**Higher Engineering Mathematics**
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**Lecture - 48**
**Two Phase Method - I**

Hello friends, welcome to my lecture on a first lecture on Two Phase Method, as an alternative to Big M method there is another method known as Two Phase Method to deal with linear programming problems, it involves involving artificial variables.
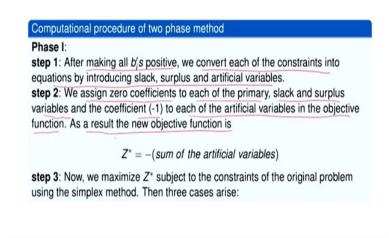
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Now, in this method the solution is obtained in two phases, in phase one all the artificial variables are eliminated from the basic variables, in phase two we use the solution from phase one as the initial basic feasible solution and use the simplex method to determine the optimal solution.

So, we in phase one all the artificial variables are eliminated from the basic variables and then the solution which we get in from phase one that is treated as initial basic feasible solution and we use the simplex method to determine the optimal solution.

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So, in phase one what we do? Let us consider step 1, after making all bi's positive we convert each of the constants into equations by introducing slack, surplus and artificial variables. In step 2 we assign zero coefficients to each of the primary, slack and surplus variables and the coefficient - 1 to each of the artificial variables in the objective function.

So, here in the here this is in this method artificial variables are assigned the value - 1 each coefficient I mean of the artificial variable is taken as - 1 as a result the new objective function is of the form $Z^* = -$ sum of artificial variables because primary, slack and surplus variables are assigned value 0, ok 0 coefficients they are assigned but the artificial variables are assigned coefficient - 1.

So, when you write the new objective function say it is $Z^*$ then it will be - sum of artificial variables. Now, we maximize $Z^*$ subject to the constants of the original problem using the simplex method and then what we will have three cases will be there.

So, these cases are if maximum of $Z^i$ is less than 0, if maximum of $Z^*$ is less than 0 and at least one artificial variable appears in the optimal optimum basis at a positive level then the given problem does not possess any feasible solution. So, when maximum of $Z^* < 0$ and one at least one artificial variable appears in the optimum basis at a positively positive level means? The value corresponding value in the B column is positive then we do then the problem does not possess any feasible solution.

If maximum of $Z^* = 0$ at least and at least one artificial variable appears in the optimum basis at zero level, zero level means? In the B column the value corresponding to the optimum artificial variable is 0, then we will proceed to phase two and in the third case when maximum of $Z^*$ is 0 and no artificial variable appears in the optimum basis then also we proceed to phase 2.

So, we proceed to phase two in two cases the two cases are when the maximum value of $Z^*$ is 0 and one at least one artificial variable appears in the optimum basis at 0 level or in the second case maximum value of $Z^*$ is 0 and no artificial variable appears in the optimum basis. So, but in the third case when maximum of $Z^* < 0$ and at least one artificial variable appears in the optimum basis at a positive level then the problem does not possess any feasible solution.

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**Phase II**

We start with the optimal solution contained in the final simplex table of the phase I. Then we assign the actual costs to the variables in the objective function and a zero cost to every surplus variable. Now we eliminate the artificial variables which are non-basic at the end of the phase I. We remove $c_j$ row values of the optimum table and replace them by $c_j$ values of the original problem. Then we apply simplex algorithm to the problem contained in the new table to obtain the optimal solution.

So, now let us go to phase 2, in phase 2 we start with the optimal solution contained in the final simplex table of phase 1, ok. So, we will start with this, final simplex table of phase 1. The solution we got there that we will start with that, so then we assigned the actual cost to the variables in the objective function and a zero cost to every surplus variable. Now, we eliminate the artificial variables which are non-basic at the end of phase one, ok so they will be eliminated artificial variables.

Now, we removes c j row values of the optimum table and replace them with c j values of the original problem, ok then we apply simplex algorithm to the problem content in the new table to obtain the optimal solution.

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**Disadvantages of Big-M method over two-phase method**

1 We can always use Big-M method to check the existence of a feasible solution but its computational procedure may be inconvenient because of the manipulation of the constant M. Two phase method eliminates the artificial variables in the beginning.
2 When we solve a problem on a digital computer we have to assign some numerical value to M which must be larger than the values $c_1, c_2, ...,$ present in objective function. But a computer have only a fixed number of digits.

Now, what is the advantage of two-phase method over Big M method? What is the advantage of two-phase method over Big M method? We can always use Big M method to check the existence of a feasible solution, but it is computational procedure may be inconvenient because of the manipulation of the constant M. Two phase method eliminates the artificial variables in the beginning in the phase one itself the artificial variables are eliminated.

Now, when we solved the problem on a digital computer we have to assign some numerical value to M which must be larger than the values $c_1$, $c_2$ present in the objective function, but a computer can have a computer has only a fixed number of digits.

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**Example**

Solve the following problem by using two-phase method

$$\min Z = 7.5x_1 - 3x_2$$

subject to

$$3x_1 - x_2 - x_3 \geq 3$$
$$x_1 - x_2 + x_3 \geq 2, \quad x_1, x_2, x_3 \geq 0.$$

Now, let us solve the following problem by using two-phase method, so we have the problem minimum of Z = 7 point $5x_1$ - $3x_2$ subject to $3x_1$ - $x_2$ - $x_3 \geq 3$, $x_1$ - $x_2$ + $x_3 \geq 2$ ; $x_1$, $x_2$, $x_3 \geq 0$. So, let us see how we do this problem so first we will convert this minimum a minimization problem to maximization problem, ok.

**Sol.** Step 1: Express the problem in standard form.
Introducing the surplus variables $s_1$, $s_2$ and artificial variables $A_1$, $A_2$, the phase I problem in standard form becomes

$$MaxZ^* = 0x_1 + 0x_2 + 0x_3 + 0s_1 + 0s_2 - A_1 - A_2$$

subject to

$$3x_1 - x_2 - x_3 - s_1 + 0s_2 + A_1 + 0A_2 = 3$$
$$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

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So, step 1, express the problem in the standard form, ok. Introducing the surplus variables, now we see this is $\geq$ type this is $\geq$ type, so we will have to add 2, we will have to consider two surplus variables and also two artificial variables, ok. So introducing surplus variables $s_1$, $s_2$ artificial variables $A_1$, $A_2$ the phase one problem in a standard form becomes maximum $Z^*$ =, now we have 0 into $x_1$, 0 into $x_2$, 0 into $x_3$, 0 and x $s_1$, 0 into $s_2$ then $A_1$ and $A_2$ the coefficients of $A_1$ and $A_2$ are taken as - 1 each, so - $A_1$ - $A_2$ we have and subject to.

Now, the questions that we have by using surplus and artificial variables are $3x_1$ - $x_2$ - $x_3$ then -$s_1$ + 0 $s_2$ + $A_1$ + 0 $A_2$ = 3 $x_1$ - $x_2$ + s $x_3$ and then 0 $s_1$ - $s_2$ + 0 $A_1$ + $A_2$ = 2, $x_1$, $x_2$, $x_3$ $s_1$, $s_2$, $A_1$ , $A_2$ $\geq$0.

**Step 2:** Find an initial basic feasible solution.
Setting $x_1 = x_2 = x_3 = s_1 = s_2 = 0$, we have $A_1 = 3, A_2 = 2$ and $Z^* = -5$. Initial simplex table is



As $C_j$ is positive under $x_1$ column, this solution is not optimal.

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subject to

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$$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

Now, we find any initial basic feasible solution, so we have how many equations are there? We have two equations and the variables are 1, 2, 3, 4, 5, 6, 7, so we have seven variables and the equations are 3, so 5 variables can be taken as 0. So, we will take $x_1, x_2, x_3, s_1, s_2 = 0$ and we get the values of $A_1$ and $A_2$, ok $A_1$ comes out to be 3, ok from here and $A_2$ comes out to be 2 from here.

So, $x_1, x_2, x_3\ s_1, s_2$ and $A_1, A_2$ are all $\geq 0$ and Z* comes out to be - 5, Z* = at $x_1, x_2, x_3, s_1, s_2$ are zeros $A_1$ is 3, $A_2$ is 2, so Z* = 5. Initial simplex table will be now, you see $x_1, x_2, x_3, s_1, s_2$, $A_1, A_2$ b column, ok and then the basic variables are basic variables are $A_1, A_2$ non basic variables are $x_1, x_2, x_1$, s 3 because they are assigned value 0 here, 0 in the objective function,

ok they are assigned 0, 0, 0, 0, 0, 0, so we have 0, 0, 0, 0, 0 there are coefficients of $x_1$, $x_2$, $x_3$, $s_1$, $s_2$ in the objective function and coefficient of $A_1$, $A_2$ in the objective function are - 1 each.
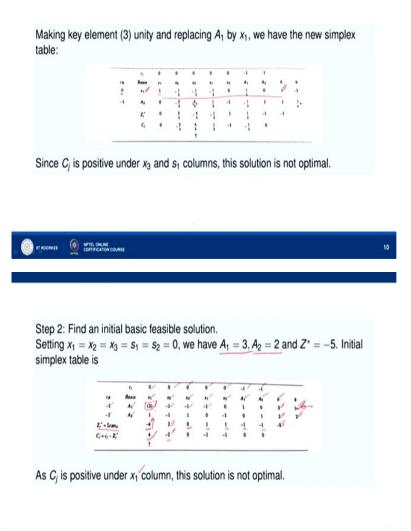
Now, coefficient of $A_1$ in the objective function is - 1, coefficient of $A_2$ in the objective function is - 1 and this is your body matrix, ok 3, - 1, - 1, - 1 yeah 3, - 1, - 1, - 1 -, 1, 1 - 4, 2, 0 - 4, 4, -, ok so this 1 these are the coefficients of $x_1$ 3, 1 (min) 3 and 1, ok the coefficients of $x_1$ are 3 & 1, coefficient of $x_2$ are - 1, - 1, coefficient of $x_3$ is - 1, + 1, ok and these are the coefficient of $s_1$, $s_2$ coefficient of $A_1$ and coefficient of $A_2$ and these are the elements in the b column 3 and 2, this one 3 and 2 this one, ok so these are coefficients in the b column.

Now, we find Z* c sigma c be $a_{ij}$ , so 3 (in) - 1 into 3 is -3, - 1 into 1 is - 1 so total is - 4, we subtract - 4 from 0 we get 4 here and then sigma, similarly - 1 into - 1 is + 1, - 1 into - 1 is + 1, 1 and 1 is 2, 2 we subtract from 0 we get - 2 and similarly we get $Zj^i$ here as 0, here $Zj^i$ is say star = 1, here 1, here - 1, here - 1 and here what we get? - 1 into 3 is -3 -, - 1 into 2 is - 2, so we got total - 5, ok.

Now, C j is = 4 here, ok C j is C j - $Zj^i$, so 0 - - 4 gets you gives you 4 and 0 - - 0 - 2 gives you - 2 here, this is 0, this is - 1, - 1, 0, 0. Now, we can see that C j is positive in this column, ok so C j is positive under $x_1$ column and therefore this solution is not optimal. So, what we will do? This is our key column, ok and we divide the elements of the b column by the corresponding elements of key column.

So, (si) 3 divided by 3 gives you 1, ok and then 2 divided by 1 gives you 2 the minimum of 1 and 2 is to 2 1, so this is our key row, ok and at the intersection of key row and key column we get key elements, so this is key element, 3 is the key element. Now, $x_1$ is the incoming variable, $A_1$ is the outgoing variable, so here we will get in the new simple new next simplex table $A_1$ between will be replaced by $x_1$ and this - 1 will be replaced by 0 the coefficient of $x_1$ , so we have this table, ok.

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Making key element (3) unity and replacing $A_1$ by $x_1$, we have the new simplex table:



Since $C_j$ is positive under $x_3$ and $s_1$ columns, this solution is not optimal.

Step 2: Find an initial basic feasible solution.
Setting $x_1 = x_2 = x_3 = s_1 = s_2 = 0$, we have $A_1 = 3, A_2 = 2$ and $Z^* = -5$. Initial simplex table is



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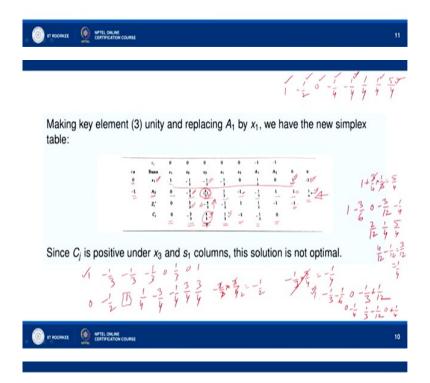The x1 comes here be, the coefficient in of $x_1$ is 0, 0 is either, ok and then what we do? We divide the elements of key row by the key element, ok so 3 by 3 we divide all the elements off key row, ok so we get 1 here, - 1 by 3, - 1 by 3, - 1 by 3, 0, 1 by 3, 0 and we get 1 here there ok, so we get this 1 -, 1 by 3, - 1 by 3, - 1 by 3, 0, 1 by 3, 0, 1, ok and then with the help of this row this new row, ok.

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Making key element $\frac{4}{3}$ unity and replacing $A_2$ by $x_3$, we obtain the revised simplex table:



Since all $C_j \leq 0$, this table gives the optimal solution. Also $Z^*_{max} = 0$ and no artificial variable appears in the basis. Thus an optimal basic feasible solution to auxiliary problem and therefore to the original problem, has been attained.

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$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

With the help of this new row we make the element this, this element at 0, ok so what we will do? We make this element 0 ok. So we have divided this row by 3 it became 1, so now this is also 1, so we simply have to subtract this new key row, a new row that we have got this row from the second row, ok so what we will get? When we have 1 - 1 we have this, ok 1 - 1, 1, 0, - 1, 0, 1 and 2 from that we subtract this row and then we get this 0 - 2 by 3, 4 by 3, 1 by 3, - 1, - 1 by 3, 1, 1 ok.

And then, now we calculate $Z_j$, $Z_j$ is 0 into 1, - 1 into 0 is 0, 0 - 0 is 0 then 0 into - 1 by 3, - 1 into - 2 by 3 is 2 by 3, 0 - 2 by 3 is - 2 by 3, so we get the new values of C j are 0, - 2 by 3, 4 by 3, 1 by 3, - 1, - 1 by 3, 0 ok and here $Z_j$ is - 1, now we can see C j is positive under $x_3$ column ok and $s_1$ column, ok.

So, maximum is 4 by 3 this is maximum, ok 4 by 3, so this is our key column, ok. So, now we find we have to find key row, so divide - 1 by 3, divide this 3 divide by 1 by - 1 by 3 we get - 3 and divide 1 by 4 by 3 we got 3 by 4 minimum positive value we have to consider here in the theta column, so minimum positive value is 3 by 4 so this is our key row, ok this key row and this is key column, so we have this as the key element, this is our key element, ok.

Now, so what will happen? Now $x_3$ will come in and $A_2$ will go out, so instead of $A_2$ now in the new term a table they will have $x_3$ here the value in the C B column will be 0, ok and what we will do then? This element of the key row will then be divided by the key element that is 4 by 3 to bring the unity here. So, we have this row this new table making key element 4 by 3 unity and replacing $A_2$ by $x_3$ we obtained it yeah, so this is our $x_3$ here, ok this is $x_3$, ok

so we have 0 in the C B column 0 for $x_1$, 0 for $x_3$ and we have here $A_1$ 8, ok so right, so what we have here?

Now, 4 by 3 we divided by 4 by 3 this elements of the key row, ok so - 2 by this will remain 0 this will become - 2 by 3 into 3 by 4, so we get - half, so this element will become - half, this element will become - half, this will become 1, this will become 1 by 3 into 3 by 4, 1 by 4, ok so you can see here - half, 1, 1 by 4 ok and this element will become what? - 3 by 4 ok - 3 by 4 and this element will become 1 into 3 by 4, so this will become 3 by 4, 3 by 4 ok so in this it will become 3 by 4 also this one 3 by 4 and this is - 1 by 4 in $A_1$ column, in $A_1$ column yeah - 1 by 3 ok divided by 4 by 3 so we get 3 by 4 so we get - 1 by 4.

So, new row that we get after dividing by 4 by 3 we get this like this, ok. Now, what we will do? This element we have made 1, ok by dividing by 4 by 3, ok now with the help of this one, ok we make this 0 and this 0, ok the other elements in the column corresponding to $x_3$ we make them as 0, so this is - 1 by 3 here we have 1, so we multiply this by 1 by 3 and (sub) add to this row, ok here it is - 4 by 3, here we have 1 by 3 it is 1, 1, so we multiplied by 4 by 3 and add to this, so we get this ok this is 0, this is 0, ok.

So, after that what happens is, ok with so this we have made as 1, here we have - 1 by 3, so we multiply the elements of the key row by 1 by 3 and add to the above row, so what we have? The above row is you see 1 - 1 by 3, - 1 by 3, ok - 1 by 3 and then we have 0, 1 by 3 and then we have 0, 1 this is 0, ok 1, - 1 by 3, - 1 by 3, - 1 by 3 0, 1 by 3, 0, 1.

Now, the row this row is what? 0 and what we have a - half, 1, 1 by 4, - 3 by 4, 1 by 4, - 3 by 4, - 1 by 4 (mi) and then 3 by 4, 3 by 4, ok. Now, what we do? With the help of this one we make this element 0, ok so we multiply this row by 1 by 3 and add to that row, ok this row, so what we will get? 1, 0 into 1 by 3 we add to this it will remain 1 then we multiply by 1 by 3 and add to that, so - 1 by 3 then we have - 1 by 6, ok.

Then 1 by 3 we add to this we get 0 here and then we multiply by 1 by 3 and add there, so - 1 by 3 + 1 by 12, ok that is what we get for this and then we multiplied by 1 by 3 and add there so we get 0 and - 3 by 4 into 1 by 3 so - 1 by 4 and what we get here? 1 by 3 we multiply by 1 by 3 and add so - 1 by 12, ok then here we multiply by 1 by 3 and add, so 0 and 1 by 4 ok and here we multiplied by 1 by 3 and add so what we will get? 1 + 3 by 4 into 1 by 3, so that gives you 5 by 4.

So, what will be the elements now? So this will remain 1 then - 1 by 3, - 1 by 6 so this is - 3 by 6, ok then we have 0 then here how much we have? - 4 by 12 so - 4 + 1, - 3 by 12, ok here we get - 1 by 4, here we get this is how much? 3 by 12, - 1 by 12 so 2 by 12 and then we get 1 by 4 and then we get 5 by 4, so what we get? 1, - 1 by 2, 0, - 1 by 4, - 1 by 4 then 1 by 6 then 1 by 4 then 5 by 4, ok so this is what we get?

Let us see, what we are getting there? 1, - half, 0, - 1 by 4, 1 - half, 0, - 1 by 4 right and then we get - 1 by 4, 1 by 4, 1 by 4, 1 - half, 0, - 1 by 4, 1 - half, 0 into this is there, this is there, this is there, this is there, then what is the next element? - 1 by 4, 1 by 4, 1 by 4, 1 by 4, 1 by 4, so this is this should be 1 by 4, ok and then last one is how much? 5 by 4, so this is ok only we have this element we have to see, so 1, 2, 3, 4, 5, 6 element so 1, 2, 3, 4, 5, 6, so here how much we have 1 by 3 - 1 by 12, so we get 4 by 12 - 1 by 12 so we get 3 by 12, so this is 1 by 4, so this 1 by 4 not 1 by 6, ok.

So, we get this table this this row, ok 1, 1 by, so we multiply it by 1 by 3 and add to the add to this row add to this row, ok we get this row and now let us find $Z j^i$, $Z j^i$ is 0, ok C j is 0, 0, 0, 0, 0 - 1, - 1 so C j is $\leq 0$ for all j and therefore this (sal) table gives us optimal solution and $Z^*$, maximum up $Z^*$ is how much? this $Z^*$, ok so maximum $Z^*$ is also 0 because $x_1$ is $x_1$ is = 5 by 4 $x_3$ is = 3 by 4, ok all other variables are non-basic variables so that means $x_2$ 0 then we have $s_1$ 0, $s_2$ 0 and $A_1$, $A_2$ 0.

So, in now maximum value of $Z^*$, Z in $Z^*$ you take $x_1$ as 5 by 4, $x_3$ as 3 by 4, $x_2$, $s_1$, $s_2$ $A_1$, $A_2$ = 0 what you get is? 0 the value of $Z^*$ has 0. Now, no artificial variable appears in the basis, ok at no artificial variable because the basis elements are $x_1$ and $x_3$, so no artificial variable appears in the basis that element that and therefore an optimal basic feasible solution to auxiliary problem and therefore to the original problem has been attend because we have seen that when no artificial variable appears in the basis and $Z^*$ is 0 then an optimal basic feasible solution to the auxiliary problem and therefore to the original problem is obtained.

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Phase II. Consider the actual costs associated with the original variables, the objective function is

$$\text{Max} Z' = -7.5x_1 + 3x_2 + 0x_3 + 0s_1 + 0s_2 - 0A_1 - 0A_2$$

subject to

$$3x_1 - x_2 - x_3 - s_1 + 0s_2 + A_1 + 0A_2 = 3,$$
$$x_1 - x_2 + x_3 + 0s_1 - s_2 + 0A_1 + A_2 = 2,$$

$$x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$$

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Solve the following problem by using two-phase method

$$\min Z = 7.5x_1 - 3x_2$$

subject to

$$3x_1 - x_2 - x_3 \geq 3$$
$$x_1 - x_2 + x_3 \geq 2, \quad x_1, x_2, x_3 \geq 0.$$

Now, let us consider phase 2, in phase to consider the actual cost associated with the original variables and the objective function is then maximum of Z dash, now Z dash is - Z, ok. So because we have the problem of minimization, ok so Z dash is - sub Z, so - of 7.5 $x_1 + 3x_2$ we consider, so maximum value of - Z we are take, so maximum value of Z dash is maximum of - 7.5, 7.5 $x_1 + 7 \, 3x_2 + 0x_3 + 0s_1 + 0s_2 + 0A_1 + 0A_2$ are subject to these constants $3x_1$ - $x_2$ - $x_3$ - $s_1$ + $0s_2 + A_1 + 0A_2 = 3$ and then $x_1$ - $x_2 + x_3$, $0s_1$ - $s_2 + 0A_1 + A_2 = 2$ and $x_1, x_2, x_3, s_1, s_2, A_1, A_2 \geq 0$.

The optimal initial feasible solution thus obtained, will be an optimal basic feasible solution to the original L. P. P.

Using final table of phase I, the initial simplex table of phase II is as follows:

| $c_j$ | | $-15/2$ | $3$ | $0$ | $0$ | $0$ | |
|---|---|---|---|---|---|---|---|
| $c_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $b$ |
| $-15/2$ | $x_1$ | $1$ | $-1/2$ | $0$ | $-1/4$ | $-1/4$ | $5/4$ |
| $0$ | $x_3$ | $0$ | $-1/2$ | $1$ | $1/4$ | $-3/4$ | $3/4$ |
| $Z_j$ | | $-15/2$ | $15/4$ | $0$ | $15/8$ | $15/8$ | $-75/8$ |
| $C_j$ | | $0$ | $-3/4$ | $0$ | $-15/8$ | $-15/8$ | |

Since all $C_j \leq 0$, this solution is optimal.
Hence an optimal basic feasible solution to the given problem is

$$x_1 = \frac{5}{4}, x_2 = 0, x_3 = \frac{3}{4} \text{ and min } .Z = \frac{75}{8}.$$

$$\min Z = -\max Z'$$
$$= -\left(-\frac{75}{8}\right) = \frac{75}{8}$$

Making key element $\frac{4}{3}$ unity and replacing $A_2$ by $x_3$, we obtain the revised simplex table:

| $c_j$ | | $0$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $-1$ | |
|---|---|---|---|---|---|---|---|---|---|
| $c_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $A_1$ | $A_2$ | $b$ |
| $0$ | $x_1$ | $1$ | $-\frac{1}{4}$ | $0$ | $-\frac{1}{4}$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{5}{4}$ |
| $0$ | $x_3$ | $0$ | $-\frac{1}{4}$ | $1$ | $\frac{1}{4}$ | $-\frac{1}{4}$ | $-\frac{1}{4}$ | $\frac{1}{4}$ | $\frac{3}{4}$ |
| $Z_j^*$ | | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ | $0$ |
| $C_j$ | | $0$ | $0$ | $0$ | $0$ | $0$ | $-1$ | $-1$ | |

$x_2 \leq 0$
$s_1 \geq 0$
$s_2 \geq 0$
$A_1 \geq 0$
$A_2 \geq 0$

Since all $C_j \leq 0$, this table gives the optimal solution. Also $Z_{max}^* = 0$ and no artificial variable appears in the basis. Thus an optimal basic feasible solution to auxiliary problem and therefore to the original problem, has been attained.

Now, the optimal initial basic feasible solution, ok thus obtained will be an optimal basic feasible solution to the original L.P.P. So what we will do? The final table of phase 1 will become now the initial simplex table of phase 2, ok. So basis is $x_1$, $x_3$ we can see $x_1$, $x_3$ is basis, ok and the coefficient of $x_1$ now, coefficient of $x_1$ is - 15 by 2, ok so we write there - 15 by 2, coefficient of $x_3$ here is 0, so we write 0 here ok and then we have 1 - half the same last table one - half 0, - 1 by 4, - 1 by 4.

So, we just copy that, you see here we just copy that and then here $x_1$, coefficient of $x_1$ is - 15 by 2, coefficient of $x_2$ is 3, coefficient of $x_3$ is 0, coefficient of $s_1$ is 0, coefficient of $s_2$ is 0, $A_1$, $A_2$ have been eliminated, ok. Now, we find Z j here, ok Z j is - 15 by 2 into 1 + 0 into 0, so - 15 by 2, capital C j therefore is small c j - Z j gives you 0 and here we get - 15 by 2 into -

1 by 2 + 0 into - half gives $Z_j$ as $s_1$ 5 by 4 then 3 - 15 by 4 is - 3 by 4, here $C_j$ comes out to be 0, here - 15 by 8, here it is - 15 by 8 and here what we get? - 15 by 2 into 5 by 8 + 0 into 3 by 4 that is - 75 by 8.

Now, let us see $C_j$ is 0 here, here negative, here 0, here negative, here negative, so $C_j \leq 0$ and therefore this solution is optimal and so an optimal basic feasible solution to the given problem is $x_1 = 5$ by 4, $x_3 = 3$ by 4, $x_2 = 0$ because $x_2$ is non-basic variable, so $x_2 = 0$ and minimum value of Z, ok.

Now, (max) this is - 75 by 8, so minimum value of $Z = -$ maximum value of your Z dash, ok which is $-(-75)$ by 8 and we get 75 by 8, so that is the minimum value of Z. So, there is solution to the linear programming problem, so that is all in this lecture, thank you very much for your attention.