EXCELing with Mathematical Modeling
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Week – 05
Lecture – 23 (Growth and decay in L-R circuit)

Hello, welcome to the course EXCELing with Mathematical Modelling.

Today we will be discussing about the dynamics of current in an L-R circuit mainly the growth and decay of the current in L-R circuit.

So, first of all what is this L-R circuit? It is called inductor resistor circuit.

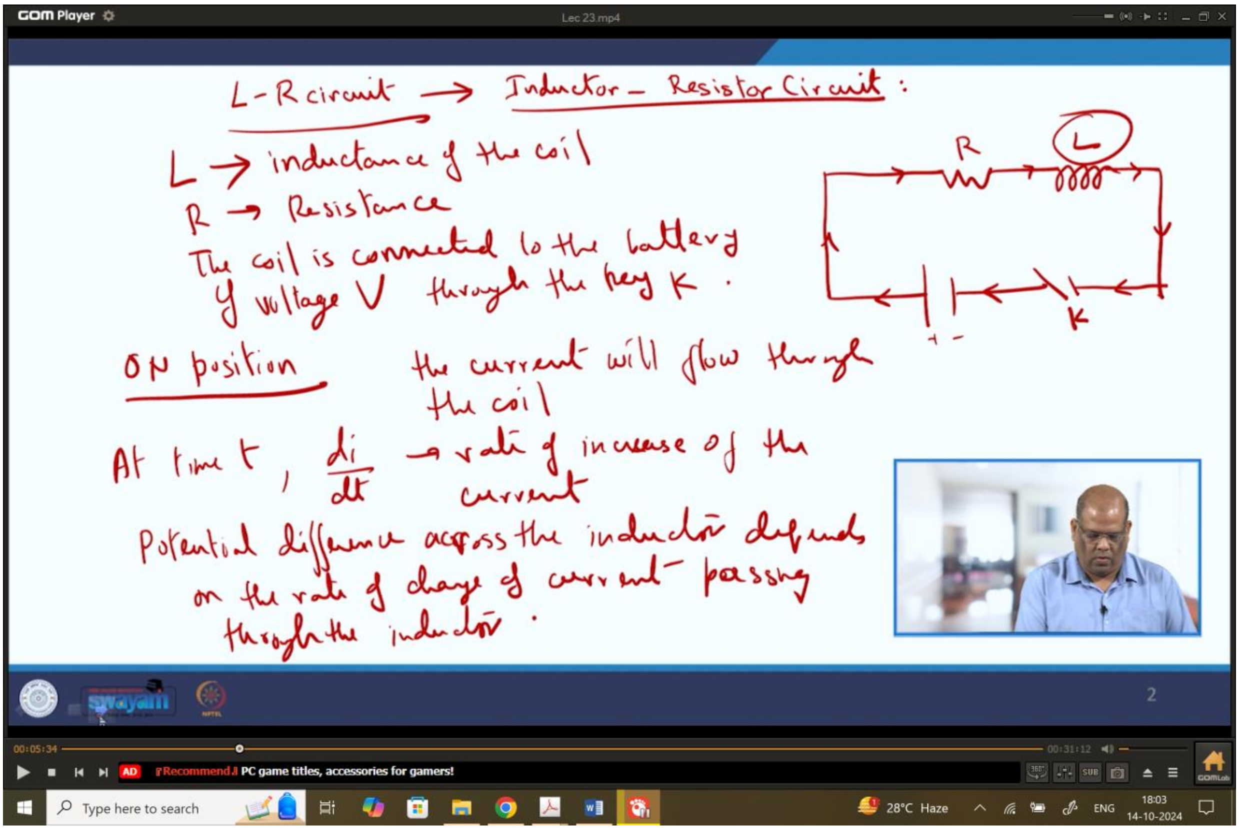
In physics, you have been familiar with this circuit diagram.

So, this one will look something like this.

You have a resistance R with an inductor L the battery from where the voltage will come and the switch which is called the key.

So, this is positive, this is negative, so the current flows like this.

So, this is a LR circuit.



So, now what is going to be the dynamics of the current in such a circuit?

So, basically when the current flows, you will see that and mathematically it will be shown that your current reaches a steady state once the switch is on and when the switch is off, the current does not instantaneously go to zero, but there is a decline and slowly it reaches to zero.

This is exactly what you may have seen that when you switch off your power source of the laptop the indicator bulb there you will see that it is still on well your switch is off and slowly at one point it then stops blinking because the power was there and slowly it decays to zero. Until it decays to zero, the indicator is on.

So, here let us start with some definitions.

So, this L is the inductance of the coil, R is the resistance, the coil is connected to the battery of voltage V through the key which can be read as switch, this is K. Now what happens in the on position?

So you put on the switch and you get what is called the on position.

So the current will flow through the coil and when this current starts to flow there will be an electromagnetic force which will induce across this L and according to this law of electromagnetic induction this EMF will oppose the voltage and as a result of which there will be a voltage drop across R and which will also oppose the applied voltage.

So, if that is the scenario, then at any time t,

$$\frac{di}{dt}$$

is the rate of increase of the current.

Now, the potential difference across the inductor depends on the rate of change of current passing through the inductor.

So, mathematically, you can write, if V_1 is the potential difference across the inductor that depends on the rate of change of current passing through the inductor,

$$V_1 = L\frac{di}{dt}$$

rate of change of current passing through the inductor and potential difference across the resistor that is equal to i ×R, which we name as some V_2, that is,

$$V_2 = i\,R.$$

So,

$$V = V_1 + V_2 = L\frac{di}{dt} + iR$$

So, you get a differential equation

$$L\frac{di}{dt} + iR = V$$

and you have to solve this differential equation.

So, this is the first order linear differential equation, either you can use integrating factor or straight away you can do the separation of variables because this V is a constant.

So,

$$-L\frac{di}{dt} = iR - V \implies \frac{di}{iR - V} = -\frac{dt}{L} \implies \frac{di}{R\left(i - \frac{V}{R}\right)} = -\frac{dt}{L}$$

We integrate both sides and get

$$\int_{i=0}^{i} \frac{di}{i - \frac{V}{R}} = \int_{t=0}^{t} -\frac{R}{L}dt \implies \ln\left(i - \frac{V}{R}\right)_0^i = -\frac{R}{L}{}_0^t$$

$$\implies \ln\left(i - \frac{V}{R}\right) - \ln\left(-\frac{V}{R}\right) = -\frac{R}{L}t$$

$$\implies \frac{\ln\left(i - \frac{V}{R}\right)}{\left(-\frac{V}{R}\right)} = -\frac{R}{L}t \implies \frac{\left(i - \frac{V}{R}\right)}{\left(-\frac{V}{R}\right)} = e^{-\frac{R}{L}t}$$

$$\implies i - \frac{V}{R} = \frac{V}{R}e^{-\frac{R}{L}t}$$

$$\implies i = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t} = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

This is your current at any time t.

Let us now look into the alternate solution. So, we have the equation of the form

$$L\frac{di}{dt} + iR = V \implies \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

Now, this is of the form

$$\frac{dy}{dx} + P(x)y = Q,$$

which is of first order linear, and we know that we have the integrating factor

$$\text{I.F} = e^{\int P(x)\, dx}, \qquad P = \frac{R}{L}, \qquad Q = \frac{V}{L}.$$

So here the value of $P = \frac{R}{L}$ and value of $Q = \frac{V}{L}$ both of them are constants.

So, your integrating factor

$$\mathrm{I.F} = e^{\int \frac{R}{L} dt} = e^{\frac{R}{L}t}$$

So the next step is you multiply this differential equation both sides by the integrating factor

$$e^{\frac{R}{L}t}\frac{di}{dt} + e^{\frac{R}{L}t}\frac{R}{L}i = \frac{V}{L}e^{\frac{R}{L}t} \Rightarrow \quad \frac{d}{dt}\left(ie^{\frac{R}{L}t}\right) = \frac{V}{L}e^{\frac{R}{L}t}$$

$$\Rightarrow \quad \int \frac{d}{dt}\left(ie^{\frac{R}{L}t}\right) = \frac{V}{L}\int e^{\frac{R}{L}t}\,dt$$

$$\Rightarrow \quad ie^{\frac{R}{L}t} = \frac{V}{L}\frac{e^{\frac{R}{L}t}}{\frac{R}{L}} + \text{constant}$$

Initial, at $t = 0$, there is no current, so $i = 0$, and this implies

$$0 = \frac{V}{R} + \text{constant} \quad \Rightarrow \quad \text{constant} = -\frac{V}{R}$$

So if I substitute back, I will get

$$ie^{\frac{R}{L}t} = \frac{V}{R}e^{\frac{R}{L}t} - \frac{V}{R} \quad \Rightarrow \quad i(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}$$

$$\Rightarrow \quad i(t) = \frac{V}{R}\left(1 - e^{\frac{R}{L}t}\right)$$

So, this is the equation which now represents the current.

So, now let us look into the numerical solution.

Let us see how the solution behaves and for that I will be using Microsoft Excel.

So, I already have the file open, you have the equation in this form.

So, I put the initial value as 0, your value of V to be 20, R is 3, L is 50 and the H is 1.

We will be using this formula, the Euler's formula

So this is your time and this is your current.

So initially at time t equal to 0, the value is 0.

Here I give an equal increment, this the value of h is 1.

So, I drag to some 100 values and to calculate this which is equal to I0 plus H times, H is a constant.

So, I put dollars multiplied by F of I0 which is this expression only I is replaced by the initial condition.

So, V which is 20 again this is a constant minus I multiplied by R which is again a constant.

So, I am inserting dollars and this whole thing is divided by L which is again have a constant value.

So, you calculate this one cell, you got the value and you drag it to the next 100 cells rather 99 cells and you get some value.

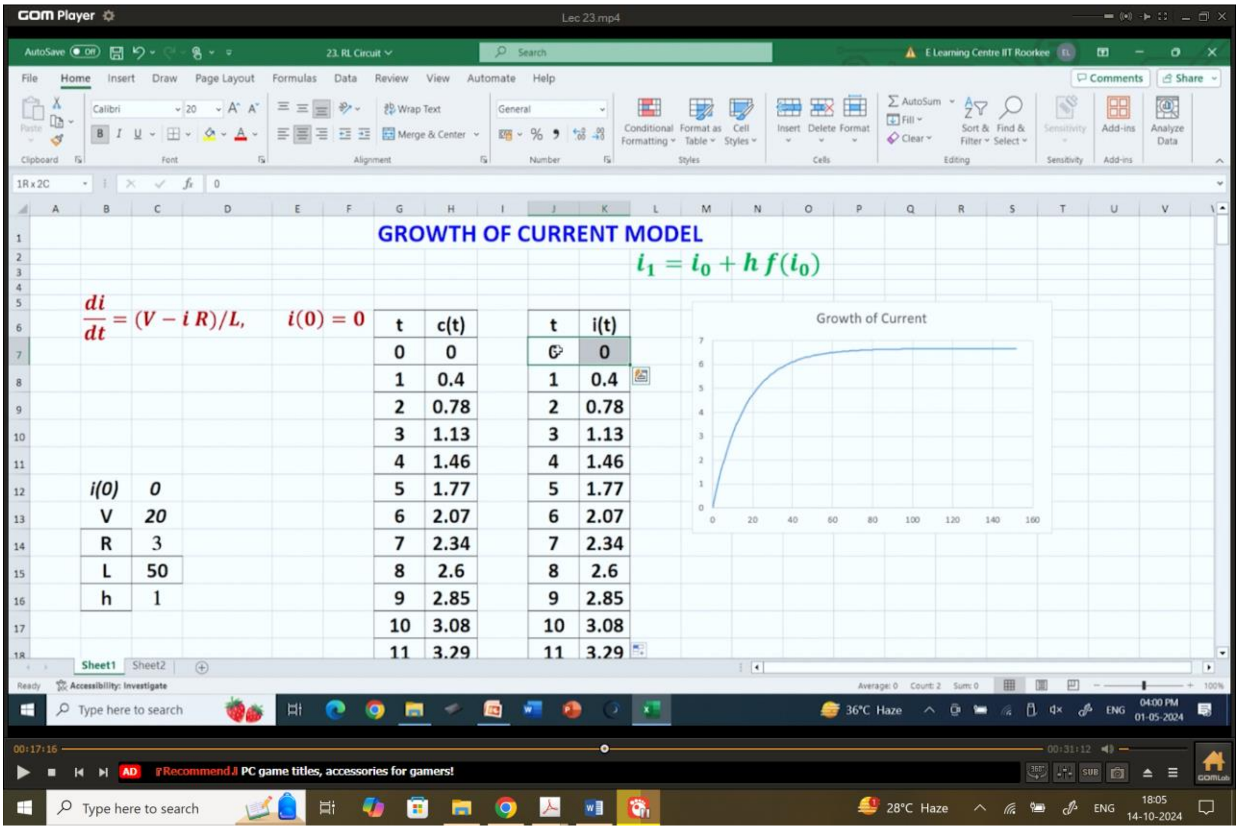
So, now you have to plot it.
So, you highlight all the calculated values go to insert, go to this chart section, find the scattered diagram and choose this.

So, this is the figure.

So, here I can just change the title to growth of current.

So you can see that it starts from initial value zero and slowly it is growing and ultimately reaching a steady state.

Now as we see the graph which gives the growth of the current which is similar as we got in our Microsoft Excel.



So if you see the growth of the current is given by

$$i(t) = \frac{V}{R}\left(1 - e^{\frac{R}{L}t}\right)$$

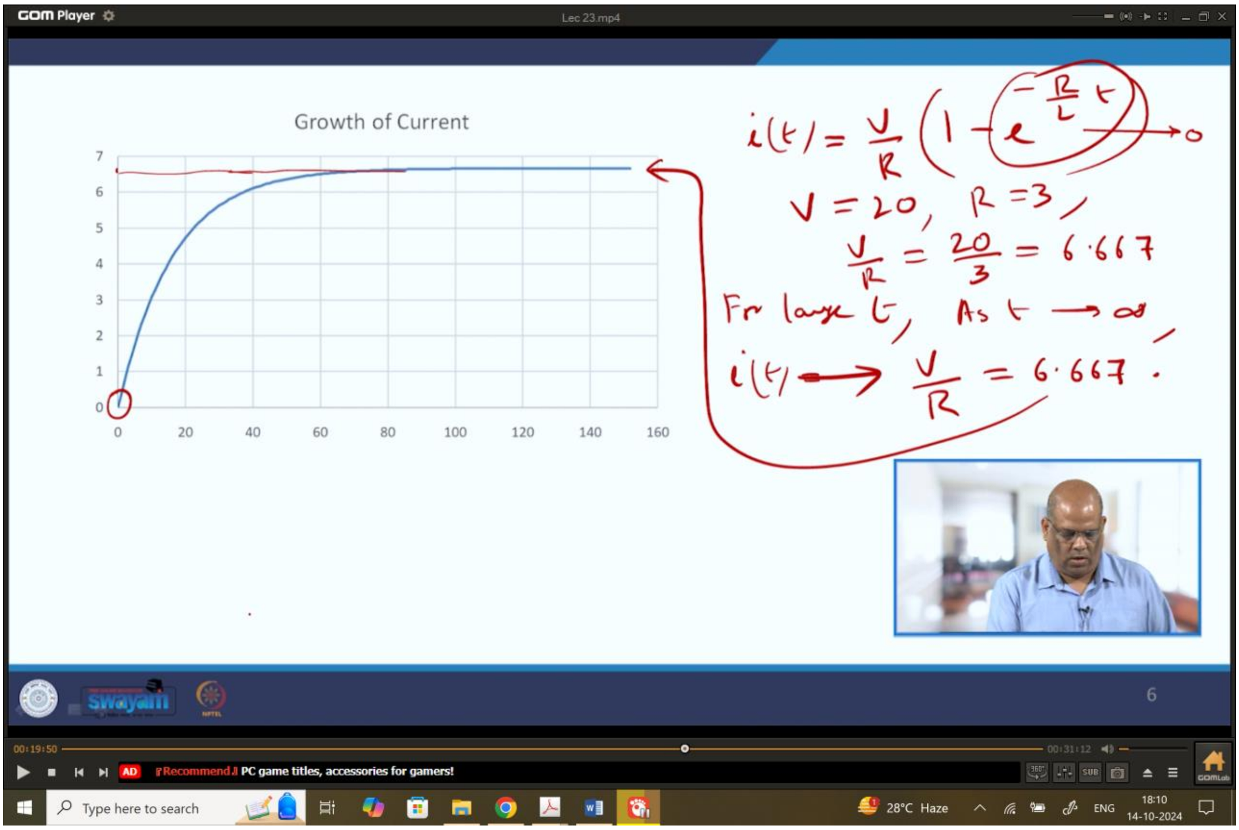
And while solving the equation using Microsoft Excel, we have taken the value of V to be 20, the value of R to be 3 and initial condition obviously it starts from i = 0.

So, if I calculate

$$\frac{V}{R} = \frac{20}{3} = 6.667$$

So, for large t, means as $t \longrightarrow \infty$,

$$e^{\frac{R}{L}t} \to 0 \Rightarrow i(t) \to \frac{V}{R} = 6.667.$$



So, which is now confirmed by the numerical graph, numerical solution represented in the graph, you can see that this value where it extends to as your time becomes large is approximately equal to 6.667.

Let us now take an example, a series L-R circuit having a resistance of 20 units and inductance of 8 units is connected to a DC voltage source of 120 units at time t equal to 0.

So, what is the current in the circuit at time 0.6 units.

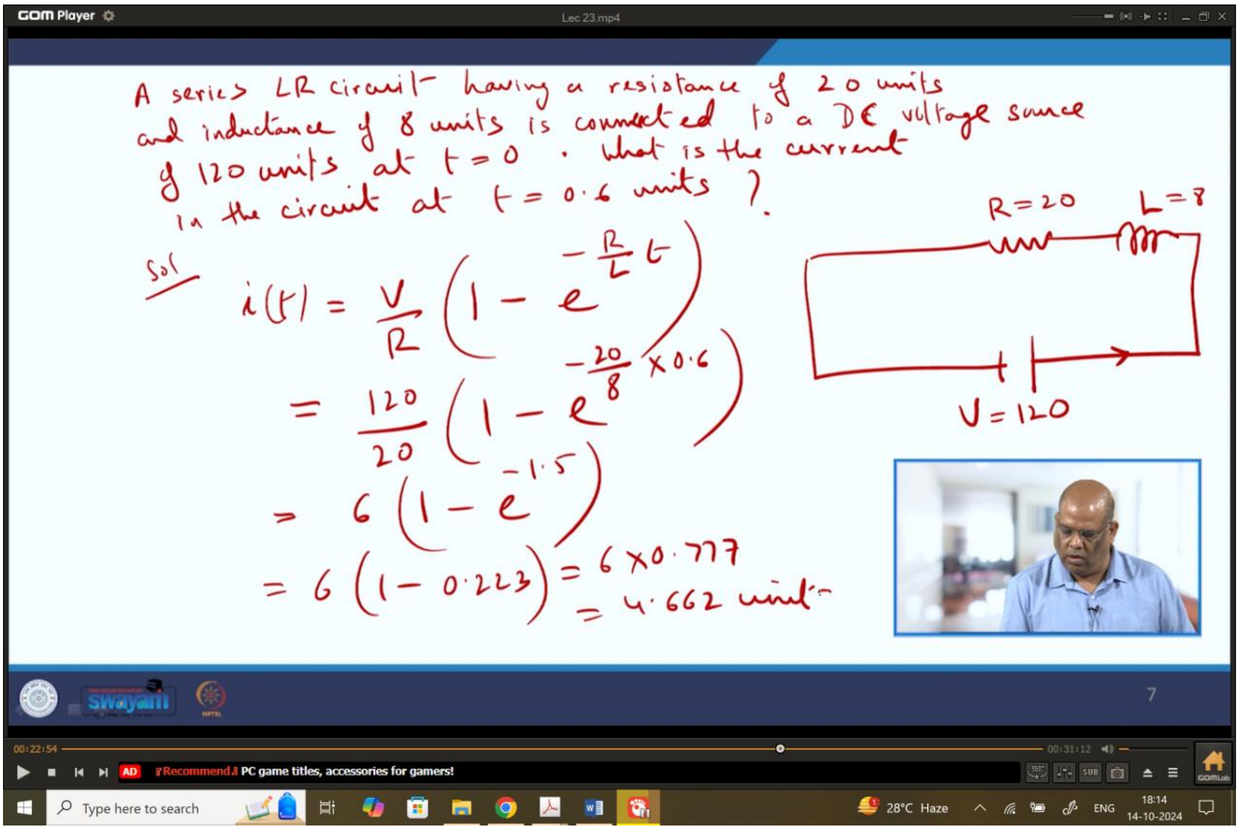
So, you have a resistance, you have a inductance and this is connected to a source, this is your R, this is your L.

and inductance is 8, your resistance is 20, this produces a 120 volt units.

So, the formula you have, you already have the formula which says that the current at any time t is given

$$i(t) = \frac{V}{R}\left(1 - e^{\frac{R}{L}t}\right) = \frac{120}{20}\left(1 - e^{\frac{20}{8}\times 0.6}\right)$$

$$= 6(1 - e^{-1.5}) = 6(1 - 0.223) = 6 \times 0.777 = 4.662 \text{ units.}$$



Let us now look what is going to be the dynamics when you switch it off.

So, now you have the key position off.

So, when you switch off the key it was from the position on position.

So, from the on position you have switched off the key.

So, your steady value was V by R.

So, what will happen?

So, we are going to find the dynamics what will happen when your current was at V by R. So, once the off positions then no current flows and the flux will reduce gradually.

This will result in the drop of voltage across the resistor of resistance R and the induced emf $L\frac{di}{dt}$ across the inductance.

So, ultimately your voltage V will come down to zero.

So, if we now use this in the differential equation, initially we have this

$$V = V_1 + V_2 = L\frac{di}{dt} + iR$$

This was in the on position. In the off position, this becomes zero.

So, your equation becomes

$$0 = L\frac{di}{dt} + iR$$

So, now you have to just solve this equation

Separation of variables gives

$$\Rightarrow L\frac{di}{dt} = -iR \Rightarrow \int_{i=I=\frac{V}{R}}^{i} \frac{di}{i} = -\int_{t=0}^{t} \frac{R}{L} dt$$

$$\Rightarrow \ln i_{\frac{V}{R}}^{i} = -\frac{R}{L}t \quad \Rightarrow \quad \ln i - \ln\frac{V}{R} = -\frac{R}{L}t$$

$$\Rightarrow \ln\frac{i}{\frac{V}{R}} = -\frac{R}{L}t \quad \Rightarrow \quad \frac{i}{\frac{V}{R}} = e^{-\frac{R}{L}t}$$

$$\Rightarrow i(t) = \frac{V}{R}e^{-\frac{R}{L}t}$$

So, this is now the solution of the differential equation when in the off position of the switch.

So as you can see from here as your time becomes large then i goes to zero.

So let us quickly see the solution of this, in the Microsoft Excel.

So we have now $\frac{di}{dt} = -\frac{R}{L}t$.

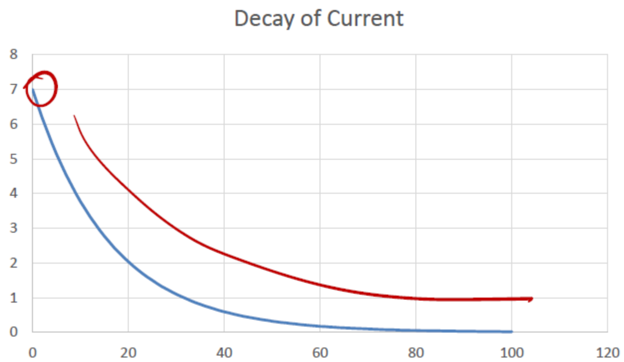
Your current, this is zero and we put some value 7 and let us calculate this one, this is equal to this plus 1 and drag it to say 100 values.

So let us make this 21 such that I equal to V by R which is 7, so initial value is 7 and then this value is equal to I0 which is this plus h times which is 1 plus, so this is a constant

so I make this the dollar multiplied by minus i_0 so a negative sign multiplied by r which is a constant divided by L and we drag to the next 100 values and if we plot this.

You press shift and highlight all the values, go to insert, go to the scattered diagram and this one.

So I will just change it to decay of current. So you can see it starts from some steady value. So, this is the point where your switch is just off and then slowly it decays to zero.



So, as I was saying that in the decay of the current, it will start from some steady value. This is the point where you just switch it off and then it slowly decays and goes to zero.

So, this is going to be the dynamics of the growth and decay of current in an L-R circuit.

In the next lecture, we will be talking about the rectilinear motion under variable forces.

Till then, bye-bye.