EXCELing with Mathematical Modeling
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Week – 01
Lecture – 04 (How to build mathematical models)

Hello.

In this lecture, we will be learning how to create or build a mathematical model from the scratch.

So, before you start modeling, you should be very very clear what you are modeling.

You have to understand the problem and your objectives.

So once that is clear, then you will be able to create this model from the very scratch.

So, the first thing you need to be very clear about is what you are modeling and what is the objective of your model.

We will start with an example, this mathematical modeling a structured process by Cynthia and Ricardo.

So, I find this example quite interesting, it came, it is published in 2015, and it is published by National Council of Teacher of Mathematics.

So, the problem is related with our day to day life.

So, the problem says that you have lost a mobile, it can be in a market, it can be in a mall and how you will approach through mathematical modeling to get an approximate location of your mobile phone.

Well, let me elaborate this problem.

So, you have a mobile phone which you have lost.

It can be in a very crowded market, it can be in a mall, but you have three mobile towers that has detected the signal of your mobile phone.

So, these are the information that is available and with the help of this information you have to formulate the model. So, you have the location of the mobile phone in terms of the coordinates.

So, this is the x coordinate, this is the y coordinate. So, (x,y) is one location of your tower.

So, it is said that that mobile tower 1 is at the position (1200, 200), please consider them in meters. So, somewhere in the first quadrant. Your second mobile tower is at the position (800, – 450), somewhere in the fourth quadrant and your third mobile tower is in the position (–100, 230) in the second quadrant.

Now tower 1, it detects the signal of the lost mobile phone at a distance of 1072 meter, tower 2, 1213 meter and tower 3, 576 meter.

So, this is the information that is available for this lost mobile phone problem.

So, the problem is that you have lost your mobile phone. The objective is to find an approximate location of that mobile phone. The information that are available is the position of the mobile towers and the distance of the mobile phone from the mobile towers.

With that, we proceed.

Step 2.

So, step 2 going to be the assumptions and please note that these assumptions, your whole modeling depends on what kind of assumptions you take.

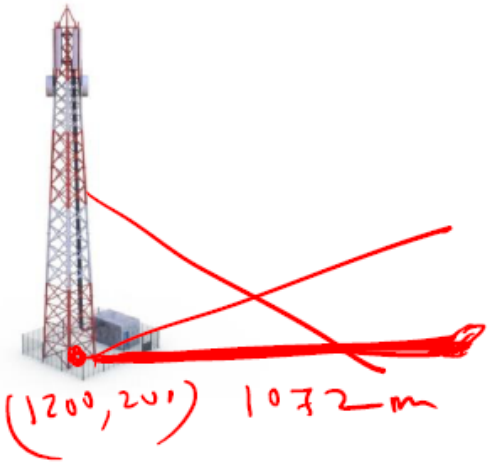
So, your assumption should be as realistic as the problem.

For example, if I consider your mobile tower in the form of a sphere, that is just will not work because we all know the shape of the mobile tower almost in the form of an Eiffel Tower or kind of pyramid.

So, the assumptions are very very important.

So, the first assumption that we take in this particular problem that the distance between from the tower to the mobile phone are horizontal, which means that if this is your mobile phone somewhere here, this distance is horizontal. It cannot be like this, it cannot be like this. So, this horizontal distance is what is given.

The second assumption, this distance which you have recorded, say 1072 meter, this has to be accurate, I mean there should not be any error in this calculation, and the third one your mobile phone is not moving, that is, it's not going from one place to the another, it is not in a bus where you have lost because then your whole dynamics changes so we assume that you have lost it in in such a place that your mobile phone is static and it is just like be there, so this is the information that we have. So basically if I take this to be the tower 1 so this position is (1200, 200), this distance is 1072 from the mobile phone to the tower and similarly for the other two towers.



The third step it is the schematic diagram or the flow charts or the flow diagrams.

So, no matter how simple your problem looks, it is always advisable that instead of just jumping to the governing equations or the model equations, you visualize what you are going to model.

And the process of this visualization can be achieved through this schematic diagram or this flow diagrams.

So, basically it provides a visual aid to your problem, and the flow diagram or a schematic diagram if you define it is just a pictorial representation depicting the flow of steps in a model.

For example, so common cold, if I have a common cold and I come across with one of another person, there is a high chance that he or she will catch the common cold.

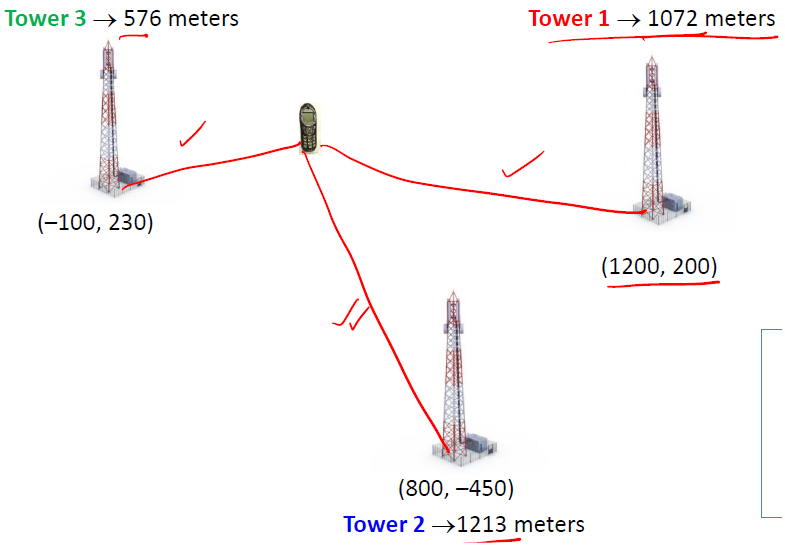
So, if I want to draw a picture or a schematic diagram to that, so I will write that, this is called susceptible means the person who can catch the disease and this is the infected.

So, when the susceptible comes in contact with the infected, they catch the disease say at a rate beta.

So this is a simple example where you draw a flow diagram or a schematic diagram showing two variables S and I, they are related with this beta and then you can form your differential equation through this simplified process of flow chart.

In our case, I have an improvised version of this schematic diagram which I find is helpful for this particular problem.

So, as you can see the schematic diagram consists of three towers.



The first tower is at the location (1200, 200) meters. The mobile phone is at a distance of 1072. So, if you see the mobile phone, it lies somewhere here.

The second tower is at the position (800, – 450) meters and tower is 1213 meter, the distance of the mobile phone and tower 3, it is at (–100, 230) at a distance of 576 meter.

So, this horizontal distance is known, this horizontal distance is known and this horizontal distance is known.

So what I have to take care is that okay if you draw an approximate diagram but make sure that this distance is 1213 and this distance is 1072, this is just 576.

So this much looks greater than this one and this one.

So once you get the schematic diagram, now you have to choose your mathematical equations.

It can be a difference equation; it can be a differential equation or it can be an optimization problem where you have to optimize a function.

But always think of the simplest thing.

So, the very first thing what we do is that you go through some literatures, you see whether somebody else has tried this kind of problem so that you can get some idea.

If it is not, then you think of the simplest way of capturing these dynamics.

Now, you can see these three concentric circles.

The reason is that if you have a mobile phone lost somewhere and you have the distance of the mobile phone from the tower, so with that as radius, if you draw a circle, so the possibility is your mobile phone is lying in that circular area.

So, that is why these circles have been involved.

Let me explain this elaborately.

So you have this mobile tower, say, this is (1200, 200). The distance is 1072 meter.

I take the coordinate of the mobile phone to be x and y. So I can find the distance using the coordinate geometry and the formula. So, that distance is 1072.

So, basically I get an equation of a circle.

So, the equation which I will get for the first mobile that is

$$(x - 1200)^2 + (y - 200)^2 = 1072^2$$

For the second one,

$$(x - 800)^2 + (y + 450)^2 = 1213^2$$

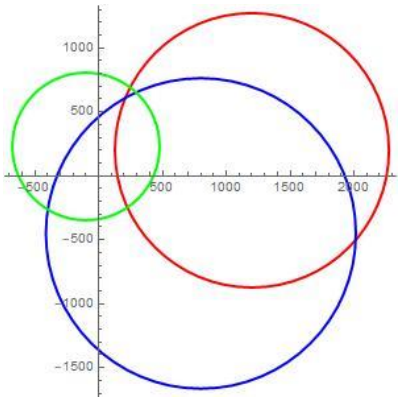
And, for the third tower,

$$(x + 100)^2 + (y - 230)^2 = 576^2$$

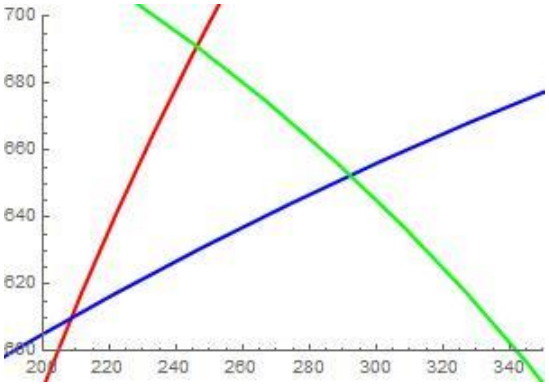
So, I name the equation as say 1, 2 and 3.

So, these three circles if I now draw, so I get the red one this is for the tower 1, the blue one for the tower 2 and the green one for the tower 3.

So, I see the common area that has been intersected by this circles, and the common area is lying here, which is quite small.



If I zoom it, I will get the area to be like this, means this particular part.



So, basically your mobile phone lies in that area.

Now, how to find that how much area is here and how to find the location in terms of this coordinates.

So to do that, I have to solve the equations 1, 2 and 3.

Now how to solve this equation?

So this comes to step 5, you have to solve the equations.

So our first equation is

$$(x - 1200)^2 + (y - 200)^2 = 1072^2$$

This is equation 1. And,

$$(x - 800)^2 + (y + 450)^2 = 1213^2$$

This is our equation number 2.

I can write 1 and 2.

So, I expand them using $(a - b)^2$ and from the first equation, I will get

$$x^2 + y^2 - 2400\,x - 400\,y = 1072^2 - 1200^2 - 200^2 = -330816 \text{ -------- (4)}$$

So this is my equation 4.

Now I expand second equation, I get

$$x^2 + y^2 - 1600\,x + 900\,y = 1213^2 - 800^2 - 450^2 = 628869 \text{ --------- (5)}$$

and for the third equation, I will be getting

$$x^2 + y^2 + 200\,x - 460\,y = 268876 \text{ ---------- (6)}$$

So, the third equation is

$$(x + 100)^2 + (y - 230)^2 = 576^2$$

This is your third equation.

So, this is your 4, this is your 5 and this is your 6.

Let us take this 4 and 5.

So, I rewrite this as

$$x^2 + y^2 - 2400x - 400y = -330816$$
$$x^2 + y^2 - 1600x + 900y = 628869$$
$$+800x + 1300y = +959685$$
$$y = \frac{959685 - 800x}{1300}$$

I subtract. This cancels and this gives me $-800\,x - 1300\,y = -959685$.

And from here I can find y or x in terms of x or y.

So, let us find y in terms of x which

$$y = \frac{959685 - 800\,x}{1300}$$

So, if you have a circle S_1 and another intersecting circle S_2, if I subtract S_1 minus S_2 then I get the straight line passing through the point of intersection of these two circles.

So, basically this gives me the straight line passing through the point of intersection of these two circles. So, if I want to solve the point of intersection all I have to do is solve this circle and this straight line or this circle and this straight line.

So, which is what we will do.

So, you have the circle,

$$x^2 + y^2 - 2400\,x - 400\,y = -330816,$$

and you have the straight line

$$800\,x + 1300\,y = 959685$$

So, I will eliminate either x or y from here.

So, we already find out that

$$y = \frac{959685 - 800\,x}{1300}$$

We substitute it here and here and if you substitute it you will get

$$x^2 + \left(\frac{959685 - 800\,x}{1300}\right)^2 - 2400\,x - 400\left(\frac{959685 - 800\,x}{1300}\right) = -330816$$

You have to simplify this, obviously you have to use, either you use hand or calculator and you will get a quadratic equation of the form

$$93200\,x^2 - 207019840\,x + 39241525569 = 0$$

Well it's a real life problem so the data which you got also a real life so you have to work a bit hard and then use calculator or a computer to get the values of this so your

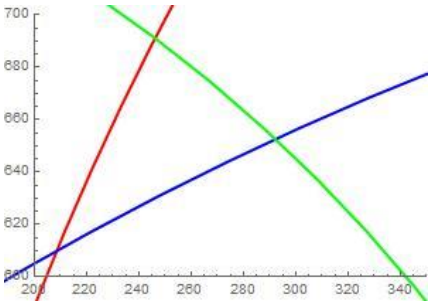
$$x = 209.27 \text{ and } 2011.97.$$

So, we got two values of x. You substitute those two values of x here and you calculate the corresponding value of y.

So, the corresponding value of y = 609.44 and 499.916.

So, you got two sets and which one to choose.

So, if you see the diagram here, everything is lying in the first quadrant.

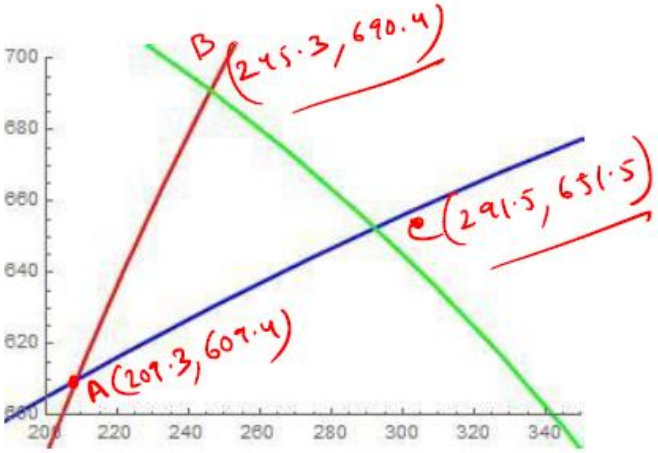


So, whatever coordinates you get, both of them has to be positive, and hence this pair is rejected and this pair is accepted, because both of them are positive.

So, you get the coordinate to be (209.27, 609.44), to be the intersection of the two circles.

So, basically 209, this is your A, this is your B and this is your C. So, you get this coordinate to be 209.3 approximately and 609.4.

In the similar manner, you get the B to be (245.3, 690.4) and (291.5, 651.5).



Once you get these coordinates, then you find the area.

So, we have this area from the coordinate geometry and if you put the values of these A, B's and C's, so this is the x coordinate x1, y1, x2, y2, x3, y3, you get approximately the area to be 2600.

$$\text{Area} = \frac{1}{2}\left[x_1\,(y_2 - y_3) + x_2\,(y_3 - y_1) + x_3\,(y_1 - y_2)\right]$$

$$= 2600\ \text{m}^2$$

Now, once you get this area, your final step is the interpretation of the result.

So, if you recall that you have posed a problem.

The problem was that you have lost a mobile and you want to find an approximate position of the mobile.

So, you have to basically now answer that question in terms of non-technical terms.

So, basically you have to say okay now my mobile lies in this particular area.

So, the answer to your problem is the location of the mobile phone is within 2600 meter square and your xi's and yi's gives you the location of the coordinates.

So, we solve the problem, but 2600 is a huge area basically it is the area of half a football

So, if you consider a busy market in a half of a football field or in a busy mall, it is almost next to impossible to find your mobile.

So, you need to you know modify, you modify your model so that this area reduces, so that you can have at least a chance, I mean to go through that or scan through that particular area and find your location of your mobile phone.

So, to modify the model, we have to go back to the drawing board.

So, our mobile tower the position is (1200, 200)

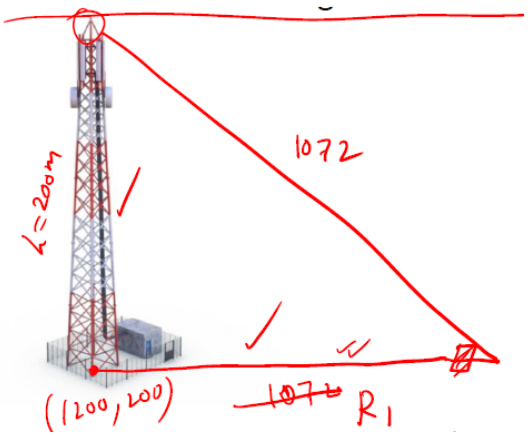
Now, we come to the assumption.

The assumption was that this distance from the mobile phone must be horizontal.

Now, tower 1 detects the signal at a distance of 1072 meter. We took that signal distance to be 1072 here. But as you know the antenna of the tower is at the top not at the bottom. So, from the top, it radiates the signal and that distance came to be 1072.

So, what we do now is, this is now I take the signal. So, this distance is 1072.

I assume that all these towers have a height of say 200 meter and now I calculate this distance, which is say, R_1.



So, if I now calculate this distance by simple Pythagoras theorem, I can see

$$R_1^2 + 200^2 = 1072^2$$

and this gives me my new distance from the base of the mobile phone as

$$R_1 = 994.6.$$

Taking this as your new distance, you can also calculate now the R_2 and the R_3 in the similar manner for the tower 2 and the tower 3.

$$R_2^2 + 200^2 = 1213^2 \Rightarrow R_2 = 1196.4 \text{ m}$$

$$R_3 + 200^2 = 576^2 \Rightarrow R_3 = 540.2 \text{ m}$$

You can now form the equation of the circles.

So, the new equation of the circles are now

$$(x-1200)^2 + (y-200)^2 = 994.6^2$$

Similarly,

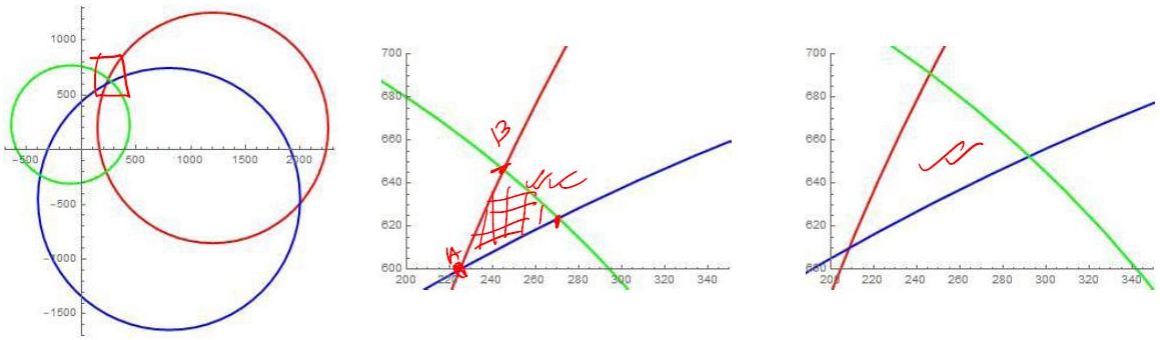
$$(x-800)^2 + (y+450)^2 = 1196.4^2$$

$$(x+100)^2 + (y-230)^2 = 540.2^2$$

So, you have a modified equation of the circle use the same technique to solve them.

So, if you now plot the circle you can see this is the common part which looks much smaller and if I zoom it this is the area now where your mobile phone lie.

This is the figure from the previous equation.



So, as you can see it is almost three times this area. So, now this becomes almost one-third.

You solve the equations, you find these points of intersection and your coordinates are now given by (225, 600) for A, (244.3 and 645.3) for B, and for C, it is (269.7, 622.7).

With this, if you calculate the area, it is now just 817 m^2 and sort of, it is much much easier than to scan threefold of this area, than the one-third of the area.

So, some sort of doable thing if you scan or look into this particular area where you have already the coordinate system, there is high chance that you will be able to locate your lost mobile phone.

So, that is all.

When a problem is given, you have to go through that problem from the step 1 till this step 5, in a meticulous manner so that you will be able to find the answer of the mathematical modelling and the question that you have posed for that particular problem where you will be using this mathematical modelling.

In our next lecture, we will be doing the hand on with Microsoft Excel.

So, in that lecture you will be learning how to use the spreadsheet, how to do addition, how to find variance, how to solve equations, how to solve differential equations, how to visualize those solutions and graphs.

Till then bye-bye.