Higher Engineering Mathematics
Professor P. N. Agrawal
Department of Mathematics
Indian Institute of Technology Roorkee
Lecture - 47
Big M Method – II (Special Cases)

Hello friends, welcome to my second lecture on Big M Method, here we shall discuss some special cases. Say for example, let us consider the following problem we shall see that this problem has no feasible solution.

So, let us consider the L.P.P. maximize Z = - x_1 - x_2 such that $3x_1 + 2x_2 \geq 30$, - $2x_1 + 3x_2 \leq -30$, $x_1 + x_2 \leq 5$ and x_1 and x_2 are non-negative.

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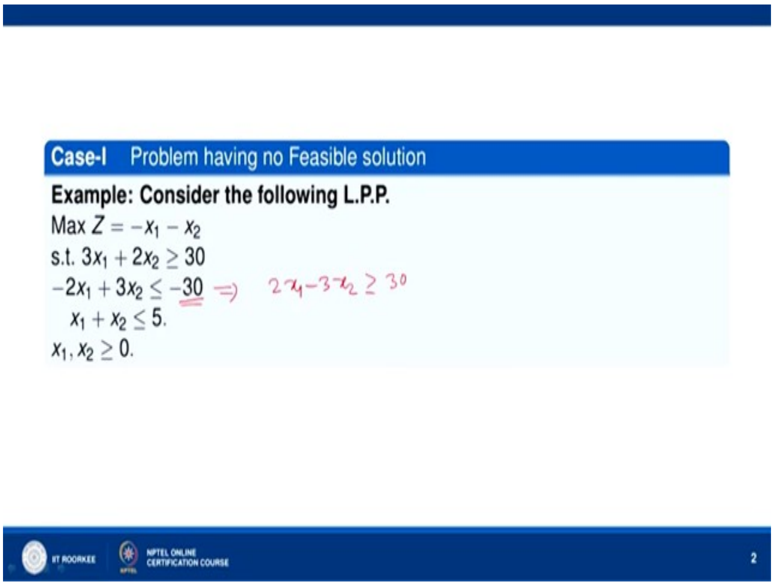

Now, what we will do in order to make the right hand side in the second constraint positive? Here, it is - 30, so in order to make it positive we multiply this in equality by - 1 and so this reduces to $2x_1$ - $3x_2 \geq 30$, ok. So, we in order to make this right hand side of the second constraint + positive value we multiplied by - 1.

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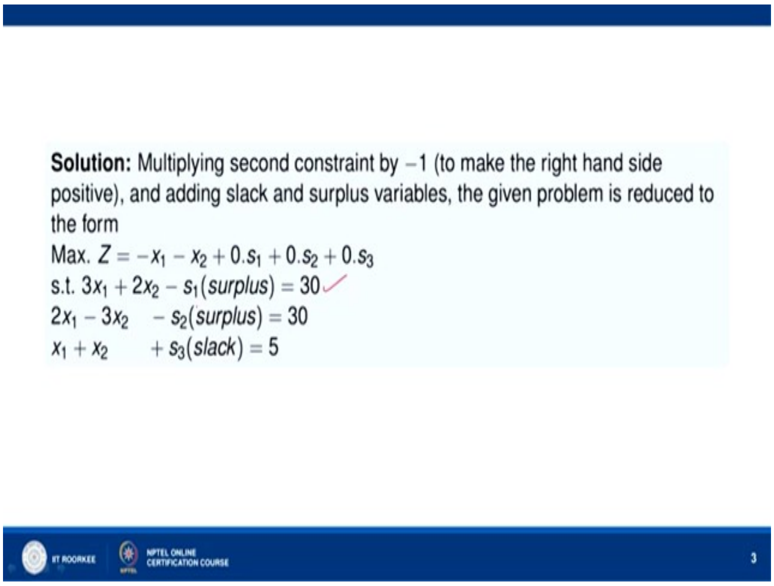

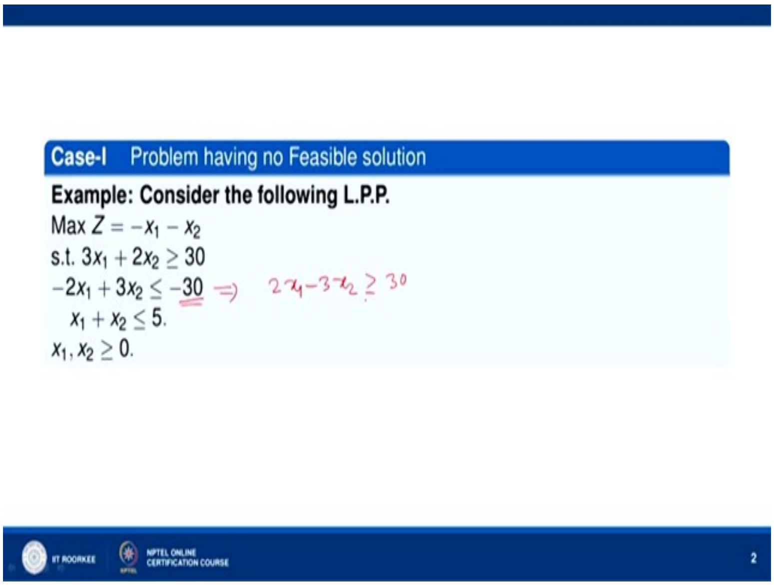

So, we have multiplying the second constraint by - 1 to make the right hand side positive and adding slack and surplus variables, so here what we will do? We have here $3x_1 + 2x_2 \geq 30$, so we add, so we subtract surplus variable here and here this is $2x_1 - 3x_2 \geq 30$, here also we subtract surplus variable and here we have $x_1 + x_2 \leq 5$.

So, we add slack variable to convert it into equality and what we will have then? Maximum of Z = $- x_1 - x_2$, ok $- x_1$, $- x_2 \geq 0$ into $s_1 \geq 0$ into $s_2 \geq 0$ into s_3 where s_1 is surplus variable (in the first equation) in the constraint we need surplus variable. So, we take surplus variable as s_1, so we get $3x_1 + 2x_2 - s_1 = 30$ and then in the second constraint also we have constraint $\geq$ type, so we use surplus variable s_2.

So, $2x_1$ - $3x_2$ - s_2 = 30 and third constraint is ≤ type x_1 + x_2 ≤ 5 we have, so we use slack variable s_3. So, we have x_1 + x_2 + s_3 = 5, so we need three variables s_1, s_2, s_3; two variables s_1, s_2 are surplus variables and the third variable s_3 is a slack variable, ok. Now, let us write the corresponding B.F.S. here, so we have here 1, 2, 3, 4, 5 variables, five variables are there, ok and the equations are three.

So, we can take any two variables as zeros, so taking x_1 = 0, x_2 = 0, ok what we get? s_1 = - 30, and here we get s_2 = - 30, and here we get s_3 = 5, so s_1, s_2 are coming out to be negative but the variables must all be non-negative in L.P.P. so we use 2 artificial variables A_1 and A_2, ok. So, we write the using two artificial variables in order to get the initial starting B.F.S. ok we write the given equations in the following form.

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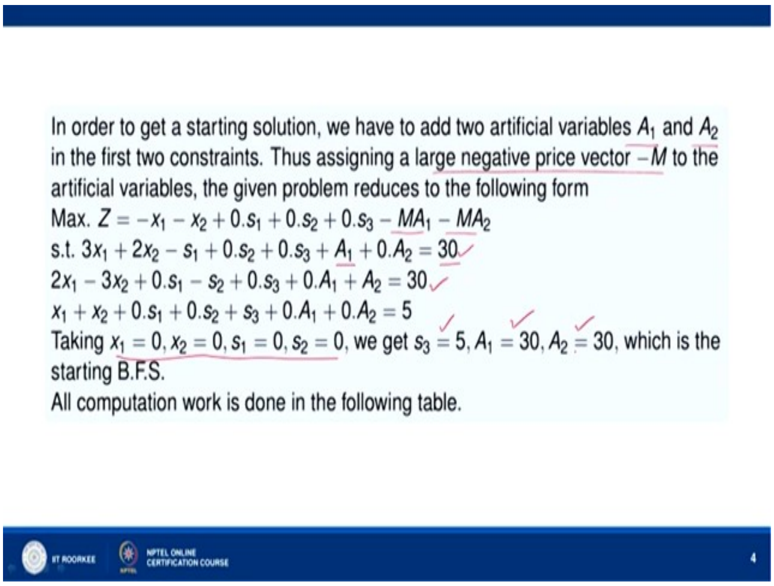

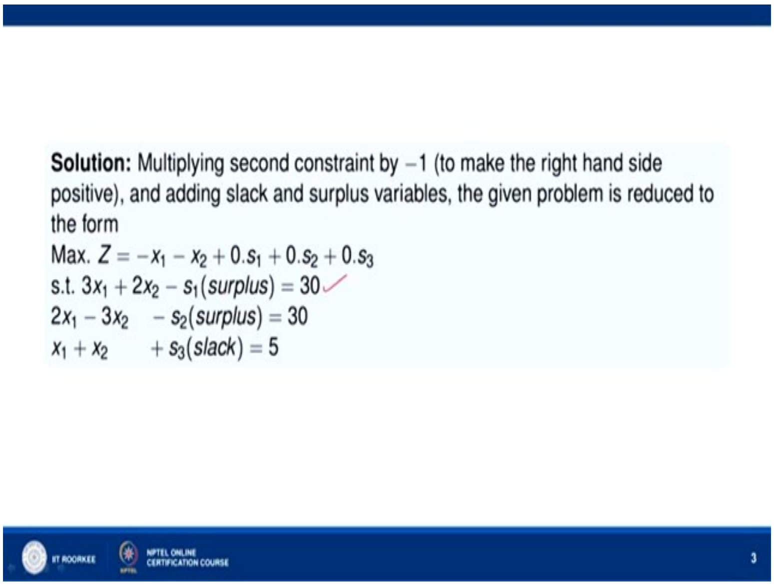

So, in order to get the starting B.F.S. we have to get two artificial variables A_1 and A_2 in the first two constraint because in the third constraint when you take x_1, x_2 = 0, s_3 is = 5. So, we do not have to do this is anything here s_3 is already in a non-negative but in the case of first and second equations s_1, s_2 are coming out to be negative, so we use artificial variables.

So, let us say artificial variables are A_1 and A_2 then we write the L.P.P. in the following form, ok maximum of Z = - x_1 - x_2+ 0s_1 +0s_2+ 0s_3 and then we use the negative price vector - M for the artificial variables. So, - MA_1 - MA_2 and here we get 3x_1 + 2x_2 - s_1 + 0s_2 + 0 s_3 + A_1 + 0 A_2 = 30. The second equation is 3x (min) 2x - 3x_2 + 0s_1 - s_2 + 0s_3 + (0s_1) 0A_1 + A_2 = 30 and then third equation is x_1 + x_2, 0s_1, 0s_2, then s_3, 0A_1 + 0A_2 = 5, ok.

Now, we have 1, 2, 3, 4, 5, 6, 7, ok seven constraints are there and the equations are three, ok so we take four constraints, four variables as zeros. So, taking x_1 = 0, x_2 = 0, s_1 = 0, s_2 = 0, taking x_1 = 0, x_2 = 0, s_1 = 0, s_2 = 0 the first equation gives us A_1 = 30, ok so A_1 = 30. Second equation gives us what? We gave we get from the second equation x_1, x_2 0 s_1, s_2 0 and we get A_2 = 30, ok.

So, we get A_2 = 30 and third equation gives x_1, x_2 0, s_1, s_2 0, so we get s_3 = 5, so we get s_3 = 5. So, thus we see that x_1, x_2, s_1, s_2, s_3, A_1, A_2 are all non-negative and therefore it can be taken as a starting basic feasible solution.

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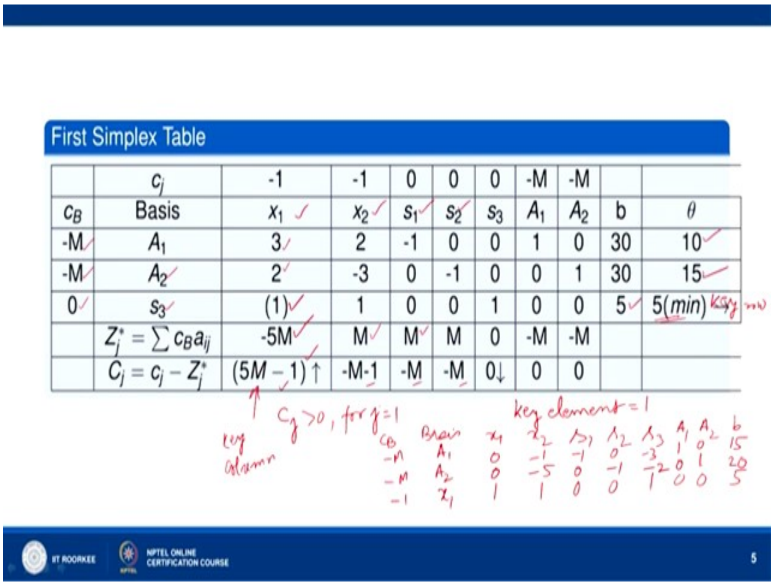
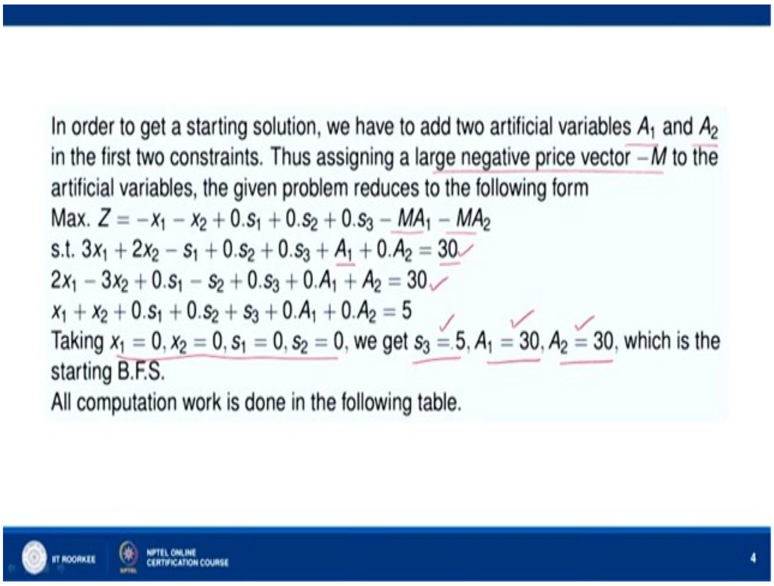


	c_j	-1	-1	0	0	0	-M	-M		
c_B	Basis	x_1	x_2	s_1	s_2	s_3	A_1	A_2	b	θ
-M	A_1	3	2	-1	0	0	1	0	30	10
-M	A_2	2	-3	0	-1	0	0	1	30	15
0	s_3	(1)	1	0	0	1	0	0	5	5(min)
	$Z_j^* = \sum c_B a_{ij}$	-5M	M	M	M	0	-M	-M		
	$C_j = c_j - Z_j^*$	$(5M-1)\uparrow$	-M-1	-M	-M	$0\downarrow$	0	0		



Now, let is consider First Simplex Table. So, cj's are coefficients of the variables in the objective function, so coefficients of x_1 is - 1, coefficient of x_2 is - 1, coefficient of s_1 is 0, coefficient of s_2 is 0, coefficient of s_3 is 0, coefficient of A_1 is - M, coefficient of A_2 is - M, ok basic vectors are A_1, A_2, s_3 and the (non-basic variables are) basic variables are A_1, A_2, A 3 these are A_1, A_2, A 3; A_1, A_2, A 3 these are basis variables and non-basic variables are x_1, x_2 s_1, s_2, s_3, ok x_1, x_2, s_1, s_2, ok x_1, x_2, s_1, s_2 they are non-basic variables, these are non-basic variables, ok.

Now, in the initial simplex table the x_1 column contains the coefficient of x_1 here, so coefficient of x_1 are 3, 2, 1, ok we get 3, 2, 1 here, coefficient of x_2 are 2, - 3, 1 so we get 2, -

3, 1 here, coefficient of s_1 is - 1, 0, 0, ok we get - 1, 0, 0 coefficient of s_2 is 0, - 1, 0 so we get 0, - 1, 0 coefficient of s_3 is 0, 0, 1, ok we get 0, 0, 1 and then coefficient of A_1 is 1, 0, 0 coefficient of A_2 is 0, 1, 0, ok so we get this and b column is the right hand side of this constraints, so 3, 30, 30, 5, ok that is b column, ok.

Now, the coefficient of A_1 in the objective function is - M, coefficient of A_2 in the objective function is - M and coefficient of s_3 is 0, so C B column contains the coefficients of the basic variables in the objective function. Now, we then find Z j star, Z j star = sigma C B a ij, so we multiply C B column to the column x_1 and get Z j (sta) Z 1 star, this is j = 1 this is first column, so j = 1 so Z 1 star will be - 3M - 2M and 0 into 1 is 0, so we get total - 5M, ok and C j is (sma) capital C j is small c j - Z j star, so from this C j value we subtract Z j star and we what it get is? 5M - 1.

Similarly, we can find (in) Z j star for the second column, so we had - M into 2 - 2M, - M into - 3 is 3M, 3M - 2M is M, ok 0 into 1 is 0, so Z j star is M. So, then C 2 capital C 2 is small c 2 - Z 2 star, so - 1 - M, so we get - M - 1. Similarly, in the column corresponding to s_1 (we) it comes out, we get Z j, Z 3 star = M and therefore C capital C 3 is 0 - M, so we get - M here.

In the column s_2, Z j star which is Z 4, j = 4 Z 4 star is M and so C 4 capital C 4 is - M and in the (c) s_3 column Z j star j = 5 is 0 and so 0 - 0 is 0, C 5 is 0 and in the column for A_1 we get Z j star = - M, so - M - - M becomes 0 and here for A_2 also Z j star is - M, so - M - - M = 0, so we get this.

Now, so 5M - 1, ok this is a positive value, this is negative, this is negative, this is negative, ok C j is greater than 0 for j = 1, ok so this is key column, this is key column let us find key row.. So divide the elements in the b column, ok by the corresponding elements in the x_1 column, so (6) 30 by 3, 30 by 3 gives you 10, then 30 by 2 gives you 15, then 5 divided by 1 gives you 5, so we get 5, ok.

Now, we have to find the minimum positive ratio here, so clearly 5 is the minimum positive ratio, so this is key row. Now, key row at the intersection of key row and the key column we get the key element, so this is our key element, key element is 1. Now, with the so, that means that s_3 will be outgoing variable, ok and x_1 will be incoming variable, so in the next simplex table basis vectors will be basis will be cover the basic variables will be A_1, A_2, s_1, s_3 will replaced by x_1, ok coefficient of A_1 is - M in the objective function, so - M here, coefficient of A_2 is - M here, coefficient of x_1 is - 1, so we put - 1 here and then we have x_1, x_2, s_1, s_2, s_3 A_1, A_2, b, ok.

Now, the key element, the key element is 1 if it is not 1, we will divide it so that it becomes 1 but it is already 1, ok so what we will do? With the help of this key row we will be making the other elements in the x_1 column 0 that is 3 and 2 we have to make 0. So, let us first write this key row we have 1, 1, 0, 0, 1, 0, 0, ok and we have 5 here, ok. Now, we multiply by 3 this key row and subtract from this first row this row, so we get 0 there, ok and we multiply it by 3.

So, 3 we subtract from 2 we get - 1 and we 3 we multiply to 0 and subtract from - 1, so it remains - 1, 3 into 0 is 0, we 2 0 - 0 is 0, then 3 into 1 is 3, so we get - 3 there and then we get 3 into 0 is 0 so we get 1 here, 3 into 0 is 0 so we get 0 here, ok we are multiplying by 3 and subtracting, so 5 into 3 is 15, 15 when we subtract from 30 we get 15, ok.

Now, second row we multiply key this row by 2 and subtract from this row, so we get 0 here, ok we are multiplying this row by 2 and subtracting from here, so 2 - 2 is 0 then we (sub) multiply by 2 and subtract from here, so - 3 - 2 is - 5, ok. Then, we are multiplying by 2 and subtracting, so 2 into 0 is 0, 0 - 0 is 0 then we 2 into 0 is 0, so it remains - 1, 2 into 1 is 2.

So, we are subtracting so - 2 there, ok and then 2 into 0 is 0, so it is remains 0, 2 into 0 is 0 so it remains 1 and then 2 into 5 is 10, 10 one when we subtract from 30 we get 20, ok so this is the next simplex table we can see.

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Second Simplex Table

	c_j	-1	-1	0	0	0	-M	-M		
c_B	Basis	x_1	x_2	s_1	s_2	s_3	A_1	A_2	b	θ
-M	A_1	0	-1	-1	0	-3	1	0	15	
-M	A_2	0	-5	0	-1	-2	0	1	20	
-1	x_1	1	1	0	0	1	0	0	5	
	$Z_j^* = \sum c_B a_{ij}$	-1	6M-1	M	M	5M-1	-M	-M		
	$C_j = c_j - Z_j^*$	0	-6M	-M	-M	-5M	0	0		

First Simplex Table

	c_j	-1	-1	0	0	0	-M	-M		
c_B	Basis	x_1	x_2	s_1	s_2	s_3	A_1	A_2	b	θ
-M	A_1	3	2	-1	0	0	1	0	30	10
-M	A_2	2	-3	0	-1	0	0	1	30	15
0	s_3	(1)	1	0	0	1	0	0	5	5(min) key row
	$Z_j^* = \sum c_B a_{ij}$	-5M	M	M	M	0	-M	-M		
	$C_j = c_j - Z_j^*$	$(5M-1)\uparrow$	-M-1	-M	-M	0↓	0	0		

key column; $C_j > 0$, for $j = 1$; key element = 1

C B is - M, - M, - 1 in the C B column, in the basis we have A_1, A_2 A_1, A_2, x_1 we can see A_1, A_2 x_1, x_1 column is 0, 0, 1; x_1 column is 0, 0, 1 x_2 column is - 1, - 5, 1; - 1, - 5, 1, s_3 column is, sorry s_1 column is - 1, 0, 0 we get - 1, 0, 0, s_2 column is 0, - 1, 0 0, - 1, 0 s_3 column - 3, - 2, 1 this is - 3, - 2, 1, ok A_1 column is 1, 0, 0 1, 0, 0 A_2 column 0, 1, 0 so we get 1, 0, 0, 1, 0, 1, 0 and b column is 15, 20, 5, ok so we get 15, 20, 5, alright.

Now, we can find Z j star, so - M into 0 is 0, - M into 0 is 0, - 1 into 1 is - 1, so we get - 1, - 1 when we subtract from - 1 we get 0 and here we get - M into - 1 so + M, - M into - 5M so + 5M, 5M and M becomes 6 M, 6M - 1 we get. So, when we subtract 6M - 1 from - 1 we get - 6M, ok and then similarly in the s_1 column Z j star becomes M, s_2 column Z j star becomes

M, s_3 column Z j star becomes 5M - 1, in the A_1 column it is - M, A_2 column it is - M, ok and the corresponding capital C j values are - M, - M, - 5M, 0, 0.

Now, you can see that all C j values are less than (zer0) or = 0, this is 0, this is - 6M, so ≤0 less than or, so all C j values are ≤0.

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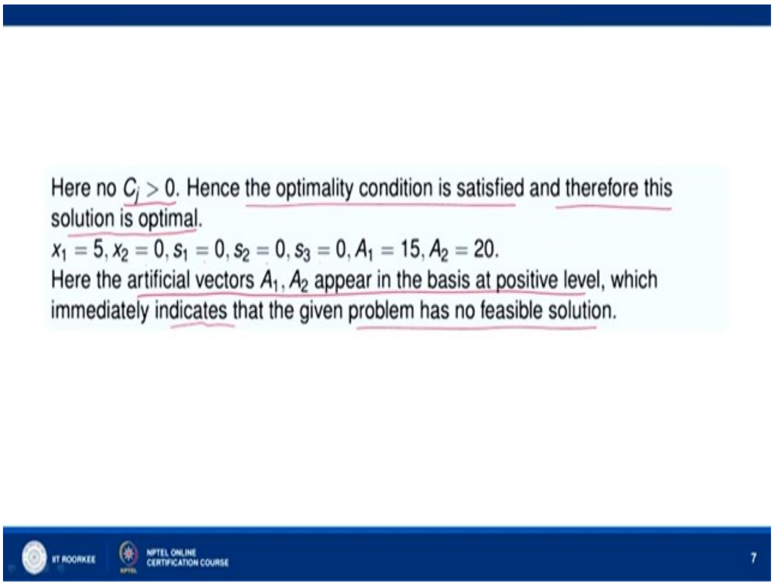

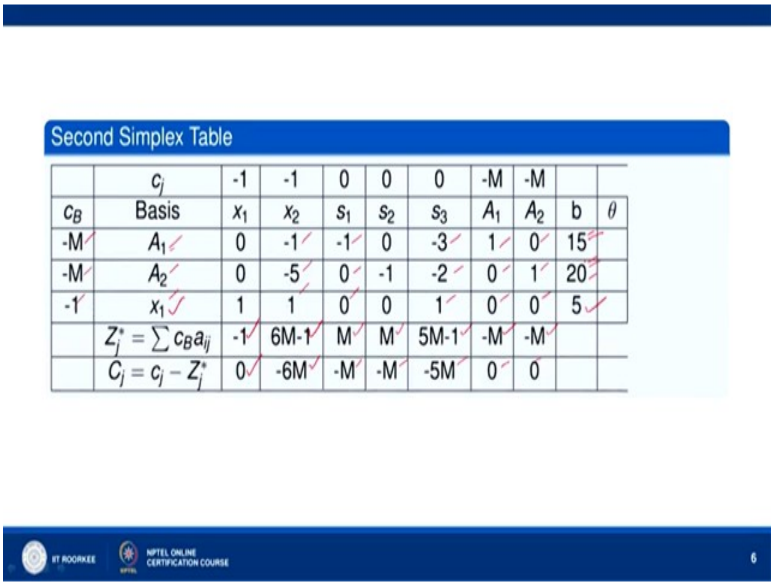
Second Simplex Table

	c_j	-1	-1	0	0	0	-M	-M		
c_B	Basis	x_1	x_2	s_1	s_2	s_3	A_1	A_2	b	θ
-M	A_1	0	-1	-1	0	-3	1	0	15	
-M	A_2	0	-5	0	-1	-2	0	1	20	
-1	x_1	1	1	0	0	1	0	0	5	
	$Z_j^* = \sum c_B a_{ij}$	-1	6M-1	M	M	5M-1	-M	-M		
	$C_j = c_j - Z_j^*$	0	-6M	-M	-M	-5M	0	0		

So, what we have? Here no C j is positive hence the optimality condition is satisfied and therefore this solution is optimal, ok and so x_1 is = 5, we can see x_1 is = 5, ok x_2 = 0, s_1 = 0, s_2 = 0, s_3 = 0 and A_1 = 15, A_2 = 20, ok. Now, so artificial vectors A_1, A_2 appear in the basis at positive level we can see A_1, A_2 occur at positive level, ok there b values are positive, so they are occur at the positive level which indicates that the given problem has no feasible solution, ok.

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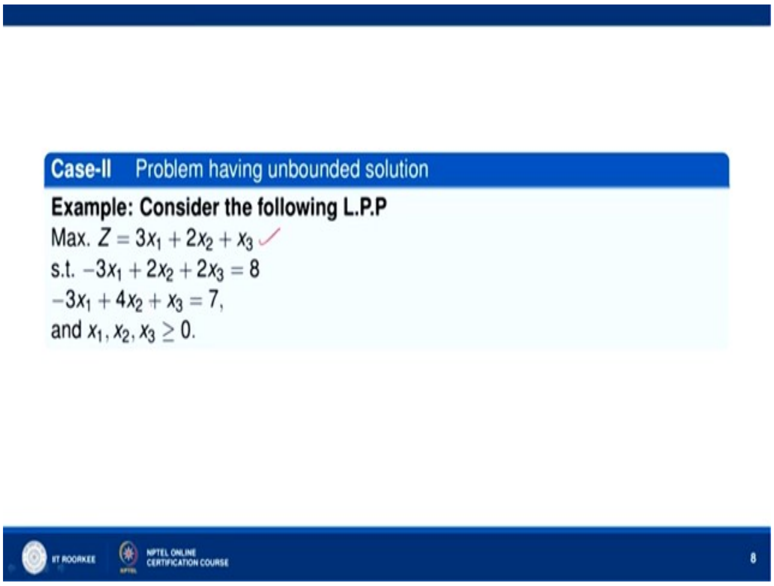

Now, let us go to problem two, consider the following L.P.P. here we will have unbounded solution, so maximum of Z we have to find Z = $3x_1 + 2x_2 + x_3$and we are given the constraints - $3x_1 + 2x_2 + 2x_3 = 8$, - $3x_1 + 4x_2 + x_3 = 7$; x_1, x_2, x_3 are non-negative.

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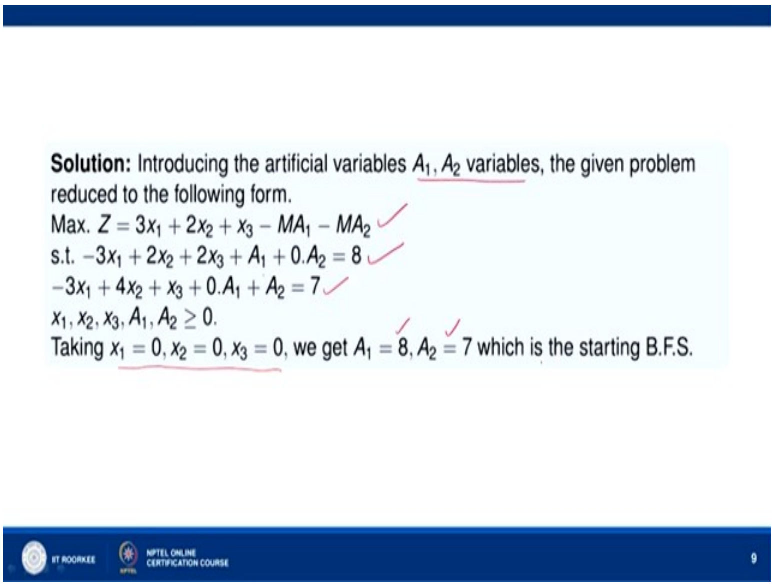

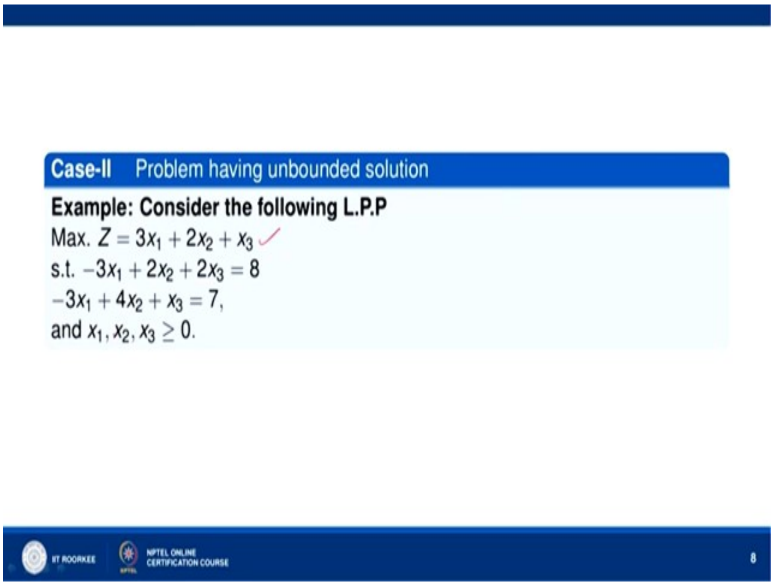

So, we will use to select two artificial variables A_1, A_2 to convert this form to the standard form, so Z = $3x_1 + 2x_2 + x_3$ - MA_1 - MA_2, ok and we have - $3x_1 + 2x_2 + 2x_3$ and then we have $A_1 + 0A_2 = 8$, ok and - $3x_1 + 4x_2 + x_3$+¿ $0A_1 + A_2 = 7$, ok so x_1, x_2, x_3 greater than or (eq) 0 and A_1, $A_2 \geq 0$.

Now, we have two equations and 5 constraints, 5 variables, ok x_1, x_2, x_3, A_1, A_2, so we take any 3 variables = 0, so let us take the variables x_1, x_2, x_3 = 0 each, ok we get the value of A_1 as 8 and the value of A_2 as 7, so this is our starting bsf B.F.S., ok $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $A_1 = 8$, $A_2 = 7$ this is our starting basic feasible solution.

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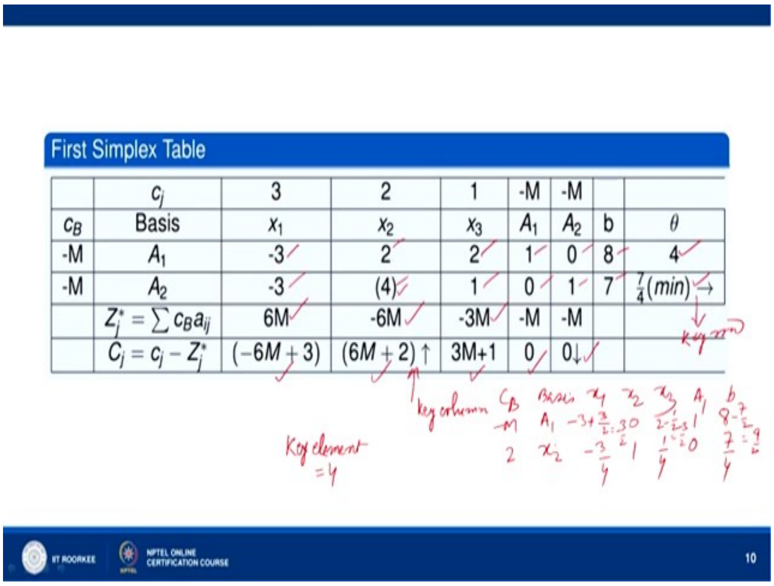

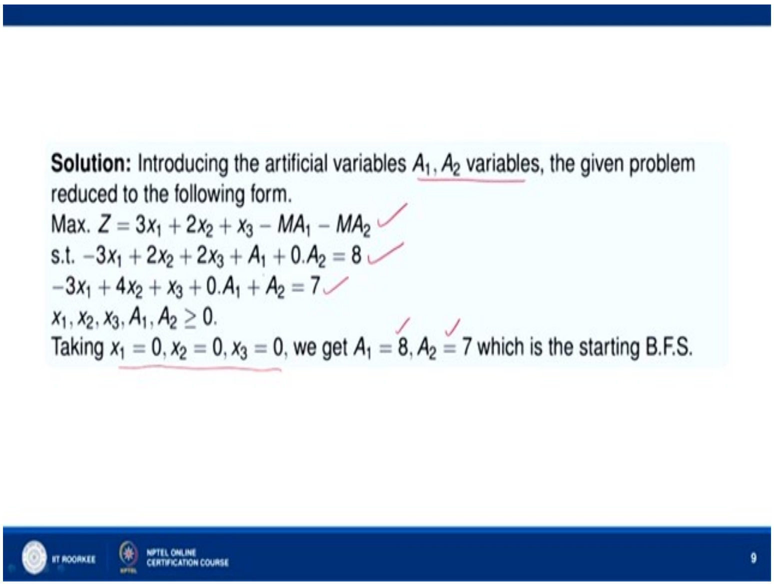

Now, let us write the first simplex table, so C j the coefficients of x_1, x_2, x_3 A_1, A_2 they are given here 3, 2, 1, - M, - M, ok x_1, x_2, x_3 are our non-basic variables A_1, A_2 are basic variables, so A_1, A_2 occur here and their coefficient in objective function are - M each, so we put them they are here and the coefficients in the x_1 column, x_1 column the coefficients are - 3, - 3 in the x_2 column we have 2, 4.

So, - 3 - 3 2, 4 and then we get x_3 column coefficients are 2, 1, ok so 2, 1 here A_1 coefficient is 1 here it is 0, so 1, 0 and then A_2 column has 0, 1, ok so 1, 0 0, 1 and b column is the equation I mean the right hand side of the equations, equation at 8 and 7, so we get 8 and 7 here.

Now, we can find Z j star, we can find Z j star so Z j star is = for j = 1, so - M into - 3 3M this is 3M, 3M + 3M is 6M, so 3 - 6M is here, ok. Similarly for the x_2 column - 2M - 4M so - 6M we get 6M + 2 that is c 2, ok capital C 2. Now - M into 2 - 2M, - M into 1 - M, so we get - 3M and then C 3 capital C 3 is 3M + 1 and similarly C 4 is 0, C 5 is 0.

Now, we can see C j is positive in the second column and C j is positive in the third column, ok but 6M + 2 is greater than 3M + 1, ok so this is our key column, ok divide the elements of the b column by the corresponding values in the key column, so (si) 8 by 2 is 4 and 7 by 4 is here, ok for this 7 divided by 4 we get 7 by 4, so out of this 4 and 7 by 4, 7 by 4 by 7 by 4 is clearly minimum, so this is our key row, ok at the intersection of key row and key column we get 4, so key element is 4, ok.

Now, so this means what? A_2 will be outgoing variable, x_2 will be incoming variable and, ok so we have C B basis, in basis we get basis (vec) basis (var) basic variable size A_1, x_2 will be replace by A_2 will be replace by x_2, coefficient of A_1 is - M, coefficient of x_2 is 2, ok and we get then x_1, x_2, x_3; A_2 column will be deleted. In the next simplex table x_1, x_2, x_3, A_1 and then we have b, so ok.

Now, what we will divide? What we will do? We will divide by key element all the elements of the key row, ok so we get - 3 by 4, ok 1 we get 1 by 4, we get 0, we get 7 by 4. Now, having made this key element 1 unity, ok we then subtract suitable multiples of this from the other rows to make the other entries in the key column zeros, ok. So, we have to make this two zero, ok that means we multiplied by 2 and subtract from the first row, so what we will get?

When you multiplied by 2 it becomes - 3 by 2, so - 3 by 2 we have to subtract from - 3, so we get - 3 + 3 by 2, ok and then we have multiplying it by 2, ok and subtracting from here, so we get 0 here, we are multiplying it by 2 so 1 by 2, 1 by 2 we subtract from 2 so 2 - 1 by 2 and then we are multiplying it by 2 and subtracting from the first row so we get 1 here and we are multiplying it by 2 so 7 by 2 we have to subtract from 8, so 8 - 7 by 2.

So, what we will have? This is - 3 + 3 by 2 it is - 3 by 2, ok and here we get 2 - half that is 3 by 2 and here we get 16 - 7, so 9 by 2. So let us see in the new simplex table we should have these values, ok.

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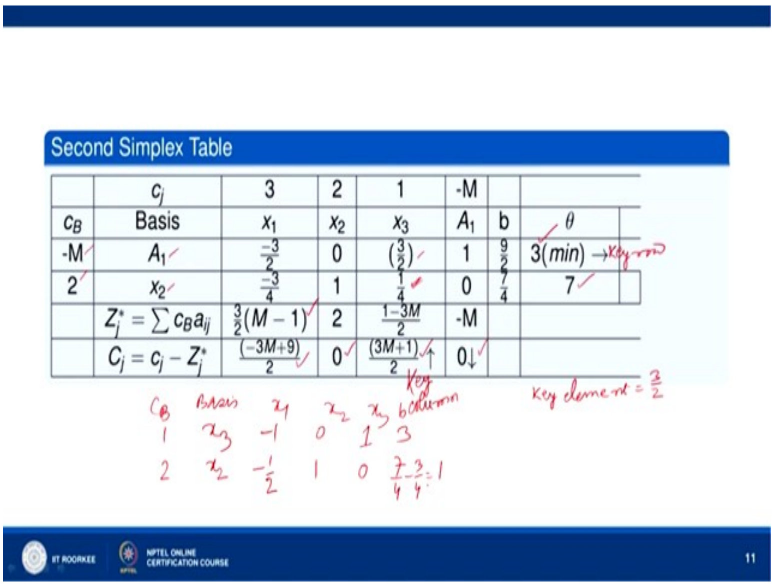
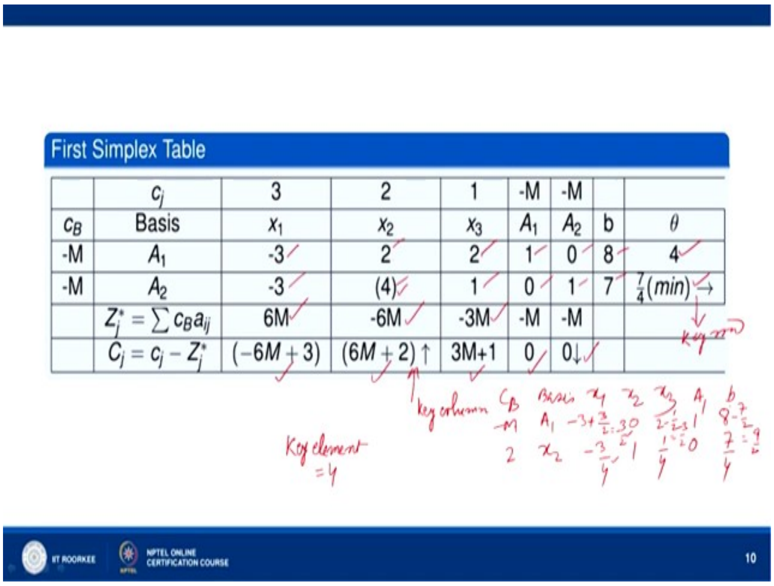


	c_j	3	2	1	-M		
c_B	Basis	x_1	x_2	x_3	A_1	b	θ
-M	A_1	$\frac{-3}{2}$	0	$(\frac{3}{2})$	1	$\frac{9}{2}$	3(min) → Key row
2	x_2	$\frac{-3}{4}$	1	$\frac{1}{4}$	0	$\frac{7}{4}$	7
	$Z_j^* = \sum c_B a_{ij}$	$\frac{3}{2}(M-1)$	2	$\frac{1-3M}{2}$	-M		
	$C_j = c_j - Z_j^*$	$\frac{(-3M+9)}{2}$	0	$\frac{(3M+1)}{2}$ ↑	0↓		



First Simplex Table

	c_j	3	2	1	-M	-M		
c_B	Basis	x_1	x_2	x_3	A_1	A_2	b	θ
-M	A_1	-3	2	2	1	0	8	4
-M	A_2	-3	(4)	1	0	1	7	$\frac{7}{4}(min) \rightarrow$
	$Z_j^* = \sum c_B a_{ij}$	6M	-6M	-3M	-M	-M		
	$C_j = c_j - Z_j^*$	$(-6M+3)$	$(6M+2)\uparrow$	3M+1	0	0↓		

So, C B column you can see, see contents C B column is - M, 2, ok - M, 2 basis variables are A_1, x_2. So, you can see A_1, x_2 x_1 column we have - 3 by 2, - 3 by 4, so - 3 by 2, - 3 by 4, x_2 column 0, 1 x_3 column 3 by 2, 1 by 4 3 by 2, 1 by 4 A_1 column 1, 0 and b column, b column is 9 by 2, 7 by 4 so we get 9 by 2 and 7 by 4, ok. Now, we go to determining the values of Z j star for each j, ok.

So, you can see here - 3 M into - 3 by 2 is 3 by 2 3M by 2 and this is two times - 3 by 2, so - 3 by 2, ok so total is 3M by 2 - 3 by 2 and when we subtract this value from 3 we get - 3M + 9 by 2, ok. And similarly in this is for j = 1, for j = 2 we get c 2 small c 2 is 2 and Z 2 star, Z 2 star is 2 because - M 1 2 0 is 0, 2 into 1 2 is 2 so we get 0 here, C 3 capital C 3 is 3M + 1 by 2 and capital C 4 is 0, now clearly 3M + 1 by 2 is positive, ok so this is our key column, ok.

Now, let us find key row? So divide the b (valu) b column values by the column values of x_3, so 9 by 2 divided by 3 by 2 so we get 3, ok 7 by 4 divided by 1 by 4 gives you 7, so minimum positive ratio is 3, so this is our key row, ok. So this key element is 3 by 2. Now, we have to so and after we have located the key element which variable will be going out or A_1 will be outgoing variable, ok and x_3 will be incoming variable.

So, in the new simplex table we will have C B column, the basis we will have (bas) vector the variables x, A_1 will be replaced by x_3, ok so x_3, x_2, coefficient of x_3 is 1, coefficient of x_2 is 2, ok and x_1, x_2, x_3 A_1 will also be going out, so one column will be deleted and so we have x_1, x_2, x_3 and b, ok. Now we divide the key row by 3 by 2 by the key element, ok so dividing by 3 by 2 it becomes - 1, 0, 1 and 1 over 3 by 2 means though this A_1 column is not there 3 by 2 we 9 by 2 divided 3 by 2 gives you 3, ok and then after we have made this element key element 1 we have to make the element 1 by 4, 0 ok.

So, we (div) multiplied by 1 by 4 and subtract from second row, so this becomes 0 and we are multiplying by 1 by 4 subtracting from second row, so - 3 by 4 and + 1 by 4 so - 2 by 4 so we get - half and then we multiplied by 1 by 4 subtract from here, so with this remains 1, ok we multiplied by 1 by 4 so we get 3 by 4, 3 by 4 we subtract from 4 7 by 4 and we get 1, ok.

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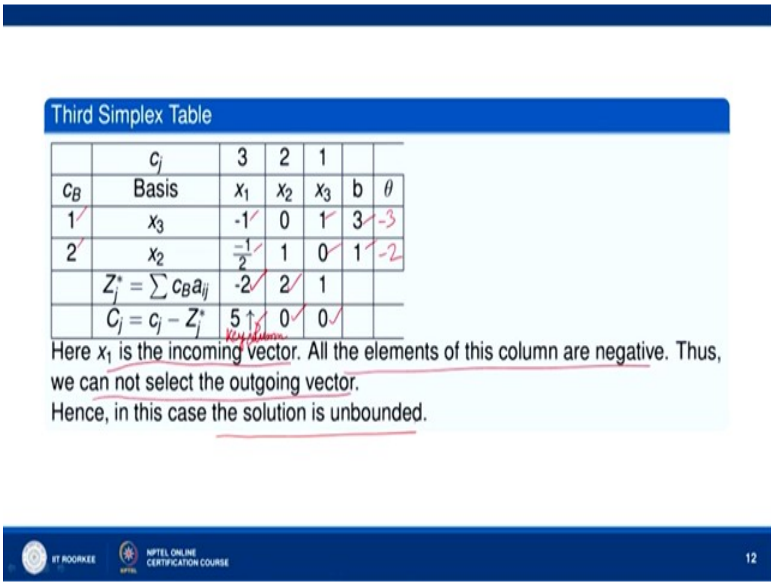
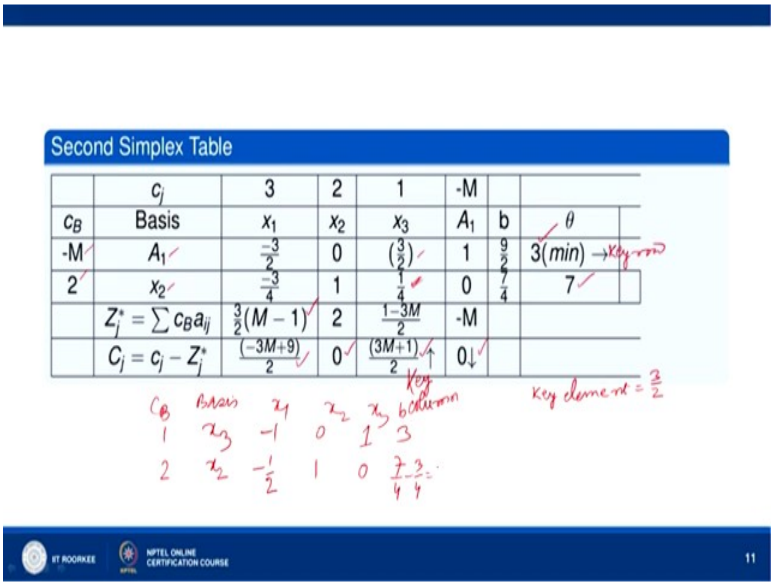

Third Simplex Table

	c_j	3	2	1		
c_B	Basis	x_1	x_2	x_3	b	θ
1	x_3	-1	0	1	3	-3
2	x_2	$\frac{-1}{2}$	1	0	1	-2
	$Z_j^* = \sum c_B a_{ij}$	-2	2	1		
	$C_j = c_j - Z_j^*$	5 ↑	0	0		

Here x_1 is the incoming vector. All the elements of this column are negative. Thus, we can not select the outgoing vector.
Hence, in this case the solution is unbounded.


Second Simplex Table

	c_j	3	2	1	-M		
c_B	Basis	x_1	x_2	x_3	A_1	b	θ
-M	A_1	$\frac{-3}{2}$	0	$(\frac{3}{2})$	1	$\frac{9}{2}$	3(min) → Key row
2	x_2	$\frac{-3}{4}$	1	$\frac{1}{4}$	0	$\frac{7}{4}$	7
	$Z_j^* = \sum c_B a_{ij}$	$\frac{3}{2}(M-1)$	2	$\frac{1-3M}{2}$	-M		
	$C_j = c_j - Z_j^*$	$\frac{(-3M+9)}{2}$	0	$\frac{(3M+1)}{2}$ ↑	0↓		

So, let us see in the new simplex table we have C B 1, 2 C B column 1, 2 basis column has x_3, x_2 so x_3, x_2 x_1 column has - 1, - half so - 1, - half x_2 column 0, 1 0, 1 x_3 column 1, 0 1, 0 x 4 column 3, 1 so 3, 1, ok so we get 3, 1 here. Now let us find Z j star, so 1 into - 1 - 1, 2 into -

half - 1, - 1 - 1 - 2, ok 3 - 2 is 5, ok and here we get 1 into 0 0, 2 into 1 2 so we get 2, 2 - 2 is 0 and then 1 into 1 is 1, 2 into 0 is 0, so 1 we subtract from 1, ok so we get 0 here, ok Z j star is 1, ok.

And then what we notice? C j is positive in for j = 1, so this is our key column, ok so x_1 is incoming vector, x_1 is incoming vector, ok but we cannot select the outgoing vector because x_1 column has all elements negative all the elements of the x_1 column are negative which is the incoming vector thus we cannot select the outgoing vector because to select the outgoing vector we find the minimum positive ratio and what we will have here? this is key column, so we divide the elements of b column by the elements of key column, so 3 divided by - 1 this will be - 3, 1 divided by - half will be - 2, ok so no positive ratio is there, theta is negative in the first row as well as in the second row. So we cannot find the outgoing vector and therefore the solution is unbounded.

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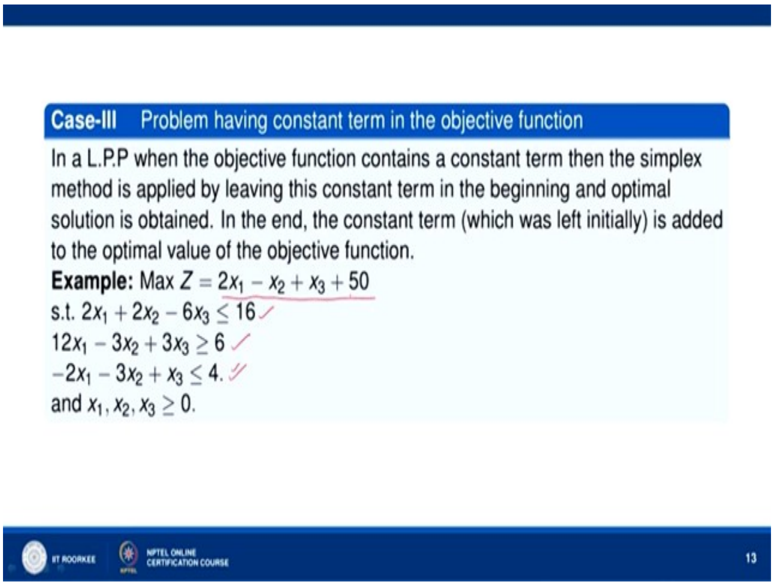

Now, let us consider the case where the problem contents constant term in the objective function. So in L.P.P. when the objective functions contains a constant term then the simplex method is applied by leaving this constant term in the beginning and optimal solution is obtained. In the end, the constant term which was left initially is added to the optimal value of the objective function.

So, let us say we have to maximize $2x_1$ - x_2 + x_3 + 50 then with subject to the constraints $2x_1$ + $2x_2$ - $6x_3$ ≤16, $12x_1$ - $3x_2$ + $3x_3$ ≥6, - $2x_1$ - $3x_2$ + x_3 ≤5, x_1, x_2, x_3 non negative then what we will do? We will consider the corresponding problem where we will drop this 50, we will simply consider the objective function $2x_1$ - x_2 + x_3 with these constraints and find the optimal solution and in the optimal value of Z we will add 50.

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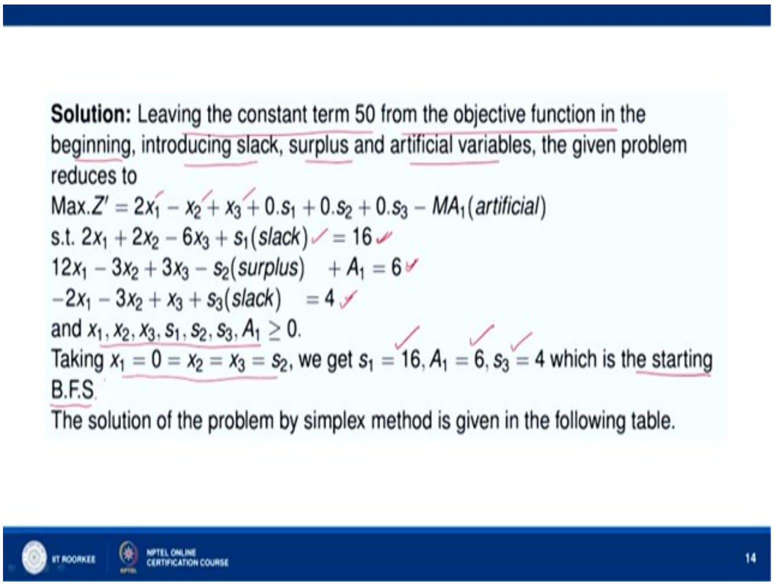

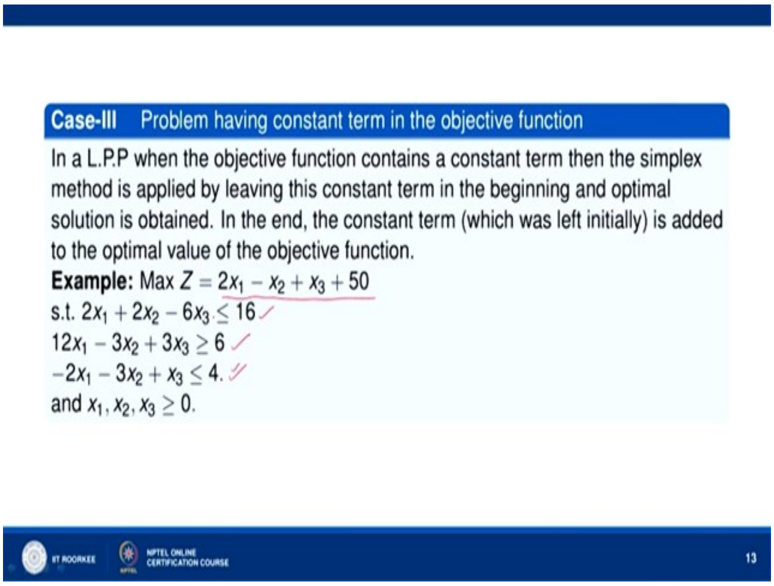

So, let us see we have leaving the constant term 50 from the objective function in the beginning, we introducing slack, surplus and artificial variables, the given problem reduces to maximum of Z dash, we will call the new Z as Z dash because we are dropping 50. So, maximum of Z dash $2x_1$ - x_2 + x_3, ok we have $2x_1$ - x_2 + x_3 and then you see first equation is first constraint is $2x_1$ + $2x_2$ - $6x_3 \leq$ 16, so we use a slack variable s_1 here, ok so $2x_1$ + $2x_2$ - 6 x_3, ok + s_1, s $s_1$1 is a slack variable = 16.

Second equation is ≥ type, so we use a surplus variable, so $12x_1 - 3x_2 + 3x_3$, ok - s_2 and then we will have to add a artificial variable able artificial variable A_1 in order to get the starting B.F.S. so artificial variable A_1 we add and we give a large penalty - M to this, so - MA_1, ok. And then third equation is $- 2x_1 - 3x_2 + x_3$ we use a slack variable here s_3, ok so $- 2x_1 - 3x_2 + x_3 + s_3 = 4$.

Now, let us take x_1, x_2, x_3 s_1, s_2 where x_1, x_2, x_3 s_1, s_2, s_3 A_1 are all non-negative, we have three equations here, ok and we have 6 7 variables, ok 7 variables so we take four variables as zeros, ok. So $x_1 = 0$, $x_2 = 0$, $x_3 = 0$ and $s_2 = 0$, ok because if you instead of s_2 you take $A_1 = 0$ we will get $s_2 = - 6$, ok so $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, $s_2 = 0$ we take, ok then what we will get? s_1 will come out to be 16, A_1 will be 6 and then s_3 will be = 4.

So, x_1, x_2, x_3, s_1, s_2, s_3 and A_1 (A) all are non-negative, ok A_1 are all non-negative so we get the starting basic feasible solution, ok. The solution of the problem by simplex method is given in the (())(32:18) in the following table.

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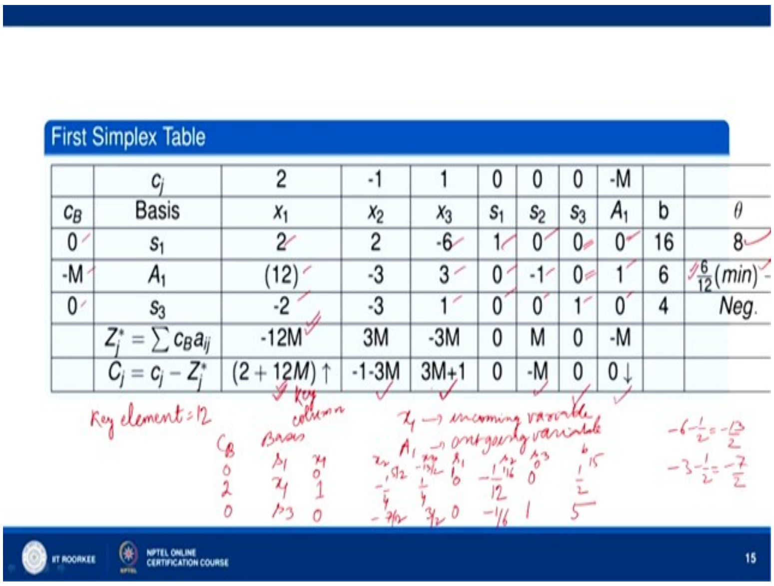
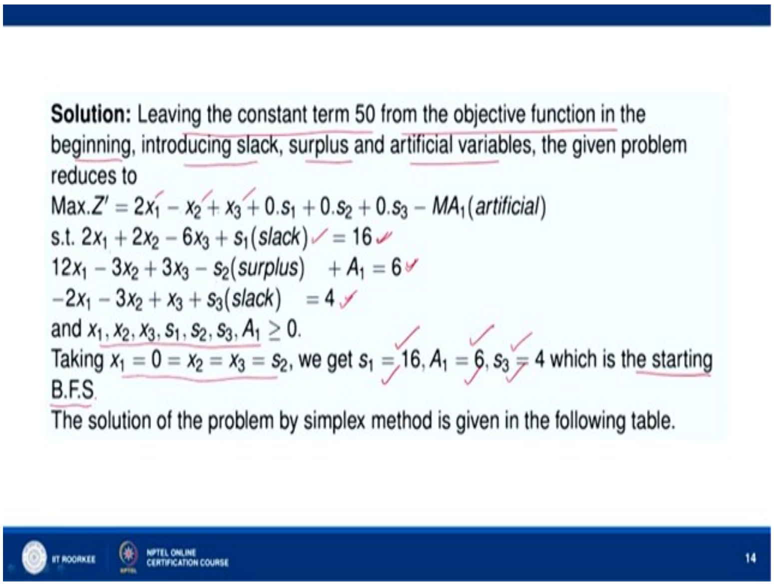


First Simplex Table

	c_j	2	-1	1	0	0	0	-M		
c_B	Basis	x_1	x_2	x_3	s_1	s_2	s_3	A_1	b	θ
0	s_1	2	2	-6	1	0	0	0	16	8
-M	A_1	(12)	-3	3	0	-1	0	1	6	$\frac{6}{12}$ (min)
0	s_3	-2	-3	1	0	0	1	0	4	Neg.
	$Z_j^* = \sum c_B a_{ij}$	-12M	3M	-3M	0	M	0	-M		
	$C_j = c_j - Z_j^*$	$(2 + 12M) \uparrow$	-1-3M	3M+1	0	-M	0	0 ↓		



Solution: Leaving the constant term 50 from the objective function in the beginning, introducing slack, surplus and artificial variables, the given problem reduces to

Max. $Z' = 2x_1 - x_2 + x_3 + 0.s_1 + 0.s_2 + 0.s_3 - MA_1$ (*artificial*)

s.t. $2x_1 + 2x_2 - 6x_3 + s_1$ (*slack*) $= 16$

$12x_1 - 3x_2 + 3x_3 - s_2$ (*surplus*) $+ A_1 = 6$

$-2x_1 - 3x_2 + x_3 + s_3$ (*slack*) $= 4$

and $x_1, x_2, x_3, s_1, s_2, s_3, A_1 \geq 0$.

Taking $x_1 = 0 = x_2 = x_3 = s_2$, we get $s_1 = 16, A_1 = 6, s_3 = 4$ which is the starting B.F.S.

The solution of the problem by simplex method is given in the following table.

So, c j we have c j 2, - 1, 1, ok 2, - 1, 1, 0, 0, 0, - M basis vectors are s_1, A_1 basis basic variables are s_1, A_1, s_3; s_1, A_1, s_3 they are basic variables, ok non-basic variables are x_1, x_2, x_3 , ok x_1, x_2, x_3 and s_2, ok they are non-basic variables and the coefficients of s_1, A_1, s_3 in the objective function are 0, - M and 0, ok in the x_1 column we get 2, 12, - 2 2, 12 - 2 x_2 column has coefficient of x_2 in the constraints.

So, 2, - 3, 2, - 3, - 3 x_3 column - 6, 3, 1 - 6, 3, 1 s_1 column 1 there is no s_1 here so 0, 1, 0 there is no s_1 here so 1, 0, 0, ok so 1, 0, 0 s_2 column 0, - 1, 0, ok so 0, - 1, 0 s_3 column 0, 0, 1, ok we get 0, 0, 1, ok A_1 column 0, 1 and then we get 0 so 0, 1, 0, ok as b column 16, 6, 4 so we get 16, 6, 4.

Now, we can find Z j star for j = 1, so 0 into 2 is 0 - M into 12 is - 12M and 0 into - 2 is 0 so we get - 12M here, we subtract this Z 1 star from C 1, so 2 + 12M we get. Similarly C 2 capital C 2 comes out to be - 1, - 3M and C 3 comes out to be 1 + 3M and then it is 0 this is 0 - M so - M here, 0 - 0 is 0 here and then we get - M - - M that is - M + M that is 0, ok.

Now, we can see the M is very large, so 2 + 12M is the maximum C j which is positive, of course C j is positive in this column also and the so C j is positive for j = 3 and j = 1 but C 1 value is more than C 3 value, so we will add this is our key column, ok divide so this means that x_1 will be incoming variable. Let us now find (out) outgoing variable, so divide the elements of b column by the elements of key column.

So, 16 divided by 2 is 8, 6 divided by 12 is half, ok this half and then (mul) 4 divided by - 2 is - 2, so we have to consider minimum positive ratio, minimum positive ratio is this half, ok so this is our key element intersection of key row and key column gives us the key element, so key element is 12, ok. Now so this means A_1 will be outgoing variable x_1 is incoming variable and A_1 is outgoing variable and in the next simplex table A_1 will not be there, the A_1 column will not be there, ok.

What we do now? So new simplex table will be C B, ok 0, (minu) A_1 is going out, A_1 is replaced by x_1, so ok. So basis so s_1 A_1 is replaced by x_1, coefficient of $x_1$is 2 here, ok and then we have s_3, s_3 coefficient is 0 here, ok and we add x_1, so x_1 is now divide the elements of key row, ok by the key element, so this means when you divide the key row by 12 the key element 12 then what is we will get? 1, - 3 by 12.

So, - 1 by 4 and then 3 by 12, so 1 by 4 and then we get 0 and then we get - 1 by 12, ok this is - 1 by 12 and then this is 0, ok A_1 column will not be there in the new simplex table, so 6 divided by 12 so that is half, ok this is the x_1, x_2, x_3, ok x_1, x_2, x_3 and then we get s_1, s_2, ok and then we get s_3 and we get here b vector, ok b column.

So, now we will have we will make with the help of this key element change to 1 will make this 0, ok first row in the this element 2, 0 in the column (condu corre) corresponding to x_1, so we multiplied by 2 subtract from there so we get 0 here, we multiplied by 2 we get - half, - half we subtract from there so we get 2 + half, so 5 by 2 and we get multiply by 2 we get 1 by 2, 1 by 2 we subtract from - 6 so we get - 13 by 2, so we get here (min) x in the x_3 column we get - 13 by 2, ok.

And then s_1 column, ok we are multiplying by 2, ok after dividing by 12, so we have we add 2 come here, ok so we so this will remain 1, this will remain 1, we multiplied by 2 we get - 1 by 6 which we subtract from this 0 so we get 1 by 6 here, ok and then we get s_3 in the s_3 column we have 0 here b multiplied by 2 subtract from 0 we get 0 here, we multiplied by 2 we get 1 when we subtract from 16 we get 15, ok and then this - 2 also has to be made 0, so we multiplied by 2, ok we multiplied by 2 and added to the third row, so this becomes 0, we multiplied by 2 so we get - half, - half we add 2 - 3, ok so - 3 - half we get - 7 by 2, so we get - 7 by 2.

And we will have to now go to (thi) x_3, x_3 means we are multiplying by 2 and adding, ok adding to this row, ok so here we have 1 here we have what? 1 by 4, so 1 by 4 we multiply by 2 1 by 2 we get 1 by 2 we add to 1 we get 3 by 2, ok 1 we multiply here by 2 this is 0 when we multiply by 2 so we get 0, 0 we add to 0 and we get 0 here then we multiplied by 2 here we get - 1 by 6, - 1 by 6 we add to 0 we get - 1 by 6 here and in the s_3 column we have 0 we multiplied by 2 and add 2 to 0 so we get 1 here and here we get 1 by 2 we multiply by 2 we get 1, 1 we add to 4 we get 5.

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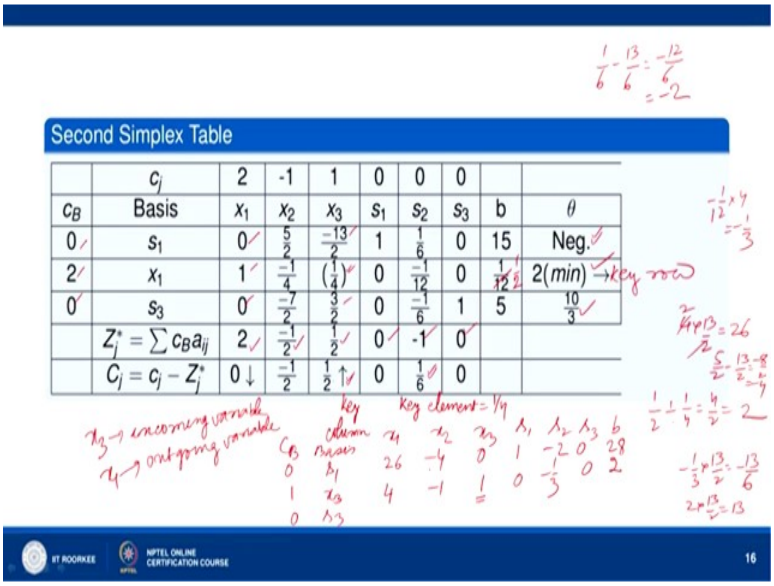

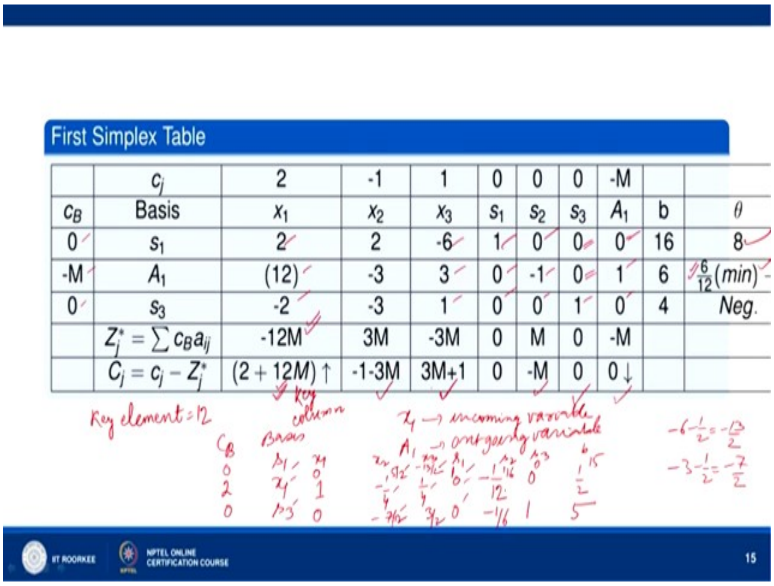

So, let us see in the new simplex table C B column has 0, 2, 0 so we get 0, 2, 0 s_1 column basis (variab) variables are s_1, x_1, s_3 so s_1, x_1, s_3, ok then the x_1 column has 0, 1, 0 values so we get 0, 1, 0 values x_2 column 5 by 2, - 1 by 4, - 7 by 2, ok so we see 5 by 2, - 1 by 4, - 7 by 2, ok so we match this x_2 column elements and then x_3 column - 13 by 2, 1 by 4, - 13 by 2, 1 by 4 and then 3 by 2, ok this, this, this.

And then s_1 column 1, 0, 0 we get 1, 0, 0 s_2 column 1 by 6 - 1 by 12, ok and - 1 by 6, so they also match and then 0, 0, 1 in the s_3 column, ok b column has 15, 1 by 12 this should be 6 divided by 12 this should be 1 by 2, ok so here 1 by 2 it should be, it should be 1 by 2, ok. Now, alright and then the, ok so and this one is 5, this one is 5, ok so they match now. Now, we can see we find Z j star, Z j star is 2 - half, half, 0, - 1, 0, ok and C j values are then 2 - 2 is 0, so 0 - half half, 0, 1 by 6, 0 there are two positive values 1 by 6, 1 by 2 of the capital C j, so 1 by 2 is greater than in the two, so this is our key column, ok.

Now, let us so this means x_3 is incoming variable, so x_3 is incoming variable let us find the outgoing variable. So we divide the elements of the b column by the values in the column x_3, so 15 divided by - 13 by 2 so we get a negative value of theta and then 1 by 2 divided by 1 by 4, ok so 1 by 2 divided by 1 by 4 is = 4 by 2 we get 2, so we get 2 here, ok and what we get? (fif) 5 divided by 3 by 2 so we get 10 by 3, so there are two positive values of theta 2 and 13, 10 by 3, so 2 is minimum so this is our key row and then add the intersection of key row and the key column we get the element 1 by 4, so key element is 1 by 4, ok 1 by 4.

Now, we divide, so this means what will happen x_3 is incoming variable, x_1 is outgoing variable, ok so now the C B column will be having 0 basis variables will be s_1, x_1 will be replaced by x_3 and we will have s_3, x_3 coefficient is 1, s_3 coefficient is 0, ok and we have x_1, x_2, x_3 s_1,s_2, s_3 and b, ok. The elements of the key row, key row are divided by key element so we divide by 1 by 4, so this when we 1 is divided by 1 by 4 we get 4, ok so we get 4 here 1 - 1 by 4 divided by 1 by 4 gives you - 1 this is 1 and this is 0 divided by 1 by 4 is 0, - 1 by 12 divided by 1 by 4.

So, 4 by - 1 by 12 divided by 1 by 4, so we get - 1 by 3, so we get here - 1 by 3 and we get 0 divided by 1 by 4 is 0 and we divide 1 by 2 divided by 1 by 4, so we get 2 here, ok. Now we have made this element 1, ok with the help of this 1, we make the elements of the element - 13 by 2 0, ok so we multiplied by 13 by 2, ok and add to the elements of the first row, so 13 by 2 4 into 13 by 2 gives you 26, ok so 13 by 2 we have multiplying to the to 1 in order to make this 0 13 by 2 multiplying 13 by 2 into 4 is 26, so 26 we add to this one 0 so we get 26.

Then, we have - 1, - 1 into 13 by 2 so - 13 by 2, - 13 by 2 we add to 5 by 2, so - 8 by 2 so we get - 4 so we get - 4 here, x_3 this is 0, ok then we multiplied by 13 by 2 and add to the first row, right. So s_1 here it is 0 when we multiplying 13 by 2 this will remain 1, ok - 1 by 3 we multiply by 13 by 2, so - 1 by 3 into 13 by 2 so what we get is? - 13 by 6, so - 13 by 6 we add to 1 by 6 so 1 by 6 - 13 by 6 and we get - 12 by 6 which is - 2, ok so we get - 2 here.

And then here what we will have? 0 will be multiply by 13 by 2 and add to 0 it will remain 0, we multiply 2 by 13 by 2, so 2 into 13 by 2 means 13, so 13 we add to 15 so 15 and we get 28. Let us see whether these this is values match, ok.

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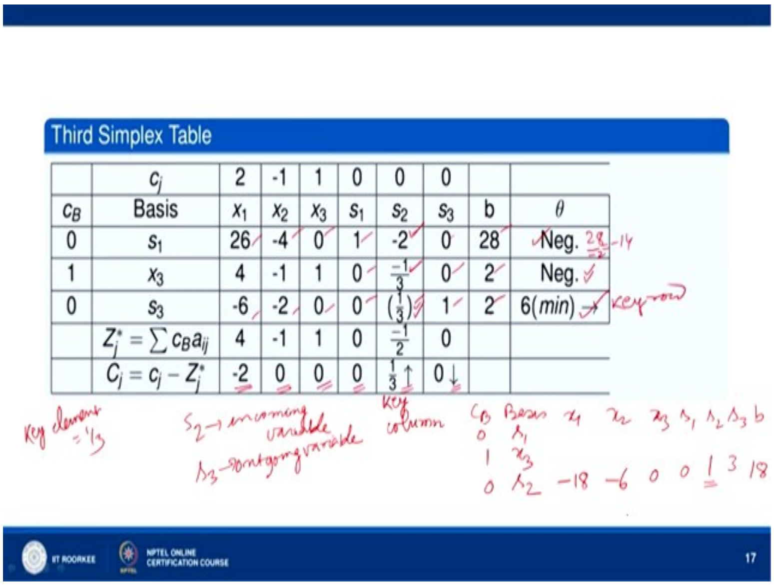

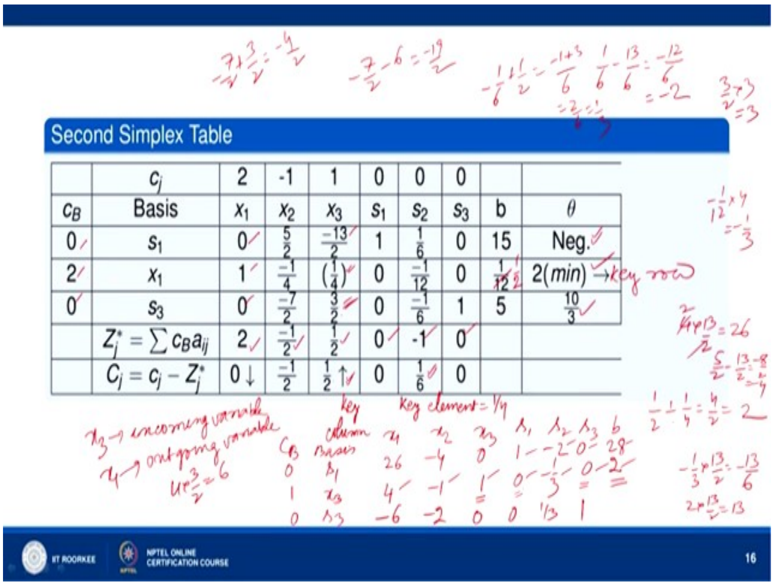

So, 26, - 4, 0 26, - 4, 0 1, - 2, 0 1, - 2, 0, 28, so 1, - 2, 0, 28 and then second row is 4, - 1, 1 so 4, - 1, 1 0, - 1 by 3 0, - 1 by 3, 0, 2 0, - 1 by 3, 0, 2, ok so here we have 4, 4 divided by 13 by 2 so 4 divided by 3 2, ok this is to be made 0, 4 into 3 by 2 is 6, ok so six we subtract from 0 and we get - 6 here, ok and then what we have - 1 - 1 into 3 by 2, so we get - 3 by 2 - 3 by 2 we subtract from - 7 by 2, so - 7 by 2 + 3 by 2 and we get here - 4 divided by 2 we get - 2, this is 0, ok.

And then we are multiplying by 3 by 2, so 0 into 3 by 2 is 0, ok we subtract that from 0, ok so we get 0 here and here what we have? - 1 by 3 we multiply by 3 by 2 so we get - half, - half we subtract from - 1 by 6 so - 1 by 6 + half and this is = what? 6 - 1 + 3, so we get 2 by 6 and that gives you 1 by 3, so we get 1 by 3 here, ok then in the s_3 column, s_3 column the element is 0 we multiplied by 3 by 2, so it becomes 0 subtracted from 1, so we get 1 here.

And then 3 by 2 into 1 by 2 3 by 2 oh no it is 2 now 3 by 2 into 2 is 3, so 3 we subtract from 5 and we get 2 here, so let us see this row 6 in the next table. So - 6, - 2, 0, 0, - 6, - 2, 0, 0 1 by 3, 1, 2 1 by 3, 1, 2, ok . So now we have found Z j star here and the corresponding C j values, so C j values are now - 2, 0, ok 0, 0, 1 by 3, 0 so this is there is still one C j value which is positive, ok so this is our key column.

So, we so that means s_2, s_2 will be incoming vector incoming variable. Now let us find outgoing variable, ok so we divide the b values, ok by the key column values, so 28 divided by - 2 that is - 14, so it is a negative value and then 2 divided by - 1 by 3, so what we get is - 6, so that is also negative then 2 divided by 1 by 3 so that is 6, ok so this is minimum positive ratio is this, so this is our key row and this is key element, so key element is 1 by 3, ok this means that s_3 will be outgoing variable, ok.

So, we will have now C B column as basis, so basis will be having a (vect) variables s_1, ok and x_3, s_3 will be going out in place of x_3 will shall have s_2, ok yeah s_1 coefficient is 0, s x_3 coefficient is 1, s_2 coefficient is 0, ok. We will have x_1, x_2, x_3 s_1, s_2, s_3 and b, ok. Now we divide the elements of the key row by key element, so we divide by 1 by 3, ok so dividing by 3 means - 6 into 3 that is - 18, so it will become - 18, ok.

And then - 2 divided by 1 by 3 will get - 6 then 0 here, 0 here, ok and this will become 1 here, ok 1 divided 1 by 3 gives you 3 and then 6 divided by 1 by 3 gives you 18, ok and then we can make with the help of this one, we can with the help of this one we can make the other entries in the s_2 column zeros, so this still - 2 and - 1 by 3 they have to be made zeros, ok.

So, we multiply this new row, new key row by 2 and add 2 - 2 there, ok so that it become 0, ok the new row we will have here and then we multiplied by 1 by 3 and add to this second row, so that it is - 1 by 3 becomes 0. So after that we get the following table.

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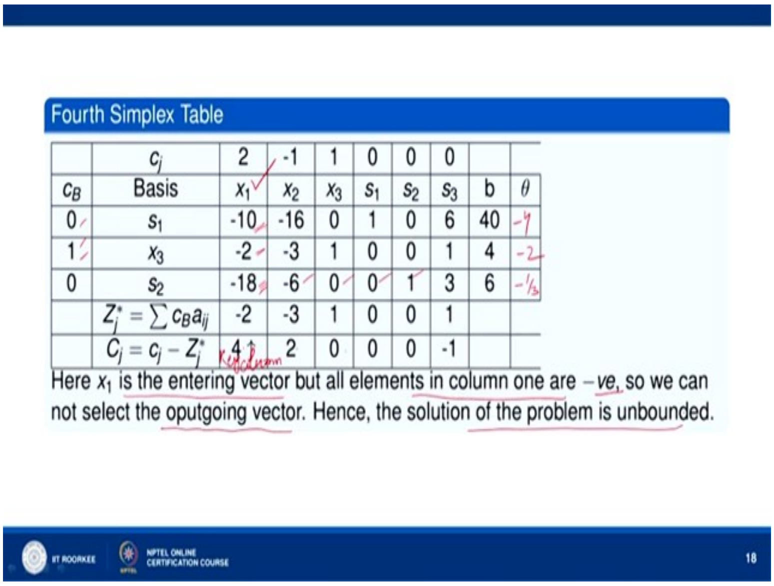
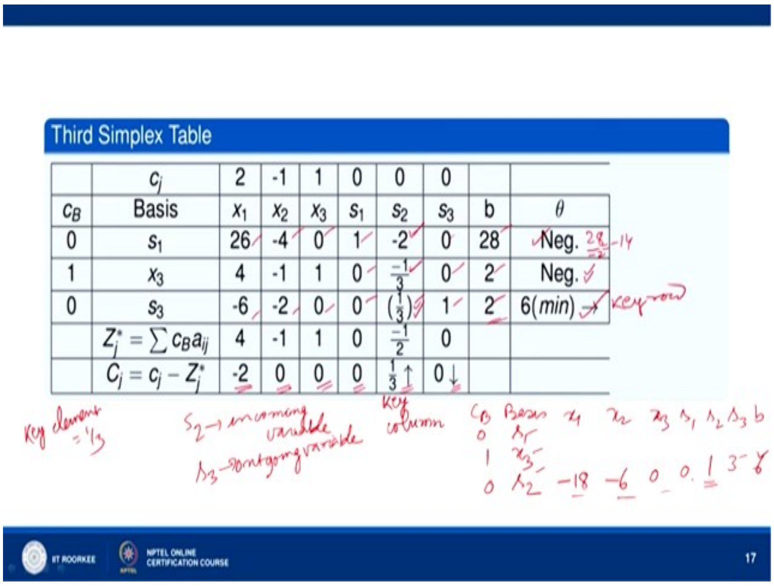


Fourth Simplex Table

	c_j	2	-1	1	0	0	0		
c_B	Basis	x_1	x_2	x_3	s_1	s_2	s_3	b	θ
0	s_1	-10	-16	0	1	0	6	40	
1	x_3	-2	-3	1	0	0	1	4	
0	s_2	-18	-6	0	0	1	3	6	
	$Z_j^* = \sum c_B a_{ij}$	-2	-3	1	0	0	1		
	$C_j = c_j - Z_j^*$	4	2	0	0	0	-1		

Here x_1 is the entering vector but all elements in column one are $-ve$, so we can not select the oputgoing vector. Hence, the solution of the problem is unbounded.



Third Simplex Table

	c_j	2	-1	1	0	0	0		
c_B	Basis	x_1	x_2	x_3	s_1	s_2	s_3	b	θ
0	s_1	26	-4	0	1	-2	0	28	Neg.
1	x_3	4	-1	1	0	$\frac{-1}{3}$	0	2	Neg.
0	s_3	-6	-2	0	0	$(\frac{1}{3})$	1	2	6(min)
	$Z_j^* = \sum c_B a_{ij}$	4	-1	1	0	$\frac{-1}{2}$	0		
	$C_j = c_j - Z_j^*$	-2	0	0	0	$\frac{1}{3}$ ↑	0 ↓		

So, we see s_1, x_3, s_2 we have s_1, x_3, s_2 we have 0, 1, 0 they are the values in the C B column, ok C B column and this is third row, third row we have written here - 18, - 6, 0, 0, 1 so - 18, - 6, 0, 0, 1 and then we have 3 , 6 3 and we have oh this two becomes 6, oh no not 18 it is 6, ok so we get match they are matching. Now, we can make these elements - 2 and - 1 by 3 0 by multiplying this row suitably an adding, ok and so we will get the following table, ok.

This table we have and in this table we have found the values of Z j star and the corresponding values of C j, C j is 4, 2, 0, 0, 0, - 1, so there are two positive values but the value 4 is the maximum, so this is our key column, ok. So that means x_1 is incoming variable, let us find outgoing variable but we notice that all the values in the x_1 column are negative, ok - 10, - 2, - 18, ok.

So, to get the outgoing vector what we have to do is? We have to divide the elements in the b column by the corresponding elements in key column, so this is - 40 divided by - 10, so that it will be - 4 here, 4 divided by - 2 will be - 2 here, 6 divided by - 18 will be - 1 by 3 here, so there is no positive ratio, ok and therefore we cannot find the outgoing vector, ok.

So, x_1 is entering vector but all elements in the column 1, ok they are negative, so we cannot select the outgoing vector, ok and hence the solution of the problem is unbounded. So we have discussed various cases which occur in the linear programming problems and we have seen how we can solve them by using the Big M Method, we have the we have discuss in a in detail the all the methods. So, that is all in the lecture, thank you very much for your attention.