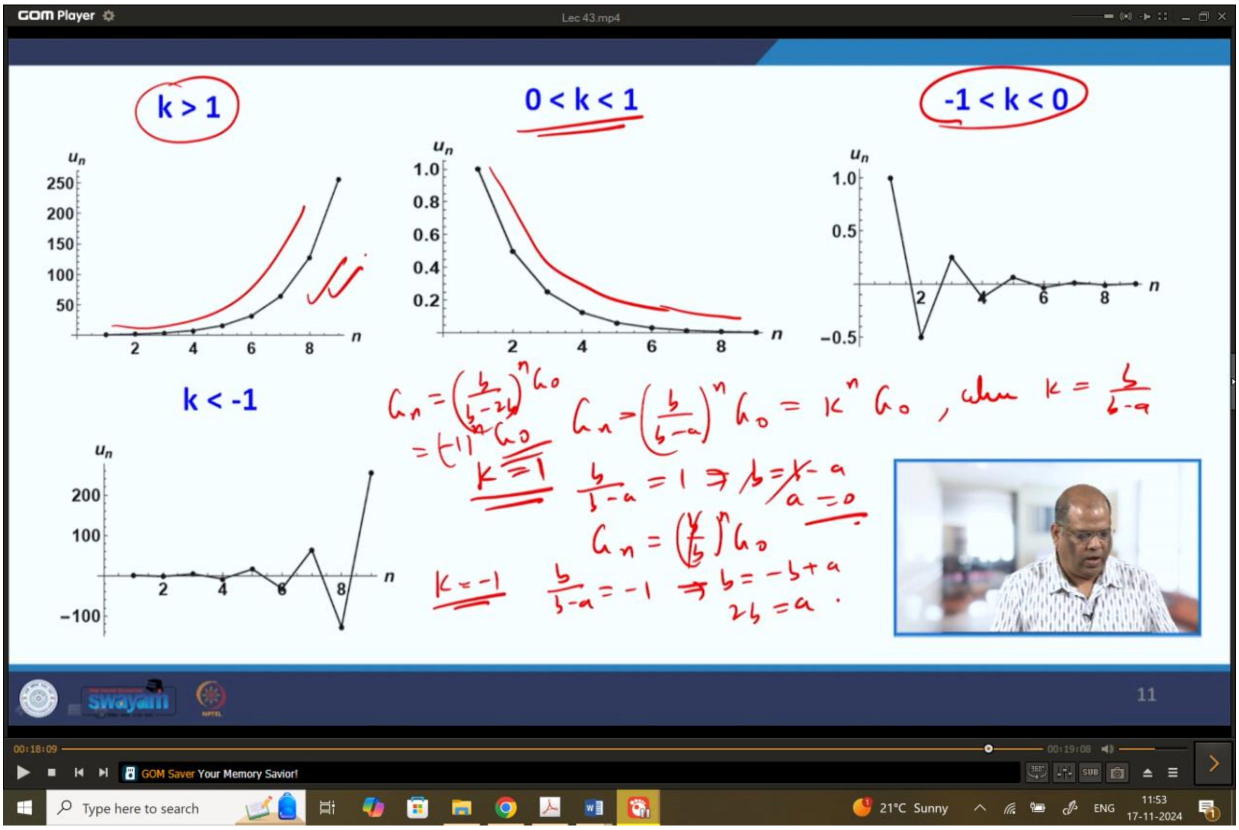
So this gives minus 1 whole to the power n g0.

So your gn will alternately becomes minus g0 and plus g0 period after period.

So for n equal to 0, it will be g0.

For n equal to 1, it will be minus g0.

So it will be oscillatory between rather a periodic kind of motion between minus g0 and plus g0.



So ultimately, this Harrods analysis, it implies that that aggregate real output must behave as in this case one, if the system is to move smoothly in such a way that the realized investment is always equal to the desired investment.

So summing up in this lecture, we have talked about an economic model, namely Harrods model.

So, from three basic assumptions, we derived the model and numerically shown various dynamics by changing the parameter values.

In my next lecture, we will be talking about late pollutant models.

Till then, bye-bye.