Higher Engineering Mathematics.
Professor P. N. Agrawal.
Department of Mathematics.
Indian Institute of Technology, Roorkee.
Lecture 6: Validity of Arguments.

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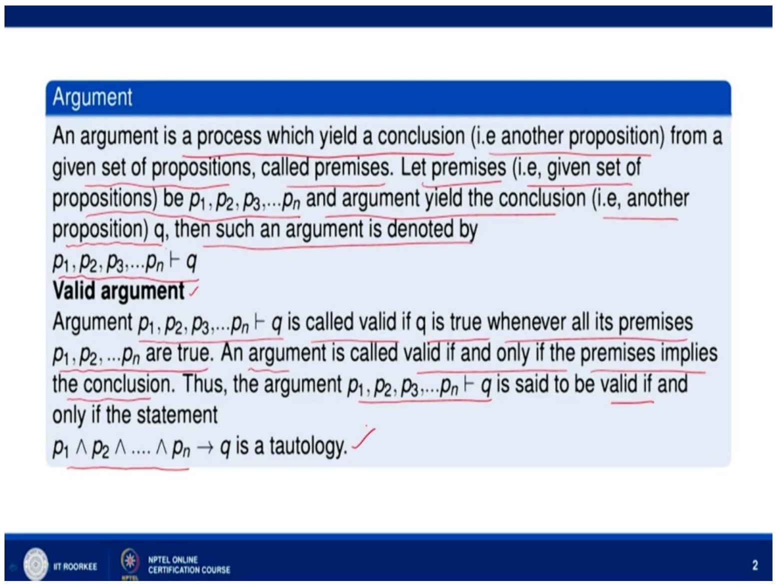

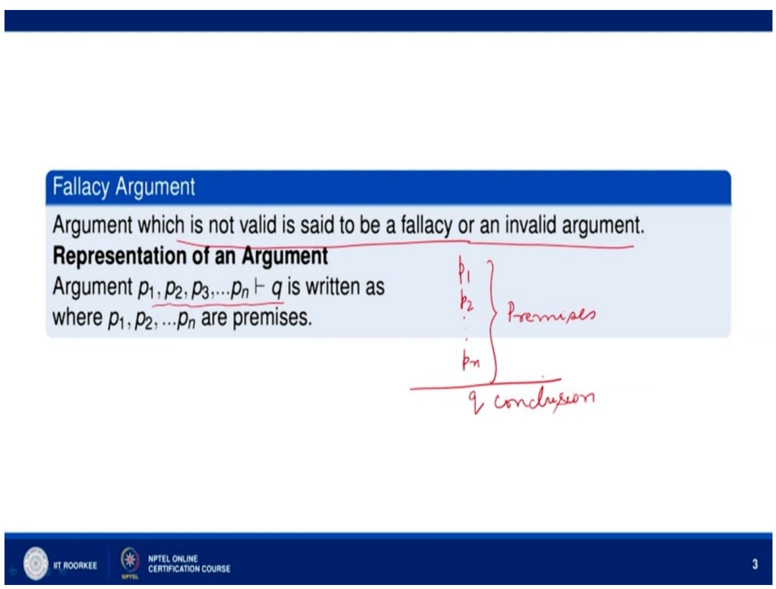

Hello friends, welcome to my lecture on Validity of Arguments. Let us first define what is an argument? An argument is a process ⟹which yield a conclusion. That is another proposition from a given set of propositions called premises. Okay, so let premises that is a given set of propositions be, $p_1, p_2, p_3, \dots, p_n$ okay. And argument yield the conclusion that is another proposition which we can denote by q, okay, then such an argument is denoted by $p_1, p_{2,} p_3, \ldots, p_n$ gives q, conclusion q, okay.

Now, valid argument, when we will call the argument to be valid. So, argument $p_1, p_{2,} p_3, \ldots, p_n$ gives q is called valid, if q is true whenever all its premises $p_1, p_{2,} p_3, \ldots, p_n$ are true. An argument is called valid if and only if the premises $\Rightarrow$ the conclusion. Thus, the argument $p_1, p_{2,} p_3, \ldots, p_n$gives q is called valid if and only if the statement p_1 and p_2 and so on and $p_n \Rightarrow$q is a tautology. That is when you form the truth table of p_1 ¿ p_2 and so on and

$p_n \Rightarrow$q, okay, the column, last column should be having all truth values T.

So, we will call the argument, A_1 argument $p_1, p_{2,} p_3, \ldots, p_n$ gives q to be a valid argument if p_1 and p_2 and so on and $p_n \Rightarrow$ q is a tautology. If it is not a tautology we will call it an invalid argument. Okay. So, argument which is not valid is said to be a fallacy or an invalid argument. Representation of an argument. Argument $p_1, p_{2,} p_3, \ldots, p_n \Rightarrow$ q is written as $p_1, p_{2,} p_3, \ldots, p_n$, these are premises and then we write a horizontal line and q, this is conclusion. So, we write premises $p_1, p_{2,} p_3, \ldots, p_n$, okay and the conclusion, above the conclusion we write this horizontal line. So, this is how we denote the argument $p_1, p_{2,} p_3, \ldots, p_n \Rightarrow$ gives q as, where q is the conclusion.

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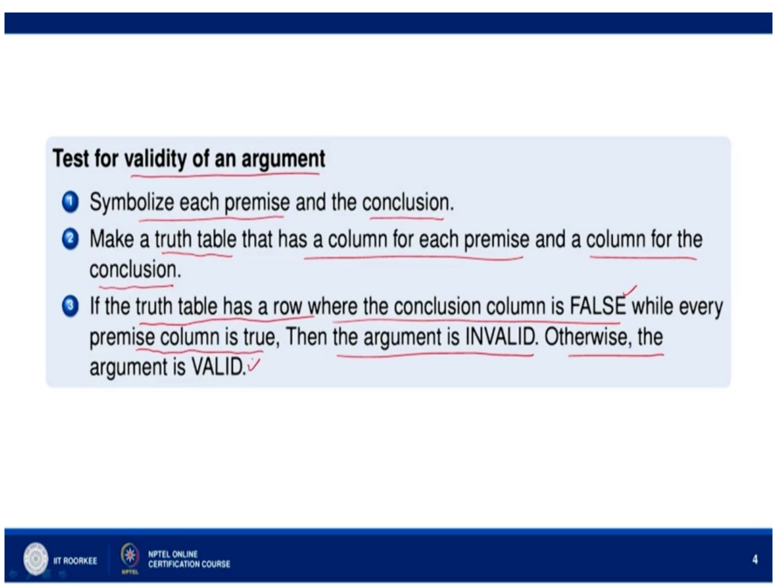

Now, let us test for validity of an argument. When we want to test the validity of an argument, symbolise each premises, okay, each premise is to be symbolised and the conclusion also has to be symbolised. Then we will make a truth table that has a column for each premise. Okay, suppose that the premises are $p_1, p_{2,} p_3, \dots, p_n$, so there will be n columns, one column for each premise and a column for the conclusion. Okay. Now if you denote the conclusion by the proposition q, then there will be a column for q.

Now, if the truth table has a row, where the conclusion column is false, okay, so if there is a row in the truth table where the conclusion column is false, while every premise column is true, okay, while every premises column is true, then the argument is invalid, otherwise the argument is valid, okay.

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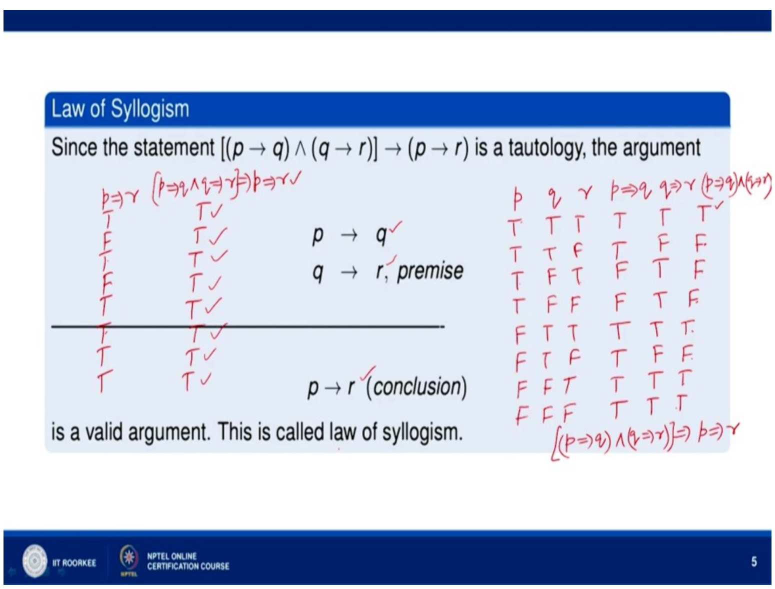

Now, let us look at law of Syllogism. So, since the statement p ⇒ q and q ⇒ r ⇒ p ⇒ r is a tautology, we can see that. Let us say, let us form the truth table p, q, r, TTT, okay, TTF, TFT, TFF, okay. Then FTT, then FTF, then FFT, then FFF, okay, so there will be 8 possibilities, 1, 2, 3, 4, 5, 5, 6, 7, 8.

Now, let us write the column corresponding to the truth values of p ⇒ q. Okay, so TT gives T, okay TT gives T, then TF gives F, TF gives F, then FT gives T, then FT gives T, then FF gives

T, then FF gives T. Okay. And q ⇒ r, let us write for q ⇒ r. Okay, so TT gives T, TF gives F, FT gives T, FF gives T, TT gives T, TF gives F, okay, FT gives T and then FF gives T. Then p ⇒ q and q ⇒ r. Let us write for q ⇒ r, okay, p ⇒ q and q ⇒ r. So, TT, T and T, T and F F, F and T F, F and T F, TT is T, TF is F, TT is T, TT is T, okay.

Then, p ⇒ q, okay, p ⇒ q and q ⇒ r, okay, ⇒ p ⇒ r. So, we have to write the column for p ⇒ r also, okay. So, let us write here, p ⇒ r. Okay, p ⇒ r, TT is T, okay, TF is F, then we have TT is T, then we have TF is F and then we have FT is T, then we have FF which is T and then we have FT which is T and then we have FF which is T. Okay. Now, let us write this column corresponding to p ⇒ q and q ⇒ r ⇒ p ⇒ r, okay.

So, we have p ⇒ q and q ⇒ r, T here, okay. So and this ⇒ this, this ⇒ this, so TT gives T, okay, then FF gives T, then FT gives T, okay, then FF gives T, then TT gives T, then FT gives T and then TT gives T and then TT gives T, okay. So, if you look at this truth table, then p ⇒ q and ⇒ r ⇒ p ⇒ r is a tautology, you can see it is a tautology and therefore, it is a valid argument. Okay, so p ⇒ q is a premise, q ⇒ r is a premise, okay and the conclusion is p ⇒ r, okay, this is known as law of syllogism.

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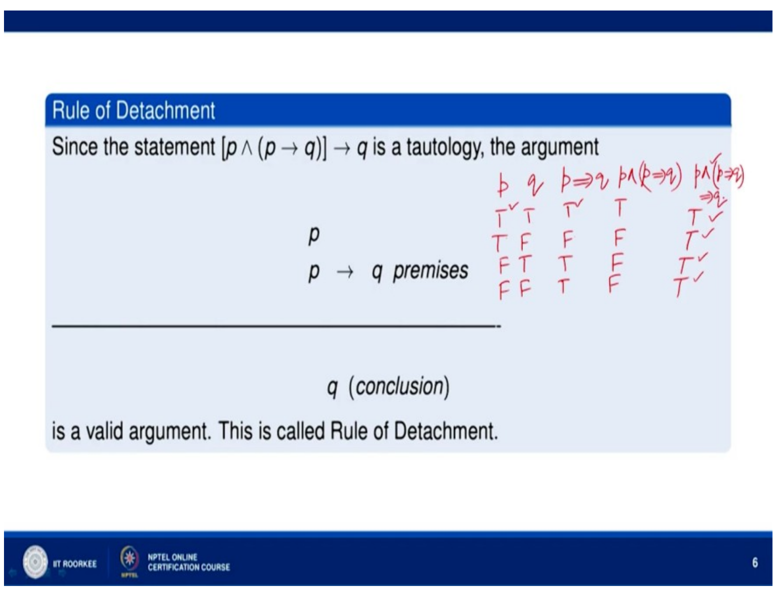

Now, let us go to rule of detachment. P and $p \Rightarrow q \Rightarrow q$ is a tautology, we can see that p, q, p and q, $p \Rightarrow q$, let us first say $p \Rightarrow q$, then we write p and $p \Rightarrow q$, okay. And then we write the column for p and $p \Rightarrow q \Rightarrow q$. So, this is T, T, T, F,F, T, F, F. Then $p \Rightarrow q$ TT gives T, TF gives F, FT gives T, FF gives T. Then p and $p \Rightarrow q$, okay. So, TT gives T, TF gives F, FT gives F, FF gives F, okay.

Now, we have taken p and $p \Rightarrow q$. Okay, now p and $p \Rightarrow q \Rightarrow q$, okay, so we have to write the truth values of this proposition. So, TT gives T, okay, then FF gives T, okay, the, FT gives T, then FF gives T, okay. So, all the entries in this column, okay, they are T, therefore p and $p \Rightarrow q \Rightarrow q$ is a tautology, okay. And so p and $p \Rightarrow q$, if they are, if they are premises, they give the conclusion q, okay, q is the conclusion. This is called the rule of detachment.

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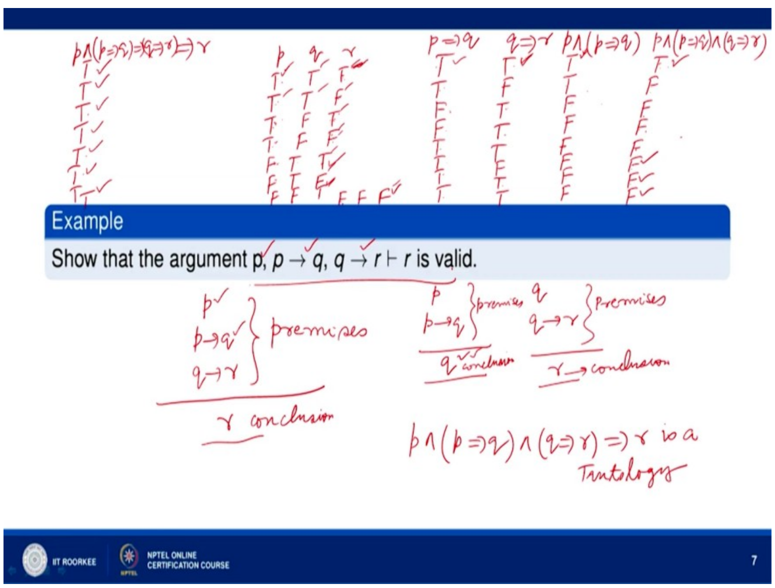

Now, we will use this rule of detachment in the, in this example you can see. We have p, $p \Rightarrow q$, $q \Rightarrow r$ and these are premises, okay, the premises are p, $p \Rightarrow q$ and $q \Rightarrow r$, these are the premises. The conclusion is r, r is the conclusion. So, let us prove this, okay, let us show this by using the law of detachment, okay. So, p and $p \Rightarrow q$, okay, let us take these 2 promises. So, by law of detachment p and $p \Rightarrow q$ gives q, okay, the conclusion q.

Now, let us take q and $q \Rightarrow r$, okay. q and $q \Rightarrow r$, so by rule of detachment again, this gives you r, okay. So, here these are premises, this is conclusion. Then we take this q and combine q with q and r, so now this becomes premises. So, p and p ⇒ q, the two1 premises give us conclusion q, then q and the, the premise q and a premise q ⇒ r gives us the conclusion r. So, without using the truth table, we can show that in this case r will be the conclusion.

We can do it by truth table also, so p q r, okay, TTT, then we have TT F, then we have TFT, then we have TFF, then we have FTT, then we have FTF,& then we have FFT, & then we have last case FFF. This is the last case, so we have 8 possibilities {3, 4, 5, 6, 7, 8} ok. And we can then find the truth value of p ⇒ q and then truth value of q ⇒ r, okay. So, p, then p and p ⇒ q and q ⇒ r we can find. Okay. And then we will find p and p ⇒ q, so that we can write here.

P and p ⇒ q and q ⇒ r goes to ⇒ r, okay, the truth value for this so p and p ⇒ q, so TT gives T, okay, TT gives T, then TF gives F,okay TF gives F, then we have FT gives T, then we have FT gives T, then we have FF gives T, then we have FF gives T, okay. Then q ⇒ r, TT gives T, TF gives F, okay FT gives T, then we have FF gives T, then we have TT gives T, then we have TF gives F and then we have FT gives T and then we have FF gives T, okay. Then p and p ⇒ q and q ⇒ r, okay.

So, we have to consider, let us first consider p and p ⇒ q, okay. So, we have to do it in steps. So, now let us write the truth values of p and p ⇒ q, okay. So, p and p ⇒ q, so p has truth value T, p ⇒ q has truth value T, so we have T here, then p has truth value T, p ⇒ q has truth value T, so we have T here. Then this is T, this F, so we get F, okay, this is T, this is F, so again we get F, okay. Then here we have F, here we have T, so we get F, here we have F, here we have T, so we get again F, then here F, here T, so, again F, and then here F, here T, so again F, okay.

Now, p and p ⇒ q and q ⇒ r, okay. So, let us come by, let us consider this column, okay. So TT is T, TF is F, okay FT is F, then we have FT again F, then we have FT, so we have F here, FF is F, then we have FT F, and then we have FT F, okay. Now p and p ⇒ q and q ⇒ r ⇒ r, okay. So, this column we have to consider and this column we have to consider. So, TT gives T, okay, FF gives T, then FT gives T, then FF gives T, then we have F here we have F here okay, F here and T here, so again T, okay, then we have F here and T here, so again T. Okay, and then we have F here and here also F, so we have T here.

So, all the values 1, 2, 3, 4, 5, 6, 7, one more value is left out, TT Gives T, FF gives T, FT gives T, then FF gives T, okay, then we have FT gives T okay, and then we have FF gives T, then FT gives T and then FF gives T. So, one more T is there. So, all entries in this column are T, and therefore p and $p \Rightarrow q$ and $q \Rightarrow r$, $q \Rightarrow r \Rightarrow r$ is a tautology. Okay, so this argument is valid, this argument is valid. So, this is another way of proving that the argument p, $p \Rightarrow q$, $q \Rightarrow r$ gives r is valid.

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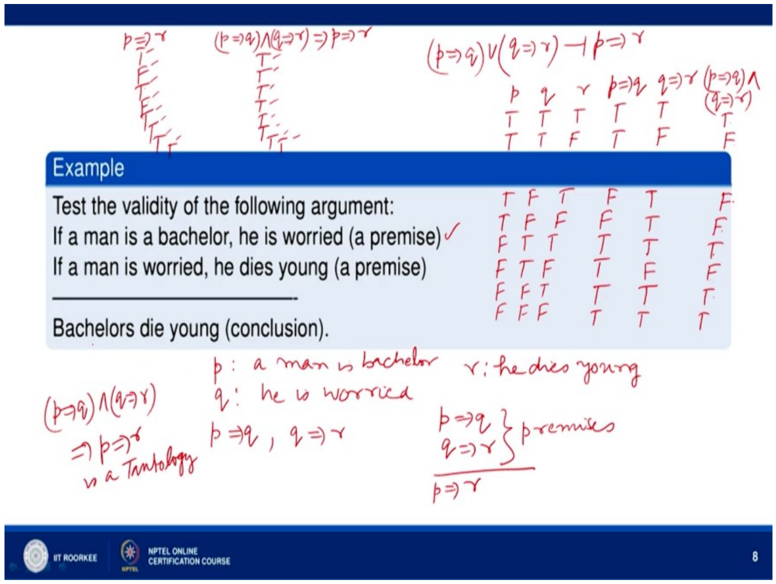

Now, let us test the validity of the following argument. If a man is bachelor, he is worried. So, let p denotes a man is bachelor and q denotes he is worried. Then this premises can be symbolically written as p ⇒ q. Now, 2nd one if a man is worried, okay, he dies young, so let r denote he dies young. He dies young, then the 2nd premise can be written as q ⇒ r, okay. So, we have to test the validity of this argument, if a man is bachelor, he is worried, that is premise, if a man is worried, he dies young. The conclusion is bachelor's die young, so let us say p ⇒ q, q ⇒ r, okay, these are premises.

Okay, then the conclusion is r, bachelors die young, okay. So, let us see p ⇒ q, so the bachelor, so this is p ⇒ r, okay, so we can show that the conclusion is p ⇒ r, that is bachelors die young. So, we have p ⇒ q and q ⇒ r, okay. The conclusion is p ⇒ r, okay. So, let us make the table for this p q r,okay so TTT, TTF, TFT, TFF, okay and then FTT, FTF, FFT, FFF, okay. So, 1, 2, 3, 4, 5, 6, 7, 8, okay, now p ⇒ q.

P ⇒ q is TT gives T, TT gives T, TF gives F, okay, TF gives F, FT gives T, FT gives T and FT gives T, FF gives T & FF gives T. And then we have q ⇒ r, okay. So, TT gives T, TF gives F, okay, FT gives T, then FF gives T, then TT gives T, TF gives F, then we have FT gives T, then we have FF gives T, okay. Now p ⇒ q and q ⇒ r, let us write truth table for this. So, truth values, so p ⇒ q and q ⇒ r, so TT gives T, TF gives F, okay, FT gives F, FT gives F, TT gives T, TF gives F and TT gives T and then TT gives T.

Okay, now we write the truth values of p ⇒ r. So, p ⇒ r, TT gives T, then we have TF gives F, then we have TT gives T, then we have TF gives F, then we have FT gives T, okay, FT gives T, FF gives T and then we have FT gives T and then we have FF gives T. So, these are 8 values, 1, 2, 3, 4, 5, 6, 7, 8, okay. Now, let us write the column for p ⇒ q and q ⇒, this is end, okay, so p ⇒ q and q ⇒ r ⇒ p ⇒ r. Okay.

So TT gives T, then FF gives T, then FT gives T, then FF gives T, then TT gives T, okay, then TT gives T, okay, then FT gives T, then TT gives T and TT gives T. So, 1, 2, 3, 4, 5, 6, 7, 8, okay so you can see all the values are T here, truth values and therefore these are tautology. So, p ⇒ q and q ⇒ r ⇒ p ⇒ r is a tautology. So, the conclusion is bachelors die young.

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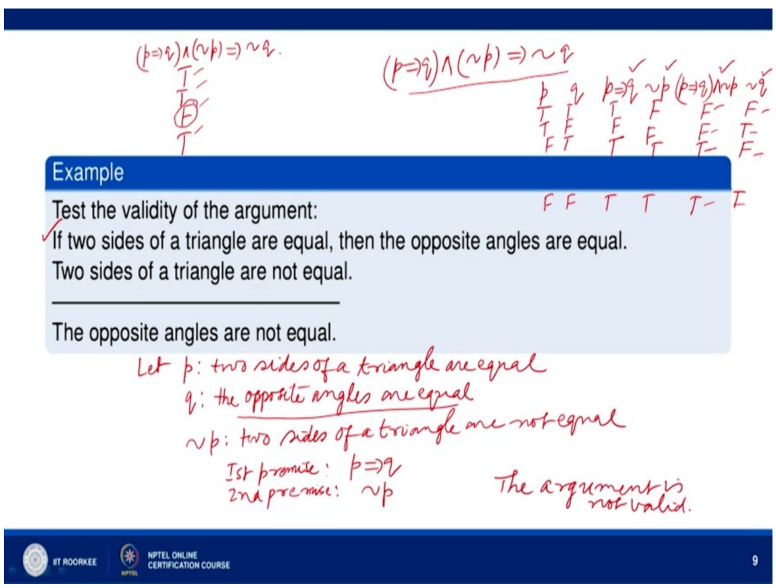

Now, let us take another example, if 2 sides of a triangle are equal, then the opposite angles are equal, 2 sides of a triangle are not equal, the conclusion is the opposite angles are not equal. So, let us say, let p denote 2 sides of a triangle are equal. 2 sides of a triangle are equal and then q denote, the preposition, the opposite angles are equal. Okay. And then 2 sides of a triangle are not equal, so 2 sides, so not p is 2 sides of a triangle are not equal. Okay, so we have p and q, p ⇒ q, okay, first premise, first premise can be symbolically written as p ⇒ q, okay. If p, then q.

And the 2nd premise is not p,okay so our conclusion is the opposite angles are not equal. Let us see the validity of the argument okay. So, p ⇒ q and not p, okay, if it is a valid argument okay, the, we should, the opposite angles are not equal, opposite angles are not equals, that means not q, okay. So, this should ⇒ not q.okay If it is a tautology, then we shall say that the argument is valid, if it is not a tautology, we shall say that the argument is not valid. So, let us write the truth values p, q, p ⇒ q, not p, okay p ⇒ q and not p, then not q and then last column will be p ⇒ q and not p ⇒ not q. Okay.

So we have T here, T here, T here, F here, F here, T here, and then FF, okay. P ⇒ q, TT will give T, TF will give F, FT will give T, FF will give T, okay. Then not be, not p will be negation of the truth values of p, so F, here F, okay, here T, and here T, okay. Then the ⇒ q and not p, p ⇒ q and not p we have to consider, so T and F Gives F, okay, F and F gives F, T and T gives T, T and T gives T, okay. Then not q, so T gives F, F Gives T, and then T gives F and F Gives T, okay.

Then p ⇒ q and not p ⇒ not q. So, we have to consider this column and this column. So, FF gives T, okay, FF gives T, FT gives T, then TF gives F, okay, TF gives F, TT gives T okay. So, we can see here that all the entries in this column are not T, okay, there is one entry which is F, okay therefore this argument is not valid because this argument will be valid only when p ⇒ q and not p ⇒ not q is a tautology. So the argument is not valid. The argument is not valid.

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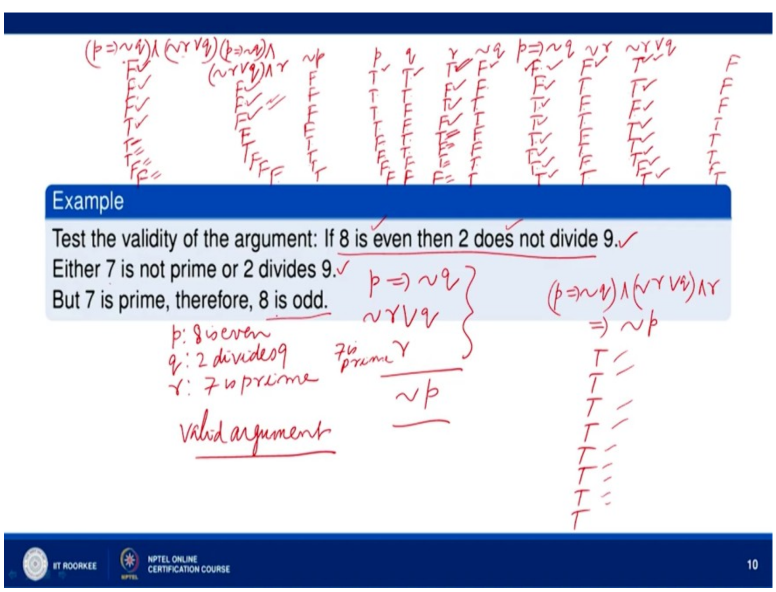

Now, let us go to this example, test the validity of the argument, if 8 is even, let us say p denotes 8 is even and q denote 2 divides 9, r denotes 7 is prime, okay, so p denotes 8 is even, q denotes 2 divides 9, r denotes 7 is prime, okay then we have to see the validity of this argument, okay. So, the first premise, the first premise is if p is even okay, if 8 is even, then 2 does not divide. So p ⇒ not q, okay, this can be symbolically written as p ⇒ not q. Now 7 is not prime or 2 divides 9 okay, either 7 is not prime, so either negation of q, so 2nd one is negation of q or 2 divides 9, 2 divides 9 means no, negation of r.

7 is not prime, so negation of r or 2 divides 9, so q. So, 2nd one can be written as negation of r or q and 7 is prime, therefore 8 is odd, okay, so p ⇒ not q and q (imp) minus r ⇒ minus, negation of r, negation of r or q, we have the 3rd premise is 7 is prime, okay, 7 is prime, so r, okay, this 7 is prime, 7 is prime. Then the negation, then we have the conclusion 8 is odd, so negation of p, okay. So, we have, the premises r, 3 premises are there, p ⇒ not q, then not r or q and r, okay.

So, when these 3 premises lead us to negation of p, we have to see whether this argument is valid, okay. So, we have p q r, okay, so TTT, TTF,, TFT, TFF, FTT, FTF, FFT, FFF, okay, then not q, so not q will be F, F, T,T then F , then F, then T, then T. Okay, now p ⇒ not q, so p ⇒ not q, so TF gives F, TF gives F, then TT gives T, then TT gives T then FF gives T, then FF gives T, then FT gives T and then FT gives T. Okay, so we got p ⇒ not q. Then not r or q, so let us write not r. Not r is F, T, F, T, F, T, F, T okay.

And not r or q, okay, so not r or r q. So, FT gives T, FT gives T, TT gives T, then FF gives F, then TF gives T, then FT gives T, then TT gives T, then FF gives F, TF gives T, okay. Not r or q. And then r, so r is there already. So, we have now to consider p ⇒ not q or not r or q and r. So, let us see, we have, we have to consider first combination of the 2 only, so, let us, see we have, we have to consider only this. So, we have to consider p ⇒ not q, this column and not r or q, this column okay.

So, F and T, so that is F okay. Then FT is F, then TF is F, then TT is T, then TT is T, then TF is F, then TT is T, okay. Now, let us combine it with r. So, p ⇒ not q and not r and q r not r or q and r, okay. So, r is this column, so TF is F, okay, then FF is F, then TF is F, then TT is T, then FT is F, then TF is F, then TT is T. There are 6 only, so let us see, this is, I think we have missed one. Just a moment, see, we are considering this column 1, 2, 3, 4, 5, 6, 7, there are 6 here only, 7 here, p ⇒ not q and not r and q. So, we have to consider, we have to consider this and this column.

So, we have here, FT gives F, okay and then FT gives F, then TF gives F, then TT gives T, then TT gives T, then TT gives T, TF gives F, TT gives T, so one is missing. FT gives F, okay, so we have taken this, we have taken this, FT gives F, then TF gives F, then we have TT gives T, then we have TT gives T, then we have TT gives T, then we have TF gives F and then we have TT gives T. So, we have written FFF, 1, 2, 3, 3F, 3T, okay, one T is missing here. So, let us write this 3F okay and 3T and we write 2F, okay.

So, let us combine this with r, okay. So, FT is F, then FF is F, then we have TF is F, then we have FT is F, FT is F, so this is F, okay. Now we have TT is T, TT is T, okay, so we have here this T, this T, okay, this T and this T, okay, it is T here and then F here and T here. So, we get F. And then we get T here and F here, so we get F here and then F here and F here, so we get F here, okay. So, these are 8 entries, 1, 2, 3, 4, 5, 6, 7, 8. So, we have got this p ⇒ not q and not r and q and r. Okay, now we have to consider the last column.

So, p ⇒ not q and not r or q and r ⇒ not p, we have to write the entries in this column. So, we have to consider this column, this column and not p column, not p column, not p we have not written. Okay, so not p we can write here, not p will be F, F, F, F, T, T, T, T. Okay. Now, let us see implication. So, FF Gives T, okay. Then FF gives T, then FF gives T, then FF gives T, okay, then TT gives T, then FT gives T, then FT gives T and then FT gives T. Okay, so all the entries in this column are T, and therefore this is a valid argument.

Okay, so 8 is odd, this is a valid argument, therefore 8 is 8. So, with this I would like to end my lecture, thank you very much for your attention.