Higher Engineering Mathematics
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Lecture - 56
Assignment Problem -II

Hello friends, welcome to my second lecture on Assignment Problem, let us discuss maximization of an assignment problem, we have earlier discuss in the previous lecture minimization problem, now we consider maximization assignment problem.

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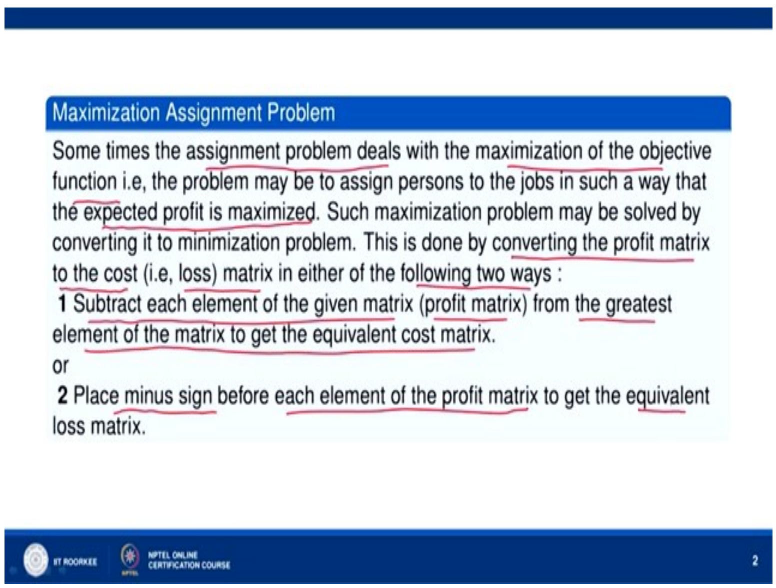

So, sometimes the assignment problem deals with the maximization of the objective function that is the problem may be to assign persons to the jobs in such the way that the expected profit is maximized. Such maximization problem can be solved by converting it to a minimization problem and it can be done by converting the profit matrix to the cost that is loss matrix.

There are two ways to do this, it can be done in two (different) ways the first way is subtract each element of the given matrix that is profit matrix from the greatest element of the matrix to get the equivalent cost that is loss matrix, the second way is place minus sign before each element of the profit matrix to get the equivalent loss matrix and then we apply the usual Hungarian method to obtain the solution of the assignment problem.

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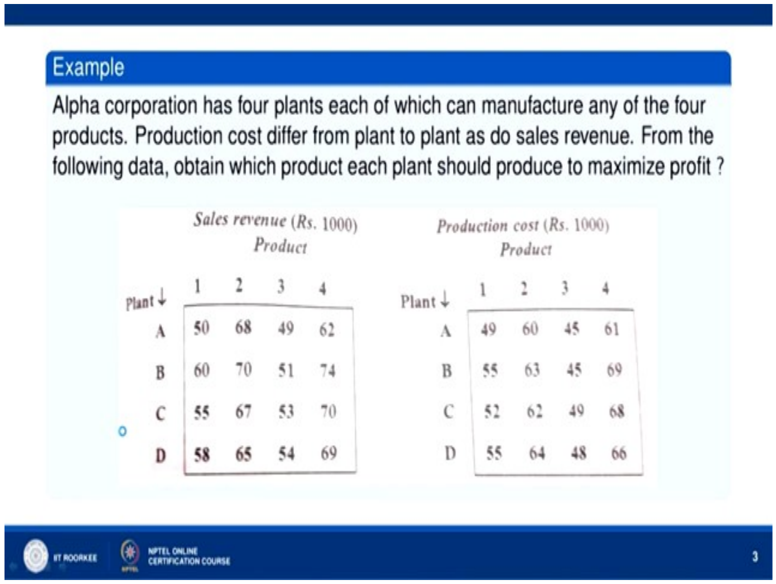

So, let us consider this problem alpha corporation has four plants each of which can manufacture any of the four products. Production cost differ from plant to plant as do sales revenue. From the following data obtain which product each plant should produce to maximize profit. So, this is, this table first table gives us the sales revenue in 1000 of rupees and the second table gives production cost in 1000 of rupees ok A B C D are the four plants and there are four products 1, 2, 3, 4 which can be which are manufactured by the four plants.

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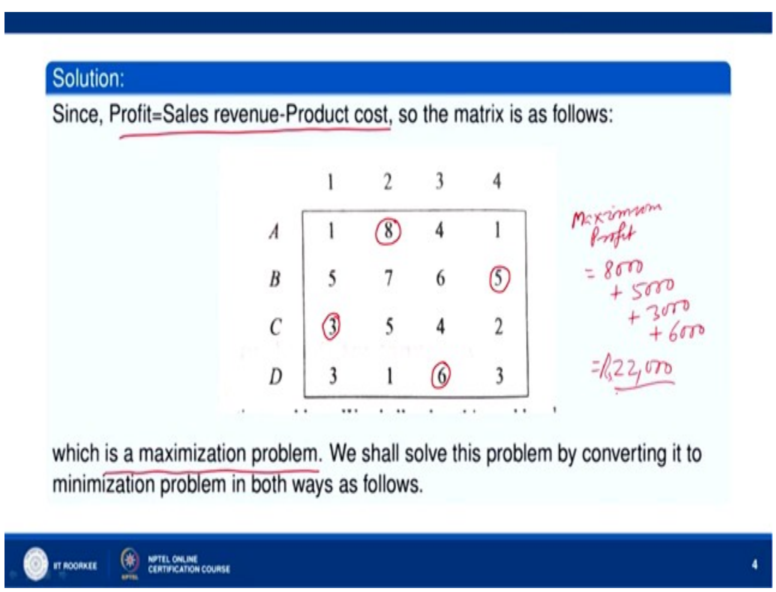

Now, we know that profit is equal to sales revenue minus product cost so the matrix of the profit matrix, the profit matrix is as follows. We can subtract from the sales revenue the product cost that is we can subtract this matrix, second matrix from the first matrix so 50-49 =1, 60-55 = 5, 55-52= 3, 58-55=3, and so on. So this matrix, from this matrix we subtract this matrix second matrix to obtain the profit matrix ok. So this is our profit matrix 1 8 4 1, 5 7 6 5, 3 5 4 2, 3 1 6 3.

Now, we have to maximize the profit so this is a maximization problem. We shall solve this problem by converting it to a minimization problem in both the ways we have discussed.

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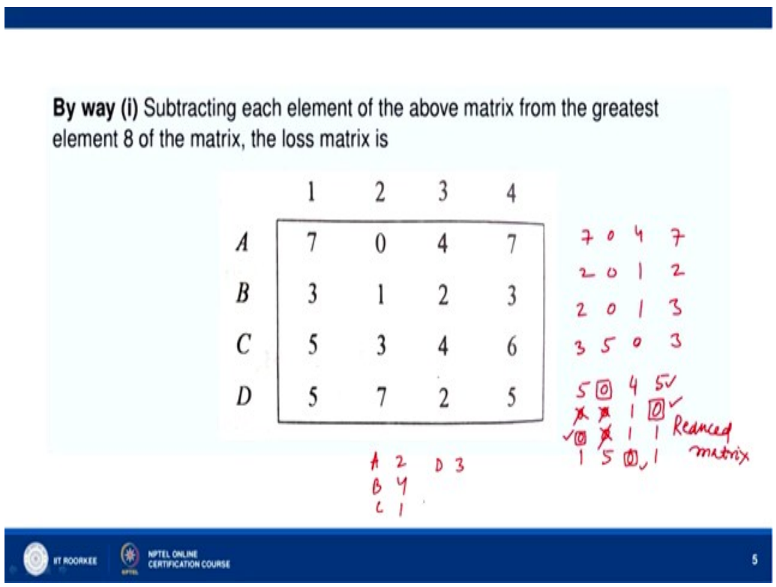

Ok let us consider the first way of converting it to a minimization problem ok so in the first way what we do we subtract each element of the profit matrix from the greatest element of the profit matrix ok. So the greatest element of the profit matrix you can see there are 4 by 4 that is 16 elements of those 16 elements the element 8 ok is the greatest element so we subtract each element of the given matrix from the greatest element ok. So 8-1=7, then 8-8=0 then we have 8-4 =4, 8-1=7 so first row is 7 0 4 7, second row 8-5= 3, 8-7=1, 8-6=2, and then 8-5=3, so we get the second row as 3 1 2 3 ok 3 1 2 3.

And then similarly third row becomes 5 3 4 and 6, so 5 3 4 and 6 and the fourth row becomes 8-3=5, 8- 1=7, 8-6 =2, and then 8-3=5, so we get 5 7 2 and then 5 ok 5 7 2 5. So this is now the loss matrix ok which we have obtained by subtracting the elements of the all the elements of the profit matrix by the greatest element of the profit matrix ok.

Now, what we will do we will apply the Hungarian method ok as we have discussed in the last lecture ok to arrive at the solution of the problem ok. So now it is a minimization problem so as we did do in the Hungarian method ok from the 7 4 0 4 we will find the smallest element of the first row. The smallest element of the first row is 0, so we subtract 0 from the all the elements of the first row and we get 7 0 4 7, so 7 0 4 7 then we subtract the smallest element of the second row from all other elements of second row so when we subtract form 3 we get 2, then we get 1 - 1 =0, then 2-1=1, then 3-1=2 and then the smallest element of the third row is 3, so 5-3=2, 3-3= 0, 4-3=1, 6-3=3 smallest element of the fourth row is 2, so we subtract 2 from 5 to get 3 and then 2 we subtract from 7 to get 5, 2-2=0 ok and 5-2=3 ok.

Now, the same thing we do for all the columns, so in the first column the smallest element is 2 ok we subtract 2 form all the elements of the first column and we get 5 0 0 and then we get 1 ok. In the second column, the smallest element is 0 so when we subtract from all other element of this column we get the same elements so 0 0 0 5 ok similarly in the third column the smallest element is 0 so subtracting 0 will not change it 4 1 1 0 ok.

In the fourth column, the smallest element is 2 subtract 2 from all the elements of the fourth column we get 5, we get 0, we get 1 we get 1 ok so this is the reduced matrix ok and then we start from the first row. In the first row we have a symbol 0 ok so we encircle it and then cross all 0s which lie in its column so we cross these two ok then we go to the second row in the second row we have two 0s, in the third row we go to third row, third row has got two 0s third row has got a single 0, so we encircle it and then cross the 0 which lie in its column so we cross this.

And then we go to the fourth row. Fourth row has a single 0 we encircle it. Now we go to the first column ok first column has been there is no 0, this 0 has been assigned, in the second column there is no 0 this zero has been assigned, in the third column 0 has been marked, in the fifth column, in the fourth column we have a 0 so make an assignment here ok.

So, now the first row has an assignment, second row has an assignment, third row has an assignment and fourth row has an assignment. so this gives the solution of the problem. So this means that A B C D plants are there ok and products are 1 2 3 4 ok. So A goes to 2 ok the plant A ok, plant A will produce product 2 ok plant A will produce product 2, plant B will give product 4 and plant C will get product 1 and plant D will get product 3 ok. So when A is given the product 2, B is given the product 4, C is given the product 1 and D is given the product 3 then the profit will be the maximum ok.

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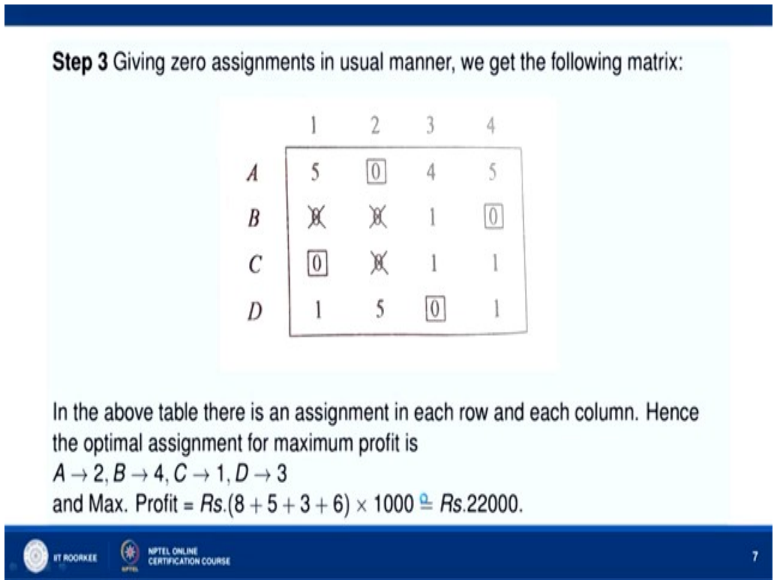

So you can see here A is getting the product 2, B is getting the product 4, C is getting the product 1, D is getting the product 3 and the maximum profit as you can see from this table A is getting the product 2. So the profit is 8 and it is in thousands ok so it will be 8000 ok so 8000 and B gets 5 this is 5 ok and C gets 1 ok C gets 1 so this is 3 and D gets a 3 so this is 6 ok so profit maximum profit will be equal to 8000+5000 + 3000+6000 so we get 22000 ok so if A is given the product 2 the plant A is given the product 2, plant B is given the product 5, C is given the product 1 and D is given the product of 3, then the profit will be the maximum and it be rupees 22000. So this is how we solve this problem of on maximization of profit.

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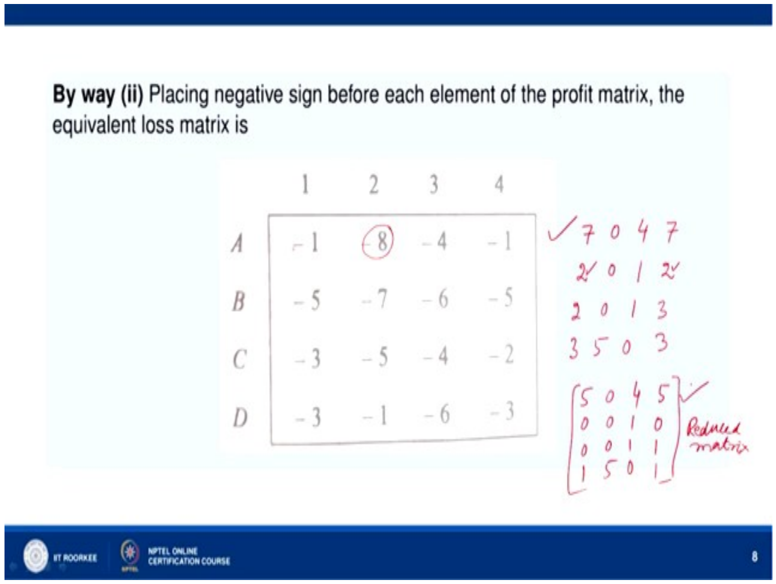

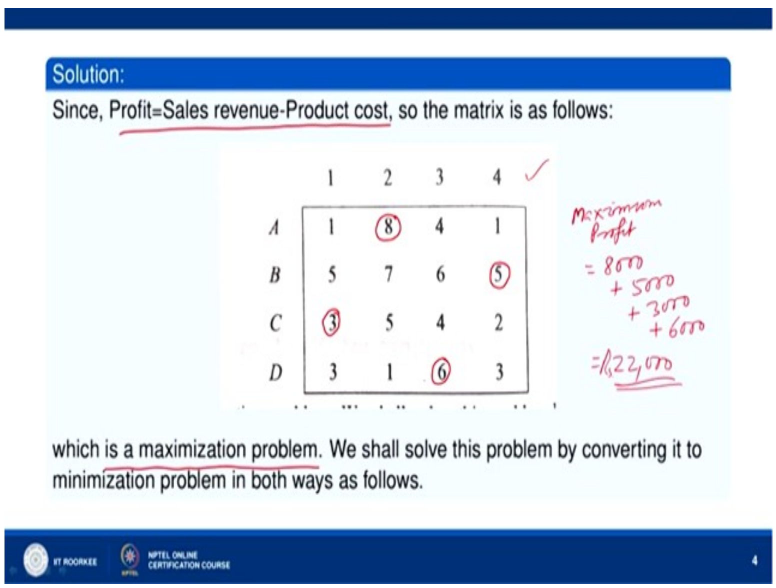

Now, let us go to the second method in the second method what we do is we put the negative sign before each element of the profit matrix, so this is our profit matrix ok this is our profit matrix we put a negative sign with each element of the profit matrix then it will be -1, -8, -4, -1, -5, -7, -6, -5, -3, -5, -4, -2, -3, -1, -6, - 3, so the equivalent loss matrix. The matrix which is the loss matrix will be this one, this one ok. After we put a negative sign with each element of the profit matrix we arrive at this matrix.

Now, what we will do? We will apply the Hungarian method so we subtract the smallest element from each element of the first row ok. So smallest element is -8, so -8 we subtract from each element of the first row so -1+8=7, - 8 (minus) when we subtract from -8 we get 0, -4 +8= 4, -1+8=7..

In the second row the smallest element is -7 so -7 we subtract from -5 we get 2, -7 when we subtract from -7 we get 0, -6 +7 =1, -5+7=2 and here the smallest element in the third row is -5 so -3+5 =2, -5 subtracted from -5 gives 0, -4+5=1, -2+5=3.

In the third row the smallest element is -6, so -3-(-6)=3 and then -1+6=5 and then -6 subtracted from -6 gives 0, -3+6=3 ok. Now, let us now subtract the smallest element from all other elements of each column ok so smallest element in the first column is 2 ok so smallest element 2 we subtract from all other element of the first column we get 5 0 0 and then we get 1 here we get 0 0 0 and 5 ok because the smallest element is 0 and 4 1 1 0 and we subtract the smallest element 2 here from all other element here 7-2=5, 2-2= 0, 3-2=1, 3-2= 1 ok so this is our reduced matrix.

Ok, and then the same reduced matrix we had got here you can see the same matrix we had got here so we follow the Hungarian method we arrive at the same solution ok so same matrix we get ok so we follow the Hungarian method and we get the same solution as in the case as in the first method that is A is given the product 2, B is given the product 4, C is given the product 1 and D is given the product 3 ok.

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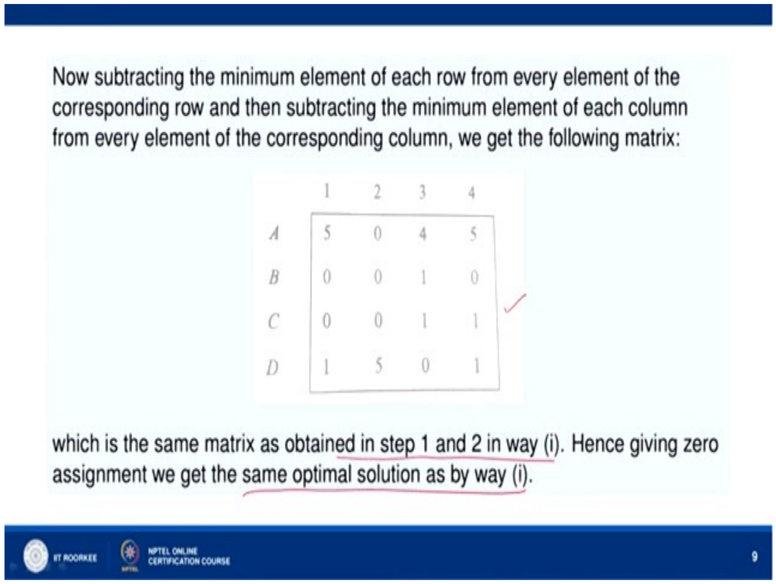

So, this is what we get which is the same matrix as obtained in step 1 and 2 in the way 1. Now hence giving 0 assignment we get the same optimal solution as by way 1 ok.

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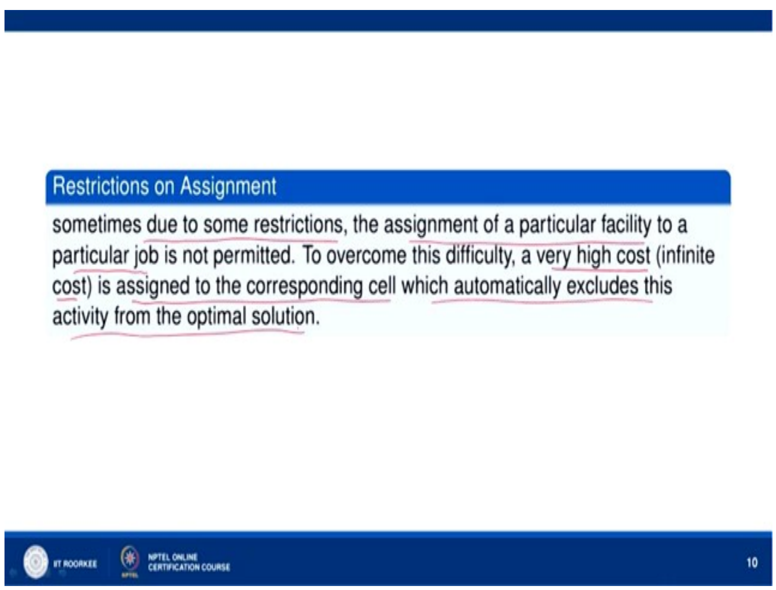

Now, restrictions on assignments. Sometimes due to some restrictions, the assignment of a particular facility to a particular job is not permitted. To overcome this difficulty a very high cost that is infinite cost is assigned to the corresponding cell which automatically excludes this activity from the optimal solution ok. So if there are some if they due to some restrictions the assignment of a particular facility to a particular job is not permitted then we will apply we will assign infinite cost to that cell ok and which will exclude that activity from the optimal solution.

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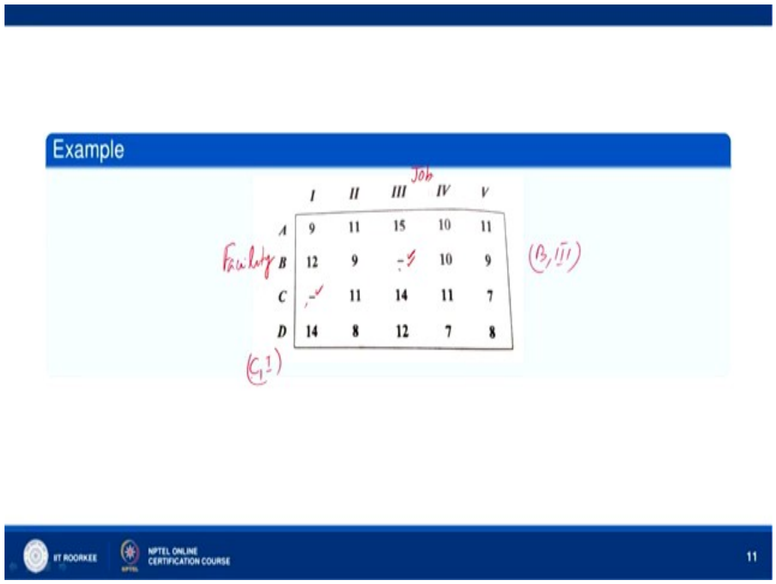

So, let see how we do this for example let us consider this problem ok so here we have a this is an unbalanced problem you can see we have this is of four row ok, 4 rows are there and 5 columns are there and due to some restrictions C and this one third this one ok this two cells this cell and this cell ok are not assigned any cost ok so they will be giving infinite cost to them ok. So this facility is not assigned to the particular facility is assigned to a particular job ok.

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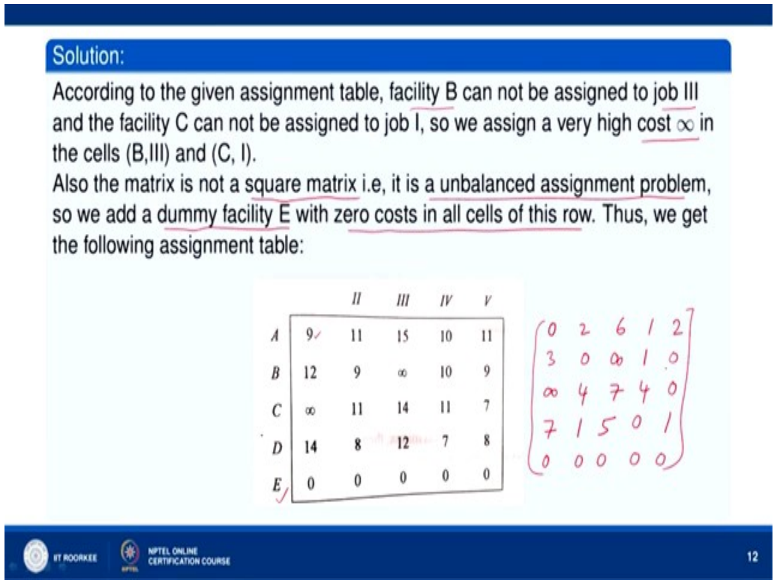

So this are blanks here we will assign infinite cost to them and to make it a balanced problem we will add one dummy according to the given assignment table facility B ok these are facilities ok these are facilities.

So facility B cannot be assigned to job 3 ok this are jobs ok so according to given assignment table facility B cannot be assigned to job 3 and facility C cannot be assigned job 1 so we can assign a very high cost that is infinity to the cells (B, III) and (C,1), this is (B, III) ok this cell is (B,III) ok and (C,I) this one is (C,I). We assign infinite cost to these cells and also the matrix is not a square matrix that is it is not it is a unbalanced assignment problem so we add a dummy facility ok we add a dummy facility E ok with zero cost in all cells of this row ok. We give zero cost to all cells in this row then we get the following assignment table ok. So now this has become a balanced problem ok.

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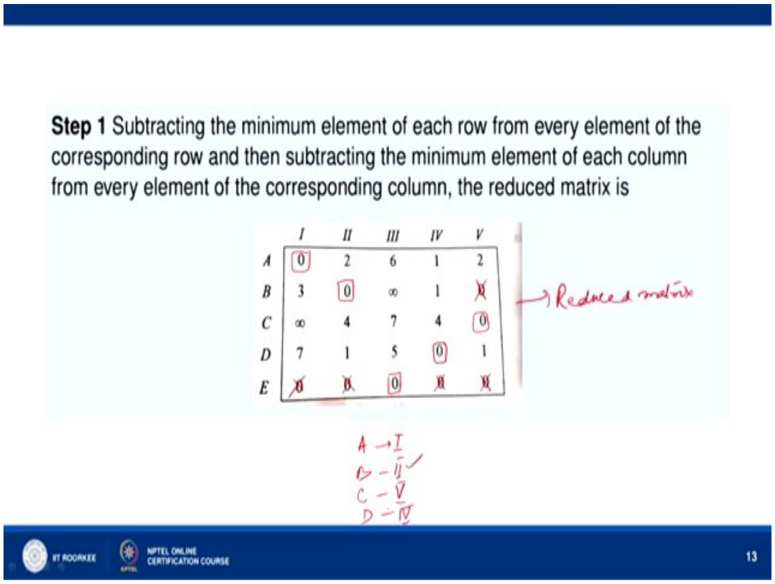

We will subtract the minimum element of each row from every element of the corresponding row and then subtract the minimum element of each column from every element of the corresponding column. So if we do this then what will happen here?

Minimum element of the first row is 9 ok so 9 we subtract from every element of the first row we get a 0 2, then we get 6 we get 1 and 11-9=2 and then here we subtract the smallest element 9 from every other element so we get 3 then 0 then we have infinity, infinity - 9 I=infinity, then 10-9=1 and then 9-9= 0, here the smallest element is 7 so infinity-7= infinity, so 11-7=4, 14 -7= 7, then 11-7=4,7-7=0, then the smallest element in the fourth row is 7, so 14-7=7, 8-7=1,12-7=5, 7-7=0, 8-7=1, and the last row is the same because smallest element is 0 subtracted from all other 0s gives 0 ok.

Now, we do the same thing for all columns so smallest element in the first column is 0 which when we subtract from another elements we get the same element here we smallest element is 0, so subtracting it from another elements does not change it, here again smallest element is 0 so when it is subtracted from another element it will not change, here also the smallest element is 0 so the column remain the same, and here also the smallest element is 0 so this is now reduced matrix ok so we get the reduced matrix as 0 2 6 1 2, 0 2 6 1 2 3 0 infinity 1 0, 3 0 infinity 1 0, infinity 4 7 4 0, infinity 4 7 4 0, 7 1 5 0 1, 7 1 5 0 1 and then last row is 0 0 0 0 0 ok so this is our reduced matrix.

Now we can begin with row 1, in the row 1 we make an assignment here because there is a single 0 here and then cross out all 0s which lying in the in its column so we cross this we go to the second row, second row has got two 0s ok so we move to the third row, third row has got a single 0 here we make an assignment here and then cross out 0 and this 0 which lie in it is column.

In the fourth row there is a single 0 here, we make an assignment here and then cross out 0 which lie in its column and then in the fifth row we are having two 0s. Now we go to the columns. Now, let us consider the columns one by one ok so in the first column there is no 0 which is left, in the second column we have two 0s. In the column we have a single 0, so we make an assignment here and then we cross out this 0 which will lie in its row and then we go to fourth column. In the fourth column there is no zero here and then in the fifth column there is no zero here ok.

So what we have in the first row we have an (assignment) so this is a 0 we have assign and now we come to the second row now we again repeat the same steps so we in the first row we have made an assignment, in the second row there is a single zero here we make an assignment here, and then in the third row we have done assignment, in fourth we have made an assignment, and fifth we have made an assignment.

So ok so A is given job 1, B is given job 2, C is given job 5, C is given job 5, D is given job 4 (D is given job 4) ok.

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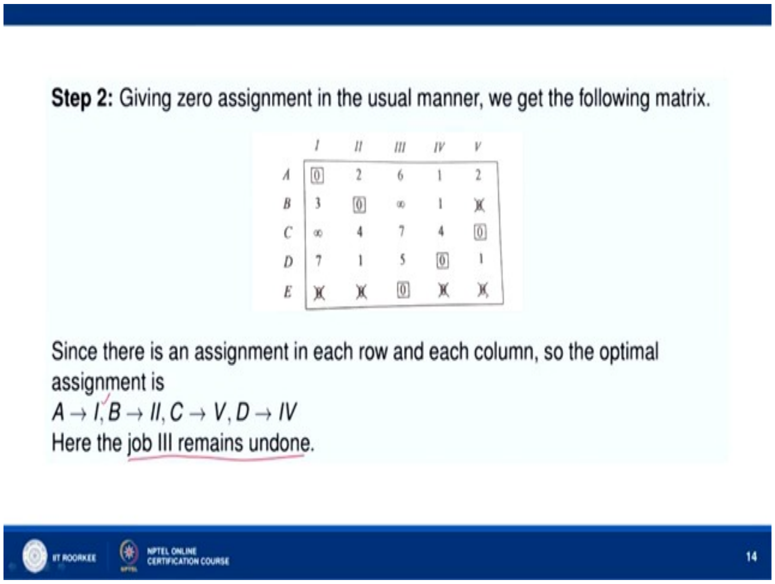

So we have this A is given job 1, B is given job 2, C is given job 5, D is given job 4 and the job 3 you can see here job 3 is undone job 3 is not given to anyone so job 3 is remains undone it is not assign to (anyone) no facility is assign to this one so this remains undone. So when we have an unbalanced problem then we will add a dummy facility ok like here we have added a dummy facility E and avoid zero cost to all elements in its row so that the problem becomes a balanced assignment problem and then we use then we follow the usual Hungarian method to solve this problem and arrive at the solution.

So here we have considered a problem where the due to some restrictions the assignment of a particular facility to a particular job is not permitted. So in such cases we assign a very high cost to the corresponding cell and that excludes this activity form the optimal solution. So that is the how we tackle such kind of a problem where restrictions on assignments are given ok, so that is all in this lecture thank you very much for your attention.