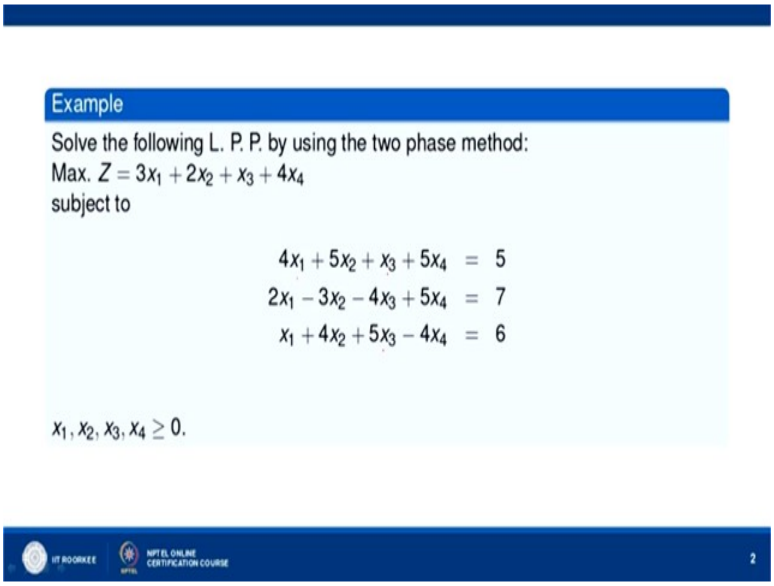
Higher Engineering Mathematics
Professor P. N. Agrawal
Department of Mathematics
Indian Institute of Technology Roorkee
Lecture - 49
Two Phase Method - II

Hello friends, welcome to my second lecture on Two Phase Method, here we shall discuss some more examples on solution of linear programming problem by Two Phase Method.

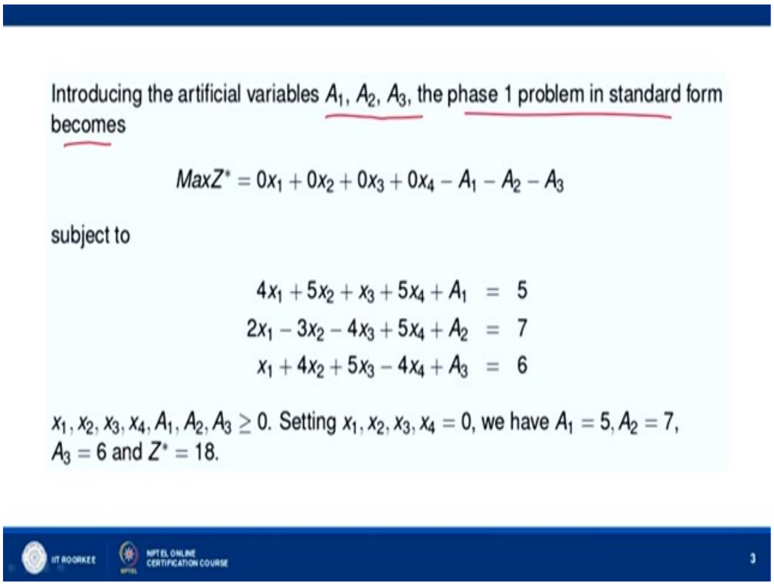
(Refer Slide Time: 00:40)

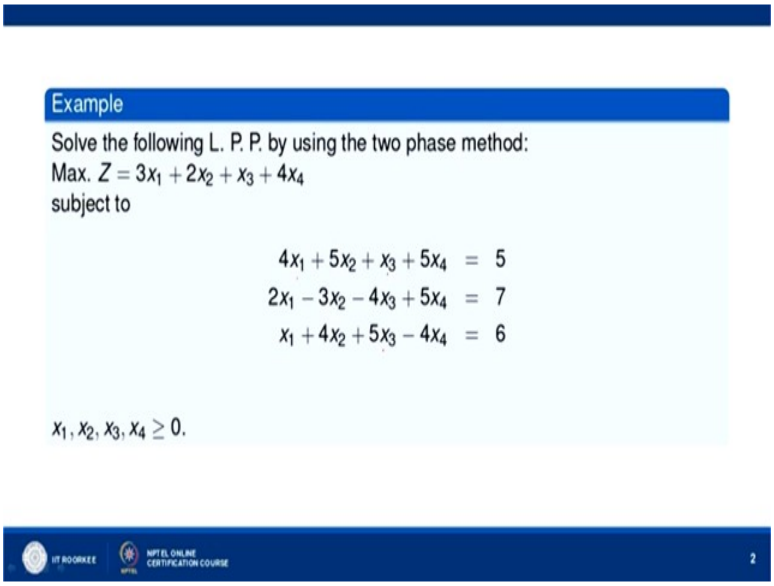


Let us consider Z = $3x_1 + 2x_2 + x_3 + 4x_4$, we have to maximize this objective function subject to the constants $4x_1 + 5x_2 + x_3 + 5x_4 = 5$, $2x_1 - 3x_2 - 4x_3 + 5x_4 = 7$,

$x_1 + 4x_2 + 5x_3 - 4x_4 = 6$, $x_1, x_2, x_3, x_4 \geq 0$.

(Refer Slide Time: 01:05)





Now, since these are constants are of equality type, ok equality here, equality here, equality here, we will consider the artificial variables A_1, A_2, A_3, so introducing the artificial variables A_1, A_2, A_3 the phase 1 problem in standard form becomes maximum of $Z^* = 0 \star x_1 + 0 \star x_2 + 0 \star x_3 + 0 \star x_4 - A_1 - A_2 - A_3$. The two the artificial variables the coefficients are taken as -1 while for the decision variablesx_1, x_2, x_3, x_4 the coefficients are taken as zeros.

So, subject to the constants are now equations are $4x_1 + 5x_2 + x_3 + 5x_4 + A_1 = 5$ and $2x_1 - 3x_2 - 4x_3 + 5x_4 + A_2 = 7$, $x_1 + 4x_2 + 5x_3 - 4x_4 + A_3 = 6$. Now, where x_1, x_2, x_3, x_4, A_1, A_2, $A_3 \geq 0$. Now setting the variables non-basic variables x_1, x_2, x_3, $x_4 = 0$ we have the basic variables A_1

, A_2, A_3 , $A_1 = 5$, $A_2 = 7$, $A_3 = 6$ and therefore $Z*$ = - A_1 - A_2 - A_3 so - 5, - 7 - 6 that is = - 18, so $Z^{¿}$ = - 18.

(Refer Slide Time: 02:40)

Solution

	c_j	0	0	0	0	-1	-1	-1		
c_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b	θ
-1	A_1	(4)	5	1	5	1	0	0	5	$\frac{5}{4}$
-1	A_2	2	-3	-4	5	0	1	0	7	$\frac{7}{2}$
-1	A_3	1	4	5	-4	0	0	1	6	6
$Z_j^* = \sum c_B a_{ij}$		-7	-6	-2	-6	-1	-1	-1	-18	
	$C_j = c_j - Z_j^*$	7↑	6	2	6	0↓	0	0		

C_j is positive under x_1, x_2, x_3 and x_4 columns, this is not an optimal solution.

Introducing the artificial variables A_1, A_2, A_3, the phase 1 problem in standard form becomes

$$MaxZ^* = 0x_1 + 0x_2 + 0x_3 + 0x_4 - A_1 - A_2 - A_3$$

subject to

$$4x_1 + 5x_2 + x_3 + 5x_4 + A_1 = 5$$
$$2x_1 - 3x_2 - 4x_3 + 5x_4 + A_2 = 7$$
$$x_1 + 4x_2 + 5x_3 - 4x_4 + A_3 = 6$$

$x_1, x_2, x_3, x_4, A_1, A_2, A_3 \geq 0$. Setting $x_1, x_2, x_3, x_4 = 0$, we have $A_1 = 5, A_2 = 7$, $A_3 = 6$ and $Z^* = 18$.

Now, let us first form the first simplex table (corr) so here C j, C j are the coefficients of the variables in the objective function, so we have 0 into x_1, 0 into x_2, 0 into x_3, 0 into x_4 - A_1, - A_2, - A_3, so we have 0, 0, 0, 0, - 1, - 1, - 1 and the basis of variables are A_1, A_2, A_3 whose coefficients are - 1, - 1, - 1 in the objective function and this is the matrix of 4, 5, 1, 5 they are the coefficients of x_1, x_2, x_3, x_4 and then the coefficient of A_1 is 1, coefficient of A_2 is 0, coefficient of A_3 0 and b = 5, ok.

And then in the second equation is the coefficient of x_1 is 2, coefficient of x_2 is - 3, coefficient of x_3 is - 4, coefficient of x_4 is 5, coefficient of A_1 is 0, coefficient of A_2 is 1, coefficient of A_3 is 0 and then the constraint on the right side is 7. Similarly, the Third equation, the coefficient of x_1 is 1, coefficient of x_2 is 4, coefficient of x_3 is 5, coefficient of x_4 - 4, the coefficients of A_1 and A_2 are zeros and coefficient of x A_3 = 1 and the constraint is 6.

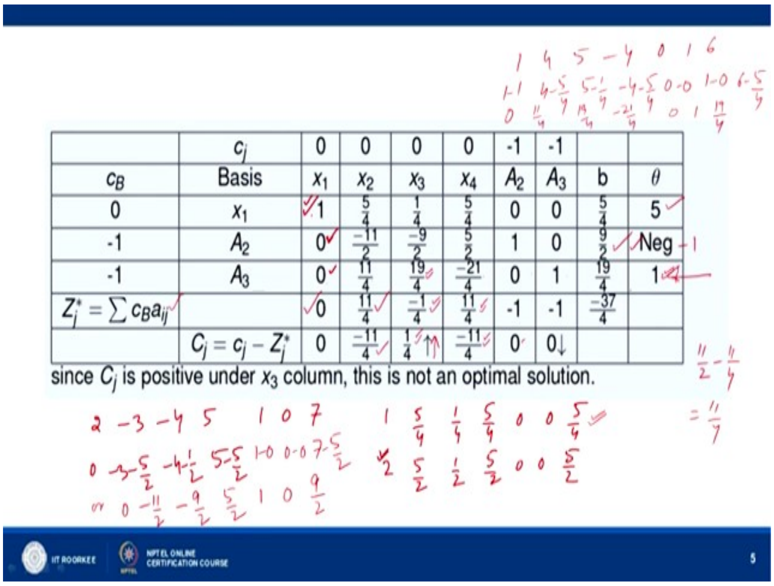
Now, we can find $Z_j^{¿}$ which is $\sum C_B\, a_{ij}$, so $\sum C_B\, a_{ij}$if we find then here you can see - 1 into 4 is - 4, - 1 into 2 is - 2, - 1 into - 1 is - 1, so - 4 - 2 - 1 becomes - 7, ok and when we get capital C j which is C j - $Z_j^{¿}$ we get 0, - - 7 so we get 7 here. Similarly, in the x_2 column we have 5, so - 1 into 5 is - 5, - 1 into - 3 is - 3, - 1 into 4 is - 4, so - 5 - 4 - 9, - 9 + 3 is - 6, so this is - 6 and when we subtract - 6 from 0 we get 6.

Similarly, we can get here and in the column corresponding to x_3 we get $Z_j^{¿}$ = - 2 and the column corresponding to x_4 we get $Z_j^{¿}$ = - 6 and in the column corresponding to A_1 we can see - 1 into 1 is - 1, - 1 into 0 is 0, - 1 into 0 is 0 so the total is - 1, when we subtract - 1 from - 1 we get 0 here. And similarly, for A_2 we get $Z_j^{¿}$ as - 1 and - 1 when we subtract from - 1 again we get 0 and the column corresponding to A_3 again we get $Z_j^{¿}$ = - 1, when we subtract - from - 1 we get 0.

Now, we can see here C j is (sta) C j is greater than 0 in the column corresponding to x_1, x_2, x_3, x_4, ok so the maximum of all these positive values is 7, so this is key column, this one is key column, ok and so the elements of the key column 4, 2, 1, ok 4, 2, 1 are there we divide the value by 5 4 to get 5 by 4, divides 7 by 2 we get 7 by 2 here, divide 6 by 1 we get 6 here and then we have to see which one is the minimum, so 5 by 4 clearly is minimum and therefore this is key row, ok so this is key row and this is key column, so their intersection is this key element 4, ok.

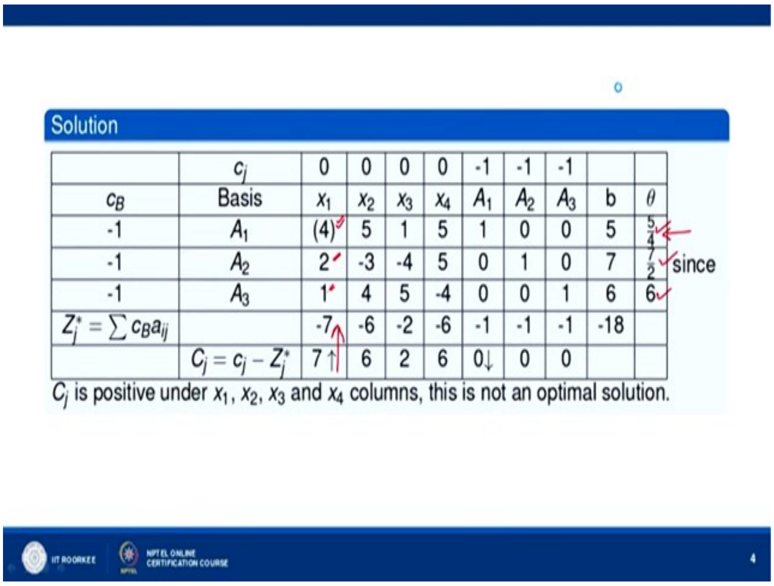
Now, once we have key element this means that x_1 is incoming variable and A_1 is outgoing variable. In the next simplex table, A_1 will be going out, ok x_1 will be coming in and the column corresponding to A_1 will also be removed, so we have the following simplex table.

(Refer Slide Time: 06:50)



	c_j	0	0	0	0	-1	-1		
c_B	Basis	x_1	x_2	x_3	x_4	A_2	A_3	b	θ
0	x_1	1	$\frac{5}{4}$	$\frac{1}{4}$	$\frac{5}{4}$	0	0	$\frac{5}{4}$	5
-1	A_2	0	$\frac{-11}{2}$	$\frac{-9}{2}$	$\frac{5}{2}$	1	0	$\frac{9}{2}$	Neg
-1	A_3	0	$\frac{11}{4}$	$\frac{19}{4}$	$\frac{-21}{4}$	0	1	$\frac{19}{4}$	1
$Z_j^* = \sum c_B a_{ij}$		0	$\frac{11}{4}$	$\frac{-1}{4}$	$\frac{11}{4}$	-1	-1	$\frac{-37}{4}$	
	$C_j = c_j - Z_j^*$	0	$\frac{-11}{4}$	$\frac{1}{4}$ ↑	$\frac{-11}{4}$	0	0↓		

since C_j is positive under x_3 column, this is not an optimal solution.



Solution

	c_j	0	0	0	0	-1	-1	-1		
c_B	Basis	x_1	x_2	x_3	x_4	A_1	A_2	A_3	b	θ
-1	A_1	(4)	5	1	5	1	0	0	5	$\frac{5}{4}$
-1	A_2	2	-3	-4	5	0	1	0	7	$\frac{7}{2}$
-1	A_3	1	4	5	-4	0	0	1	6	6
$Z_j^* = \sum c_B a_{ij}$		-7	-6	-2	-6	-1	-1	-1	-18	
	$C_j = c_j - Z_j^*$	7↑	6	2	6	0↓	0	0		

since

C_j is positive under x_1, x_2, x_3 and x_4 columns, this is not an optimal solution.

We see here, we have instead of A_1 we have x_1 here, the coefficient of x_1 is 0, so we put here 0 A_2, A_3 are as such and then what we do is A_1 the column corresponding to A_1 is deleted here and now what we do? The key row elements are divided by the key element, ok. So we divide key row elements by 4 and we get 1, 5 by 4, 1 by 4, 5 by 4 and we get here 5 by 4, ok so the new row is 1, 5 by 4, 1 by 4, 5 by 4, A_1 column is not there 0, 0, 5 by 4, ok.

Now, with the help of this new row, ok new row where we have made the key element 1 by dividing by 4 with the help of this we make the entries in the key column other entries in the key column as zeros, so other key (co) entries in the key column are 2, here we have 2, here we have 1, ok this means that we multiply the row this row by 2 and subtract from the second row and then to make the this entry 0 we just subtract the we just subtract this row from the third row, ok.

So, let us multiply the elements 1, 5 by 4, 1 by 4 and 5 by 4 and we get 0, 0, 5 by 4, ok these elements we multiply by 2 and subtract from the second row, so when we multiply these by 2, ok we get 2, 5 by 2 and when we multiply by 2 we get 1 by 2 and then we get 5 by 2, 0, 0, 5 by 2, ok. Now, this is this we subtract from the second row elements, so second row elements are 2, - 3, - 4, 4, 5, 2, - 3, - 4, 5 0, 1, 0, 7, ok.

So, 5, 0, 1, 0, 7, ok now let us see, so we subtract these values, ok from the corresponding values here, so when we subtract this 2 from this 2 here we get 0, ok so we get 0 here and then 5 by 2 we subtract from - 3, so - 3 - 5 by 2, ok then - 4 we subtract (min) half, so - 4 - half then 5 - 5 by 2 then 0 - 0 then 1 - 0, ok this element will not be there, so what we will do? 2 - 2 is 0, - 3 - 5 by 2 then - 4 - 1 by 2 then 5 - 5 by 2, ok then 1 - 0, ok and then we get 0 - 0 and then we get 7 - 5 by 2, ok

Alright, so we get 0 and then we have - 11 by 2, - 9 by 2 and then we get 5 by 2, 1, 0 and then we get 9 by 2, so you can see here 0, - 11 by 2, - 9 by 2, 5 by 2, 1, 0, 9 by 2, ok and these from the third row we subtract just this row, this one, ok and third row is what ? 1, 4, 5, - 4, ok A_1 we have not do not have to take, so 1, 4, 5, - 4, A_1 is deleted 1, 4, 5, - 4, 0, 1, 6 we have 0, 1, 6.

So, from this we simply subtract the first row that is 1 5 by 4, ok so 1 - 1, 4 - 5 by 4 then we get 5 - 1 by 4 then we get - 4 - 5 by 4 then we get 0 - 0, 1 - 0, 6 - 5 by 4 and what we will get here? 0 and this will be 16 - 5, so we get 11 11 by 4, ok, so 0, 11 by 4 a rear 5 into 4 is 20 20 - 1 so 19 by 4 and here we get - 16 - 5 - 21 by 4 here 0, here 1, here 24 - 5 so 19 by 4 so we get this 0, 11 by 4, 19 by 4, - 21 by 4, 0, 1, 19 by 4, ok.

Now, we find that $Z_j^{¿}$, $Z_j^{¿}$ is $\sum C_B a_{ij}$, so 0 into 1 0, - 1 into 0 0, - 1 into 0 is 0, so we get 0, 0 - 0 0, so this is 0, ok then 0 into 5 by 4 is 0, - 1 into - 11 by 2 is 11 by 2 and then we get - 1 into 11 by 4 so - 11 by 4 and then 11 by 2 - 11 by 4 is 11 by 4, so we get 11 by 4 here, 0 - 11 by 4 is - 11 by 4.

And similarly, we can calculate $Z_j^{¿}$ here for x_3 column, it is - 1 by 4 when we subtract from 0 it becomes 1 by 4 and here in the x_4 column $Z_j^{¿}$ is 11 by 4 when we subtract from 0 we get -

11 by 4, here in the A_2 column it is - 1 when we subtract from - 1 it is 0, here in the A_3 column it is again - 1 when we subtract from - 1 we get 0, ok.

Now, so the C j is positive in the column corresponding to x_3 in the column (ek) corresponding to x_4 you know in the column corresponding to x_3 it is positive in the other column it is ≤ 0, so C j is greater than 0 in the x_3 column, so this will be key column, this is key column, ok.

Now, divide the elements in the b column by the corresponding values in the column that is key column, so 1 by 5 by 4 is divided by 1 by 4 we get 5, 9 by 2 when we divide by - 9 by 2 we get - 1 that is negative value, this is - 1, ok negative value and then we divide 19 by 4 by 19 by 4 and we get 1 here, ok.

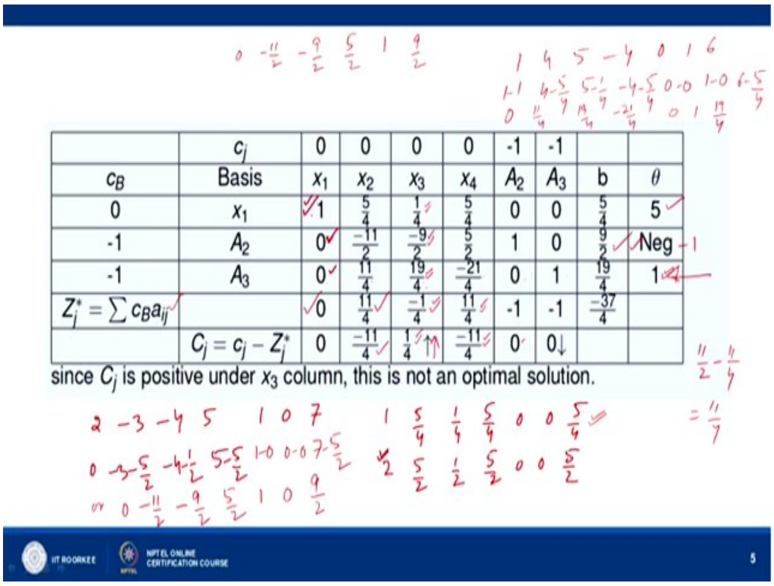
So, we have to now consider minimum value of among the positive values in the theta column, so minimum value is 1 in the among the positive values they are 5 and 1, so minimum value is 1, so this is key row and at the intersection of key row and the key column we get 19 by 4, ok. So 19 by 4 we divide, ok this means that x_3 will be incoming variable and A_3 will be outgoing variable in place of A_3 we shall now have x_3 in the new (tab) in the next table and the coefficient of x_3 is 0, so - 1 here will be replace by 0, ok.

After that we will make the key element which is 19 by 4 as unity by dividing the entire key row by 19 by 4, ok. So, let us see what happens in the next table.

(Refer Slide Time: 14:35)

	c_j	0	0	0	0	-1		
c_B	Basis	x_1	x_2	x_3	x_4	A_2	b	θ
0	x_1	1	$\frac{21}{19}$	0	$\frac{29}{19}$	0	1	
-1	A_2	0	$\frac{-55}{19}$	0	$\frac{-47}{19}$	1	9	
0	x_3	0	$\frac{11}{19}$	1	$\frac{-21}{19}$	0	1	
$Z_j^* = \sum c_B a_{ij}$		0	$\frac{55}{19}$	0	$\frac{47}{19}$	-1	-9	
	$C_j = c_j - Z_j^*$	0	$\frac{-55}{19}$	0	$\frac{-47}{19}$	0		

since all $C_j \leq 0$, therefore an optimal B. F. S. to the auxiliary problem has been attained. But the artificial variable vector appears in the optimal basic at a positive level. Hence, the auxiliary as well as the original L. P. P. does not posses any feasible solution.



	c_j	0	0	0	0	-1	-1		
c_B	Basis	x_1	x_2	x_3	x_4	A_2	A_3	b	θ
0	x_1	1	$\frac{5}{4}$	$\frac{1}{4}$	$\frac{5}{4}$	0	0	$\frac{5}{4}$	5
-1	A_2	0	$\frac{-11}{2}$	$\frac{-9}{2}$	$\frac{5}{2}$	1	0	$\frac{9}{2}$	Neg
-1	A_3	0	$\frac{11}{4}$	$\frac{19}{4}$	$\frac{-21}{4}$	0	1	$\frac{19}{4}$	1
$Z_j^* = \sum c_B a_{ij}$		0	$\frac{11}{4}$	$\frac{-1}{4}$	$\frac{11}{4}$	-1	-1	$\frac{-37}{4}$	
	$C_j = c_j - Z_j^*$	0	$\frac{-11}{4}$	$\frac{1}{4}$	$\frac{-11}{4}$	0	0		

since C_j is positive under x_3 column, this is not an optimal solution.

So, we have C B 0, - 1 we have 0, - 1 A_3 is now becoming x_3 and the here the value becomes 0, so we get 0 here and we are get x_3 here and, ok and we get x_1, x_2, x_3 and A_3, A_3 is now (del) removed the column corresponding to A_3 is now removed, we have just column corresponding to the artificial variable A_2.

Now, we divide the key row elements by key element, so key row elements are divided by 19 by 4, so here we will get 0, here we will get 11 by 19, here we will get - 21 by 19, ok and here we get 0, here we get 4 by 19 here we get 1.

So, let us see, so the column the key row now new row becomes this 0, 11 by 19, so this is 11 by 19, this is 1, ok this is - 21 by 19, this is 0, this is 4 by 19, ok because A_2, A_3 is not there, ok so this is no longer there, so this is 0 then we come to this. So 19 by 4 divided by 19 by 4 becomes 1, so this is equal becoming 1, ok A_2 A_3 column is not there. Now, with the help of this one, ok the this is key column with the help of this one we make this entry and this entry zeros, ok.

So, we multiply the new row this one, the new row is this, this (ne) row, ok we multiply by 9 by 2 and add to the second row, ok now what are the elements of this row? 0, 11 by 19 and then we get 1, we get - 21 by 19, ok we get 0, we get 1, ok and what are the elements there? We do not have to consider A_3 column, ok so A_3 column we do not have to consider, so 0, 9 - 11 by 2, ok - 9 by 2 and we get 5 by 2, we get 1 here, this we do not have to consider and we get 9 by 2 here, ok.

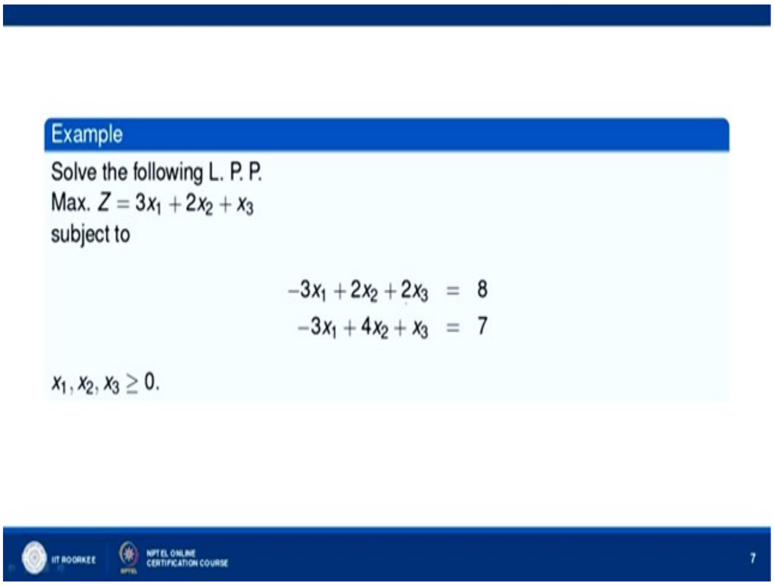
So, 0, - 11 by 2, - 9 by 2, 5 by 2, 1, 9 by 2, ok so we will have to make we have to make this entry 0, ok this entry 0 we have to make, so we multiplied by 9 by 2 and add to this when we multiply by 9 by 2 what it becomes? 0, 11 by 19 into 9 by 2, ok and we get 9 by 2 there and we get - 21 by 19 into 9 by 2 here and we get 0, we get 9 by 2 here. Now, we add this to this row, ok so when you add this row to this row what you will get? 0 + 0 is 0, - 11 by 2 and we get 99 by 38, ok 9 by 2 + 9 by - 9 by 2 is 0, this is 9 - 189 by 38 + 5 by 2 and then we get 0 + 1 and then we get 9 by 2 + 9 by 2, so we get 9, ok.

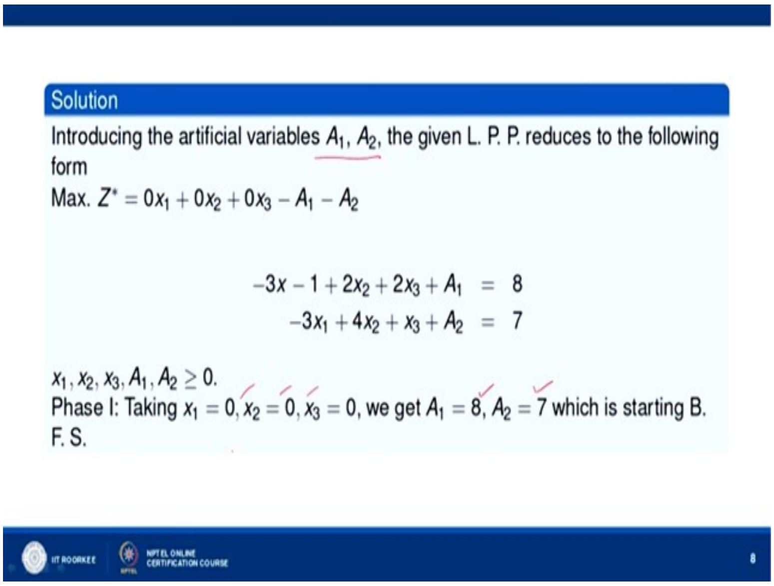
So, what is this? So 0, this is 38, so 19 into 11 so 11 into 19, so how much is that? 209, ok so - 209 + 99 then 0 here we get 38 and we get - 189 and then we get 19 (five za) 19 into (five za) 95 1 9, so how much is that? 0, - 110 and here we get, ok so what we will get? Yes, 0 - 55 by 19 9 - 55 by 19 0, this is - 47 by 19, this is 1, this is 9, ok so that is how we get this and then we make this entry 0, this to make this entry 0, what we have to do? We have 1 by 4 here, ok so we multiply this row by 1 by 4 and add here, no subtract there this is 1 by 4, so we multiply by 1 by 4 and subtract.

So, we multiply this by 1 by 4, this row 1 by 4 and subtract from the elements of this row, remember that A_3 is not there, A_3 column, ok. So then we will get the new row as this 1, 21 by 19, 0, 29 by 19, 0, 1, ok. Now we find C $Z_j^{¿}$, $Z_j^{¿}$ is for x_1 it is 0, for x_2 it is 55 by 19, for x_3 it is 0, for x_4 column it is 47 by 19 for A_2 it is - 1 and for b it is - 9, ok when we consider C j capital C j we get C j - $Z_j^{¿}$, so it is 0, - 55 by 19, 0, - 47 by 19 and it is 0, ok.

So, what we see? All C j's are ≤ 0 therefore an optimal bfm B.F.S. to the auxiliary problem has been attend, ok but the artificial vector you can see A_2, artificial vector A_2 appears in the optimal basic at a positive level we can see in the b column, the value is 9, so at a positive level it is occurring hence the auxiliary as well as the original L.P.P. does not possess any feasible solution in this case, ok.

(Refer Slide Time: 21:20)





Now, let us consider second problem, Z = $3x_1 + 2x_2 + x_3$ we have to maximize this objective function subject to the constants of - $3x_1 + 2x_2 + 2x_3 = 8$, - $3x_1 + 4x_2 + x_3$ is = 7, x_1, x_2, x_3 $\geq$0, ok. So what we will do? We will consider two artificial variables here, ok A_1 and A_2, ok and given L.P.P. then will become Z maximum of $Z^* = 0x_1 + 0x_2 + 0x_3 - A_1 - A_2$ and we will have here - $3x_1 + 2x_2 + 2x_3 + A_1 = 8$ and this equation will become - $3x_1 + 4x_2 + x_3 + A_2 = 7$, ok so x_1, x_2, x_3, A_1, A_2 ≥ 0.

Now, let us take the non-basic variables x_1, x_2, x_3 to be = 0, ok what we get A_1 is = 8, $A_2 = 7$ which is starting B.F.S. ok.

(Refer Slide Time: 22:32)

	c_j	0	0	0	-1	-1		
c_B	Basis	x_1	x_2	x_3	A_1	A_2	b	θ
-1	A_1	-3	2	2	1	0	8	4
-1	A_2	-3	(4)	1	0	1	7	$\frac{7}{4}$
$Z_j^* = \sum c_B a_{ij}$		-6	-6	-3	-1	-1	-15	
	$C_j = c_j - Z_j^*$	6	6 ↑	3	0	0 ↓		

since C_j is positive for x_1, x_2, x_3, the solution is not optimal.

IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 9

Solution

Introducing the artificial variables A_1, A_2, the given L. P. P. reduces to the following form

Max. $Z^* = 0x_1 + 0x_2 + 0x_3 - A_1 - A_2$

$$-3x-1+2x_2+2x_3+A_1 = 8$$
$$-3x_1+4x_2+x_3+A_2 = 7$$

$x_1, x_2, x_3, A_1, A_2 \geq 0.$

Phase I: Taking $x_1 = 0, x_2 = 0, x_3 = 0$, we get $A_1 = 8, A_2 = 7$ which is starting B. F. S.

IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 8

So, C j is now coefficients of the x_1, x_2, x_3, A_1, A_2 so they are 0, 0, 0, - 1, - 1, ok this is C j, C j the decision variables are x_1, x_2, x_3 basic variables are A_1, A_2, ok so the coefficients of A_1, A_2 here are - 1, - 1 and this is your column corresponding to x_1 - 3, - 3 column corresponding to x_2 2, 4, for x_3 it is 2, 1 for A_1 it is 1, 0 for A_2 it is 0, 1 and column of constants is 8, 7, ok so 8, 7 we have, ok.

So, now what we notice is that - 1 into - 3 is 3, - 1 into - 3 is 3, 3 + 3 is 6, so we get Z_j^* = 6, to C j = 0, - 6 is - 6, ok and then here we get - 1 into 2 is - 2, - 1 into 4 is - 4, so Z_j^* becomes - 6, when we subtract - 6 from 0 we get 6 here in the column for x_3 we get Z j C j = 3 and the column for A_1 we get Z C j = 0, for the column A_2 we get Z C j = 0, so C j is positive in the column x_2 and in the column x_3 but the maximum value is 6 of this 2 positive values, so this column is the key column.

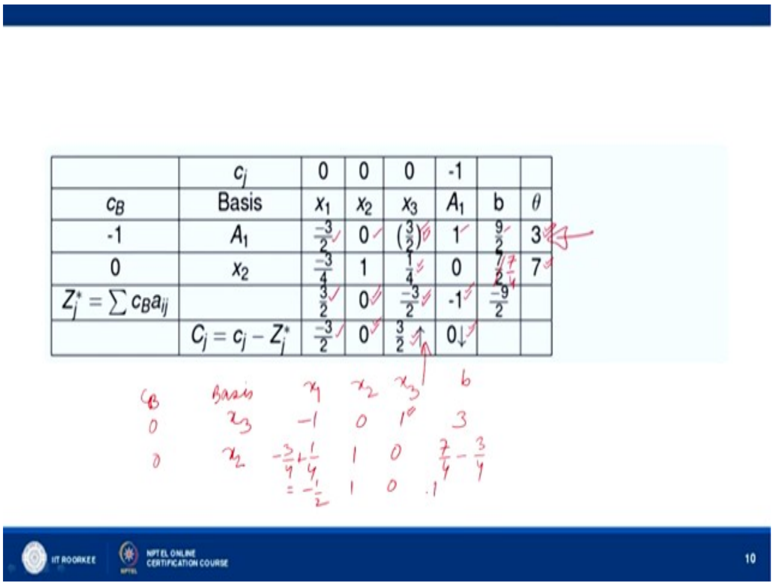
Now, we have to find the key row, so (due) that of divide the elements of the B column by the corresponding values in the key column, so 8 by 2 gives 4, 7 by 4 gives 7 by 4, ok. Now of this two values of theta we see that 7, 5, 4 is the minimum, so this row is the key row and the intersection of key row and the key column is this element, so this element is the key element, key element is 4, x_2 is then the incoming variable A_2 is the outgoing variable.

So, this new in the new table what we will have? C B will be - 1, ok basis will be A_1 and A_2 will be replaced by x_2, ok coefficient of x_2 is 0, ok and what we have here? The key row elements, key row elements will be divided by 4 to make this key element unity, so we will have here - 3 by 4, we will have here 1, we will have here 1 by 4 and we will have here now A_2 is going out, ok so A_1 (ele) in this will become 0, A_2 will not be there b will becomes 7 by 4, ok this is right, so this is x_1, x_2, x_3 and then we get A_1 and we get b, ok.

So, now with the help of this one we will make this element 0, ok so we multiply it by 2 and subtract from this row, ok so when we multiplied by 2 this becomes - 3 by 2 and we subtract from - 3, so what we get? - 3 + 3 by 2, ok and this becomes 0 and here what we will get? 2 - 2 we are multiplying 2 1 by 4, 1 by 2 we are subtracting from 2, so 2 - 1 by 2 and here we will get 1 - 0 into sorry 0 1 - 0 into 2 so that will be 0 1.

And then here, what we will get? 8 - we are subtracting 7 by 2, we are dividing it by 4 then multiplying by 2, so we are dividing 8 - 7 by 2, so this will be how much? This will become - 3 by 2, so this will become - 3 by 2, this will become - 3 by 2, then we will have 0, then we have 3 by 2 and then we have 1 here then we have 16 - 7 9 by 2 here, so this is x_1, x_2, x_3 , A_1 and b, ok, so let us see now.

(Refer Slide Time: 27:10)



So, we can see this is x_1, x_2, x_3 , A_1, b and these are the elements in the new row modified after (oun cure) key row (divide) after we have divided by 4. So - 3 by 4, 1, 1 by 4, 0, 7 by 4, so this is 7 by 4.

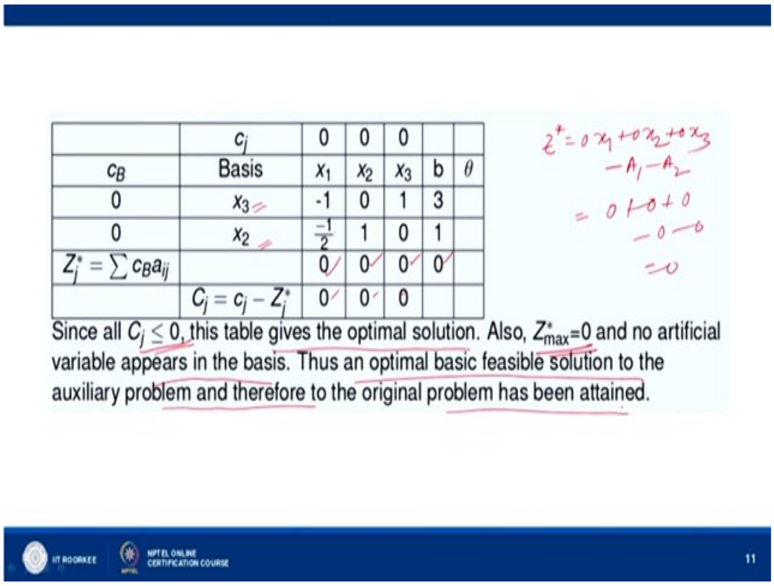
Now, we can see we again get C $Z_j^{¿}$, $Z_j^{¿}$ now will be - 1 into - 3 by 2 3 by 2 and this is 0, ok so we get 3 by 2 here, 0, - 3 by 2 gives you - 3 by 2 then - 1 into 0 is 0, 0 into 1 is 0 so we get 0 here, 0 - 0 is 0 then we get - 3 by 2 and - 3 by 2 is this, so when we subtract from 0 it gives 3 by 2 and - 1 into 1 is 1, 0 into 0 is 0, so this is - 1, - 1 subtracted from - 1 gives 0.

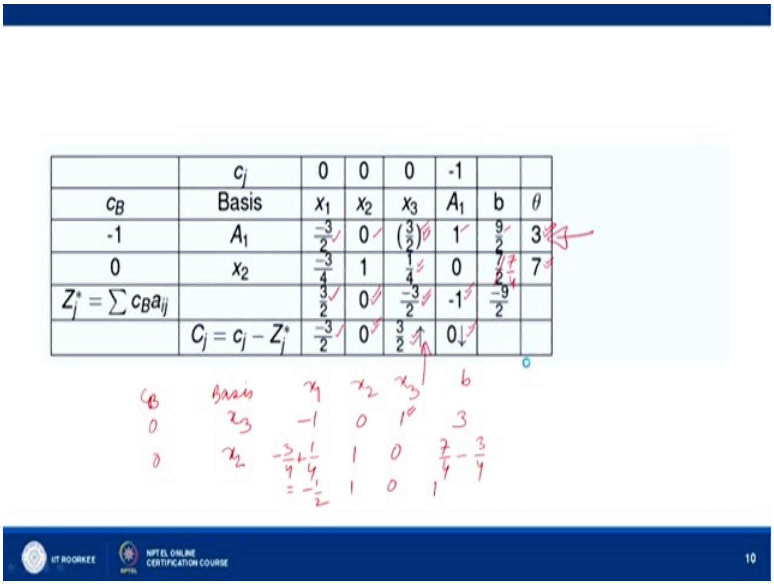
Now, we can see C j is positive in this column, ok so this is key column and 3 by 2, so let us find the key element. Now, we divide by 3 by 2 this 9 by 2, 9 by 2 over 3 by 2 gives 3 and 7 by 4 divided by 1 by 4 gives you 7, so minimum is 3, so this is key row, key row so this is key element, so we divide by key element, ok (bore) before that what we notice? That now x_3 will be incoming, A_1 will be going out.

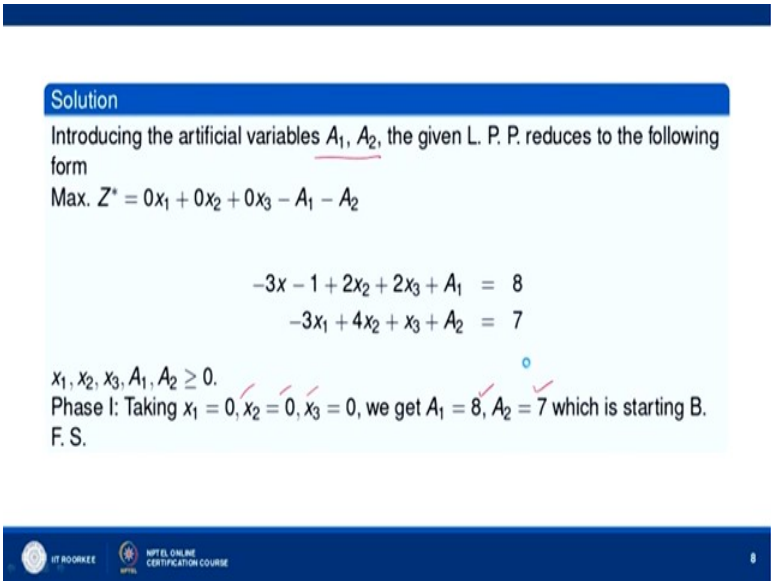
So, what we will have? The in the new table C B, basis so here a in place of A_1 now we will have, so x_3 will be coming in, A_1 will be going out, ok so x_3 and the coefficient of x_3 is 0, x_2 the coefficient of x_2 is 0 then we have x_1, x_2, x_3, ok A_1 will not be there anymore, so b and then we will have get dividing by 3 by 2, ok so 3 by 2 means this is - 1, 0 then 1 and then we divide by 3 by 2, so it will become 9 by 2 over 3 by 2, so it will become 3, ok and with the help of this element now 1 we will make this element 0, ok so we multiplied by 1 by 4 this row and subtract from this row.

So, when we multiply by 1 by 4 this becomes - 1 by 4, - 1 by 4 we subtract here, so - 3 by 4 + 1 by 4 we get and here what we will get? We are multiplying it by 1 by 4 and subtracting from this, so we get this will remain 1, this will become 0 and this will become 7 by 4 - we are multiplying by 1 by 4, so 3 by 4 so how much is that? This is - 2 over 4, ok - 2 over 4 this is - half 1, 0 and then this 7 - 3 4 by 4, so this is 1, ok so what we get? Let us see.

(Refer Slide Time: 30:25)







- 1, 0, 1, 3, ok - 1, 0, 1, 3 then - half 1, 0, 1 - half 1, 0, 1, ok. Now let us find $Z_j^{¿}$, so $Z_j^{¿}$ 0 into - 1 0 this is 0, so this is 0, 0 will be into 0, 0 into 1 so 1, so these are the all zeros, ok and then C j -$Z_j^{¿}$, so 0 - 0 is 0, 0, 0, ok. Now, C j is ≤ 0, so this table gives us the optimal solution, ok and Z maximum star, Z maximum Z^* is what? Z^* is this one, ok x_1 into 0, x_2 into 0, x_3 - A_1, - A_2, ok.

So, from $Z^* = 0 \star x_1$, $0 \star x_2$, $0 \star x_3$, - A_1, - A_2, now A_1, A_2 are zeros, ok x_1, x_2, ok so x_3 is (zer) 6 and x_2 is 1, x_1 is 0, so we get 0 x_1 is 0, x_2 is 1 so 0 into 1 is 0, x_3 is 6, so again we x_3 is 3 so we get 0 A_1, A_2 are also zeros, so $Z_j^{¿}$ is 0, so this is 0 and no artificial variable you can see, no artificial variable appears in the basis, thus an optimal basic feasible solution to the auxiliary problem and therefore to the original problem has been attend, ok so what we do?

(Refer Slide Time: 31:59)

Considering the actual costs associated with the original variables Max. $Z = Z = 3x_1 + 2x_2 + x_3 + 0A_1 + 0A_2$ and the basic feasible solution found at the end of phase I is used as starting solution for the original problem in this phase.

	c_j	3	2	1	0	0
c_B	Basis	x_1	x_2	x_3	b	θ
1	x_3	-1	0	1	3	
2	x_2	$\frac{-1}{2}$	1	0	1	
$Z = \sum c_B a_{ij}$		-2	2	1	0	
	$C_j = c_j - Z$	5↑	0	0		

Here, x_1 is the incoming vector. All the elements of this column are negative. Thus we can not select the outgoing vector. Hence in this case the solution is unbounded.

	c_j	0	0	0		
c_B	Basis	x_1	x_2	x_3	b	θ
0	x_3	-1	0	1	3	
0	x_2	$\frac{-1}{2}$	1	0	1	
$Z_j^* = \sum c_B a_{ij}$		0	0	0	0	
	$C_j = c_j - Z_j^*$	0	0	0		

$Z^* = 0x_1 + 0x_2 + 0x_3 - A_1 - A_2 = 0 + 0 + 0 - 0 - 0 = 0$

Since all $C_j \leq 0$, this table gives the optimal solution. Also, $Z^*_{max}=0$ and no artificial variable appears in the basis. Thus an optimal basic feasible solution to the auxiliary problem and therefore to the original problem has been attained.

Example

Solve the following L. P. P.
Max. $Z = 3x_1 + 2x_2 + x_3$
subject to

$$\begin{aligned} -3x_1 + 2x_2 + 2x_3 &= 8 \\ -3x_1 + 4x_2 + x_3 &= 7 \end{aligned}$$

$x_1, x_2, x_3 \geq 0.$

Now, considering the actual cost associated with the original variables, ok we find maximum value of Z, Z = $3x_1 + 2x_2 + x_3$ this was Z = $3x_1 + 2x_2 + x_3$ and then we get 0 into A_1, 0 into A_2 and the basic feasible solution found at the end of phase one is used as staring solution for the original problem, ok. So basis, basis or x_3, x_2, ok this x_3, x_2, ok the coefficient of x_3, x_2 are now will be written here in the C B column, ok.

So, they are x_3 coefficient is 1, x_2 coefficient is 2 and the remaining table this will be remaining as such x_1, x_2, x_3 - 1, 0, 1, 3 - half, 1, 0, 1 and then let us find now Z*, so Z, ok Z is $\sum C_B a_{ij}$, so 1 into - 1 is - 1, 2 into - half is - 1 so we get Z as - 2 and then here 1 into 0 is 0, 2 into 1 is 2, ok then 1 into 1 is 1, 2 into 0 is 2 0 so we get 1 here 1 into 3 is 3, ok and then Z 2 into 0 2 into 1 is 2, so 2 + 3 = 5, 1 into 3 is 3, 2 into 1 is 2 so here we should have 5, ok.

So, now what we will have? C j = C j - Z, so we will 3 - - 2 that is means 3 + 2 so we get 5 here, 2 - 2 is 0 and 1 - 1 is 0, ok. Now, we can see a C j is positive in this column, ok so this is key column, ok we divide by - 1 this 3, so we get - 3 and we divide 1 by - half, so we get - half, so these are negative values, ok these are negative values and therefore because of this 5 being positive x_1 is incoming vector but all the elements of this column are negative as we can see, ok.

All the elements are negative, so we cannot select the outgoing vector, ok because theta (min) outgoing for it is selective outgoing vector we should have positive value of theta, ok from which we get the that the minimum value positive value of theta, so there is no minimum positive value of theta, so we cannot select outgoing vector hence in this case the solution is unbounded, ok. So, that is all in this lecture, thank you very much for your attention.