Higher Engineering Mathematics
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Lecture 46 – Big M Method - I

Hello friends! Welcome to my lecture on Big M Method, first lecture on Big M Method. This method was given by A. Charnes and it consists of the following steps:

First we express the problem in the standard form. So by expressing the problem in the standard form means we first put the problem in the maximization form, and then the second thing is that we ensure that all the b_i, s in the constraints are non-negative. Third thing that we have to do is that we have to express all the constraints in the form of equations by using slack or surplus variables. The fourth thing is that we have to see that all the variables in the problem are non-negative.

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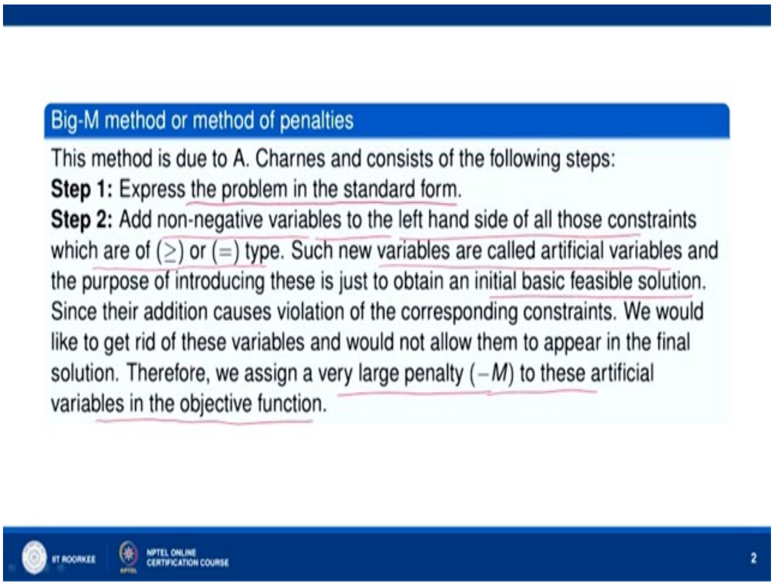

So first we put the problem in the standard form. Then the second step we do is that we add non-negative variables to the left hand side of all those constraints which are of $\geq$ type. Such new variables are called artificial variables and the purpose of introducing these variables is just to obtain an initial basic feasible solution.

So these variables are causing violation of the corresponding constraints, so therefore we would like to get rid of these variables and would not allow them to appear in the final solution. And that is why we assign a very large penalty, - M to these artificial variables in the objective function. So purpose of introducing objective….artificial variables is just to obtain a initial basic feasible solution. But they violate the corresponding constraints, therefore we would like to get rid of them and also would not allow them to appear in the final solution.

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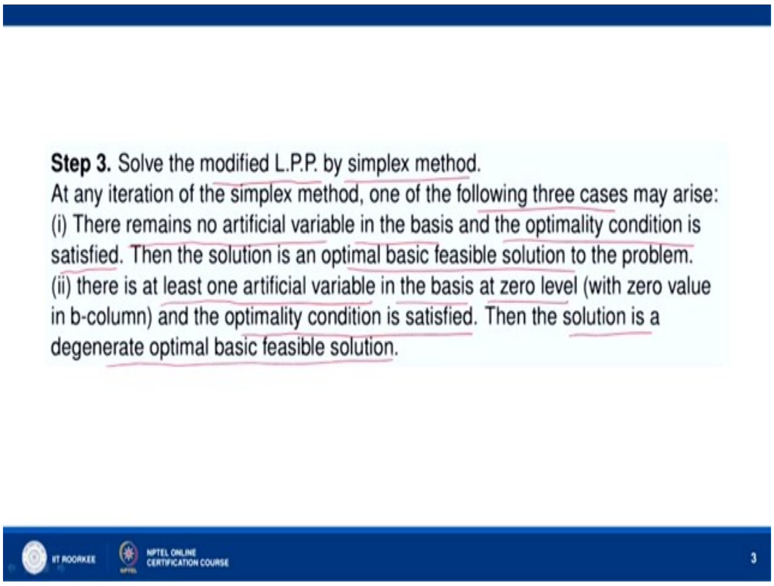

Now what we do is let us solve the modified LPP by using simplex method. At any iteration of the simplex method, the following three cases may arise: First case is that, there remains no artificial variable in the basis and the optimality condition is satisfied. Then the solution is an optimal basic feasible solution to the problem.

The other case could be, there is at least one artificial variable in the basis at zero level, that means zero value in the b-column. The artificial variable having zero value in the b-column and the optimality condition is satisfied, then we say that the solution is a degenerate optimal basic feasible solution.

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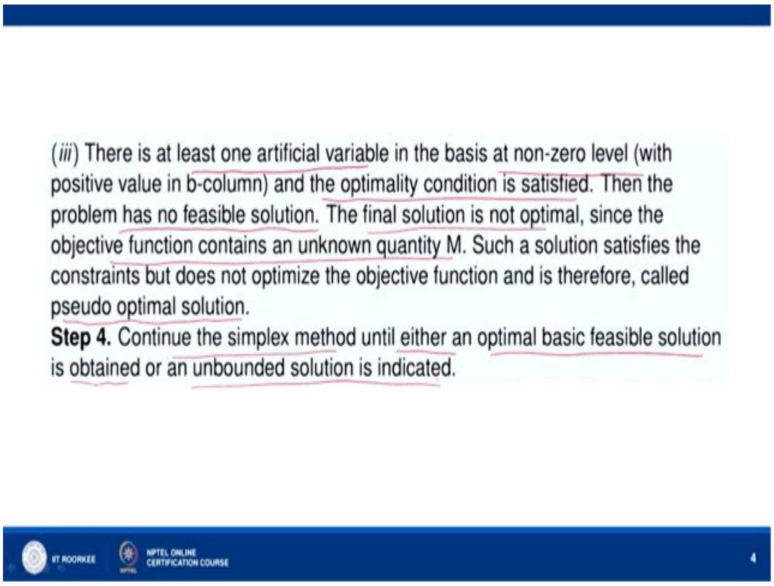

Now the third case is there is at least one artificial variable in the basis at non-zero level. That means with the positive value in the b-column and the optimality condition is satisfied, then the problem has no feasible solution because the artificial variable, there is at least one artificial variable in the basis at non-zero level and so the final solution is not optimal because the objective function contains an unknown quantity M.

Such a solution satisfies the constraints but does not optimize the objective function and therefore is called pseudo optimal solution. Now in the step 4, we continue the simplex method until either an optimal basic feasible solution is obtained or an unbounded solution is indicated.

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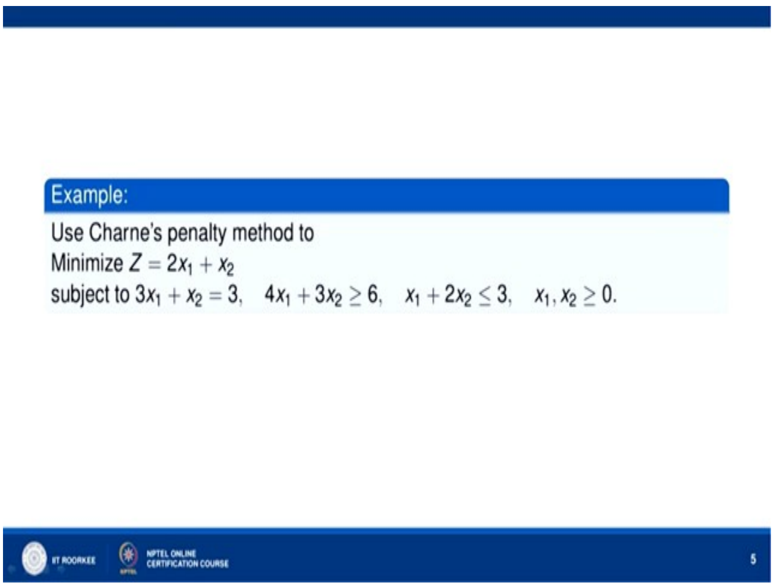

Now let us consider this example and solve this example by using Charne's method. So use Charne's penalty method to minimize Z = $2x_1 + x_2$, subject to $3x_1 + x_2 = 3$; $4x_1 + 3x_2 \geq 6$; $x_1 + 2x_2 \leq 3$; $x_1, x_2 \geq 0$. Since this equation, this LPP is of minimization type, first we convert it to maximization type and therefore consider, we consider maximum of Z'.

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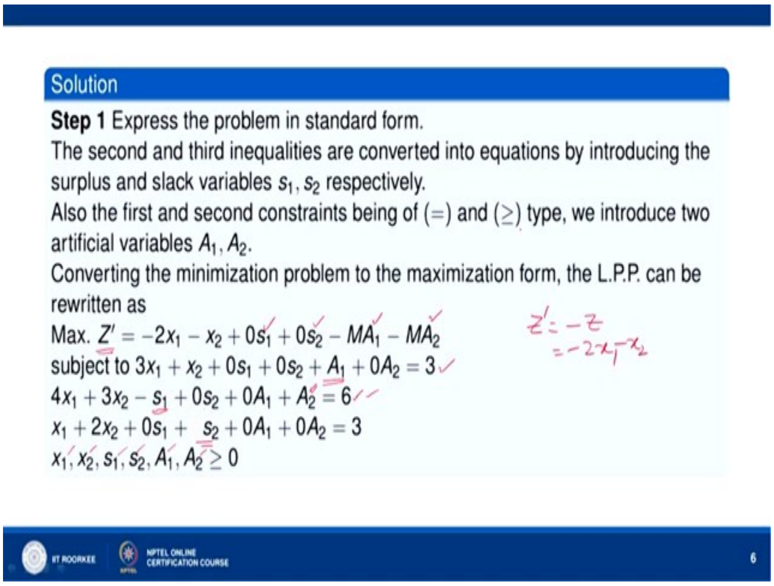

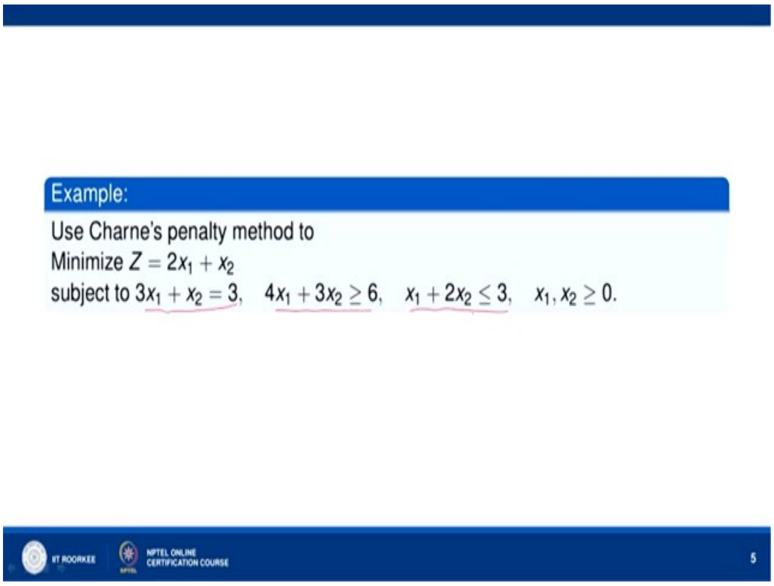

Z' is - Z. So Z = $2x_1 + x_2$, so this is $-2x_1 - x_2$. And then we assign 0 values to this, because the first constraint is of the type of equality. $3x_1 + x_2 = 3$, this is first constraint. Second is $4x_1 + 3x_2 \geq 6$; $x_1 + 2x_2 \leq 3$. Now this is of equality type, so we have to add an artificial variable here. And here this is of $\geq$ type, so we add one, we subtract surplus variable as s_1 let us say and add an artificial variable here. And here we add a slack variable.

So let us say we have, this is your surplus variable s_1, this is your slack variable s_2. And A_1 and A_2 are artificial variables which have been given the value - 1, that is the penalty - M. So the equations, first equation, 3 x_1 + x_2 = 3 can then be written as $3x_1$+ x_2+ $0s_1$, $0s_2$ + A_1 + $0A_2$ = 3. We are adding artificial variable here. Now second constant is $4x_1$+ $3x_2$ ≥ 6, so we consider, we take a surplus variable and we subtract the surplus variable here.

So 4 x_1+ $3x_2$ - s_1+ $0s_2$+ $0A_1$+ A_2. Because it is ≥ type, so we also have to take artificial variable which we can take as A_2, A_2= 6. And then third constant is ≤ type, so x_1 + 2 x_2 ≤ 3, so x_1+¿ 2 x_2+0 s_1 + s_2 + 0 A_1 + 0 A_2 = 3. So s_2 is the slack variable here. And we take x_1, x_2, s_1, s_2, A_1, A_2 to be all non-negative.

So thus we consider the corresponding maximization problem where we have to maximize Z' = - 2 x_1 - x_2 + 0 s_1 + $0s_2$ - MA_1 - MA_2 , subject to these equations, these three equations, where the variables x_1, x_2, s_1, s_2, A_1, $A_2$are all ≥ 0. So the second and third inequalities are converted into equations by introducing surplus variable and slack variable. And first and second constraints, this one and this one, they were of equality and ≥ type, so we have introduced two artificial variables, $A_1$and A_2.

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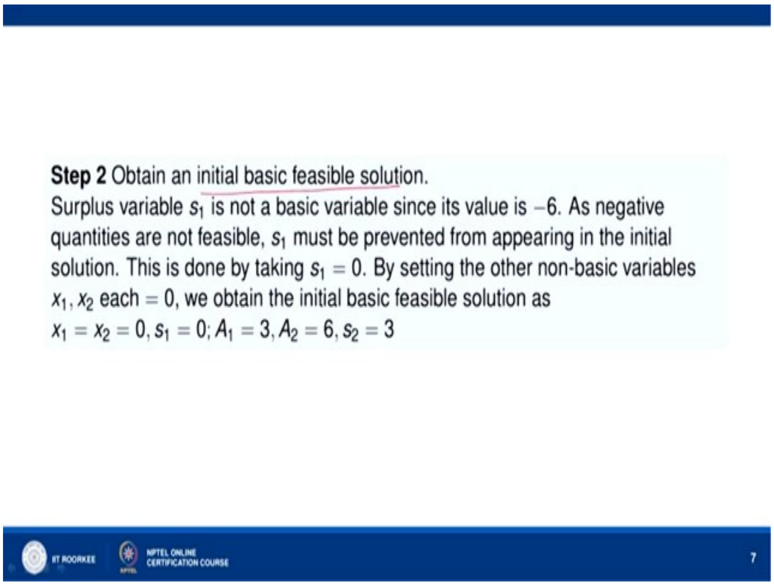

Now let us go to finding initial basic feasible solution. So surplus, now let us note that surplus variables_1 is not a basic variable here.

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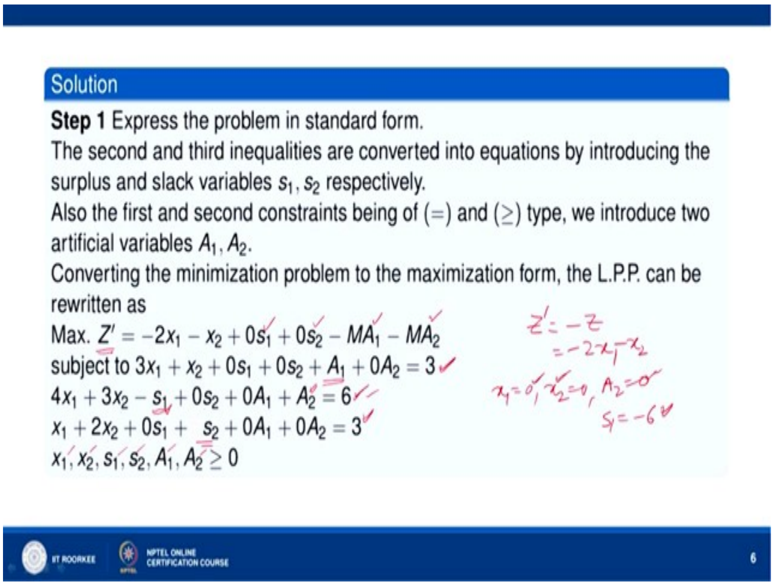

This surplus variable s_1 is not a basic variable here because if we take x_1, x_2, s_2, A_1, A_2= 0, if we take x_1=0, x_2 = 0, and s_2 = 0, A_1 = 0, then A_2 = 0, then what we get? s_1 = - 6. So taking x_1 = 0, x_2 = 0, s_2 = 0, we have three equations here and there are how many constraints? 1, 2, 3, 4, 5, 6 constraints. So we need to take three constraints =0. So we can take x_1, x_2 =0 and A_2 =0.

See, we have three equations here, 1, 2, 3. So to obtain initial basic feasible solution we need to take three constraints =0. So that means let us take x_1 =0, x_2 =0, A_2 =0. If we take x_1 =0, x_2 =0, and A_2 =0, then we arrive at s_1 =- 6. But s_1 is ≥ 0, so what we have?

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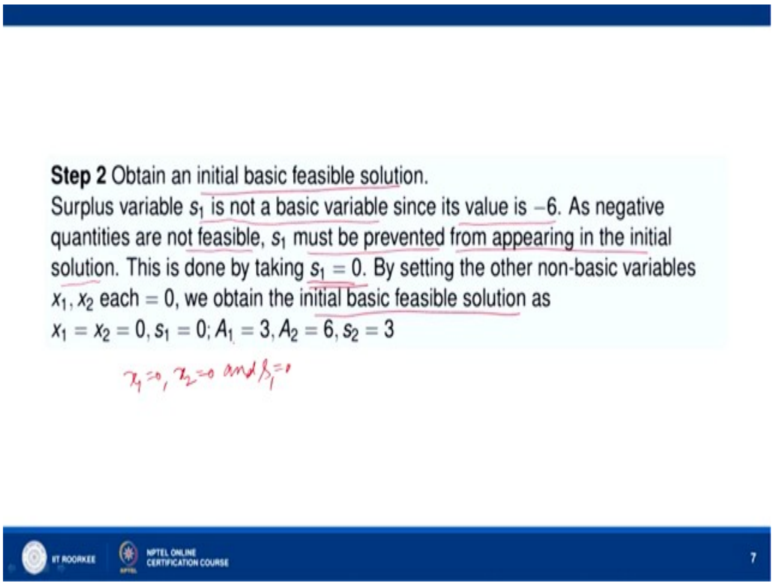

s 1 will not be a basic variable because its value is - 6. So as negative quantities are not feasible, s_1 must be prevented from appearing in the initial solution. For a feasible solution all the variables must be non-negative. So here s_1 must be prevented from appearing in the initial solution and this we can do by taking s_1 = 0. So by setting the other non-basic variables x_1, x_2 each to = 0, so what we do now?

We take x_1 = 0, x_2 = 0, and s_1 = 0. x_1 = 0, x_2 = 0, and s_1 = 0, we obtain the initial basic feasible solution as x_1 = 0, x_2 = 0, s_1 = 0, and then we get A_1 = 3.

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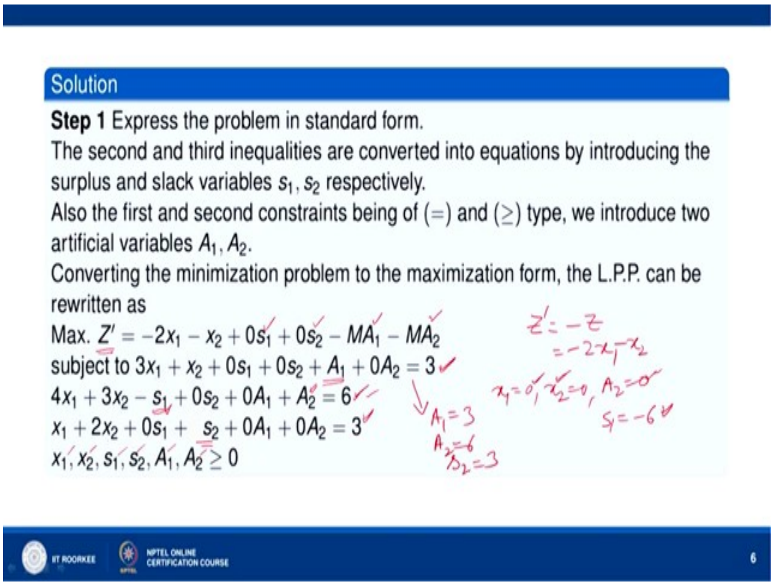

A_1 equal to…here you see, x_1 is 0, x_2 is 0 and then we are taking $s_1 = 0$. So we get $A_1 = 3$, we get this equation. This equation gives you $A_1 = 3$. And here we get x_1, 0; x_2, 0; s_1, 0, so $A_2 = 6$. And we get x_1, 0; x_2, 0 and yeah, so x_1, 0; x_2, 0 and this what we get? Here we have $s_2 = 3$. $x_1 + 2x_2 + 0s_1 + s_2 + 0A_1 + 0A_2 = 3$ gives $s_2 = 3$.

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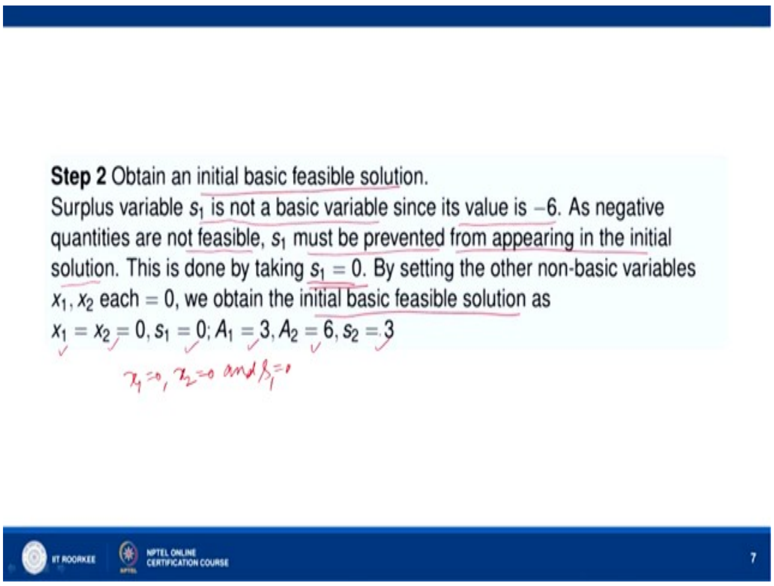

So what we have is this: The initial basic feasible solution is $x_1 = 0$, $x_2 = 0$, $s_1 = 0$, $A_1 = 3$, $A_2 = 6$, $s_2 = 3$.

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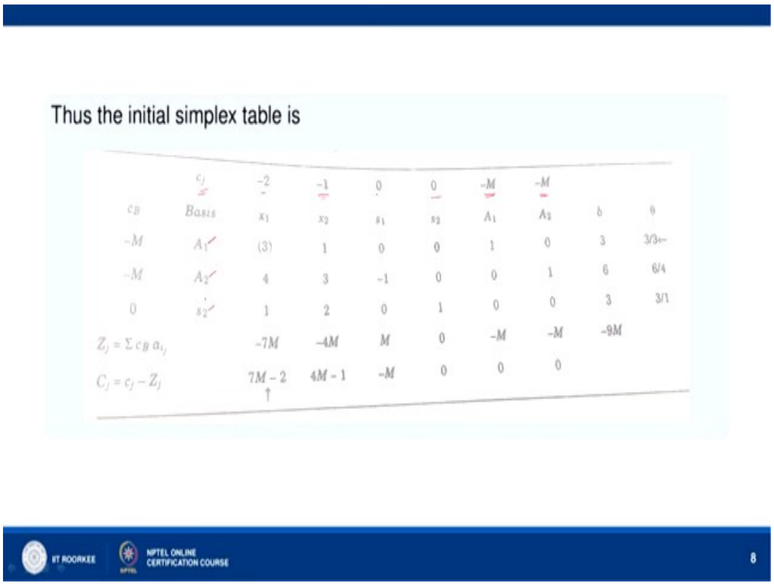

	c_j	-2	-1	0	0	$-M$	$-M$		
c_B	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ
$-M$	A_1	(3)	1	0	0	1	0	3	3/3←
$-M$	A_2	4	3	-1	0	0	1	6	6/4
0	s_2	1	2	0	1	0	0	3	3/1
$Z_j = \Sigma c_B a_{ij}$		$-7M$	$-4M$	M	0	$-M$	$-M$	$-9M$	
$C_j = c_j - Z_j$		$7M-2$ ↑	$4M-1$	$-M$	0	0	0		

And we then have the initial simplex table as: So these are C js, C js are the coefficients of the variables in the objective function. So x_1, coefficient of x_1 is - 2, coefficient of x_2 is - 1, coefficient ofs_1 is 0, coefficient of s_2 is 0, coefficient of A_1 is - M, coefficient of A_2 is - M. These are the coefficients of x_1, x_2,s_1, s_2, A_1, A_2 in the objective function. The basis is A_1, A_2, s_2. Basis is A_1, A_2, s_2.

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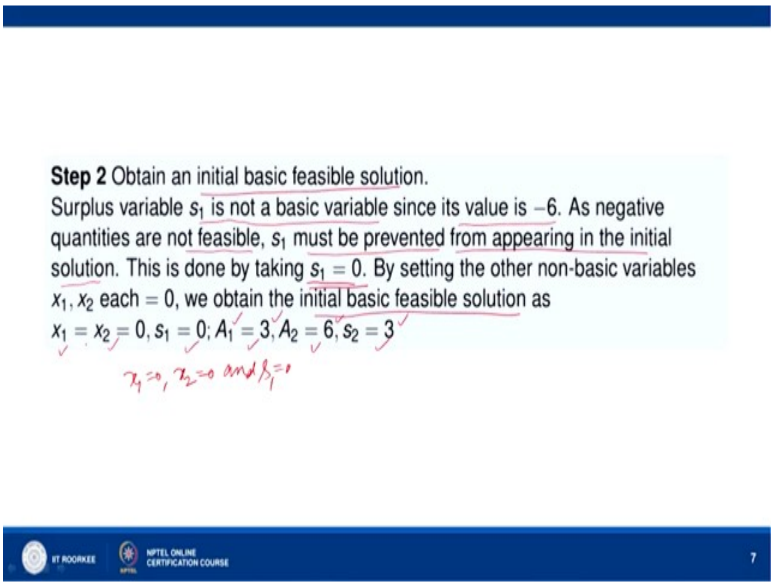

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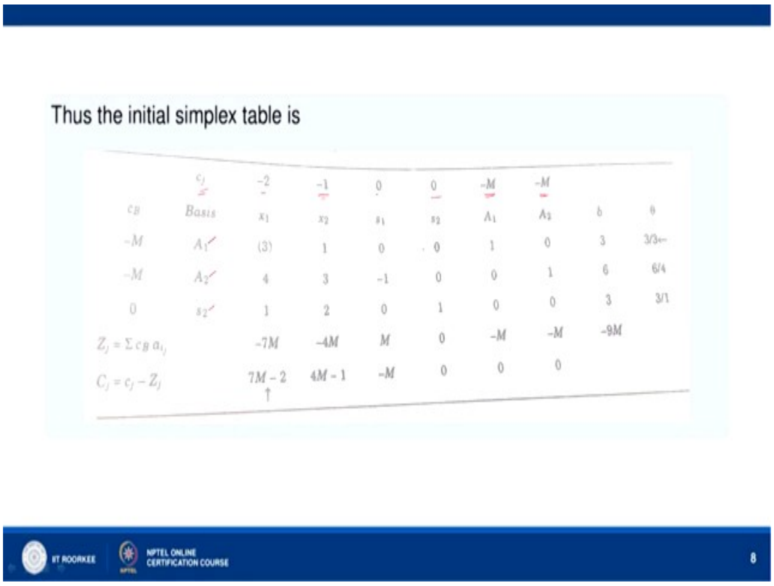
Thus the initial simplex table is

	c_j	-2	-1	0	0	-M	-M		
c_B	Basis	x_1	x_2	s_1	s_2	A_1	A_2	b	θ
-M	A_1	(3)	1	0	0	1	0	3	3/3←
-M	A_2	4	3	-1	0	0	1	6	6/4
0	s_2	1	2	0	1	0	0	3	3/1
$Z_j = \Sigma c_B a_{ij}$		-7M	-4M	M	0	-M	-M	-9M	
$C_j = c_j - Z_j$		7M − 2 ↑	4M − 1	-M	0	0	0		

This one, basic variables are A_1, A_2 and s_2. And non-basic variables are x_1, x_2,s_1. So non-basic variables are x_1, x_2,s_1. Okay, basic variables are s_2, A_1, A_2, and these are the coefficients which we have written from the constraints, the equations.

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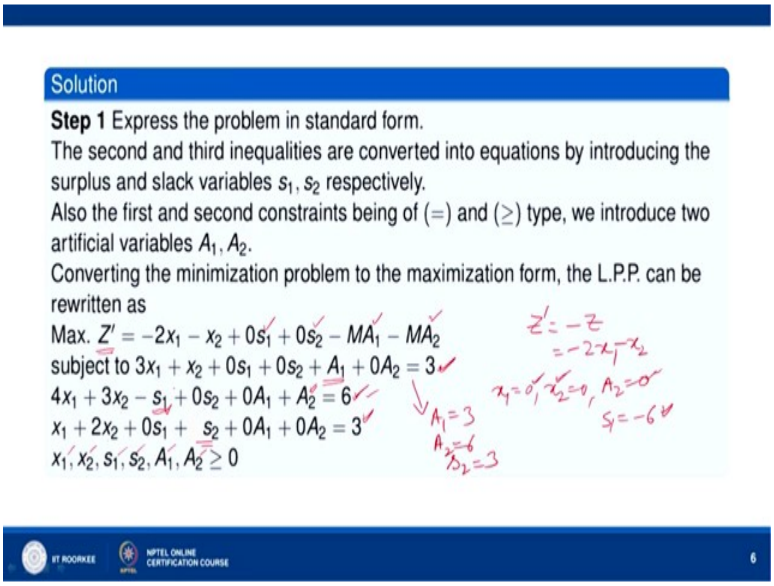

These equations. This equation, this equation and this equation: first equation, second equation, third equation.

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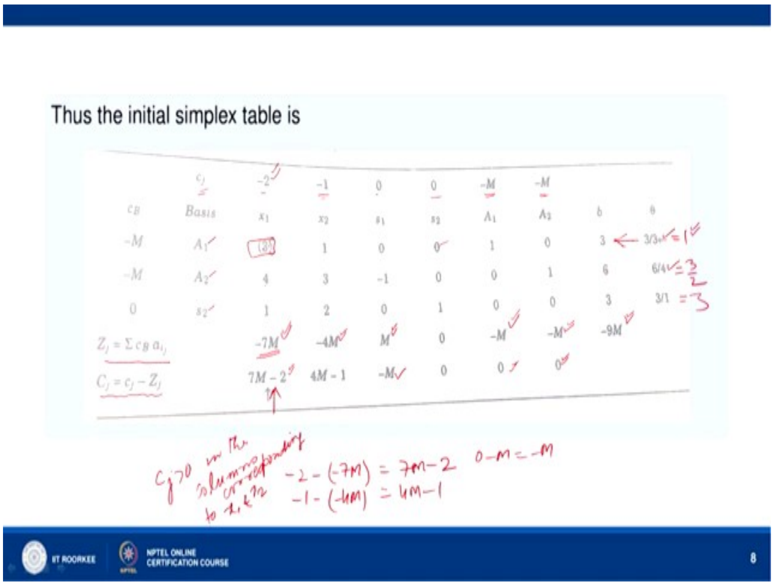

So the coefficients of x_1, x_2,s_1, s_2, A_1, A_2 are written here: 3, 1, 0, 0, 1, 0, 4, 3, - 1, 0, 0, 1, 1, 2, 0, 1, 0, 0. And these are the…This is the b column. b column is your 3, 6, 3, that is b column. Now the coefficient of A_1 in the objective function is - M. So in the CB column we write the coefficients of the basic variables, so - M, the coefficient of A_2 is - M and coefficient of s_2 is 0.

Now we calculate Z j. Z j =Sigma CB a i j. And so then let us see. This is your CB column. So - M into 3, so we get - 3 M. - M into 4, we get - 4 M. And then 0 into 1 is 0. So we get Sigma CB a i j is =- 7 M. Then Sigma CB a i j we find for the second column. So - M into 1, - M. - M into 3, - 3 M. And 0 into 2 is 0. So we get total - 4 M. So this is - 4 M.

And then we write similarly. - M into 0 is 0. - M into 1 is - M. This is 0 into 0, so we get….- M into - 1 is M. So we get M here. Similarly this column s_2, - M into 0 is 0. - M into 0 is 0. 0 into 1 is 0. So total sum is =0. And then - M into 1 is 0, - M into 1 is - M. - M into 0 is 0. 0 into 0 is 0. So sum is - M. So we get here - M.

And then - M into 0, - M into 1, 0 into 0; so we get sum as - M. And then here when you multiply - M, 0 to the b column we get - 3 M, - 6 M and then 0. So total is - 9 M. Now we find C j. Capital C j =small c j - Z j. So this is your C j. And this is here, this is our Z j. So - 2 - of - 7 M gives you 7 M - 2. So - 2 - - of 7 M gives us 7 M - 2.

Similarly - 1, - of - 4 M gives us 4 M - 1. And 0 - M, 0 - M is - M, so we get - M here. Then 0 - 0 is 0. Then - M - of - M is 0, so we get 0 here. Then - M - of - M is 0, so we get 0 here.

Now let us see. So C j is =c j - Z j. And here it is 7 M - 2, here it is 4 M - 1. Here it is - M. M is a very large quantity, so we see that C j is positive. C j is positive in this column corresponding to x_1 and column corresponding to x_2.

But since M is large, 7 M - 2 is more than 4 M - 1. So maximum C j is 7 M - 2. C j is positive in the two columns, in the columns corresponding to x_1 and x_2. But since M is large 7 M - 2 is more than 4 M - 1. So 7 M - 2 is our key column, this is our key column. And then what we have? We have to find the key row. So the elements of key column we divide to the elements of b, the b column. The elements of b column are divided by the corresponding elements of key column.

So 3 over 3 gives you 1 and then 6 over 4 gives you 3 by 2. So this is 1, this is 3 by 2 and then 3 divided by 1 is =3. Now we have to find minimum of these three values. So minimum is clearly 1. And therefore this row, this row is key row. This row is a key row. This arrow shows the key row. And at the intersection of key column and key row we get the key element, so this is our key element.

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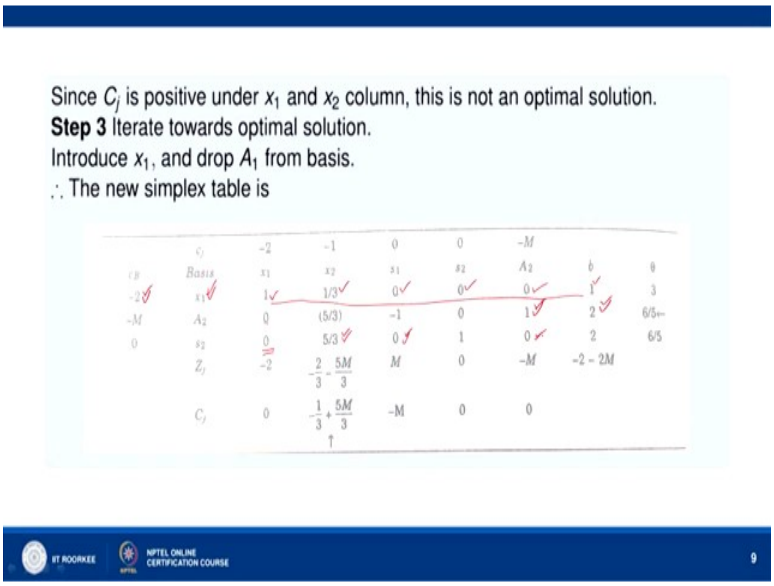

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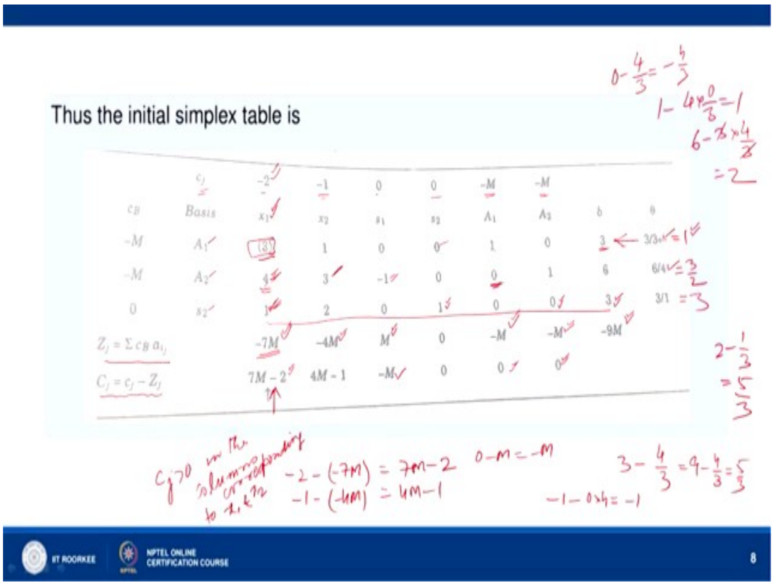

So we have the following: Since C j is positive under x_1, x_2 column, this is not an optimal solution. We therefore iterate towards finding optimal solution. And what we do then? We have found the key element. Now key element, we divide by the key element all the elements of the key row. So to get here 1, so we divide this key row by 3. We get 1 here, 1 by 3 here, 0 here, 0 here, 1 by 3 here and 1 here. So we get that.

1, 1 by 3, 0, 0, 0, 1. So we divide the elements of the key row by the key element to obtain this key element as 1. And then we use elementary row operations to make all the other entries in the key column =0. So by using….now this is what? This 4. So we multiply this row by 4, key row by 4 and subtract from the second row to have 0 here. And then to get 0 here, we subtract the key row, this key row, this row from the third row. So what we do?

After dividing this key row by 3, we make the element 3 as 1 here and then with the help of this 1 we make the elements in the key column, that is 4 and 1, 0, using elementary row operations. To make 4, 0 we have to multiply the elements of this key row by 4 and subtract from the second row. And then to make this 1, 0 we subtract the elements of the key row by, I mean elements of the key row from the elements of this third row after the key element has been made 1.

So that we have to do here. So key row after dividing by 3 becomes 1, 1 by 3, 0, 0, 0, 1. And then we subtract four times of this from the second row. We get 0, 5 by 3; how we get 5 by 3? We can see. See, this is 1 by 3. After dividing by 3 this becomes 1 by 3. And then we multiply it by 4 and subtract from this. So we will get what? 3 - 4 by 3. So that means 9 - 4 by 3. So 5 by 3. So this element, this will become 0, this will become 5 by 3.

Here we will get - 1, - 0 into we are dividing by 3, so 0 by 3 is 0. And then multiplying by 4, so 0 into 4, so we get - 1. So this element will remain - 1. This element here will also remain 0 because we are dividing this row by 3 and then multiplying by 4 and subtracting from this row. But here what will happen? This is 0. We are dividing it by 3, we are dividing it by 3 and then multiplying by 4 and subtracting, so 0 - 4 by 3.

So this will be - 4 by 3. So this A_1 element here will be - 4 by 3. So what we do is…now what will happen is that this is incoming variable, x_1 is incoming variable and A_1 is outgoing variable. So in the new simplex table A_1 will be replaced by x_1 and the coefficient of x_1 that is - 2 will come in place of - M.

So here we get - 2 and then A_1 is replaced by x_1. A_1 is replaced by x_1 and the column corresponding to A_1 is deleted. There is no column in the new simplex table corresponding to A_1. A_1 is the artificial variable. We want to get rid of A_1 as quickly as possible. So after writing this incoming variable x_1 and the corresponding value in the CB column, we make this element 1.

We make this element…key row…key element 1 and then subtract it from the….using elementary row operations from the second and third rows to make this 0 and this 0. And what we get is this: Yeah, so we have….column corresponding to A_1 is not there. A_2, A_2 we have to look into, 1 - 0 by 3 into 4, 4 into 0 by 3, so we get 1 here. So we get 1 here. And then b column, in the b column what we will get? In the b column we have 6 -, we divided it by 3 and multiply it by 4. So 3 into 4 by 3, so we get 2. So we get 2 here.

And similarly, we subtract this row, this row we subtract, we subtract it from the third row. Third row is this one, 1, 2, 0, 1, 0, 0, 3. From this we subtract. So what we will get? We are subtracting, so this will become 0, right? This will become 0. Yeah, this will become 0 here. And then we will have here, okay, here we have 2. We have 2 here and after dividing by 3 it became 1 by 3. So 2 - 1 by 3 we will have. So that will be 5 by 3.

So that is 5 by 3 here. And then here what we will have? This is 0. Because 0 - 0 is 0 here, so we have 0 here. Then in the s_2 what we have? s_2 column here we have 1. So 1 - 0 will remain

1, so that is 1 here. Under A_2 column what we will have? A_2 column here it is 0. And here what we did was we divided by 3 and then after dividing by 3 we are subtracting. So this is 0. So this will remain 0. And then what we will have here? Here we have 3. After dividing by 3 it became 1, so 3 - 1 is =2.

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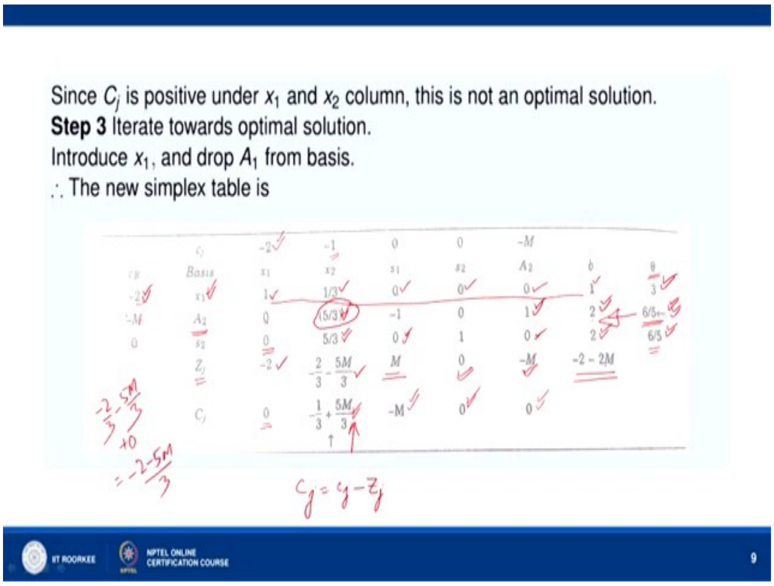

So we get 2 here. Now what we have? Let us find Z j. Z j =CB into x j. So - 2 into 1 is - 2. - M into 0 is 0. 0 into 0 is 0. So total will be - 2. So this is - 2 here. Then - 2 into 1 by 3, - 2 by 3 we have. And then we have - 5 M by 3. And then 0 into 5 by 3, so we have 0. So total is - 2 - 5 M by 3, so we get this.

And then similarly, - 2 into 0 is 0. - M into - 1 is + M. 0 into 0, 0. So we get M here and then this is 0, this is 0. This is 0, so we get 0 here. And then we have - 2 into 0, - M into 1 - M, 0 into 0, so we get - M here. Then - 2 into 1 is - 2. - M into 2 is - 2 M. And 0 into 2 is 0, so we get - 2 into, - 2 - 2 M.

Now let us see. We find C j, C j =c j - Z j. So C j is - 2, when we subtract Z j which is also - 2, we get 0 here. And then - 1 - of this quantity, - of Z j makes it - 1 + 2 by 3. So - 1 by 3 + 5 M by 3, and then you have 0 - M, so we get - M. 0 - 0 is 0. And then we get - M and we subtract - M. So we get - M + M. So that is we get 0 here.

And we notice now that since M is large, - 1 by 3 + 5 M by 3, this is a positive quantity. So C j is greater than 0 in the column corresponding to x_2. And therefore this is our key column. This is our key column. And the elements of key column are 1 by 3, 5 by 3, 5 by 3. So we divide the elements of b column by the corresponding elements of x_2 column. So 1 divided by 1 by 3 gives you 3. 2 divided by 5 by 3 gives you 6 by 5. And 2 divided by 5 by 3 again gives you 6 by 5.

Now we see that we have to find that value here in the column corresponding to theta which is the minimum. But 6 by 5 and 6 by 5 there is a tie here. These two values are both equal and they are less than 3. So which row we will take as the key row? Because we want to get rid of the artificial variable A_2, so we have to take this row as key row, this one. Because if we take this as key row, then A_2 will be, it will be going out and we will get at the intersection of A_2, x_2 will be coming in and this will be our key element.

This will be our key element. x_2 will come in place of A_2 and A_2 will go out. The column corresponding to A_2 will be, will no longer appear now. So now this new simplex table, in the new simplex table what we will have? We will have x_1. In place of A_2 we will have x_2, then we will have s_2 here. The coefficient of x_2 is - 1, so - M will be replaced by - 1.

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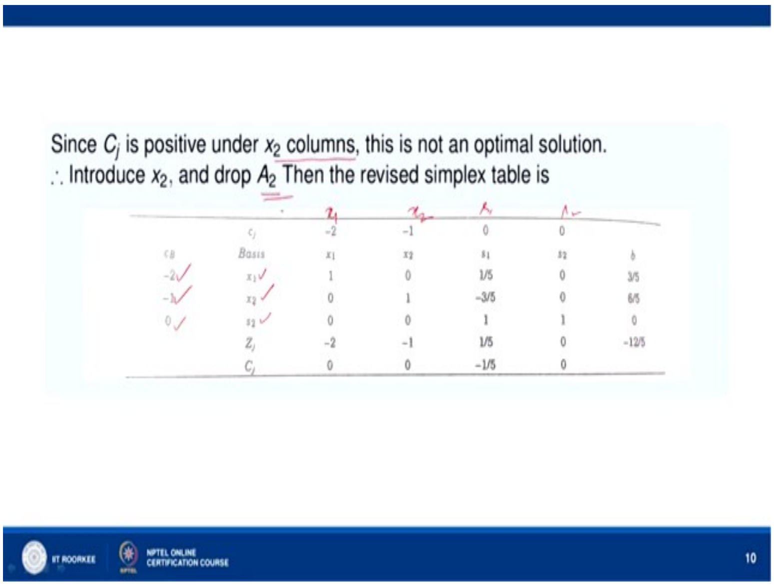

So let us see the new…since C j is positive under x_2 column, this is not an optimal solution. We introduce x_2 and drop A_2. And then revised simplex table is like this: So x_1, x_2, s_2; the coefficient of x_1 is - 2. Coefficient of x_2 is - 1, coefficient of s_2 is 0. So now our basic variables are x_1, x_2 and s_2. This is x_1, yeah, this is x_2. This one iss_1, s_2. So A_1, A_2, the two basic variables, the columns corresponding to the artificial variables are not there. Now let us see, we have the new simplex table. In the new simplex table we have 1, 1; okay, so we see what we have now.

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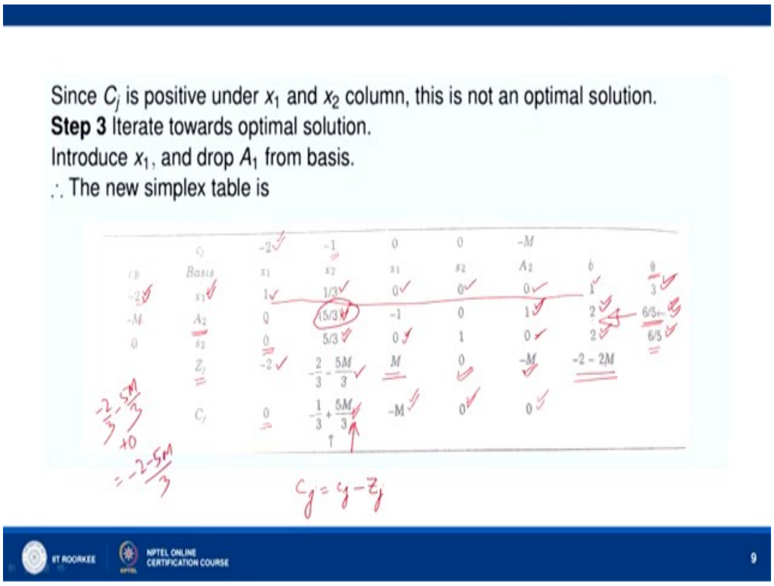

Yeah. So this is our 5 by 3. We divide this equation by 5 by 3. This row, this row we divide by 5 by 3 to bring unity here, and then subtract, after multiplying this by 1 by 3 we subtract it from this row. And subtract, multiplying by 5 by 3 we subtract it from this row to make this element 0, this element 0. That is, we are doing elementary row operation. So let us first divide this key row by 5 by 3. When we divide key row by 5 by 3, what we get?

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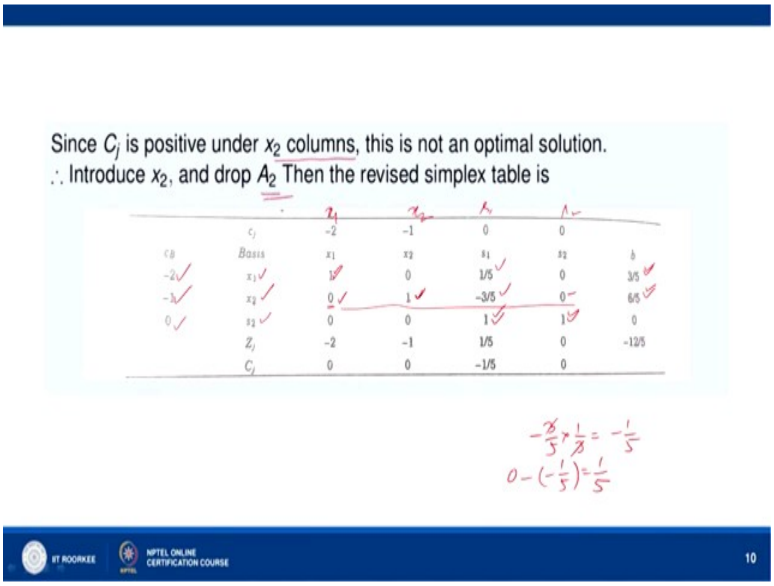

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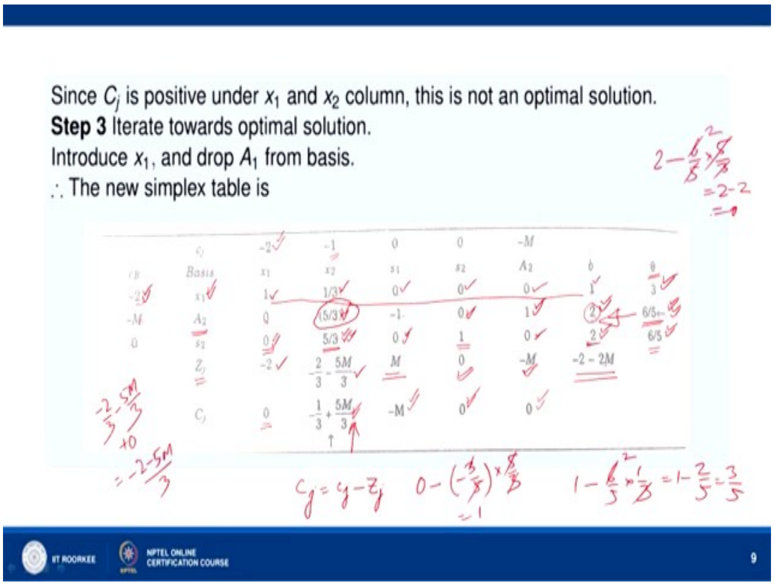

We get this one: 0, 1, okay. And 5 by 3 we are dividing. So we will get here - 3 by 5. So that is - 3 by 5 here. And then this one is 0. And then here 2, we are dividing by 5 by 3, so it will become 6 by 5. So this is 6 by 5. Now we subtract this row. This row is the row which will be…with the help of which we will make the entries, these ones: 1 by 3 and 5 by 3 as zeroes.

So what we do? We multiply it by 1 by 3 and subtract it from the this row, from this row. So what we will have? We have 1 here. And here we have 0, so we are multiplying by 1 by 3 and subtracting. So 1 - 1 by 3, this is 0, 0 into 1 by 3 is 0, so we get 1 here. It is unaffected. Here what we will have? This is 1 by 3. We are multiplying after making this entry as 1.

After making the key element as 1, we are multiplying by 1 by 3 and subtracting, so we get 0 there. And here what we will have? This became - 3 by 5. So we are multiplying by 1 by 3 and subtracting. So - 3 by 5 into 1 by 3 =- 1 by 5. Now this quantity we have to subtract from 0. So what we get? 0 -, - 1 by 5, which is =1 by 5, so we get 1 by 5 here.

And in the s_2 column let us see what we get here. We are dividing by 3 by 5, sorry 5 by 3, so this become 3 by 5. This become 3 by 5. No, A_2 is not there. We come to this one. So are dividing by 5 by 3. This becomes 6 by 5. And then we are multiplying by 1 by 3 and subtracting from this row. So we get 1 -…what we have? 1 - 6 by 5 into 1 by 3. 6 by 5 into 1 by 3, so we get 1 - 2 by 5 which gives us 3 by 5. So this is 3 by 5.

Now what we have to do? So now we have to make this entry 0. So we have to multiply this row…we have to make this entry 0, so we have to multiply this row by 5 by 3 and subtract it from the third row. So let us see what we will get? So here this is one. We are multiplying by 5 by 3 and subtracting. So what will happen?

This will remain 0, this will become 0 and here what we will have? 0 -, we divided it by 5 by 3, so it was - 3 by 5. - 3 by 5 into, we are multiplying by 5 by 3 and subtracting, so this, this and this this, what we have here? 0 + 1. So this is = 1, I am getting. In the column s_1 this is 1. And in the column s_2 what we will get here? This is 1, okay, we divided it by 5 by 3, we divided it by 5 by 3, this become 0. This is 0, and so what we get here?

Now why we are multiplying by 5 by 3? We are multiplying by 5 by 3 and subtracting. So 5 by 3 when you multiply and subtract what we will get? This is 0 here. So multiplying by 5 by 3 and subtracting from 0 will give 1. So this will remain 1. And what we will have there? This element will become how much? We will have 2 -, this was 6 by 5. 6 by 5 into 5 by 3, so we get this 2 - 2 =0.

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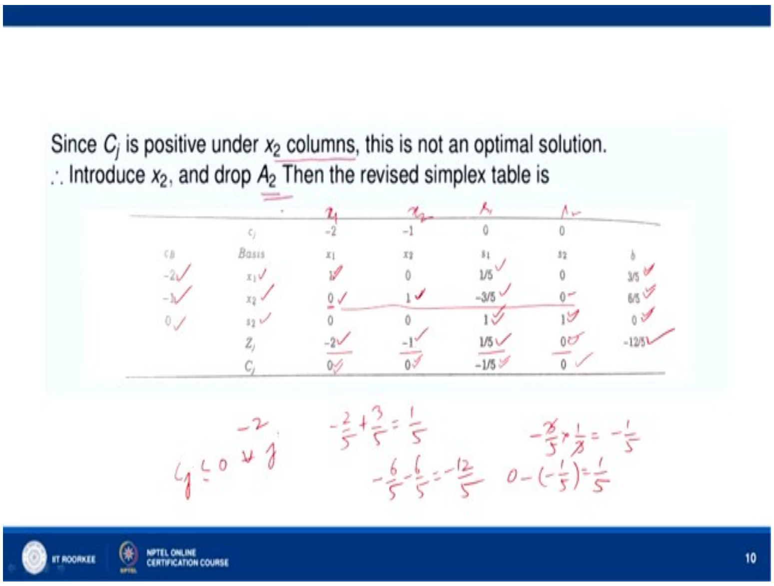

So this becomes 0. Now let us find Z j. Z j =- 2 into 1 - 2. - 1 into 0 is 0. 0 into 0 is 0. So total is - 2. Then - 2 into 0 is 0. - 1 into 1 is - 1. 0 into 0 is 0. So we get - 1 here. - 2 into 1 by 5, so we get - 2 by 5. - 1 into - 3 by 5 is 3 by 5, and we get 0 into 1, so we get 3 by 5 - 2 by 5 is 1 by 5. So we get 1 by 5 here. And then - 2 into 0 is 0. - 1 into 0 is 0. 0 into 1 is 0. So we get 0 here.

And then - 2 into, okay, so now let us see, so we have got these values. Now in the b column what we will get? - 2 into - 3 by 5, so - 6 by 5. And then - 1 into 6 by 5, so - 6 by 5. And then we get 0 into 0, so we get - 12 by 5. So this is - 12 by 5.

Now let us see Z j, we subtract it from c j. So - 2 - of - 2, that gives 0. - 1 - of - 1 gives 0. 0 - 1 by 5, - 1 by 5 we get. And then we have 0, - 0 is 0. Now we can see C j is ≤0 for all j. C j is ≤0 for all j.

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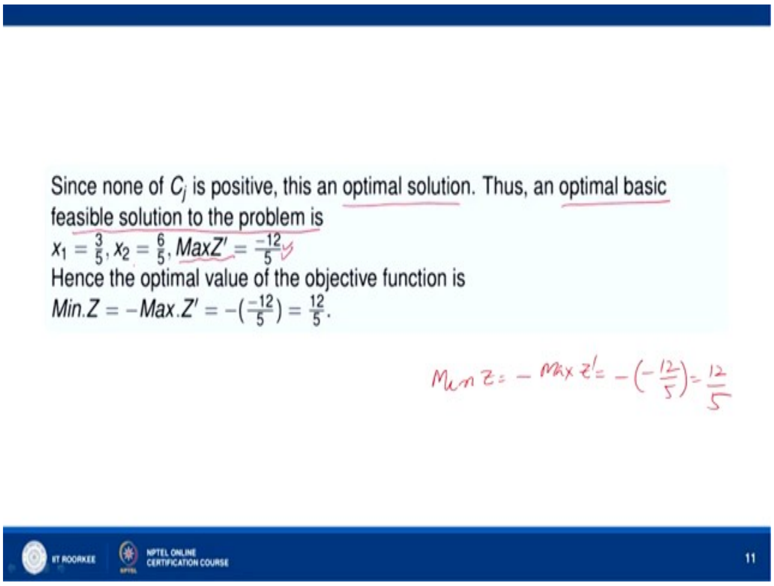

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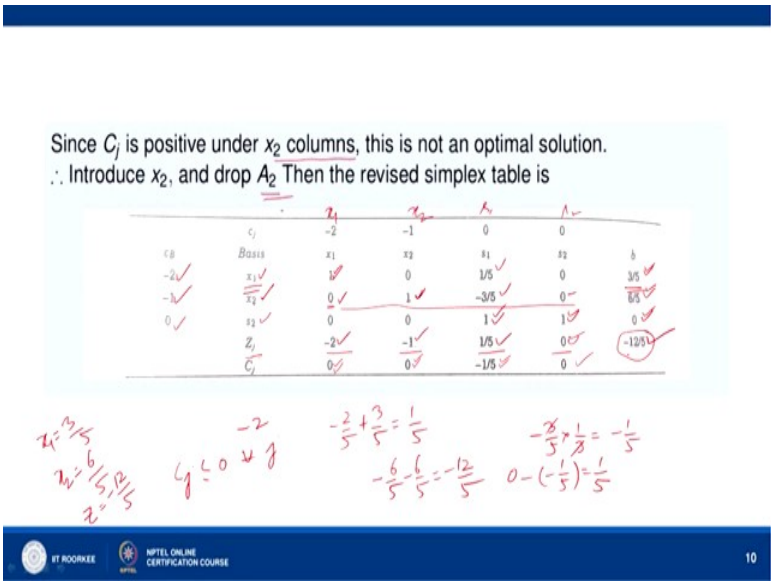

So none of C j is positive and therefore we get the optimal solution. An optimal basic feasible solution to the problem is therefore x_1, what is x_1? x_1 =3 by 5, so x_1 =3 by 5, x_2 =6 by 5 and z =- 12 by 5. z =- 12 by 5. So now maximum of Z dash is - 12 by 5, that is maximum value of

Z dash. So - 12 by 5. And minimum of Z is what ? Minimum of Z = - of maximum of Z dash, so - of - 12 by 5.

This value is nothing but maximum value of Z dash because we have converted the problem to maximization. So after C j is ≤ 0 for all j, the value of Z j gives the maximum value of Z. so we get maximum value of Z dash is - 12 by 5, and that is equal to, I mean the minimum value of Z is - of maximum of Z dash which is 12 by 5. Thus, we get the solution of the given problem, where we find that x_1 is 3 by 5, x2 is 6 by 5 and the minimum value of the objective function is 12 by 5.

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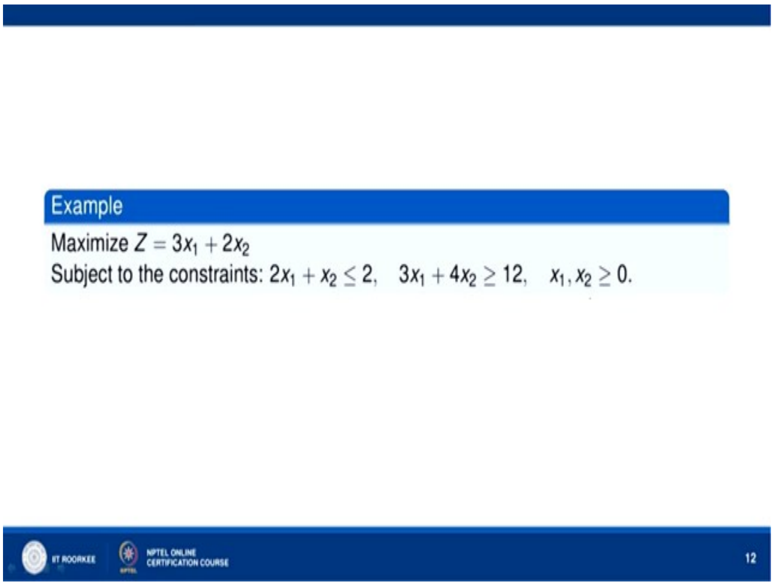

Let us take another problem. Maximum value of Z =3 x_1 + 2 x_2, subject to the constraints: 2 x_1 + x_2 ≤2, 3 x_1 + 4 x_2 ≥12; x_1, x_2 ≥0.

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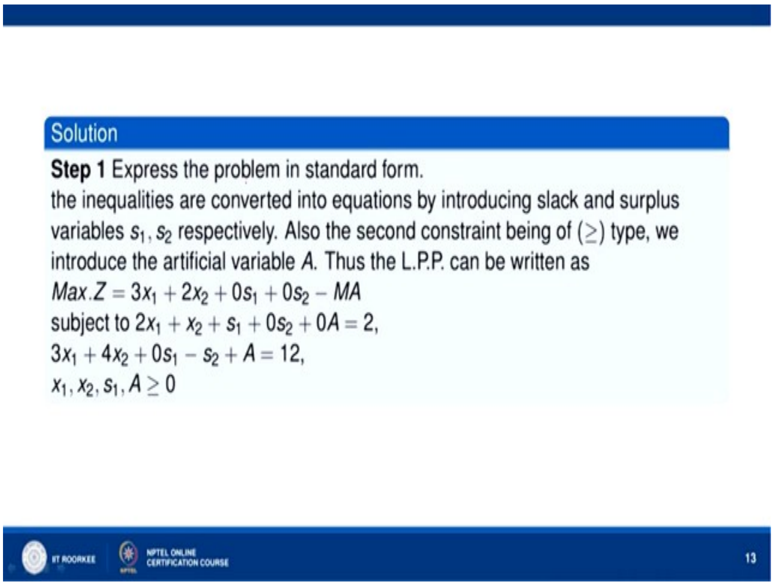

Now we have to express the problem in the extended form.

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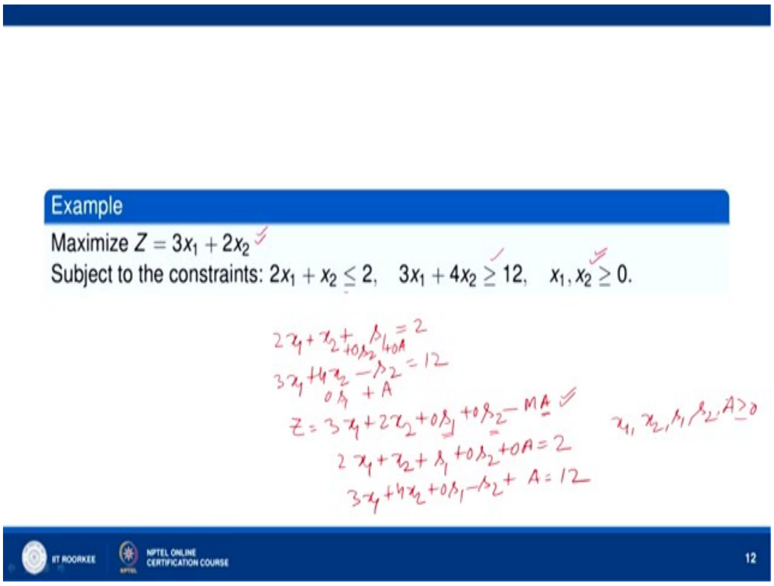

Since it is already a maximization problem, so it will remain as such: x_1, x_2 are ≥ 0; 2 x_1 + x_2 ≤ 2, so this constraint is ≤ type. So we will write one, we will convert it into equality by writing 2 x_1 + x_2 + 0 into, we have to consider a slack variable, so 0 intos_1 and then = 2 it becomes.

But here we have 3 x_1 + 4 x_2 +…≥ type it is, ≥ type, so we get this iss_1, so we have to add a slack variable here. 2 x_1 + x_2 +s_1 =2. 3 x_1 + 4 x_2; now it is ≥ 2, so we write - s_2. - s_2 here, - s_2. We have to subtract the surplus variable. So - s_2 =12. Now we have to add…In order to get initial basic feasible solution, we have to consider the artificial variables also.

So we will write z =3 x_1 + 2 x_2 + 0s_1 + 0 s_2. And then we have to add one artificial variable here because this is ≥type. So we get one artificial variable here, + 0 into A. So artificial variable is given as - M penalty, so - M into A we will have. So objective function Z =3 x_1 + 2 x_2 will be written as Z =3 x_1 + 2 x_2 + 0s_1 + 0 s_2 - M into A.

And here we will have 2 x_1 + x_2 +s_1 and then 0 s_2. Here we will have 0s_1 and 0 A here. So these equations will now become 2 x_1 + x_2 +s_1 + 0 s_2 + 0 A =2 and 3 x_1 + 4 x_2 + 0s_1 - s_2 + 0 into A equal to, no, not 0 into A, we have to add A, artificial variable we have to add, so A, =12. Now the objective function will be written as 3 x_1 + 2 x_2; we will have two variabless_1, s_2. One is…s 1 is slack variable, s_2 is surplus variable and one artificial variable A we have, which is given penalty - M.

Now this artificial variable comes because of the fact that the second inequality is greater or =type. So the first inequality is ≤type. So we have 2 x_1 + x_2 +s_1 =2. But s_2 and A are also there, so we get 0 s_2 + 0 into A, we add them. And here 3 x_1 + 4 x_2 and then 0s_1 and we subtract surplus variable s_2, so - s_2, + artificial variable A =12. So now here we can say x_1, x_2, s_1, s_2 and A all are ≥0.

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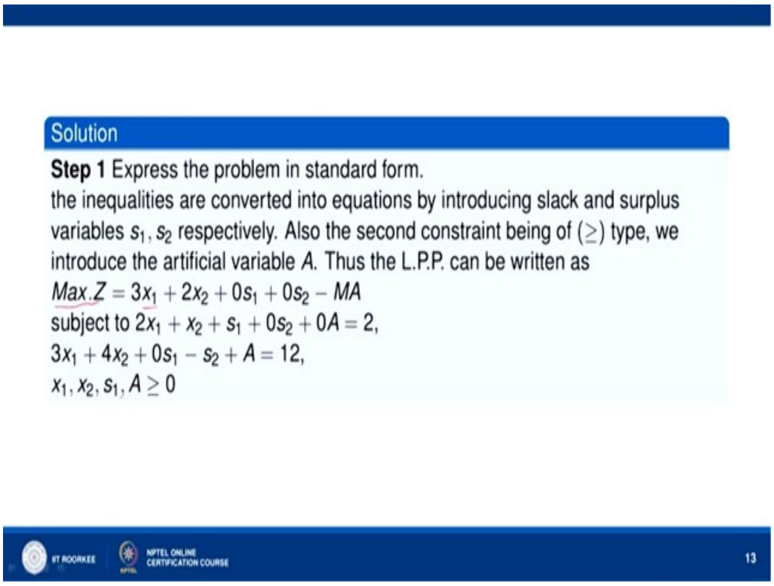

So we get this equation, this maximum value of Z, 3 x_1 + 2 x_2+0s_1 + 0 s_2 - MA, subject to 2x_1 + x_2 +s_1+ 0s_2+ 0 A = 2; 3x_1 + 4x_2+ 0s_1 - s_2 + A =12; x_1, x_2,s_1, s_2, and A $\geq$ 0.

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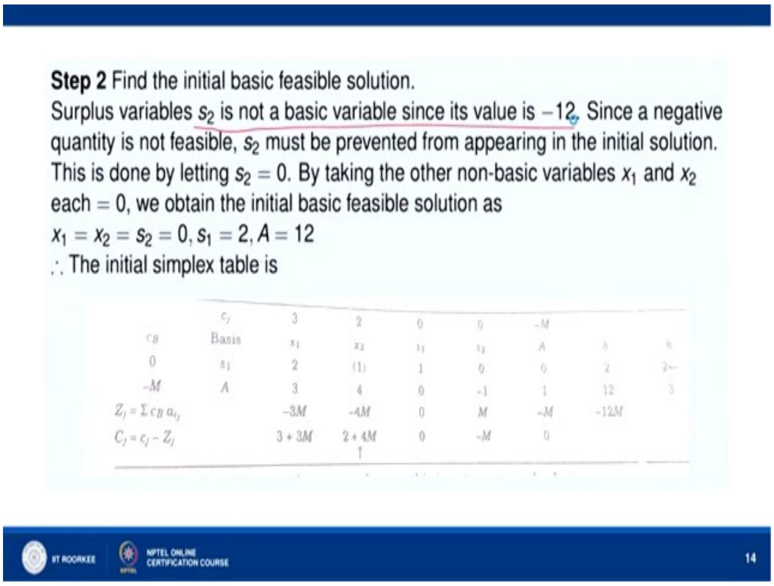

	c_j	3	2	0	0	$-M$		
c_B	Basis	x_1	x_2	s_1	s_2	A	b	θ
0	s_1	2	(1)	1	0	0	2	2←
$-M$	A	3	4	0	-1	1	12	3
$Z_j = \Sigma c_B a_{ij}$		$-3M$	$-4M$	0	M	$-M$	$-12M$	
$C_j = c_j - Z_j$		$3+3M$	$2+4M$ ↑	0	$-M$	0		

Now we will find the initial basic feasible solution. Now s_2 is not a basic variable because its value is - 12.

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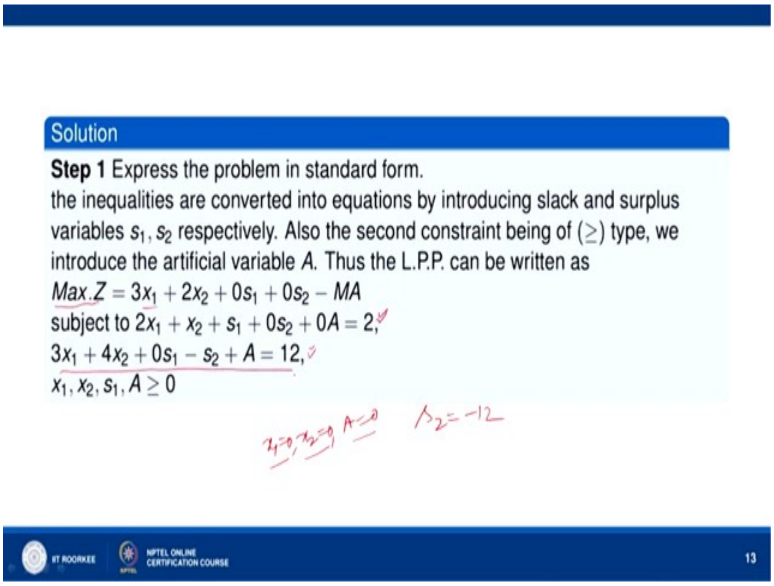

We can see here, there are two equations, 1, 2 and 1, 2, 3, 4, 5 variables are there: x_1, x_2,s_1, s_2 and A. So three variables can be chosen arbitrarily. So if we choose x_1, x_2; x_1 =0, x_2 =0 and A =0, if we take x_1 =0, x_2 =0, A =0, then from this equation what do we get? s_2 =- 12. So if choose variables x_1, x_2 and A =0, we get s_2 =- 12 but s_2 cannot be - 12.

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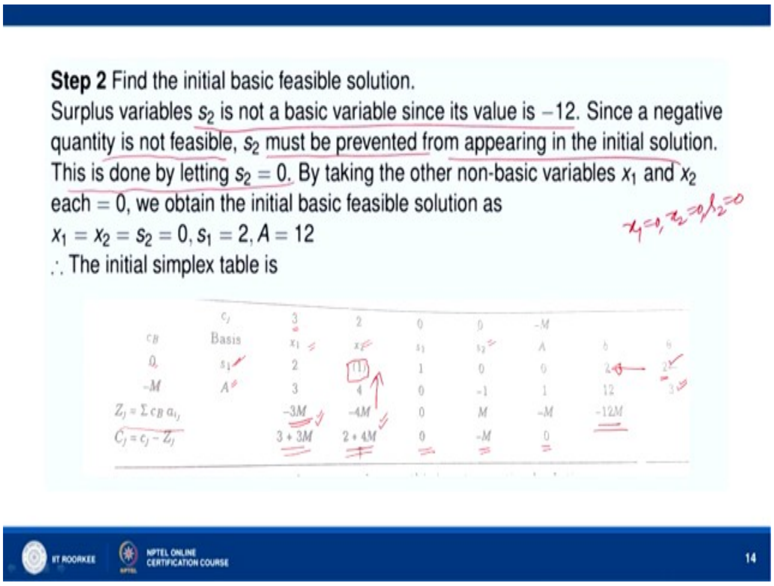

s_2 is, it is not a basic variable because its value is - 12. A negative quantity is not feasible, so s_2 must be prevented from appearing in the final solution. And therefore we take s_2 =0, from the initial solution, so we take s_2 =0. So what we do? We consider x_1 =0, x_2 =0, and s_2 =0. And when we do that, we have this, initial simplex table. C js are coefficients of x_1, x_2,s_1, s_2 and A in the objective function. This is CB column where now basis variables ares_1 and A.

And non-basic variables are x_1, x_2, and s_2; x_1, x_2, and s_2. So the coefficient of s_1 in the objective function is 0, coefficient of A in the objective function is 0. We then calculate Z j. Z j =Sigma CB a i j. 0 into 2 is 0. - M into 3 is - 3 M. The total is - 3 M. Then c 3 - of 3 M is 3 + 3 M here. And this becomes 2 + 4 M here. Here it is 0. Here it is - M, here it is 0.

And this is 0 into 2 is 0. - M into 12 is - 12 M, so this is - 12 M. Now we can see C j is positive in the column corresponding to x_1, in the column corresponding to x_2. But M is large, so we can say 2 + 4 M is greater than 3 + 4 M. So this is our key column. Now here what do we do? We divide the elements of b column by the corresponding elements of x_2 column. So 1 when divides 2, what we get is 2 and 4 when divides 12 we get 3. So minimum value is 2. So this is our key row. This is our key row.

And when this, so this is our key element. Now x_2 will, s_1 will be replaced by x_2. x_1 will be outgoing variable, s_2 will be incoming variable. So in place of s_1 we will have x_2. And here the coefficient of x_2 will come, that is 2.

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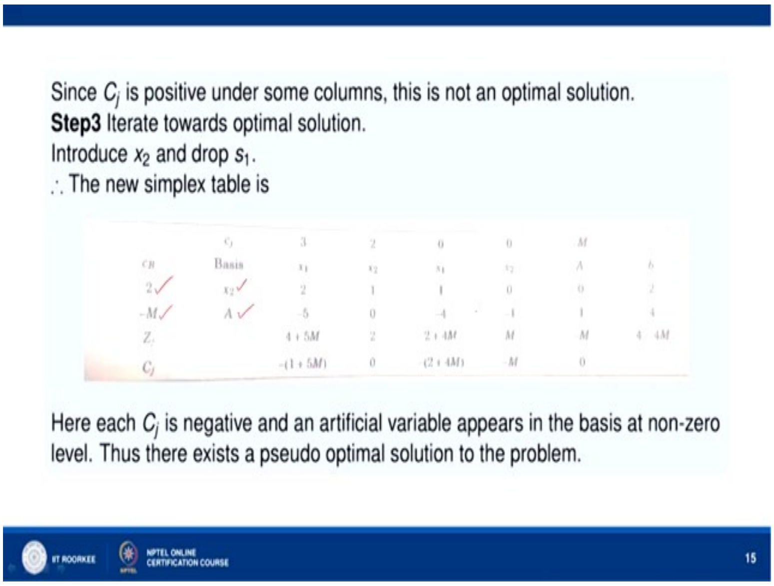

And we get this table: 2, x_2, - M, A, and then we get this table. So 2, 1, okay...

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Step 2 Find the initial basic feasible solution.

Surplus variables s_2 is not a basic variable since its value is -12. Since a negative quantity is not feasible, s_2 must be prevented from appearing in the initial solution. This is done by letting $s_2 = 0$. By taking the other non-basic variables x_1 and x_2 each $= 0$, we obtain the initial basic feasible solution as

$x_1 = x_2 = s_2 = 0, s_1 = 2, A = 12$

$\therefore$ The initial simplex table is

	c_j	3	2	0	0	$-M$		
c_B	Basis	x_1	x_2	s_1	s_2	A	b	θ
0	s_1	2	(1)	1	0	0	2	2
$-M$	A	3	4	0	-1	1	12	3
$Z_j = \Sigma c_B a_{ij}$		$-3M$	$-4M$	0	M	$-M$	$-12M$	
$C_j = c_j - Z_j$		$3 + 3M$	$2 + 4M$	0	$-M$	0		

Now what we do? We divide the element…the key row elements by this key element. So when we do that, it is 1, 1 divides these elements, so they are unaffected.

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Since C_j is positive under some columns, this is not an optimal solution.

Step3 Iterate towards optimal solution.

Introduce x_2 and drop s_1.

$\therefore$ The new simplex table is

	c_j	3	2	0	0	M	
c_B	Basis	x_1	x_2	s_1	s_2	A	b
2	x_2	2	(1)	1	0	0	2
$-M$	A	-5	0	-4	-1	1	4
Z_j		$4 + 5M$	2	$2 + 4M$	M	M	$4 - 4M$
C_j		$-(1 + 5M)$	0	$-(2 + 4M)$	$-M$	0	

Here each C_j is negative and an artificial variable appears in the basis at non-zero level. Thus there exists a pseudo optimal solution to the problem.

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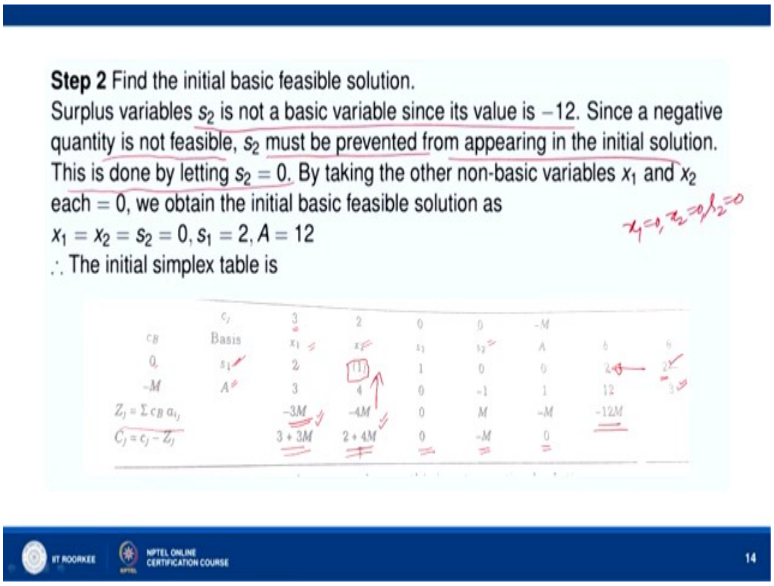

So we get the same elements. We get the same elements. And then with the help of this key element, with the help of this element 1, we make this element 0. To make this element 0, what we do? We have multiply this row, the elements of this row by 4 and subtract from this row. So we have to multiply it by 4 and subtract from the elements of the second row.

Now coefficient of x_1 is 2. And here in the next coefficient of x_1 is 3. So 3 - 4, we are multiplying by 4, so 2 into 4 is 8, so 3 - 4 is - 5. So we get - 5 here. This is 0 and this we get as what? We get here, we multiply by 4 and subtract, so - 4 we will get. So this is - 4.

And here what we will get? We multiply it by 4 and subtract from here. So 2 into 4 is 8, 8 when subtracted from 12 gives you 4, so we get 4. And then we again calculate Z j. Z j is 4 + 5 M. This is 2 here, this is 2 + 4 M. This is M, this is - M and here we get 2 into 2 is 4, - M into 4 is - 4 M, so we get 4 - 4 M. Now this is c j - Z j. We find c j - Z j. This is - 1 + 5 M. This is 0, this is - 2 + 4 M. This is - M. So C j < 0 for all j.

So C j is negative and an artificial variable…but what do we notice? An artificial variable A appears at non-zero level. You see in the b column, this is 4. So this artificial variable appears at non-zero level, therefore there exists a pseudo optimal solution to the problem. So that is all in this lecture. Thank you very much for your attention.