Higher Engineering Mathematics

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Transportation Problem - II
Mod11_Lec54

Hello friends, welcome to my second lecture on Transportation Problem. In the previous lecture on transportation problem we had considered two methods, they are North West corner rule and then the second one was least cost method, to determine the basic feasible solution for the transportation problem.

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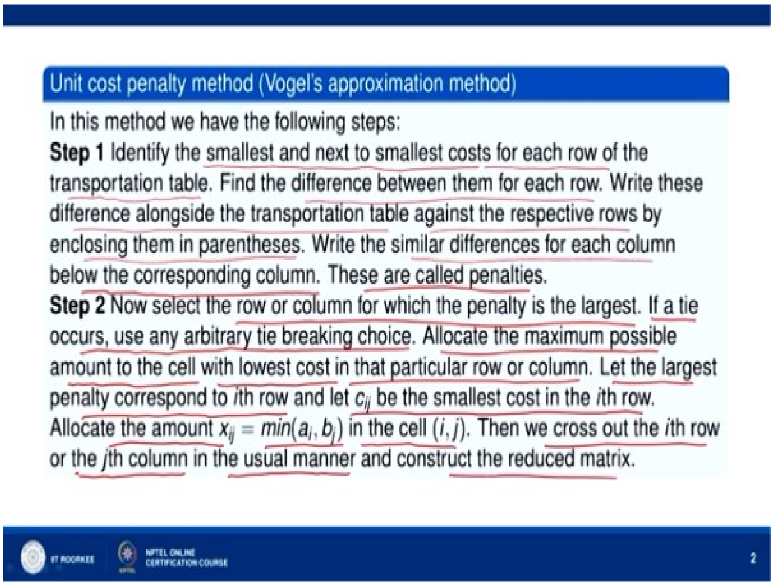

Now, we have third method, which is knows as Vogel's approximation method or unit cost penalty method. So, this method is most effective method to find basic feasible solution. Let us discuss this method, so in this method we first step is to identify the smallest and next to smallest cost for each row of the transportation table. So, (we prepare the transportation tab) we have the transportation table, we will identify the smallest and the next to smallest cost for each row of the transportation table and then we find the difference between them for each row.

We write these differences alongside the transportation table against the respective rows by enclosing them in parentheses. We write similar differences for each column below the corresponding column. They are called penalties. Okay, these differences are known as penalties. Now, we select the row or column for which the penalty is the largest.

So we inspect all the values, the differences which we found for rows and the differences which we found for penalties we found for columns, okay that is we inspect all the penalties, then we identify the penalty which is the largest, if a tie occurs, we use any arbitrary tie breaking choice.

Suppose if there is tie between two penalties and both penalties are same and they are largest then we can choose either one, either penalty to begin with. Now allocate the maximum possible amount to the cell with lowest cost in that particular row or column, so we identify the penalty which is the largest, then in that particular row we identify the column which has the lowest cost, so in that cell we allocate the maximum possible amount.

Suppose the maximum penalty occurs in the i-th row, for example, the penalty occurs in the i-th row, then and the lowest cost cell is the cell where the cost is c_{ij}, so then to that cell which is having lowest cost cij we shall allocate the maximum possible amount that is the minimum of a_i and b_j, where ai are the amounts which are available, in that row and then the b_j is the maximum demand for that.

So, a_i, b_j will take the minimum value of that, so let the largest penalty correspond to i-th row, largest penalty corresponds to the i-th row and c_{ij} be the smallest cost in the ith row, then we allocate the amount x_{ij}which is the minimum of a_i, b_j in the cell i,j. Then we cross out the i-th row or the j-th column in the usual manner and construct the reduced matrix.

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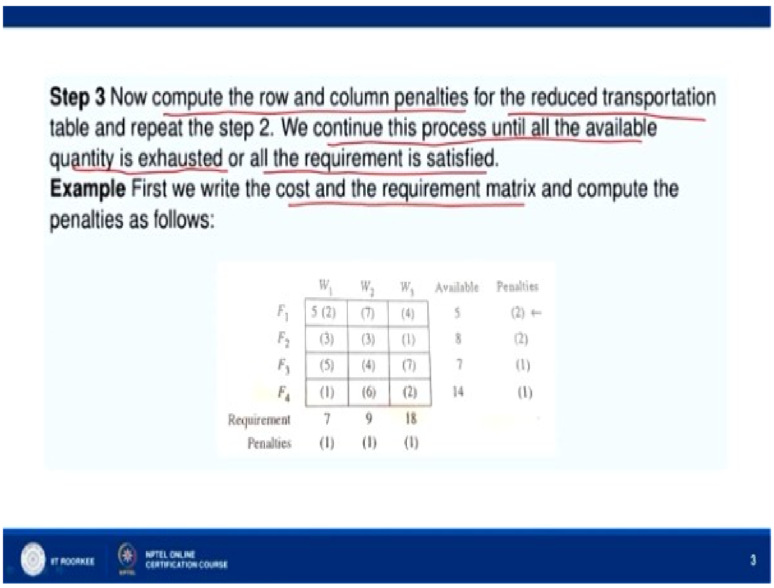

	W_1	W_2	W_3	Available	Penalties
F_1	5 (2)	(7)	(4)	5	(2) ←
F_2	(3)	(3)	(1)	8	(2)
F_3	(5)	(4)	(7)	7	(1)
F_4	(1)	(6)	(2)	14	(1)
Requirement	7	9	18		
Penalties	(1)	(1)	(1)		

So, in the third step, we compute the row and column penalties for the reduced transportation table because we have deleted the i-th row of the j-th column, now we have the reduced transportation table, so we again calculate the penalty and then identify the row or column which has got the maximum penalty and in that row or column we then identify the cell which has got the lowest cost, and in that we allocate the amount x_{ij} which is the minimum of a_i, $b_j$for that cell. So, we continue this process until all the available quantity is exhausted or all the requirement is satisfied.

Now, lets us see, this is our transportation table, the cost and requirement matrix, okay, this is our cost and requirement matric, in the parentheses you can see we have the cost for each cell, 2 is the cost of this cell, 7 is the cost for this cell, 4 is the cost here and 3 and 3 1 they are the cost in this row, 5 4 7 are the cost in third row and 1 6 2 are the cost in the fourth row and with F_1, F_2, F_3, F_4 and 5, 8, 7, 14 units are available and the requirement at the warehouse W_1, W_2, W_3 are 7, 9 and 18.

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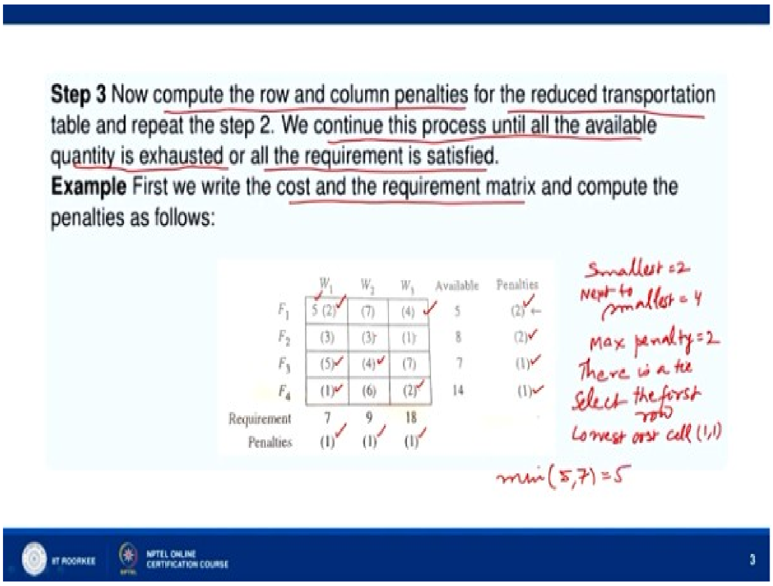

So, let us first find the penalties, so how do find the penalties? Let us go to this, we find the difference between the smallest and next to smallest cost for each row, so here smallest is what? Let us take this first row and the smallest is 2 and next to smallest is 4, so the difference is 2, so we write this 2 in the bracket. In the second row the smallest is 1 and the next to smallest is 3 so this difference is 2. In the third row the smallest is 4 and next to smallest is 5 and the difference is 1, so we write 1 here in the bracket and then here smallest is 1 and next to smallest is 2 and the difference is 1, so we write 1 here.

So, we have found the penalties for all the 4 rows, for first row its 2, for the second row its 2, the third row it is 1 and fourth row it is 1, now for the first column, what is the penalty? So, the least cost is 1 and then the next smallest is 2, so the difference is 1, so we have 1 here and here the smallest is 3, next to smallest is 4 again the difference is 1, here smallest is 1, next to smallest is 2 so again the difference is 1, so the penalties row wise are 2, 2, 1, 1 and column wise they are 1, 1, 1 and 1.

Now, let us find the maximum penalty, so maximum penalty occurs along first and second rows, maximum penalty is 2 and it is occurring along first row and second row, so there is a tie. So we have to break this tie, we can select either first row or the second row to begin with, suppose we select the first row, so select the first row, now then we will find the cell in the first row which has got the lowest cost, so this cell, this cell has got the lowest cost and the lowest cost is 2.

So the lowest cost cell is 1,1 and the cost is 2, so to this cell which has got the lowest cost we give the maximum possible amount, now with F1 5 units are available and the requirement of W is 7, so we can allocate minimum of 5 and 7 that is 5, so allocate 5 here to this cell. Now we can cross out first row, after we have allocated 5, the capacity of F_1 is adjusted, we have delivered all 5 available units with F_1 to the warehouse W_1, so 5 is also, we will cross out the first row and then we will have three rows, second, third and fourth row, then three columns.

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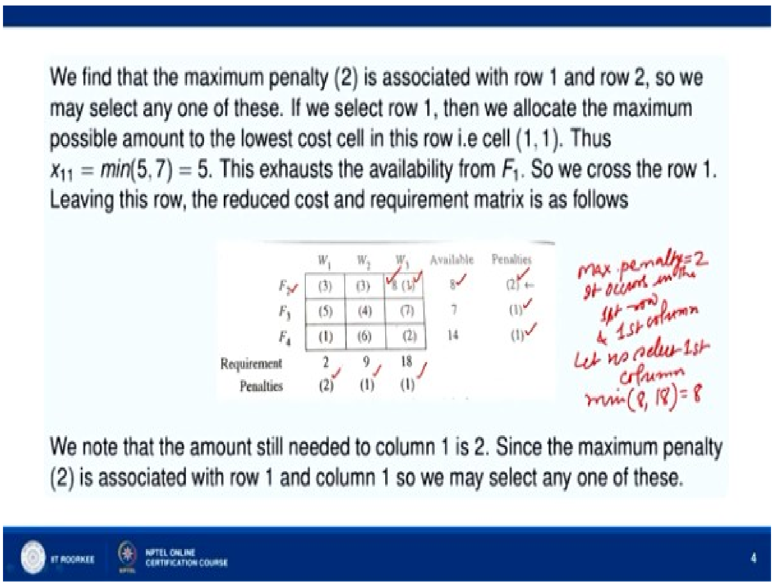

So, then the next transportation table is this one, so 3, 3, 1 and 5, 4, 7 and 1, 6, 2 that is the next table after we have deleted the first row, remove the first row. Again we calculate the penalties, so here the lowest cost is 1 and next to smallest is 3, so the difference is 2, here lowest cost is 4, next to lowest is 5, so we have difference is 1, here smallest cost is 1, next to smallest is 2, so we have difference is 1. So, column wise let us find the penalties, so smallest is 1 next to smallest is 3, so the difference is 2, smallest is 3 here next to smallest is 4 so difference is 1 here, smallest is 1 here next to smallest is 2 so the difference is 1 here.

So, now let us find again the maximum penalty, the maximum penalty is 2 and it occurs in the first row and the first column, so it occurs in the first row, first row and first column. So we can select either the first row or we can select the first column, so let us select first column. Now then we will identify the cell which has go the lowest cost, this cell has got the lowest cost and to this cell we shall allocate the maximum possible amount that is available.

So with F_2 8 units available and the requirement of W_3 is 18, so we can allocate maximum possible amount with F_2 that is 8, so minimum of 8 and 18, that is 8, so we can allocate 8 to W_3. Now all the available units with F_2 have been allocated, so we will cross out the second row and then we shall have the remaining two rows 5,4,7, 1,6,2 so that will be our next transportation table.

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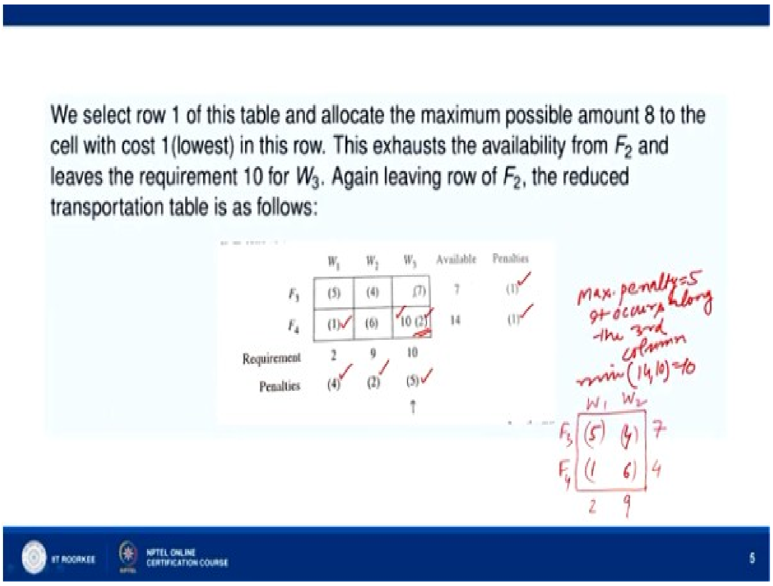

So, this is our next transportation table 5,4,7, 1,6,2, we will again find the penalties, so penalties now are, least, lowest, smallest cost is 4, next to smallest is 5, so difference is 1, smallest cost here is 2, no smallest cost is 1 next to smallest is 2, so the difference is 1 and column wise 1 is here and 5 is there, so smallest is 1 and next smallest is 5, so difference is 4 and here smallest is 4 and next to smallest is 6, so the difference here is 2, here the difference between the smallest and next to smallest is 5, so maximum penalties is now 5 and it occurs along the third column.

So, let us now find the cell which has got the lowest cost and this cell has got the lowest cost 2, so we allocate the minimum of 14 and 10, so 10 units are allocated to the cell with the cost 2, now what is left with us here? So we have allocated 10, so then after that we cross out this cell because we have already allocated, the capacity of W_3 is now exhausted, W_3 requirement was 10 and we have allocated 10 units to W_3, so the third column is now removed and we have the transportation table containing two rows, this one 5, 4, 1, 6 and two columns, so this one 5, 4, 1, 6, so this is now reduces, this transportation table reduces to this one, this is W1, this is W_2, this is F_3, F_4.

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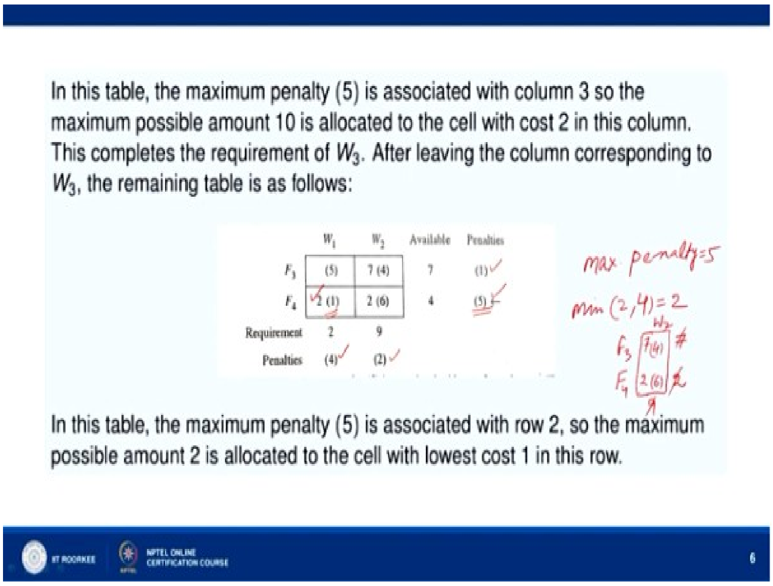

Alright, now we go to the next table. So out of 14, 10 have been given so this is now reduced to 4,F_4 now has 4 units available and F_3 has 7 and here this is 2, this is W_2 2, 9, we have this transportation table, so let us go to this one. We can see again the lowest cost is 4 and next to lowest is 5, so the difference is 1, here the smallest cost is 1 next to smallest is 6, so the difference is 5, here the smallest is 1 next to smallest is 5, so the difference is 4, here smallest is 4 next to smallest is 6, so the difference is 2.

So, what is the maximum penalty? The maximum penalty is 5, it occurs along the second row, so let us identify the cell which has the least cost, so this cell has the lowest cost 1, so we will then allocate minimum of 2 and 7, that is 2, so we allocate 2 to this cell. Now the requirement of W_1 was 2 and we have allocated 2 units to W_1, so we can cross out this column and then what are we left with? F_3 and F_4.

So, we have, from 4 we have given 1, 2, no, we are taking minimum of 2 and, we are giving 2 2, minimum of 2, why 2 and 7, maximum penalty was 5 and this cell has got the lowest cost 1, we take the requirement of W_1 is 2 and the availability of F_4 is 4, so minimum of 2 and 4 we have to take, so we allocate 2 here, so then the requirement of W_1 is fulfilled, we can cross out W_1 and with F_4, now 2 units are available and F_3 has got 7 and the total requirement of W_2 is 9, so we allocate 7 to F_3, F_3 has this and got cost 4, so we allocate 7 to this and we allocate 6 to this and we cross out. So, this is what we do, we had only 7 units available with F_3, which we have given to W_2 where the cost is 4 and we have given 2 units W_2, where the cost is 2.

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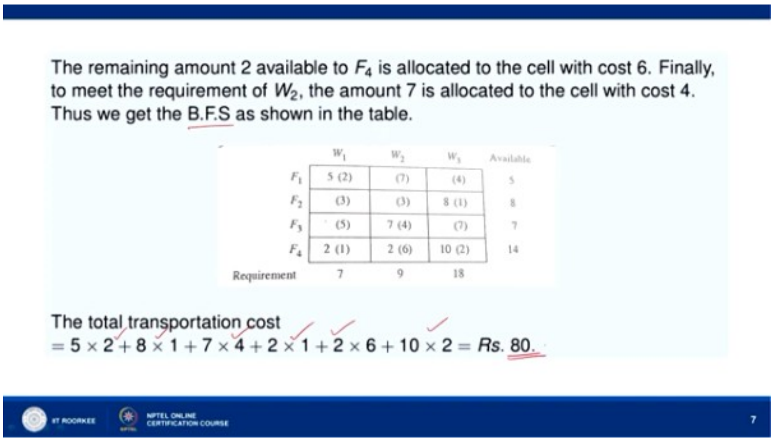

So ultimately we have this table, 5 from F_1 source we have given all 5 to W_1 with cost 2 and we summarized and then we have given all the 8 that was available with F_2 to W_3 and we have given out of 7 to W_2 which cost 4 and then we have given out of 14 which was available with F_4, 2 to W_1 and 2 to W_2 and 10 to W_3, this we can see from here, so we give 5 from F_1 to W_1, we gave out of 8, all 8 to W3 and we have given from F_4 out of 14, 10 to W_3 and then we have given out of 7 which was available with F_3 to W_2 with cost 4, 2 to W_1 have been given from F_4 and 2 to W_2 have been given from F4, so this is how we have allocated the units that were available with F_1, F_2 and F_4 to the warehouses W_1, W_2, W_3 and W_4.

Now, we can calculate…so this is B.F.S of which we have got and the total transportation cost is 5 into 2 then 8 into 1 then 7 into 4 then 2 into 1, two into 6 and then 10 into 2 so that is 80 Rs, so that is the transportation cost which we have by the Vogel Approximation Method.

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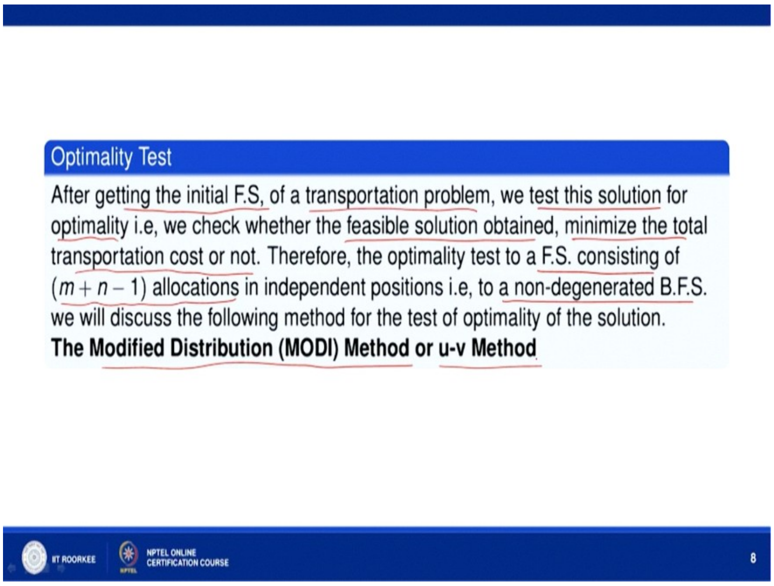

Now, let us discuss the Optimality test, after getting the initial feasible solution of the transportation problem we then test this solution.

We then find, we did not test this solution for optimality okay, let us test this solution for optimality, which I gather the feasible solution obtained, minimise the total transportation cost or not. Okay, therefore the optimality test for a feasible solution consisting of M + N - 1 allocation, see you can see how many allocations we have made, see 1, 2, 3, 4, 5, 6, six allocation we have made and M + N, there are 4 rows and 3 columns.

So M is equal to 4 and N is equal to 3, so M + N - 1 equal to 6. Okay, so we have made 6 allocations okay, so let us test whether this solution which we have got, this one, this solution is optimal or not, and if it is not we will improve this to get this optimal allocation okay, so therefore the optimality test to a feasible solution consisting of M + N - 1 allocations in independent position that is to a non-degenerated because when there are M + N -1 allocations in independent positions we get non-degenerated BFS okay, so we will discuss this method MODI method okay, or we also call it as U-V method to test the optimality of the solution which we have got from Vogel approximation method okay.

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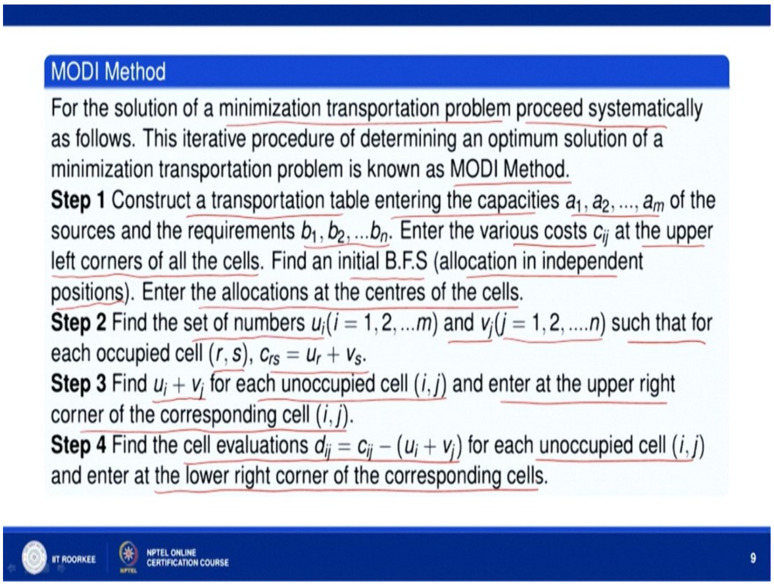

So for the solution of a minimization transportation problem proceeds, let us proceed systematically as follows. Okay, the iterative procedure of determining an optimum solution of a minimization transportation problem is known as MODI method. Now let us, first we construct a transportation table entering the capacities a_1, a_2,…, a_m, are the capacities of the sources, F_1, F_2,…, F_m and the requirements, b_1, b_2,…, b_n they are the requirements of the N warehouses, W_1, W_2,…, W_n.

Now, we enter the various cost c_{ij} at the upper left corner, so we have follow system of allocation here, writing values in the various cells, will write the cost c_{ij} of in each cell at the upper left corner. Okay, find an initial BFS that is allocation in independent positions. Okay and then enter the allocations at the centre of the cells, so the problem just now we have solved okay, in that problem the allocation that we have made they will be written in the centre of the cell. Okay and the cost c_{ij} of each cell will be written in the upper left corner.

Now, let us find the set of numbers u_i and v_j, i runs from 1 to m, j runs from 1 to n. Okay, such that for each occupied cell, occupied cell means the cell which has been, where the allocation has been made okay and not occupied cell means this cell where no allocation has been made okay, so such that for each occupied cell rr okay, c_{rs}= u_r + v_s, c_{rs} is the cost of the rs cell okay, so $c_{rs} = u_r + v_s$.

Now find u_i + v_j for each unoccupied cell okay, for each unoccupied cell, or the cell where the no allocation has been made, so for each such cell we had to find u_i + v_j and enter the upper right corner of the corresponding cell i j okay, so $u_i + v_j$ we shall find and u_i and v_j we shall find and $u_i + v_j$ we shall enter at the upper right corner. Okay.

Now find, then we will find the cell evaluations d_{ij}, d_{ij} equal to $c_{ij} -(u_i + v_j)$, c_{ij} is the entry in the ij cell at the upper left corner and $u_i + v_j$ is the entry at the upper right corner, so we will find the difference of c_{ij} and $u_i + v_j$ for each unoccupied cell okay and enter at the lower right corner of the corresponding cells okay, so the values of the d_{ij} will be found for each unoccupied cell and the values of d_{ij}will be written at the lower right corner of the unoccupied cell.

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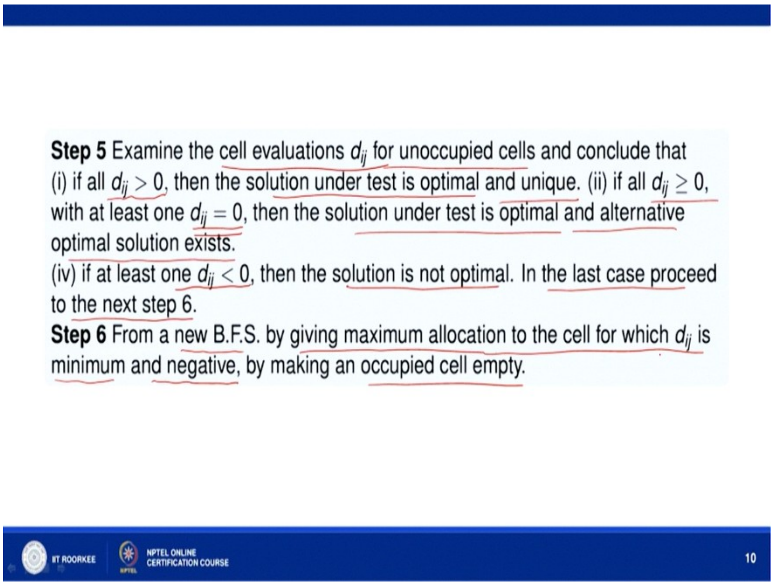

Now, we will examine the cell evaluations d_{ij}okay, for all unoccupied cells, if it happens that all $d_{ij} > 0$. Okay, $d_{ij}> 0$ for every unoccupied cell, then the solution under test is optimal okay and unique. Okay, so we found the unique optimal solution, if d_{ij} turns out to be greater than 0 for all i and j and they belong to unoccupied cells, now if $d_{ij} \geq 0$, with at least one $d_{ij} = 0$. Okay, then the solution under test is optimal and alternative optimal solution exists. Okay, so solution under test is optimal, but it is not unique, alternative optimal solution exists.

Now if at least one $d_{ij} < 0$, suppose there is one d_{ij}, at least one which is less than 0, then the solution is not optimal and we have to proceed to the step 6. Okay, this step 6, so we from a new BFS okay, by giving maximum allocation to the cell for which d_{ij} is minimum okay, we will form a new a BFS okay, by giving maximum allocation to the cell for which d_{ij} is minimum and negative.

Okay, so this step 6 will go to step 6, if there is at least one d_{ij} with negative value. Okay, less than 0. Okay, so maximum allocation will be made to the cell for which d_{ij} is minimum and negative. Okay, so will take that d_{ij} which is minimum and negative by making an occupied cell empty and occupied cell will be made empty and the allocation will be made to di, this cell, maximum allocation will be made to this cell okay, where d_{ij} is minimum and negative.

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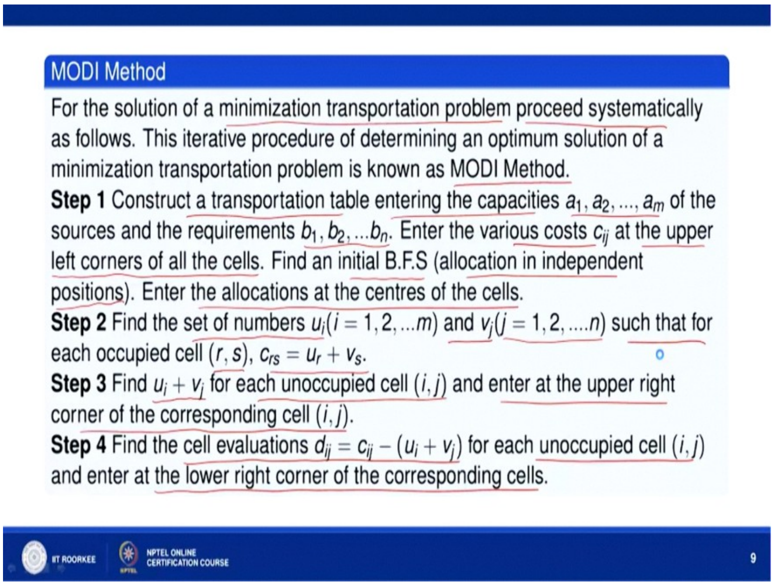

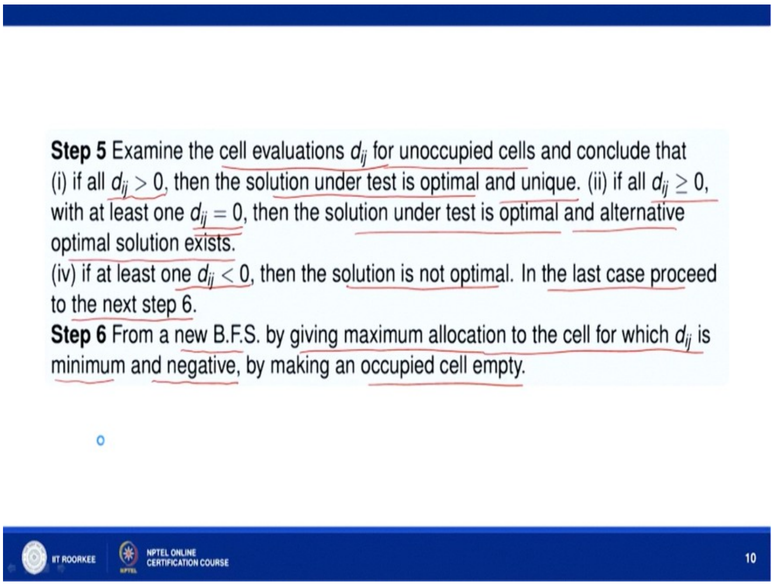

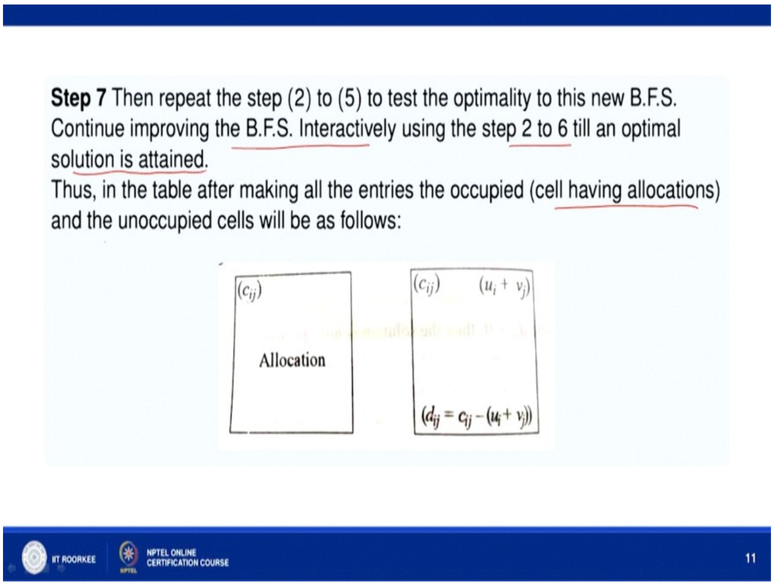

Nowm then we will repeat the step 2 to 5. Okay, 2 to 5 means this steps okay, we will repeat these steps again. Okay, this 2, 3, 4 and 5, these steps will be repeated okay and then we will again the test optimality that is we will again check the value of d_{ij} okay, we will continue new to improve the this BFS okay, using the step 2 to 6 till an optimal solution is attained. Thus, in the table after making all the entries the occupied cell, that is cell having allocations and the unoccupied cells will be as follows.

Let us see, so this is our first table, where c_{ij} is are the cost of the ij-th cell, they are written in upper left corner. Okay, so allocations are there in this table. Okay, now this $u_i + v_j$ they are the values, $u_i + v_j$ is the value sum of u_i and v_j which is written each unoccupied cell upper right corner. Okay, so upper right corner it is written and in the lower right corner, they are writing $d_{ij} = c_{ij} - (u_i + v_j)$ and we have to check the sign of d_{ij} for the optimality.

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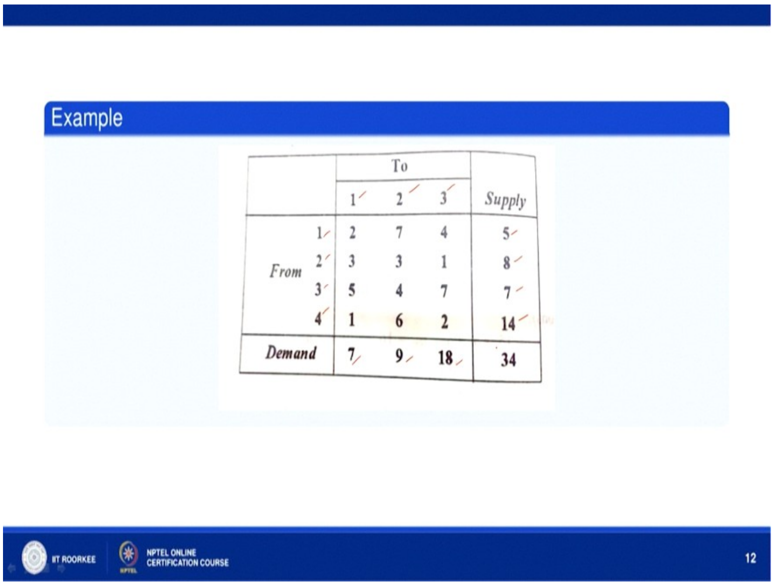

So let us say for example we have this table. Okay, this table is given we are sharing 4 sources 1, 2, 3, 4, 4 sources and 3 warehouses 1, 2, 3 and the availability with the 4 sources is 5, 8, 7, 14 and the demand at warehouse 1, 2, 3 are 7, 9, 18 okay, total is 34, so it is a balance transportation problem, now we have got its solution, solution of this problem, we have already got by Vogel approximation method and we have seen the allocations that have been made to the cells here.

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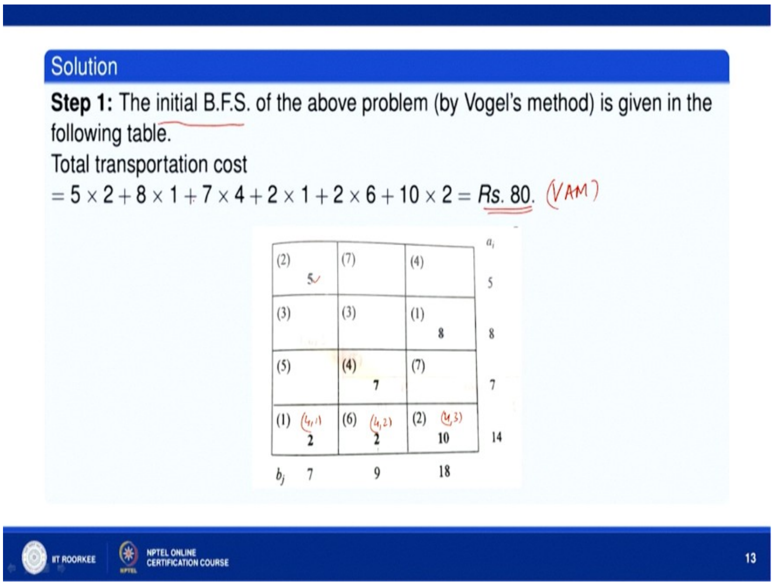

So let us use that initial BFS okay, by VAM method Vogel approximation method, so many applied Vogel approximation method we got the allocation 5 to the cell with cost 2, this is cell 11 okay and then we allocated 8 to this cell, with cost 1 that is second row and third column. Okay, we allocated cost 7 okay, units to be column with cost 4 to the cell with cost 4 that is third row and second column cell okay, 3, 2 column, 3, 2 cell okay and then we have allocated out of 14 that were available with source 4 to be allocated to the cell which lies at third row and first column.

2 units to the cell which lies at fourth row and second column, so this is of 4, 1 position, these 4, 1 position, this 4, 1 of cell, this is 4, 2 of cell, this is 4, 3 cell okay, cell 4, 1, cell 4, 2, cell 4, 3, so 4 1, 4 2, 4 3 have been given 2, 2 and 10 units out of 14, so this allocation we have made by VAM method. Okay, so we will use this allocation and see whether the solution, this, that we have got, the solution is optimal or not. Okay.

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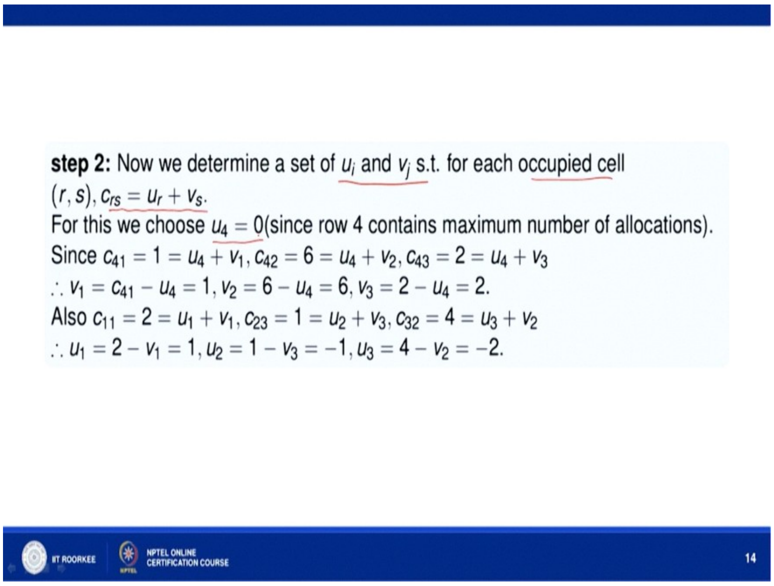

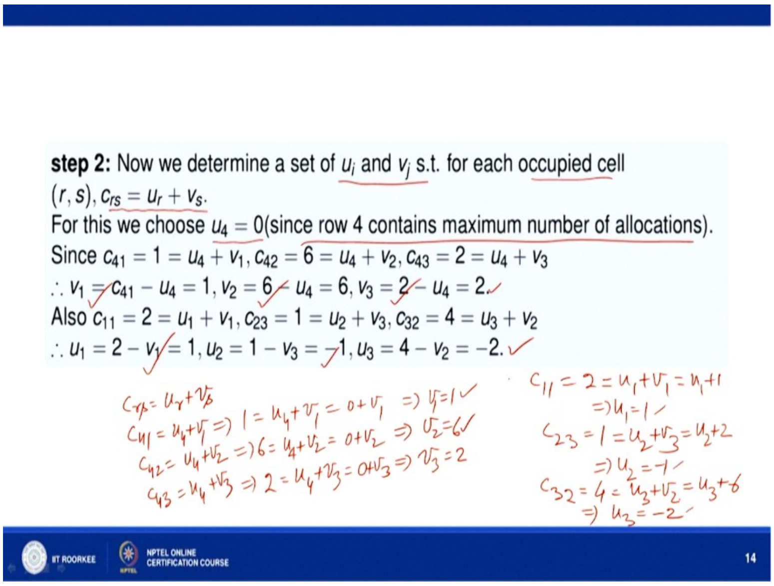

So, let us find u_i and v_j okay, u_i and v_j such that for each occupied cell rs, $c_{rs} = u_r + v_s$. Now we have to choose this u_4 okay, u_4 we have to choose, so u_4, see let us inspect all rows and columns. Okay, which have got various allocations, first row has got allocation of 5 units, second row has got allocation of 8 units, third row has got allocation of 7 units and fourth row has got allocation of 14 units.

So the row that has got the maximum allocation, that we have to consider, so this fourth row has got maximum allocation, so we will consider $u_4 = 0$, in fact suppose ith row has got the maximum allocation we shall consider $u_i = 0$, so her fourth row has got the maximum allocation, so we consider $u_4 = 0$, row 4 contains maximum number of allocations, so we take $u_4 = 0$.

Now, making use of, $c_{rs} = u_r + v_s$, $c_{rs} = u_r + v_s$you see, this cell has got 2 units okay, the cell has got unit, so it is occupied cell okay, for each occupied cell c_{rs} should be equal to $u_r + v_s$, so this is $c_{41} = u_4 + v_1$ we have, c_{41} okay, $c_{41} = u_4 + v_1$ and $c_{42} = u_4 + v_2$, $c_{43} = u_4 + v_3$, $c_{41} = 1$, you see cost is 1 okay, $c_{42} = 6$, c_{43} is 2 okay, so we have this gives you c_{41} is 1, $1 = u_4 + v_1$, but $u_4 = 0$, so we get $v_1 = 1$ okay.

Then c_{42} is 6, so this gives you $6 = u_4 + v_2$and this equal to $0 + v_2$, so we get $v_2 =$ ¿6 and then c_{43}, $c_{43}$is 2 okay, so $2 = u_4 + v_3$so we get $v_3 =$ ¿2. Okay, now, so we got the values of v_1, v_2, v_3 okay, now we another, other cells which are occupied are c_{11}, c_{11} has been allotted 5 units okay, so c_{11}, so c_{11} the cost of associated with c_{11}, $c_{11} = 2$ right, so we have $c_{11} =$ ¿2, so $2 = u_1 + v_1$, $c_{11} = u_1 + v_1$ and v_1 is equal to how much? $v_1 =$ ¿1 okay, so $u_1 + 1$, so this gives you $u_1 = 1$ okay, we got $u_1 = 1$ here and then the other occupied cells are let us see.

So this cell okay, this cell we have, this cell we can see, we have, this is third row second column okay, third row second column, so we have okay, let us go to this one. Okay, after first row we go to second row. Okay, so this is second row third column, so c_{23}, $c_{23} =$ ¿1 okay, $1 = u_2 + v_3$ okay and $v_3 =$ ¿2. Okay, so we get $u_2 + 2$, so we get $u_2 =$ ¿- 1 okay, so we get $u_2 =$ ¿- 1 here and then third one is this 7 okay.

So, this is third row second column, so third row second column says third row second column, so $c_{32} = 4$, so $4 = u_3 + v_2$okay and this is $u_3 + v_2 = 4$. Okay, $v_2 =$ ¿ 6, so 6 here, so this gives you $u_3=$¿ - 2, so we get this - 2. Okay, so we got the values of v_1, v_2, v_3 okay, v_1 is 6, v_2 is 6, v_1 is 1, v_2 is 6, v_3 is 2, u_1 is 1, u_2 is - 1, u_3 is - 2. Okay.

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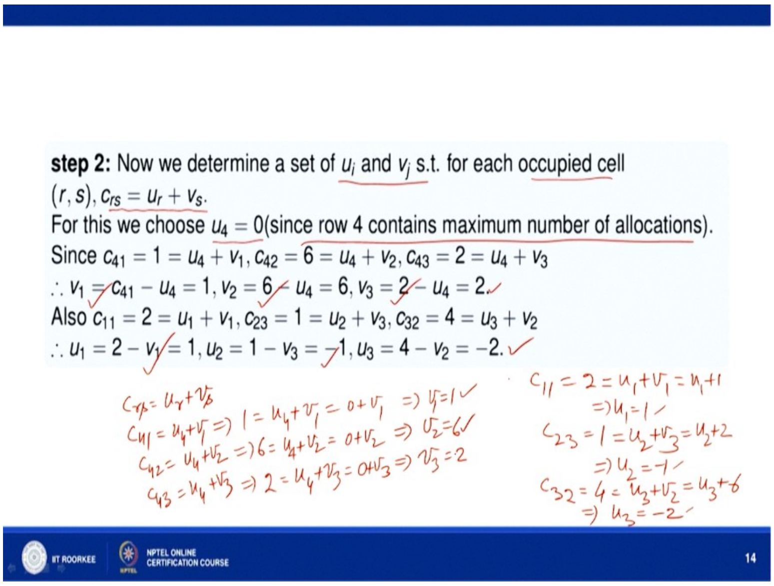

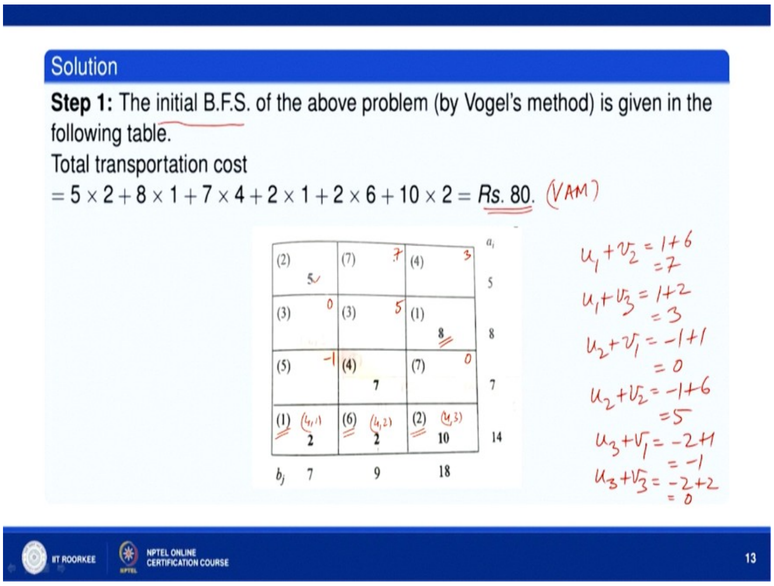

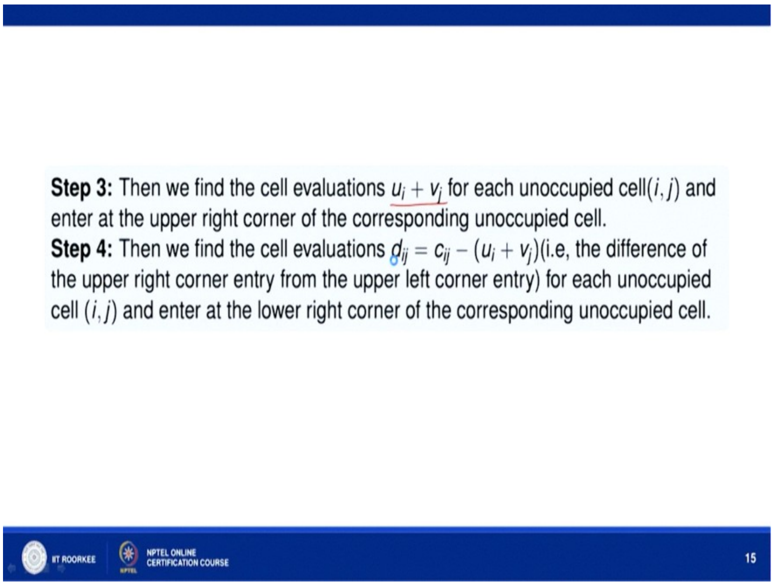

Now let us see, we then can write $u_i + v_j$ for each unoccupied cell okay, so $u_i + v_j$ for each unoccupied cells, so say for example, this is occupied cell we have allocated 5 units here okay, this is unoccupied cell, so we can allocate, so here we have first row that is $u_1 + v_2$okay, first or second column, so $u_1 + v_2$ will be how much? $u_1 =$¿ 1 and $v_2 =$¿ 6, so we will have $u_1 + v_2 =$¿1 + 6 = 7, so this will be, 7 will be written here and then this is also unoccupied cell.

So we have $u_1 + v_3$ okay, first row third column, so $u_1 + v_3$ =3, $u_1 =$¿ 1, $v_3 =$¿ 2, so we have 1 + 2 = 3, so 1 + 2 = 3, so 3 here we shall write, there in the second row first column okay, so $u_2 + v_1$okay, $u_2 + v_1$, $u_2 =$¿ - 1 and $u_2 + v_1$, $v_1 =$¿ 1, so - 1 + 1 = 0, so we get - 1 + 1 = 0. Okay, then, so this will be 0 here and then we will have this one, so $u_2 + v_2$, second row second column $u_2 + v_2$ is how much? $u_2 =$¿- 1, $v_2 =$¿ 6, so - 1 + 6, so we have 5, so will write 5 here.

Now, this is occupied cell we go to left unoccupied cell, so $u_3 + v_1$, $u_3 + v_1$, so what is u_3? $u_3 =$¿ - 2 and $v_1 =$¿ 1, so - 2 + 1 that is equal to - 1 okay, so - 2 + 1 okay, so will get - 1 here and then this is occupied cell, next unoccupied cell is $u_3 + v_3$, so $u_3 + v_3$is how much? $u_3 =$¿- 2, $v_3 =$¿2, so - 2 + 2 = 0, so we get 0 here and then this, this, they are all occupied cells okay, so we have got the values of $u_i + v_j$ okay, so this $u_i + v_j$we have got.

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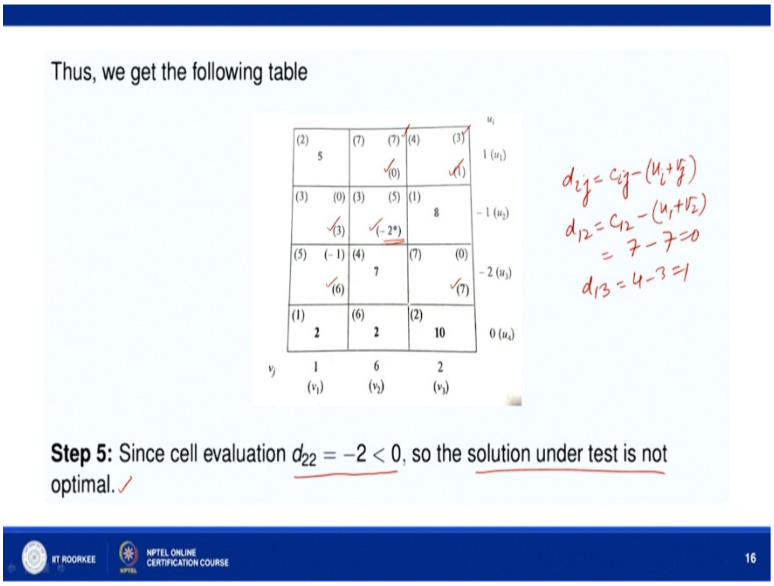

And these are the values 7, 3. Okay, this unoccupied cell, this is unoccupied cell okay, this also unoccupied cell we have got 0 here, these is also unoccupied cell we got 5 here, this unoccupied cell we got - 1 here, this is unoccupied cell we got 0 here, these all are occupied cells okay, so we got the values.

Now we calculate $d_{ij} = c_{ij} - (u_i + v_j)$ for each unoccupied cell okay, so unoccupied cell is this one, this is first unoccupied cell, so we get the d_{12} okay, first row second column, so c_{12} - (u_1 + v_2) okay and c_{12} =7, so 7 - 7 which is equal to 0, so we got 0 here, this is also unoccupied cell, so we take the difference of 4 and 3. Okay, so d_{13}, first row third column, so 4 – 3=1, so we get 1 here and then here for this cell 3 - 0 equal to 3, 4 this cell 3 - 5 is - 2. Okay, this is occupied cells, so we will leave it, for this cell, which is unoccupied the difference is 5 - 1, so 6 here and this is occupied cells will leave it, then 7 - 0 is 7 okay.

So we have got d_{ij} values for all unoccupied cells, now let us check their sign. Okay, so here d_{ij}=¿0, here d_{ij} is 1, here it is 3, here it is - 2, here it is 6, here it is 7, so there is one d_{ij}, which is negative. Okay, if d_{ij}<¿0, for at least 1 i, j okay, then the solution obtain is not optimal okay and we have to then proceed further, so since cell evaluation d_{22} = - 2 < 0, so the solution under test is not optimal okay.

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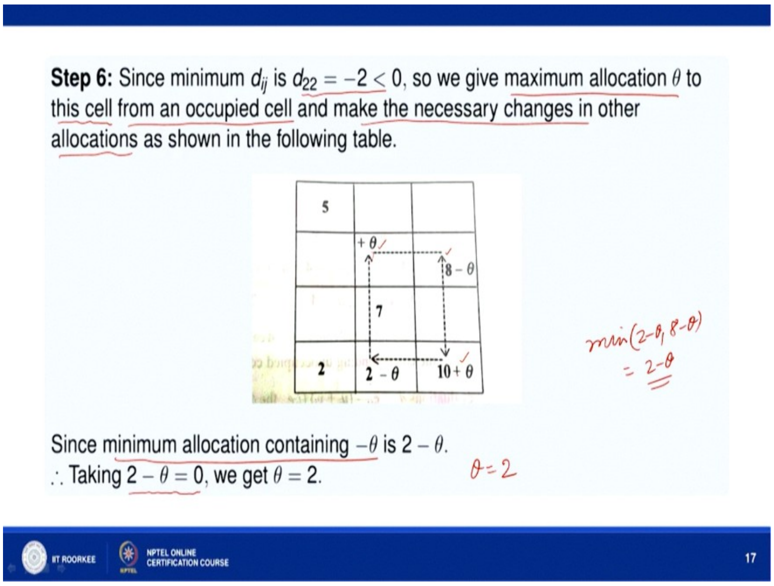

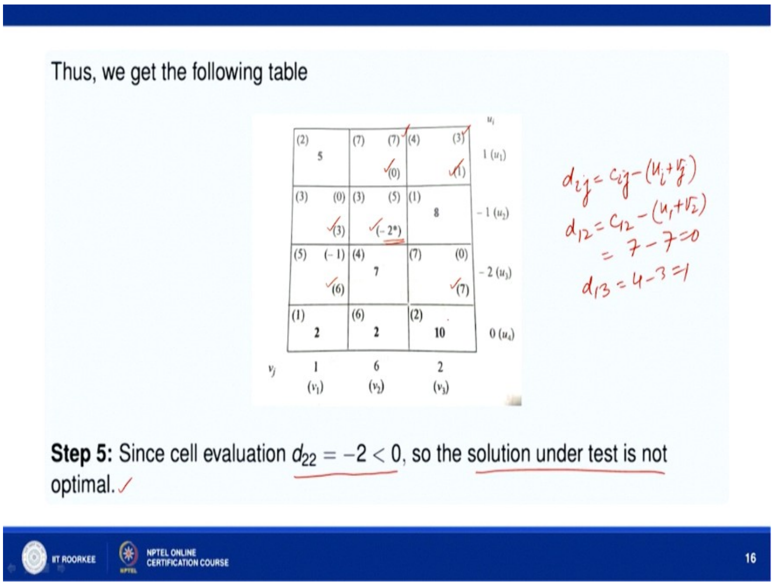

So what we will do, since minimum d_{ij} is d_{22} = - 2, so we give maximum allocation θ to this cell okay, maximum allocation θ to this cell from an occupied cell okay, now let us see, let us look at this yes, so we have this - 2, 8, 2, 10 okay, so we take maximum allocation to this cell from an occupied cell okay, so and make the necessary changes in other allocations.

Now what we will do, we have this loop okay, so we make, suppose we make allocation θ to this cell okay, where d_{22} = - 2, then we have 2 subtract θ from this 2 okay, this will become 2 - θ, when it becomes 2 - θ total has to be 14 for this fourth source, so subtracting θ here means we have to add θ to this allocation that is 10, so we get 10 + θ here and then we have added θ to this 10, we have 2 subtract θ from this allocation 8, so we get 8 - θ okay, so we get θ, if we have θ here we have to make 2 - θ here, here 10 + θ, here 8 - θ and then we consider any allocation containing - θ okay.

So containing - θ we have 2 allocations, 2 - θ, 8 - θ, so minimum of 2 - θ and 8 - θ we have to considered and this is to - data. Okay, so we have to put T - θ = 0, minimum allocation is then put is equal to 0, and then we get it θ = 2, so we get θ =2 means 2 here and this is 6, this is 0, so this occupied cell is now becomes 0 and here we get 12, so this is the modification that we make.

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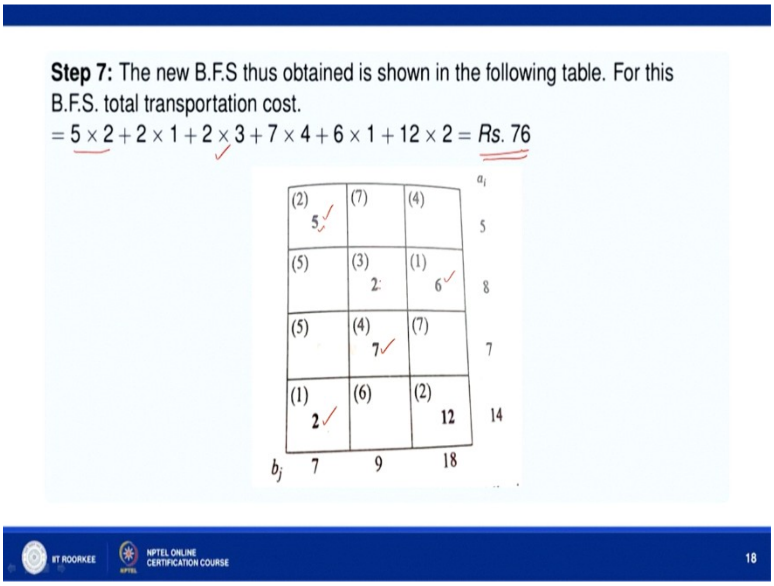

And after that what we will do have, this is 5 now. Okay to this cell θ is 2, so we get to allocation here and this is 8 - θ so we get 6 here, this becomes 0, this cell allocation become 0, here it was 2 and here 10 becomes 12 okay, so we get now transportation cost as 5 into 2. Okay 2×1, this is 2 × 1, yes, this one. 2 × 1, so 5 × 2, 2 × 1, then 2 × 3, then 7 × 4, then 6 × 1, then 12 × 2 so we get Rs. 76. Okay.

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step 2: Now we determine a set of u_i and v_j s.t. for each occupied cell (r, s), $c_{rs} = u_r + v_s$.

For this we choose $u_4 = 0$(since row 4 contains maximum number of allocations).

Since $c_{41} = 1 = u_4 + v_1$, $c_{42} = 6 = u_4 + v_2$, $c_{43} = 2 = u_4 + v_3$

$\therefore v_1 = c_{41} - u_4 = 1$, $v_2 = 6 - u_4 = 6$, $v_3 = 2 - u_4 = 2.$

Also $c_{11} = 2 = u_1 + v_1$, $c_{23} = 1 = u_2 + v_3$, $c_{32} = 4 = u_3 + v_2$

$\therefore u_1 = 2 - v_1 = 1$, $u_2 = 1 - v_3 = -1$, $u_3 = 4 - v_2 = -2.$

Step 7: The new B.F.S thus obtained is shown in the following table. For this B.F.S. total transportation cost.

$= 5 \times 2 + 2 \times 1 + 2 \times 3 + 7 \times 4 + 6 \times 1 + 12 \times 2 = Rs.\ 76$

				a_i
	(2) 5	(7)	(4)	5
	(5)	(3) 2	(1) 6	8
	(5)	(4) 7	(7)	7
	(1) 2	(6)	(2) 12	14
b_j	7	9	18	

Step 8: Proceeding as in step 2, 3, and 4 we get the following table

				u_i
	(2) 5	(7) (5) (2)	(4) (3) (1)	1 (u_1)
	(3) (0) (3)	(3) 2	(1) 6	−1 (u_2)
	(5) (1) (4)	(4) 7	(7) (2) (5)	0 (u_3)
	(1) 2	(6) (4) (2)	(2) 12	0 (u_4)
v_j	1 (v_1)	4 (v_2)	2 (v_3)	

Since, all $d_{ij} > 0$ hence, the B.F.S. shown by the table in step 8 is an optimal solution which is also unique and the total transportation $cost = Rs.\ 76$.

So now we have to check whether this solution that we have got is optimal or not. Okay, so what we will do, let us proceed again as in step 2, 3, 4 okay and what we will do then, proceed in this step 2, 3, 4, so 2, what is the step 2? Now we determine a set of u_i and $v_j$such that for each occupied cell rs, c_{rs}= u_r + v_s, so let us do that. Okay, so we come here to this table. Okay.

So allocations we have to see okay, first row has got allocation 5 okay, second has got allocation 8, third has got allocation 7 and fourth has got allocation 14 okay, so total allocation is maximum alone the fourth row. Okay, so taking u_4 = 0. Okay, let us take $u_4=¿0$. Okay, then as we have done earlier. Okay, we have to use c_{rs}= u_r + $v_s$okay, so first let us say which one is the occupied cell, occupied cell is this one we are given 2 here, 12 here okay, so this is fourth row first column, so c_{41}. Okay equal to u_4 + $v_1$okay, $c_{41}=¿$ 1, so 1 equal to u_4 is 0 + v_1, so we get $v_1=¿1$ okay.

Then we are giving 12 units to fourth row third column so this means c_{43}, c_{43} = 2. Okay, u_4 + v_3, so 2 equal to 0 + v_3, so we get v_3 = 2 okay, now then we are giving 5 to c_{11} okay, so $c_{11}=¿$ u_1+ 5, 5 we are giving to cell 11, so $c_{11}=¿$ 2, so 2= u_1 + v_1 right, so $v_1=¿1$ okay, so this gives $u_1=¿2$ - v_1 that is 2 - 1 = 1 okay and then we are giving 2 units to second row second column okay, c_{22}. Okay, $c_{22}=¿$ 33 = u_2 + v_2, we are giving 6 to second row third column, so c_{23} =u_2 + v_3 okay, so what is it, you $c_{23}=¿$ 1, 1 = u_2 + v_3, $v_3=¿2$, so $u_2=¿$- 1, when u_2 = - 1, v_2 =4.

Since $u_2=¿$- 1, we get $v_2=¿4$ okay, yes, then we have this one, so c_{32}, third row second column c_{32}, $c_{32}=¿4$, u_3 is equal to, we have to find u_3?, $v_2=¿4$, so $u_3=¿0$. Okay, so we have used this one. Okay and we have used this two also, have we used fourth row and column 3. Okay, we have that one also right, so now let us find u_i + $v_j$for each unoccupied cell okay, so this is unoccupied cell okay.

So u_1 + v_2 okay, because it lies in the first row and second column, so u_1 + v_2, $u_1=¿$ 1 okay, so 1 + v_2, $v_2=¿4$, so 1 + 4 = 5, so this is 5 here okay, then there is also an unoccupied cell, it lies in the third row, first row and third column, so u_1 + v_3, u_1 is equal to 1 and V3 equal to 2, so we get 3 here okay, then her second row first column, so u_2 + v_1, u_2 is equal to - 1 okay and V1 equal to 1, so we get 0, so this is 0 here and this is occupied column, this is occupied cell, this is also occupied cell, this is unoccupied cell, so we have here third row first column okay.

So u_3 + v_1, u_3=¿0. Okay and v_1, v_1=¿1, so we get 1 here. Okay, there is occupied cell, here we have third row and third column, so u_3 + v_3, u_3=¿0, v_3=¿2, so that gives you 2 here, now this is occupied cell, this is unoccupied cell, so fourth row and second column, so u_4 + v_2, u_4 = 0. Okay, u_4=¿0, v_2=¿ 4, so we get 4, so this is 4 okay, now so we have got the u_i + $v_j$for all unoccupied cells.

Let us find d_{ij} okay, so d_{ij}=¿ u_{ij} – (u_i + v_j) for all unoccupied cells, so here you see 7 - 5 is 2 okay, 4 - 3 is 1, here we get 3 - 0 3, here we get 5 - 1 4, 7 - 2 5 and 6 - 4 2, so we can see this is 2, this is 1, this is 3, this is 4 okay, this is 5, this is 2, so d_{ij} > 0, for all I and J which are unoccupied cells okay, so d_{ij} > 0. Okay and therefore the BFS shown by the table in step 8. Okay, this BFS is an optimal solution. Okay, which is also unique and total transportation costs we can find.

Total transportation costs is 5 into 2 okay, because we are allocated in 5 here and cost is 2, 5 into 2 and then we have 2 into 3 okay, 2 into 3, 6 into 1 and then we have 7 into 4 okay and then we have 2 into 1 and then 12 into 2, so this is 24 + 2 26, 26 and 28 54, 54 + 6 60, 60 + 10 70, 70 + 6 is 76. Okay, so this is the total transportation cost, so this is how we can use this Modis method to get the optimal solution to the problem where we have already found the initial, physical solution by the VM method and then we can find the d_{ij} at each step as soon as we get d_{ij} is to be positive for all unoccupied cells we get the optimal solution to the problem, so that is all in this lecture. Thank you very much for your attention.