Higher Engineering Mathematics
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Lecture - 53
Transportation Problem - 1

Hello friends, welcome to my lecture on Transportation Problem. This first lecture on this topic, let us first see "what is the transportation problem?". The transportation problem is a special cause of linear programming problem that deals with shipping commodity from source that factory to destination which is a warehouse the objective is to determine the shipping schedule that minimizes the total shipping cost while satisfying supply and demand limits. The general problem is represented by the network as shown in figure one here this figure.

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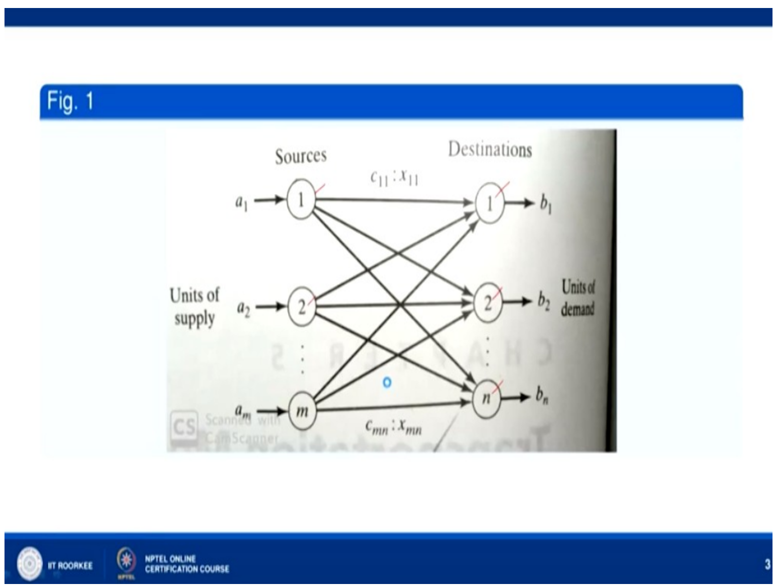

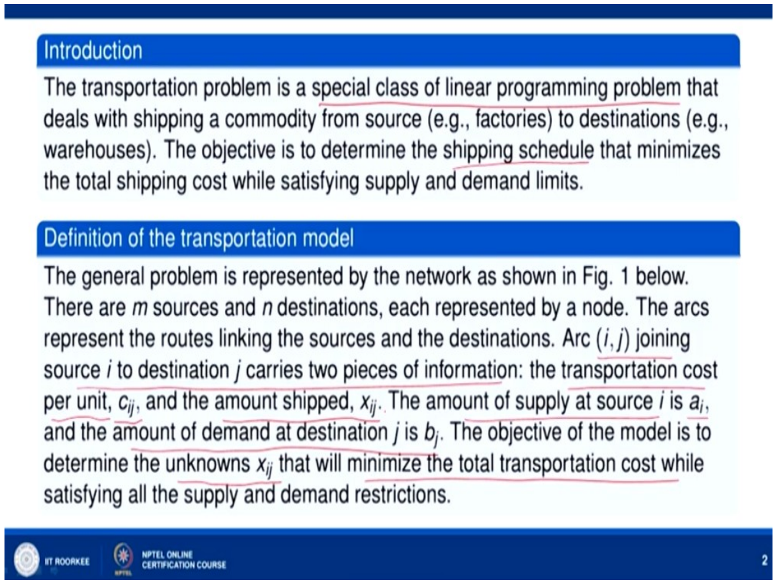

So there are M sources and N destinations, you can see there M sources 1 2 and shown up to M there are M sources and N destination, 1 2 and so on up to N each represented by a node, each source N destinations are represented by a node. Now the arcs represent the routes linking the sources in the destinations, so these arcs, these arcs between sources and destinations represent the route that linking the destinations arc i, j joining the source i to destination j carries two pieces of information, so if you consider the node i and the destination j anode i and the destination j than the arc i j the arc i j with joins the source i to the destination j it carries two pieces of information the transportation cost per unit c_{ij} and the amount shipped that is x_{ij}. b_j, so here at the source i the amount that is available is Ai and at the destination j the demand is given by b_j, so amount of supply at source I is aj, and the amount of demand at destination j is b_j, the objective of the model is to determine the unknown x_{ij}. that will minimize the total transportation cost while satisfying all the supply and demand restrictions, so x_{ij} is the amount that is shipped from ith location to jth destination, from the i-th source to the j-th destination, so the x_{ij} are unknowns we have to determine the values of x_{ij} that the amount that is to be shipped from the i-th source to the j-th destination for each i and j. Now so this is the network that represent this transportation problem.

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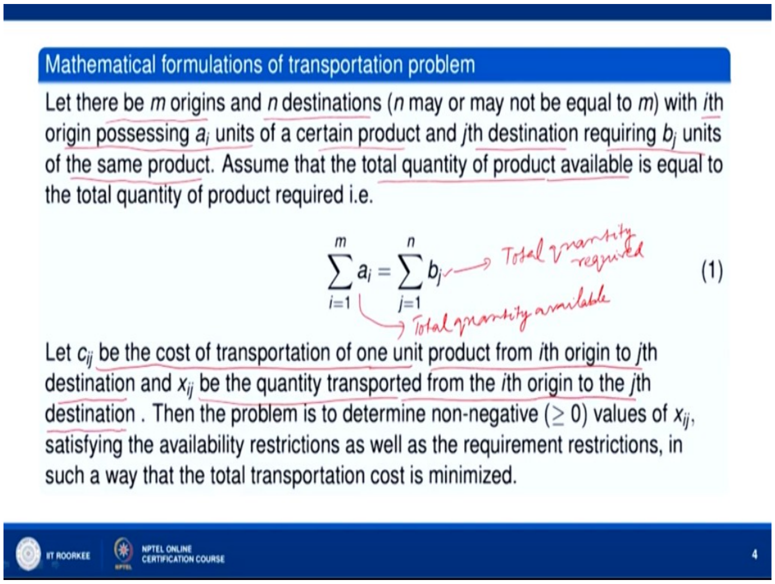

Now, we let us consider it is mathematical formulation, so let there be m origins and n destinations n may or may not be equal to m number of sources may not be equal to number of destinations, now i-th origin possessing a_i units of a certain products and ith destination requiring bj units of the same product, so there m origins and n destinations n may not be equal to m and at the i-th origin therea_i unit available and at the j-th destination we require b_j units of the same products,. Let us assume that total quality of products available, total quantity of product available is what at the i-th location we have units available, so and i is equal to 1 to m because there are m source.

So total quantity available is $\sum_{i=1}^{m} a_i$ this is total quantity available and at the j-th destination we require b_j units of the same products j is from 1 to n because there n destinations, so total quantity that is required is $\sum_{j=1}^{n} b_j$ so this is total quantity required. So, let us assume that total quantity that is available to us for shipment is equal to total quantity that is require to us so $\sum_{i=1}^{m} a_i$ = $\sum_{j=1}^{n} b_j$.

Let Cij be the cost of transportation of one unit product from i-th origin to j-th destination and x_{ij} be the quantity transported from the i-th origin to the j-th destination. Then the problem is to determine non-negative values of x_{ij} satisfying the availability restrictions as well as the requirement restrictions, in such a way that total transportation cost is minimized, so that is our problem.

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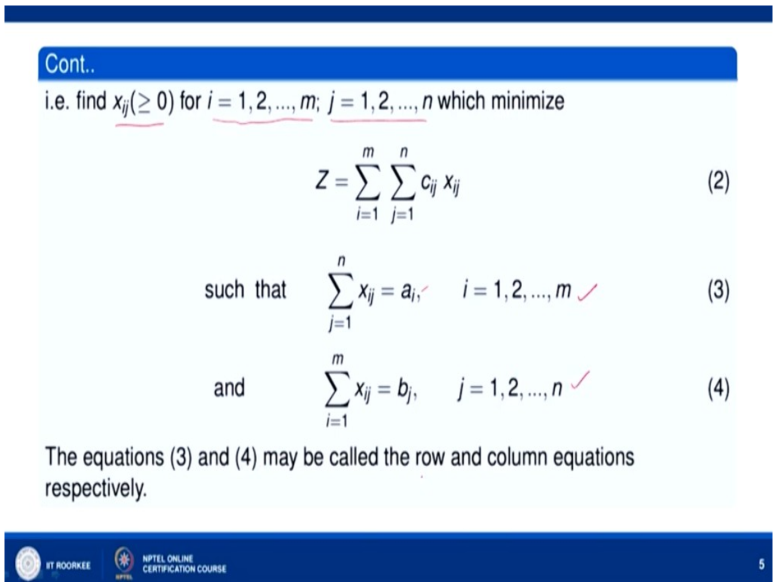

Now, that means we have to find x_{ij}. now unknown x_{ij}., non-negative values of x_{ij}. for i equal to 1 2 and so on up to m j is equal to 1 2 and so on up to n which minimized the total cost, total cost is $\sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$. is the cost per unit of transporting from i-th source to j-th location and x_{ij} is the quantity that is shipped from i-th location to j-th destination, so c_{ij} into x_{ij} is the cost incurred in transporting x_{ij} units from ith source to jth destination and i runs from 1 to m, j goes from 1 to n, so when we take this summation we get the total cost and sigma j equal to 1 to n sigma j equal to 1 to n is means.

Now, a_i is the quantity available at the i-th source from i-th source the quantity is being transported to the n locations where that means j equal to 1 to n and destination so the total sum of x_{ij} j equal to 1 to n should be equal to a_i because at the source we have a_i units available with us so those a_i units are transported to the jth end destination, so the total of x_{ij} for j equal to 1 to n for the i-th source will be equal to a_i and $\sum_{i=1}^{m} x_{ij} = b_j$ at the j-th destination we require b_j units of the product, so that b_j is units of the product are being made available by the m sources, so $\sum_{i=1}^{m} x_{ij}$.

Now, some of the x_{ij} is may be 0 there may not be any shipment from some sources and there may be some from positive x_{ij} value will be there so x_{ij} is here for some i's could be 1 0 that means from those sources there is no shipment of the product to the j-th location, so $\sum_{i=1}^{m} x_{ij} = b_j$ so these are the constants on x_{ij}. Now the equations 3 and 4, these equations 3 and 4 are called as row and column equations respectively.

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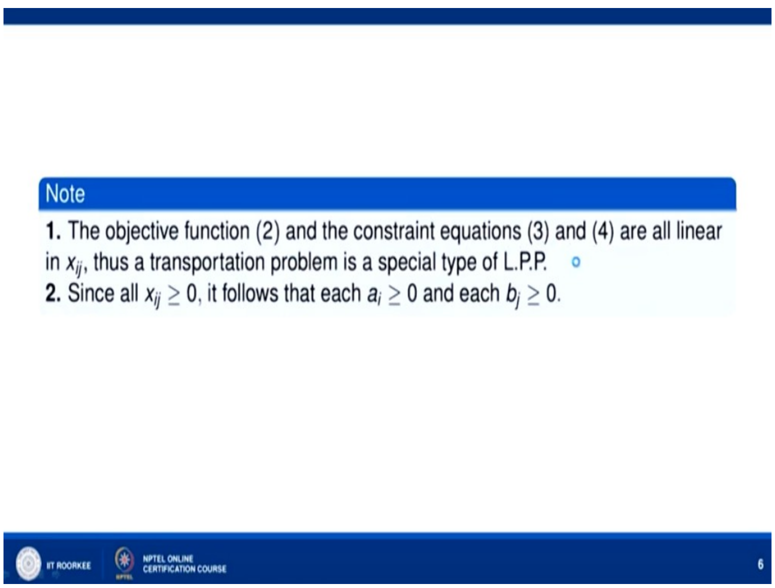

Now, the objective function 2 and the constraint equation 3 and 4 are all linear in x_{ij}, thus you can see here this this they are all they are linear in x_{ij}, so it is the linear programming problem it is a special type of linear programming problem and since all $x_{ij} \geq 0$, since all $x_{ij} \geq 0$,$a_i \geq 0$ and bj ≥ 0 for every i and j, so each $a_i \geq 0$ and each $b_j \geq 0$.

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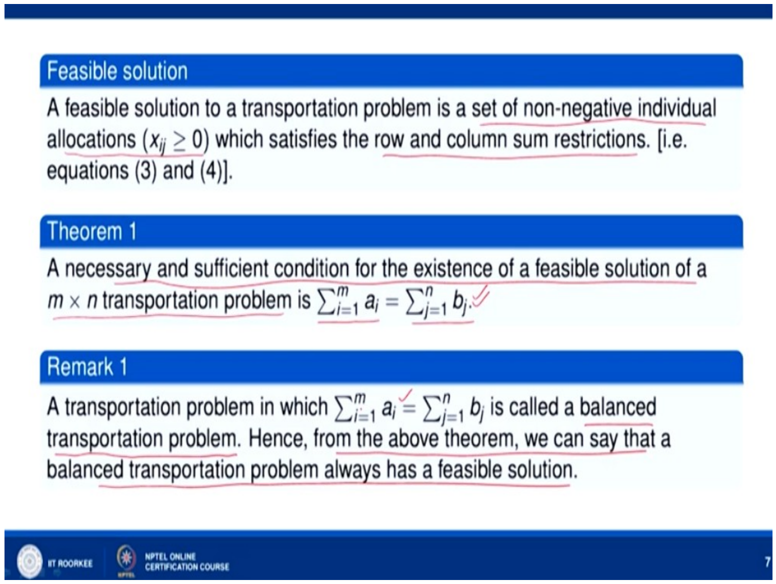

Now, let us see what is a feasible solution a feasible solution to a transportation problem is a set of non-negative individual allocation $x_{ij} \geq 0$ which satisfies the row and column sum restrictions that is $\sum_{i=1}^{m} x_{ij} = b_j$, $\sum_{j=1}^{n} x_{ij} = a_i$, so a set of non-negative values of x_{ij} which satisfies the row and column sum restrictions is called a feasible solution to the transportation problem. Now a necessary and sufficient condition for the existence of a feasible solution of a m by n transportation problem is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

So we have the transportation problem here it is m by n transportation problem because you can see here i runs from 1 to m j runs from 1 to n there total m+ n equations here, these are m equations these are n equations, this is m × n transportation problem it will have a feasible solution if and only if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, so this condition is necessary and sufficient for the existence of a feasible solution to a transportation problem, so this condition we have assumed and therefore our problem will have a feasible solution.

Now it transportation problem in which this condition $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ holds to is called a balanced transportation problem. So we are considering the transportation problem as balanced because we have assumed that $\sum a_i = \sum b_j$ where i runs from 1 to m j runs from 1 to n, so hence from the above theorem we can say that a balanced transportation problem always has a unique solution, so in the transportation problem if total of total quantity of product available is equal to total quantity of product that is required than such a transportation problem is called balanced transportation problem and it will always have a feasible solution that is valeted by this theorem 1.

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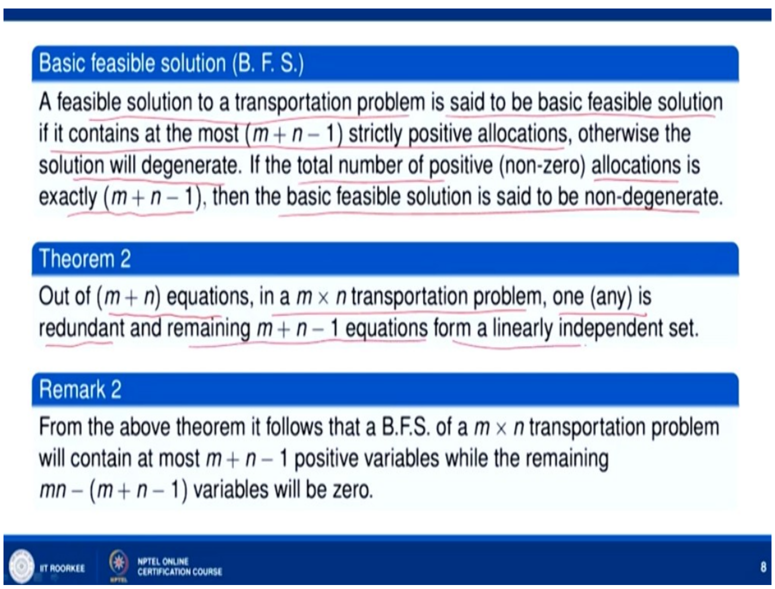

Feasible solution to a transportation problem is said to be basic feasible solution if it contains at the most m + n - 1 strictly positive allocation. So there m + n equations there are m + n equation total variables are m ×n, x_{ij} i runs from 1 to m j runs from 1 to n, so total variables are m into n, transportation problem will be said to have basic feasible solutions, if it contains at the most m + n - strictly positive allocations otherwise the solution will degenerate. If the total number of positive allocations is exactly m + n - 1 than the basic feasible solution is said to be non-degenerate.

See, now out of m + n equations, there are out of, there are m + n equations here you can see there are m equation here n equation here, so total of m + n equation involving mn variables, so there out of m + n equation in a m by n transportation problem 1 equation is redundant and remaining m + n - 1 equations are linearly independents sets.

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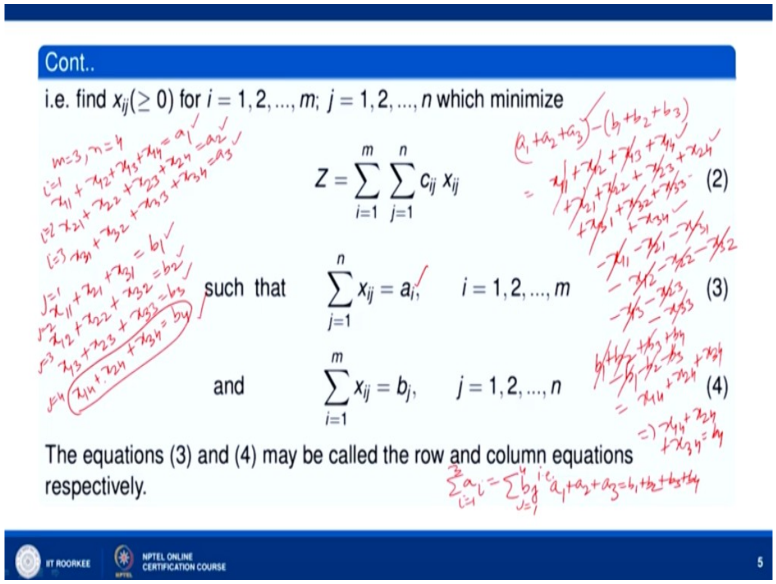

So, let us shows that this follows from here, so let us take m = 3 for example let us take m =3 and n = 4, so then when m = 3 and n = 4 let us see we will have this equation i is taking value from 1 to 3, so i is taking value from 1 to 3 so i =1 will give $x_{11} + x_{12} + x_{13} + x_{14} = a_1$ then i = 2 will give $x_{21} + x_{22} + x_{23} + x_{24} = a_2$, i = 3 will give you $x_{31} + x_{32} + x_{33} + x_{34} = a_3$ so these are three equations as a result of this one and there will be 4 equations as a result of j = 1, 2, 3, 4, so j = 1 so we will get x_{11} , x_{21} x_{31}, i is taking values from 1 to 3 so $x_{11} + x_{21} + x_{31} = b_1$, there will be 4 equations are result on this one.

Then j = 2 so we will get x_{12} or j = 2, $x_{12} + x_{22} + x_{32} = b_2$ and then j equal to 3 will give you $x_{13} + x_{23} + x_{33} = b_3$ and j = 4 so $x_{14} + x_{24} + x_{34} = b_4$. Now, let us consider $(a_1 + a_2 + a_3) - (b_1 + b_2 + b_3)$, see what we will get, so $a_1 + a_2 + a_3$, we are adding this one this one and this one and subtracting $b_1 + b_2 + b_3$, so what we will get, so $x_{11} + x_{12} + x_{13} + x_{14}$+¿ $x_{21} + x_{22} + x_{23} + x_{24}$+¿ $x_{31} + x_{32} + x_{33} + x_{34}$ so these are first three equations give us, this then $-x_{11} - x_{21} - x_{31}$−¿ $x_{12} - x_{22} - x_{32}$ and $-x_{13} - x_{23} - x_{33}$, so $a_1 + a_2 + a_3 - b_1 - b_2 - b_3$.

So, x_{11} cancels with this one x_{11} x_{21} cancels with the x_{21} x_{31} cancels with the x_{31} x_{12} with x_{12} x_{22} with x_{22} x_{32} with x_{32} here x_{13} with x_{13} x_{23} with x_{23} x_{33} with x_{33}, so we get $x_{14} + x_{24} + x_{34}$. Now we have been given that transportation problem is balanced, so $\sum a_i$, i = 1, 2, 3 equal to $\sum b_j$ j = 1, 2, 3, 4 that is $(a_1 + a_2 + a_3) = (b_1 + b_2 + b_3)$, this is given to us, so $a_1 + a_2 + a_3$ this is equal to $b_1 + b_2 + b_3 + b_4 - b_1 - b_2 - b_3 = x_{14} + x_{24} + x_{34}$.

So we get so this gives us $x_{14} + x_{24} + x_{34} = b_4$ which is the this equation seventh equation, so out this 7 equations 1 equation this equation can be obtained from the other 6 equations, so this equation is redundant, so out of the m + n equations any one equation is redundant and remaining 6 equations are independent they form a linearly independent set.

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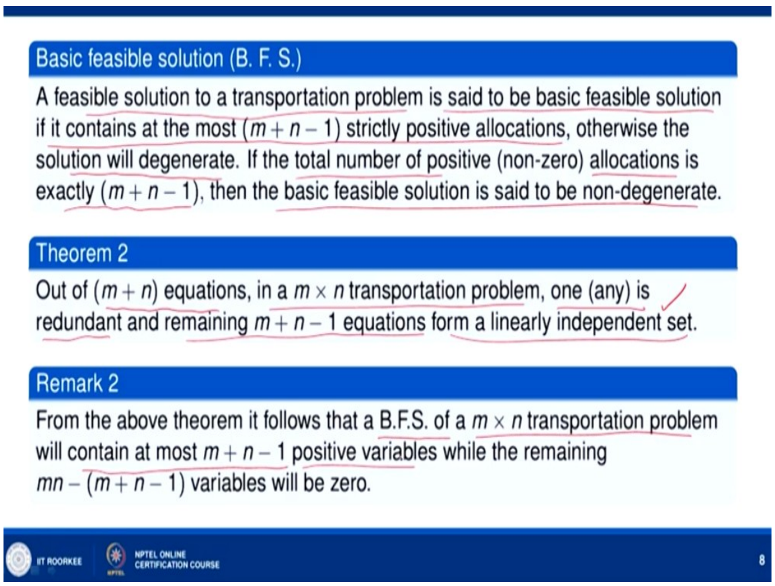

So that is the that is the verification for this theorem. Now from this above theorem it follows that a basic feasible solution of an m by n transportation problem will contain at most m + n-1 positive variables while the remaining m×n –(m + n- 1) variables will be 0, so taking m + n-1 taking m×n –(m + n- 1) variables 0 the remaining m + n- 1variables can be determined from the m + n- 1variables, so that is why it says that there will be at the most m + n- 1 positive variables while the remaining m + n- 1 variables will be 0, so a basic feasible solution can be obtained from the this m + n- 1equations.

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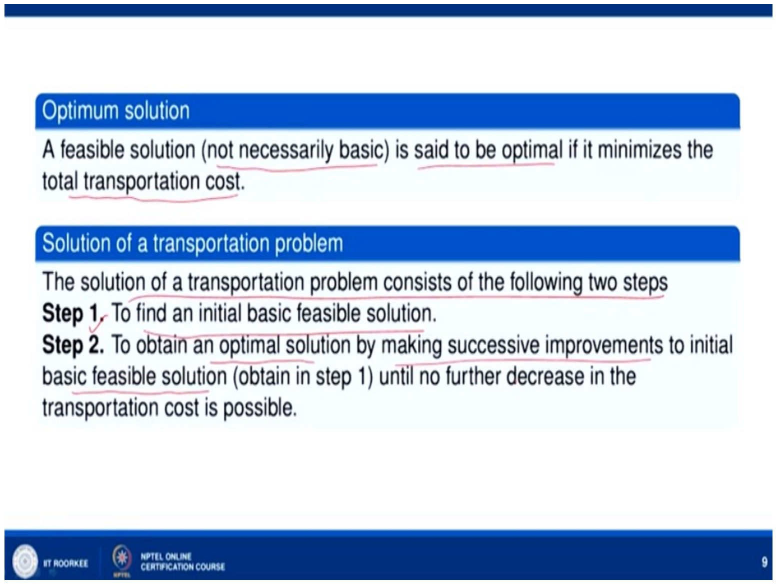

Now, a feasible solution not necessarily basic feasible solution is called optimal if it minimizes the total transportation cost. Now, let us determine the basic feasible solution so the solution of a transportation problem consist of following two steps first we find an initial basic feasible solution and then we will find an optimal solution by making successive improvements to initial basic feasible solution that is obtained in the step 1 until no further decrease in the transportation cost is possible, so let us first go to step one and try to determine an initial basic feasible solution then we shall find an optimal solution corresponding to the step 2.

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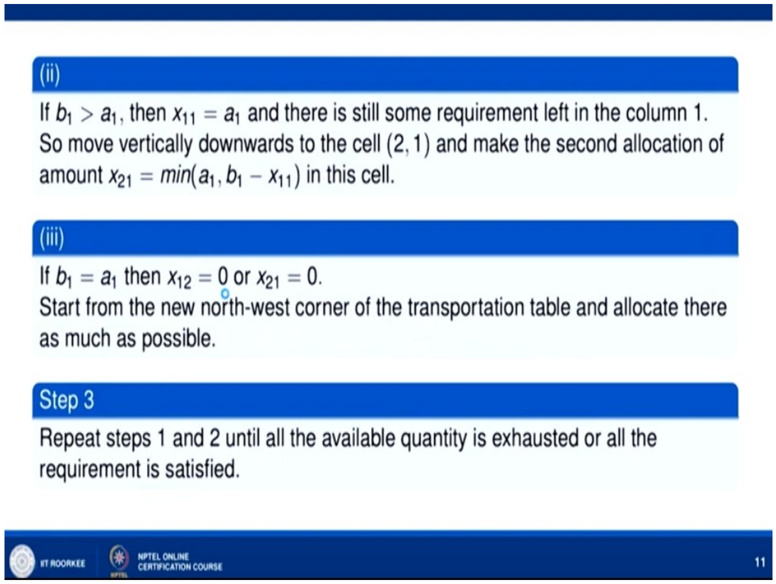

Now using these steps, now there are three methods which we use to find an initial basic feasible solution the north-west corner rule that is one method the other method is lest cost method and the third method is Vogel approximation method. So let us first discuss the method one North-West corner rule in this rule we have the following steps, so form the transportation problem table in the table in the cell 1 1 let us see what kind of table we will have.

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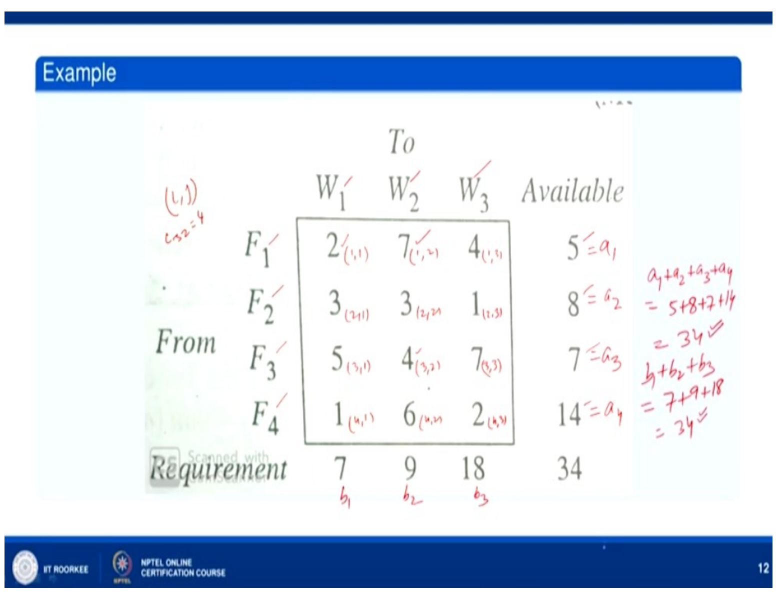

This kind of table we will have here F_1 F_2 F_3 F_4 they are sources W_1 W_2 W_3 are the destinations, so destinations are written here sources are written here and these are transportation cost per unit from source F_1 to destination W_1 the transportation cost per unit is per unit of the product is 2 from the source F1 to the destination W_2 it is 7, from the source F_1 to the destination W_3 it is 4 and so on, so these values 2 7 4 3 3 1 5 4 7 1 6 2 they are the transportation cost from the sources to the destination these values 5 8 7 14 they are the units available at the source F_1 F_2 F_3 F_4.

So 5 units of the products are available for shipment at the source F_1 8 units are available at the source F_2 for shipment 7 are available at the source F_3 for shipment and 14 are available at F_4 for shipment. Now what are the requirement of the three sources W_1 W_2 W_3 the source means warehouses at the warehouse W_1 the requirement of 7 units of the product is there at the warehouse W_2 the 9 units of the product are required and at the warehouse W_3 18 units of the product is required. Now, we have assumed that so this are the values this a_1, a_2, a_3, a_4 the quantity that is available at the sources F_1 F_2 F_3 F_4 and these 7 9 18 they are the values of b_1,b_2, b_3.

Now, we have assume that $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$, so here we have right so $a_1 + a_2 + a_3 + a_4$, let us see how much is that so total quantity that is available is 5 +8 + 7 + 14, so that is equal to 13+ 7+ 20 + 34 and what is the quantity that is required at the three warehouses b1 + b2 + b3 that is equal to 7 + 9 + 18 so this is also 34, so the quantity that is available for shipment is 34 the quantity that is required for at the 3 warehouses is also 34.

So it is a balanced transportation problem and therefore, we can find basic initial feasible solution. Now these are cells this is 1 1 position this is 1 2 position this is 1 3, 2 1, 2 2, 2 3, 3 1, 3 2, 3 3, 4 1, 4 2, 4 3, so at the i j-th i j-th entry means i j-th entry means the this is i j-th entry the entry in the third row second column it tell us the transportation cost this is C_{ij} the transportation cost from ith source to location so let we say for the particular value of i and j so i could say that C_{32}= 4 the transportation cost from the source F_3 to the location W_2 per unit is 4 right.

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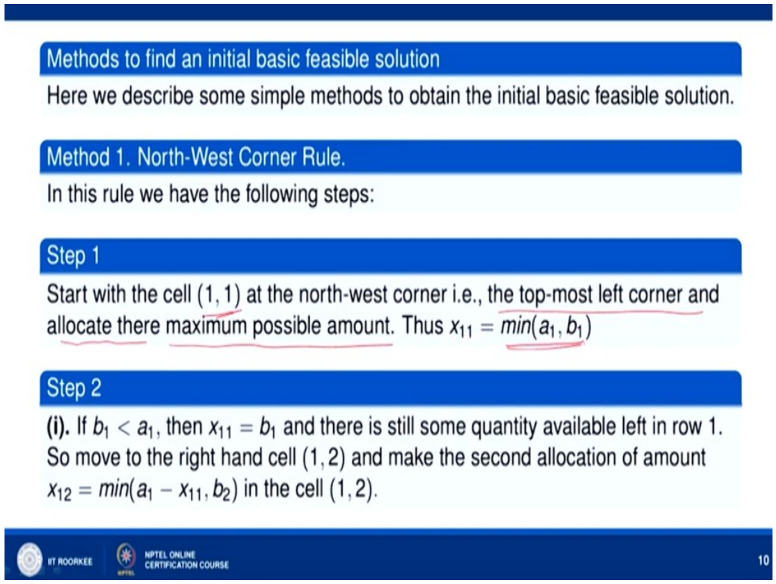

So now in the cell let us see what is north-west corner rule, so starts with the cell 1 this is cell 1 1 this is 1 1 cell, so start with the cell 1 1 what we will do and you can see this 1 1 cell is also the north-west corner, so this is north-west corner of the table and the top most left corner and the so at the north-west corner is the top most left corner you can see this is top most of this table this is the top most left corner, so what we will do here allocate there maximum possible amount to the top most left corner allocate there maximum possible amount, now how much maximum possible amount we can allocate that we have to see see xyz is the quantity that is to be allocate so so here x_{11} so what should be the value of x_{11} should be equal to minimum of a_1 b_1.

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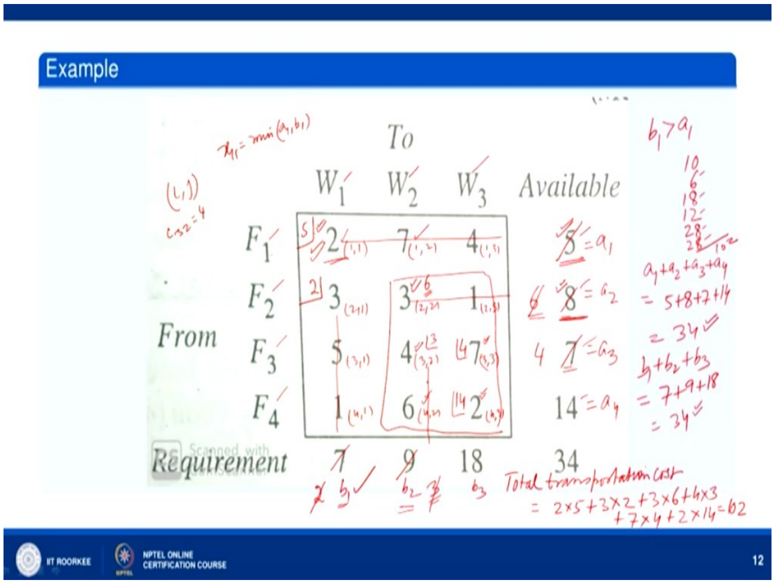

Why it should be minimum of a_1 b_1 you see this is a_1 and this is b_1 at the warehouse W_1 we needs 7 units of the product but here at the source F_1 only 5 units are available 5 units of product are available for shipment, so we can at the most allocate 5 units to the cell W to the cell 1 1 the top left corner to the top left corner we can at the most allocate 5, so that is why we say if the requirement is of the maximum allocation could be only 5 5 7 minimum of 5 and 7 is 5 so we will allocate 5 to the top most left corner of this transportation table so x_{11} x_{11} will be equal to minimum of a1 b_1 minimum of a_1 and b_1 should be allocated to the top most left corner of the table in this north-west corner rule.

So x_{11} is minimum of a_1 and b_1 so we will allocate 5 to this column to this cell, now then if $b_1 < a_1$ let us go to step 2 if $b_1 < a_1$ then if $b_1 < a_1$ than minimum of a_1 b_1 will be b_1 so x_{11} may be equal to b_1 and there still some quantity left, now see here it is $b_1 > a_1$ you can see in this problem $b_1 > a_{1,}$ $b_1 > a_1$, so if $b_1 < a_1$ then some quantity will be available in left row in left in the left row one by because here if this a_1 is more it is 5 here but if a_1 suppose was 8 and it was 7 then minimum of 7 and 8 would have been 7.

So we would have allocated 7 here only in the cell 1 1and then there out of 8 units 7 units we have allocated to the cell 1 1 we would still have 1 unit to allocate so if b_1 is less than a_1 than some quantity will be left for allocation in the first row and therefore there will be some quantity left in the row 1 so move to the right hand cell 1 2 then we will move to the right hand cell 1 2, so suppose it is say here it was 8 it was 7 here 7 we allocated here 1 unit will be left then 2 allocate that 1 unit we will move to the cell W_2 in the cell W_2 we need at the warehouse W_2 we need 9 units and we still have 1 units which can be allocated so that 1 unit we will allocate to cell 1 2 and so on.

So what we will do here there is still some quantity available in left in row 1 so move to the right hand cell 1 2 and make the second allocation of amount. Now, how much will be that amount x12 will be left over a_1 - x_{11} we have allocated b_1 the left over is a_1 - x a_1 - b_1 that is a_1 - x_{11} so a_1 - x_{11} and b_2 that minimum of that we have to take and that is to be allocated to cell 1 2, Now, in case $b_1 > a_1$ like in our problem here you can see this b1 is 7 a1 is 5, so what we will have we have allocate 5 here still 2 units are need for the warehouse W_1.

So we moved down to the cell 2 1 and in the cell 2 1 we have we can allocate 2 units which are left over here because we needed 7 we have allocated 5 here 2 more are required those 2 can be given to the cell 2 1 requirement is 8 and at the location F_2 we sorry at the location F_2at the source F_2 we have 8 units available so from 8 we will give 2 take 2 and give 2 this cell 2 1, so that total availability of W_1 total quantity available the warehouse W_1 is exhausted we get 5 here from the source F_1 and 2 here from the source F_2, so if $b_1 > a_1$then x_{11} will be equal to a_1 and there is still some requirement left in the column 1.

So move vertically downwards to the cell 2 1 and make the second allocation of the amount x_{21} equal to minimum of a_2 and this should be a_2 minimum of a_2 and minimum a_2 we have to take minimum of a_2 and b_1 - a_1 because a_1 we have already allocated so from b_1 we will subtract a_1 so b_1 - a_1 and a2 and this should be a_2, so minimum of a_2 and b_1 - x_{11} that will be allocated 2 1 cell 2 1. Now if $b_1 = a_1$ suppose suppose this quantity a_1= b_1 here than what we will do we will allocate this value and then we will do not have to go this we do not have allocate further 2 in the first row or in the first column.

We can omit first row or we can omit first column because we have already allocated the amount that is required to the warehouse W_1 and the amount that is available in the source 1 that is also to be exhausted we will cross out the first row I will cross out the first column we will not cross out both, so if $b_1 = a_1$ then x_{12} or x_{21} equal to 0, so start from the new north-west corner of the transportation table and allocate there as much as possible, so suppose we have cross out the first then we will have this table second row third row forth row so this is our new table, so we will again consider the north-west corner of this table that is we will the 2 1 cell will now be the new north-west corner of the table and we will repeat this steps.

So repeat steps 1 and 2 until all the available quantity is exhausted or all the requirement is satisfied, so let us see how we do this here so you can see this is north-west corner we at the source F1 quantity available is 5 and at the warehouse W_1 we need the 7 units of the product, so we take minimum of 5 and 7 allocate 5 to this cell W_1 so 5 is allocated here, now so 5 we will cross out here 5 will cross out here and now there nothing more which can be allocated from the source F_1 so we cross out the first row, now here 5 already been allocated so it will become now 2 2 units more are to be allocated.

So move vertically down come to this one now here we are at the source F_2 we have 8 units available so we can allocate 2 out of 8 to this 2 1 cell so that this becomes 0 here and here 8 - 2 it becomes 6 here now we have allocated all we have the requirement of the first warehouse W1 is fulfilled so we cross out the first column and now what is left with us this table is left with us this table this is left with us so north west corner is this one north west corner is this one, now we take the minimum of 6 and 9 is 6 so we allocate 6 to this column, this cell so 6 is allocated here to this cell so this becomes 0 and here we get 9 - 6 so we get 3 now from the warehouse F_2 we have allocated all the 8 units so we cross out this row and now what is left is this table this one 4 7 6 2.

So, this is left with us now the north-west corner is 4 this corner 3 2 this north west corner, so here what is the availability availability F_3 is 7 and we have 3 more units are needed at the warehouse W_2 so 3 and 7, the minimum is 3 so we allocate 3 here to this, so then this column is now crossed out because we have allocated we have W_2 has been allocated all the 9 units so 3 we subtract from here and we get 4 4 are stilled to be allocated, now from the source F_3 we have to allocate 4 at 4 units are available at 3 and we have W_1 and W_2 have been allocated and W_3 remains to be allocated.

So from F_3, W_3 will now receive 4 so this will receive 4 this one and from F_4 this one will receive 14 the 4 3 cell F_3 will from F_3 W_3 will receive remaining 4 and from F_4 W_3 will receive this 14, so what we have so the allocations are to the cell 1 1 we have allocated 5 units to the cell 2 1 we have allocated 2 units to the cell 2 2 we have allocated 6 to the cell 3 2 we have allocated 3 and to the cell 3 3 we have allocated 4 to the cell 4 2 no not 4 2 to the cell 4 3 we have allocated 14.

So, how many allocations are there 1 2 3 4 5 6 6 allocations are there where the all 5 units from F_1 are allocated to the warehouse W_1 out of 8 units available at the source F_2 2 have been allocated to the warehouse W_1 6 have been allocated to the warehouse W_2 from the source F_3 3 units have been allocated to the warehouse W_2 and 4 have been allocated to the warehouse W3 and from F4 all 14 have been allocated to the warehouse W_2 and the total transportation cost is now you can see 5 × 5 then we have 3 × 2 and then we have 3×6 and then we have 4 × 3 then we have 7 × 4 and then we have 2×14, so how much is that 10 6 than we have 18 than we have 12 here than we have 28 and then we have 28, so which is 102 so this is equal to 102 so the total transportation cost is 102 rupees.

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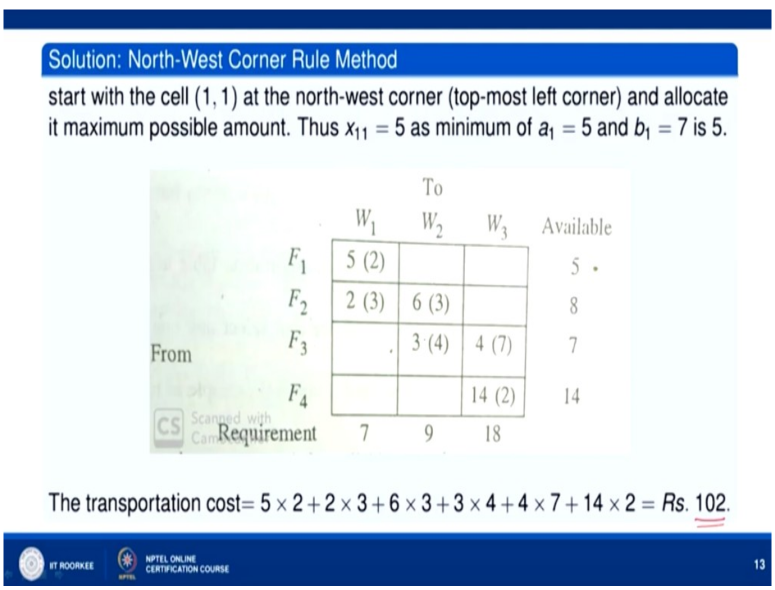

This is the total transportation cost and this is the new table from F_1 we have allocated all 5 quantities to the cell 1 1 to the cell 2 1 from F_2 we allocate 2 and to the warehouse W_2 we allocate 6 from F_2 we allocate 3 to the warehouse W_2 4 to the warehouse W_3 and from F_4 all the 14 into the warehouse W_3, so that is the total transportation cost so this is north-west corner rule.

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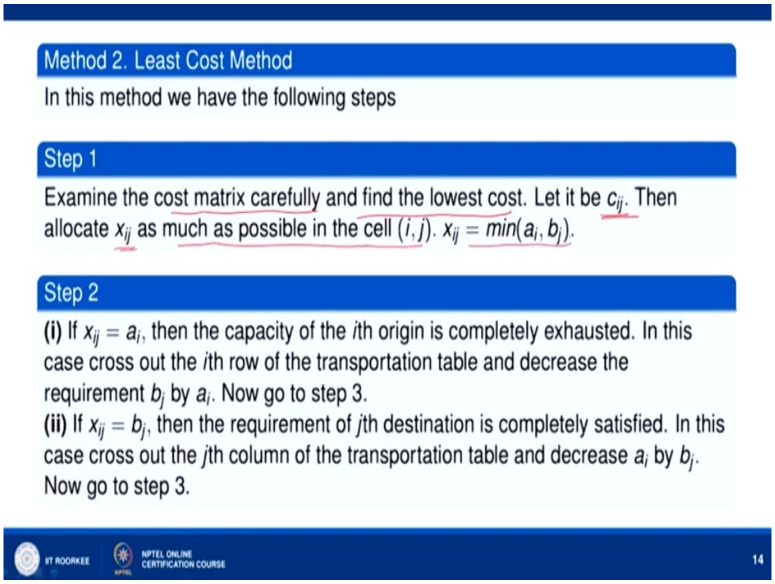

Now, let us go to the next method which is least cost method in this method we have following steps examine the cost matrix carefully and find the lowest cost. Let us say it is c_{ij}. Then allocate xij as much as possible in the cell ij so x_{ij} is equal to minimum of a_i and b_j, so we will the whole cost matrix and whichever cell has got the lowest cost to that cell we shall allocate maximum possible value that is maximum possible will be minimum of a_i and b_j, so because ai is the amount available at the ith source and b_j amount required at the j-th destination.

So minimum of a_{ij} that much is the maximum value which we can allocate so minimum of a_{ij} will be x_{ij} that will be allocated to the i j-th cell, now if $x_{ij} = a_i$ suppose $a_i < b_j$ if $x_{ij} = a_i$ then the capacity of the i-th origin is completely exhausted because a_i is the quantity that is available at the i-th source so i-th origin is completely exhausted in this case cross out the ith row of the transportation table and decrease the requirement of b_j by a_i so b_j will be reduce by a_i.

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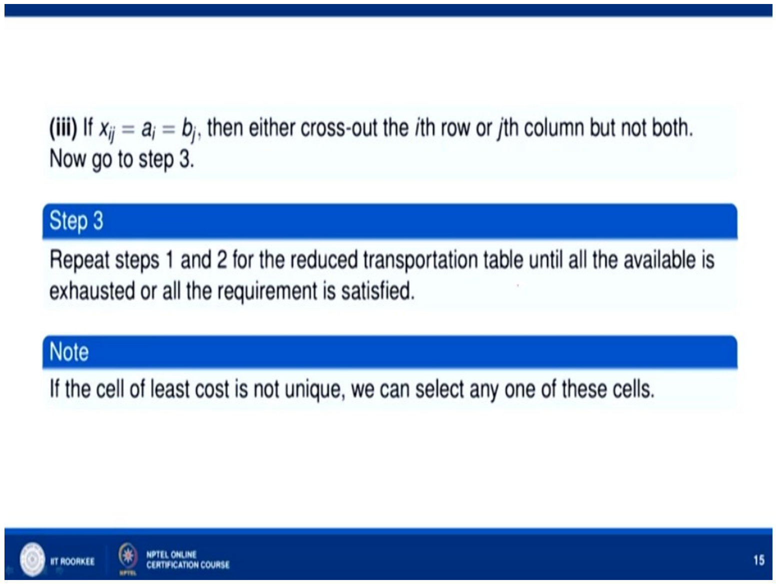

Now, we will go to step 3 so what is step 3 repeat steps 1 and 2 for the so after that we will repeat steps 1 and 2 until all the available is exhausted and all the requirements is satisfied.

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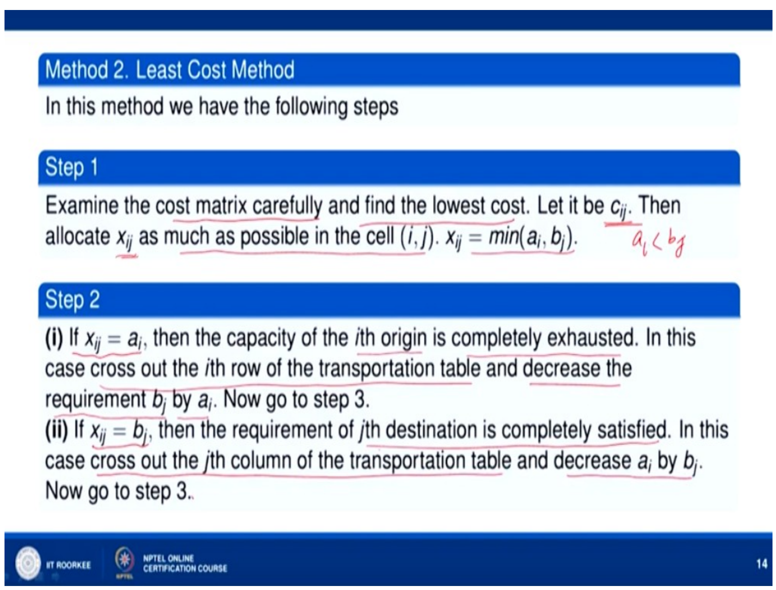

Now, else what we will do if $x_{ij} = b_j$ if x_{ij} is not equal to i but b_j that is $b_j < a_i$ then the requirement of jth destination is completely exhausted completely satisfied, so in this case cross out the j-th column we will cross out the j-th column of the transportation table and decrease a_i by b_j and then we will go to step 3.

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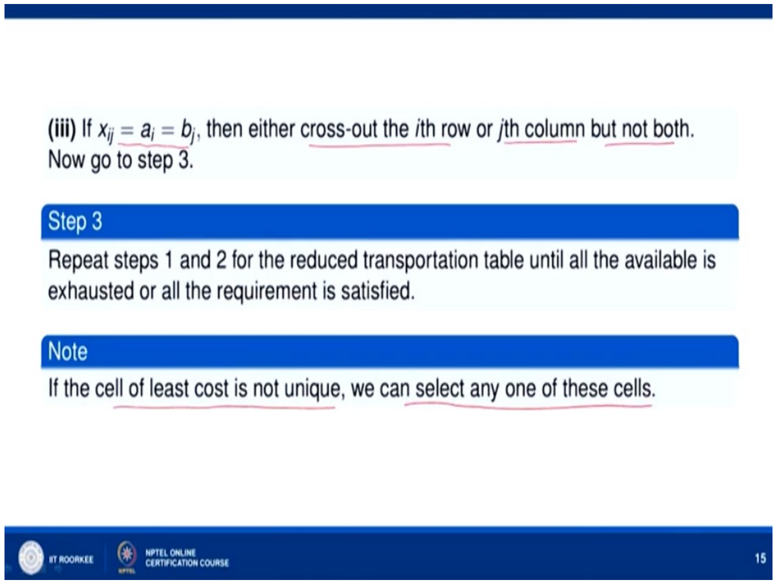

Now if $x_{ij} = a_i = b_j$ then either cross out the i-th row or j-th column but not both and go to step 3 so step 3 this where we will repeat step 1 and 2. Now if the cell of least cost is not unique suppose there are more than one cell where the cost is given to be least it is not unique cell that is just one cell where least costs is there more than one cells are where least cost is given then we can select of this cells to begin.

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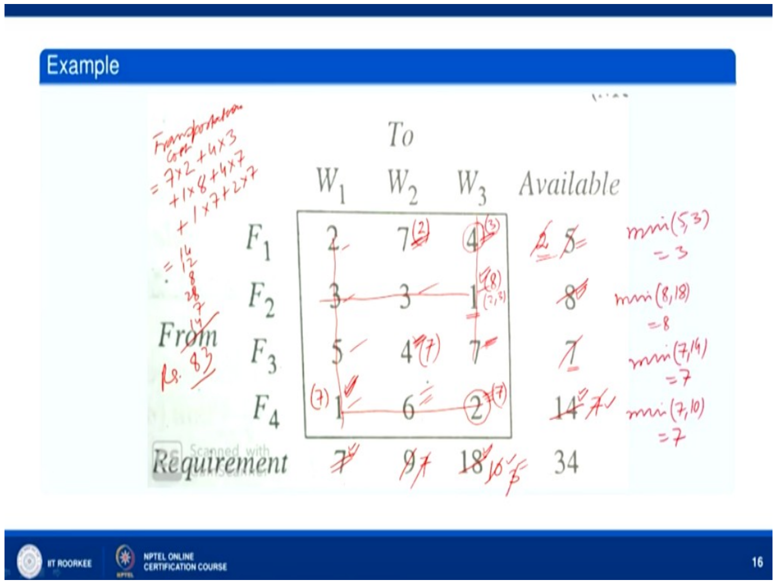

Now, let us see we again have the same example which have taken in the case north west corner so let us apply the least cost rule here now we see the cost matrix so we have 2 7 4 3 3 1 5 4 7 1 6 2 they are the cost of transporting unit from one source to another warehouse from F_1 to W_1 it is 2 per unit fromF_1 W_2 it is 7 per unit from F_1 to W_3 it is 4 per unit and so on, so let us see minimum the least value among all this values is 1 2 7 4 3 3 1 5 4 7 1 6 2 so 1 is the least among all this values and 1 occur at 2 places here as well here we can select any one cell to begin so let say suppose I choose this cell that is the cell where we have second row and third column.

So this is 2 3, cell 2 3, in the cell 2 3 the cost is 1 rupee for transporting unit of product from the source F_2 to the warehouse W_3 we could have also consider this cell which is cell 4 1, forth row and first column suppose I have selected this one, then we to this cell we will allocate the maximum quantity and maximum quantity will be the minimum of this the quantity available that the source F_2 and the required at the warehouse W_3, so at the warehouse W_3 we required 18 units so we take minimum of 8 and 18, so minimum of 8 and 18 that is equal to 8 so we allocate 8 here so after we have allocated the 8 units the availability of F_2 at F_2 we had 8 units available so that is exhausted.

So we will cross out this row, we will cross out and after that now what is the table available with us 2 7 4 5 4 7 1 6 2 at 8 have gone at already been allotted allocated to W_3 so it will become 10 now, now 2 7 4 5 4 7 1 6 2 so we again fine the cell which has got the least cost, so this cell this cell has got the least cost now so we will allocate minimum of a_i $b_j$now here what we have 14 14 units are available at source 2 7 are requires at W_1so minimum of 7 and 14 is equal to 7 so we allocate 7 here to this so we have allocated 7 so now the requirement of the availability at F_4 is reduce to 7 7 units are remaining to be allocated and here W_1 has got all the 7 units that are needed.

So this so what we will do so this will cross this column because the requirement of $W_1$is satisfied, so we will cross-out this column now what is left with us 7 4 4 7 and 6 2 so which is minimum 7 4 4 7 6 2 least cost is 2 least cost is 2 and what is the minimum now we have to take minimum of 7 and 10 minimum of 7 and 10 we have to take, so minimum of 7 and 10 is 7 so we allocate 7 here we allocate 7 here so 7 has this 7 this 7 has gone now so we cross-out this row and this will remain now 3 3 more or to be made available to W_3, now what is left with us so we have 7 4 4 7 so minimum is 4.

Now we can take either this 4 or we can take this 4 so let us take this 4 for example we take this 4 so we then consider minimum of 5 and 3, so minimum of 5 and 3 that is equal to 3 so we allocate 3 here so we now left with 2 here and here this 3 has gone, so we will cross-out this column we will cross out this column, now so we have here now so 2 this 2 can now F_1 from F_1 W_2 can be given this 2 and then from F_3 this 7 7 can be given to W_2 and then 1 we give W2 from F_1 this 9 became 7 and then we gave 7 from this F_3 to W_2 it become 0.

So what we have done from F_1 2 units are given to W_2 and 3 units are given to W_3 from F_2 all 8 units to W_3, from F_3 we are giving all 7 units to W_2 from F4 we are given 7 units to W1 and 7 units to W_3, so what is the transportation cost is equal to 7 into 2 then 4 into 3 than we 1 into 8 and then we have 4 into 7 than we have 1 into 7 and then we have 2 into 7 so this is 14 + 12 + 8 + 28 + 7 + 14 this is 83 so we get transportation cost to be rupees 83, so by applying this least cost method the transportations cost comes out to be rupees 83 while in the case of the first method that is north-west corner rule we got the transportation to be 102.

So this that is better because we take into consideration the least cost of the cost matrix and allocate maximum of quantity that is available that we say x_{ij} is taken again as minimum a_i and

b_j and that is allocated minimum of a_{ij} and b_j that is x_{ij} is allocate to the cell ij where the least cost c_{ij} occur so that is why it give as a better result than the north-west corner rule.

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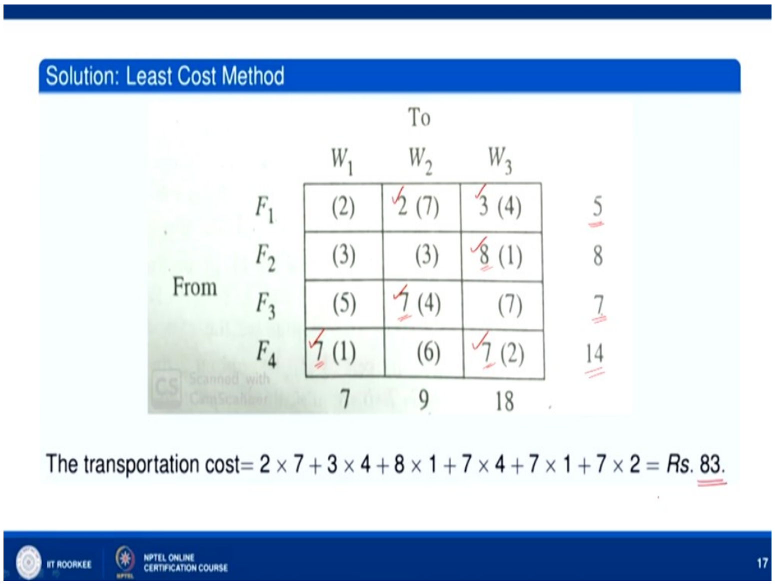

So this is the matrix where you can see from F_1 we have allocated 2 to the we have allocated 2 to second cell so this are allocations this are allocations this are allocations in the brackets we are writing the cost per unit while the while you that are written without bracket they are the allocations the xij this are this x_{12} this is x_{13} this is $x_{32}$and this x_{32} so and this x_{41} x_{43} so they are this values 2 3 8 7 7 7 they are the allocations to the warehouse from this sourceF_1 F_2 F_3 F_4 so this 5 units that were available at F_1 out of that 5 2 have been given to W_2 3 have been given to W_3 out of 8 available at F_2 all 8 have been given to W_3 out of 7 all 7 have been given to W_2 out of 14 7 have been given to W_1 and remaining 7 have been given to W_3 and the total cost of transportation is 83.

So this is the least cost method in the next lecture we shall discuss the Vogel approximation method which when which one you will see is still better than this 2 so we will find the transportation cost there and then we shall discuss the how to determine an optimal solution of the transportation problem. So with that I would like to end this lecture, thank you very much for your attention.