Higher Engineering Mathematics
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Lecture No 45
Simplex Method - II

Hello friends welcome to my 2nd lecture on Simplex Method. So here we shall consider some examples where we will have an unbounded solution or we will have more than one optimal solution. Let us consider the case of $Z=5x_1+4x_2$ which we have to maximise subject to the conditions $x_1 \le 7, x_1 - x_2 \le 8, x_1, x_2 \ge 0$ we shall see that. In the case of this LPP we have got an unbounded solution.

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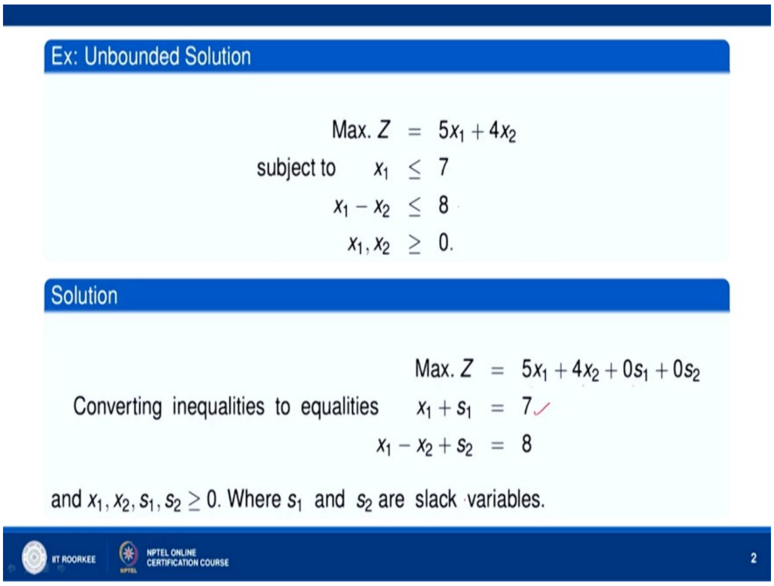

So we will convert it into the standard form maximum of $Z=5x_1+4x_2+0s_1+0s_2$

and s_1, s_2 are 2 slack variable which are being used to convert the constants into equations, so $x_1+s_1=7$ and $x_1-x_2+s_2=8$ okay $x_1, x_2, s_1, s_2 \ge 0$ s_1, s_2 are two slack variables.

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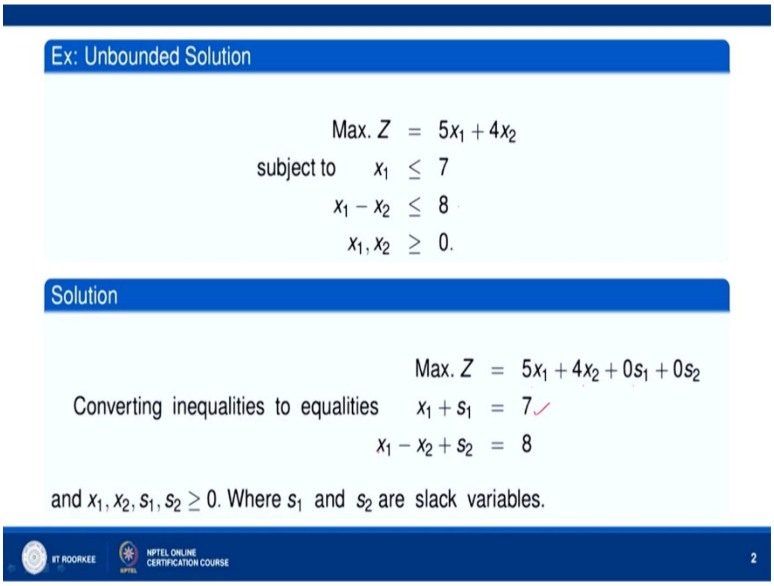

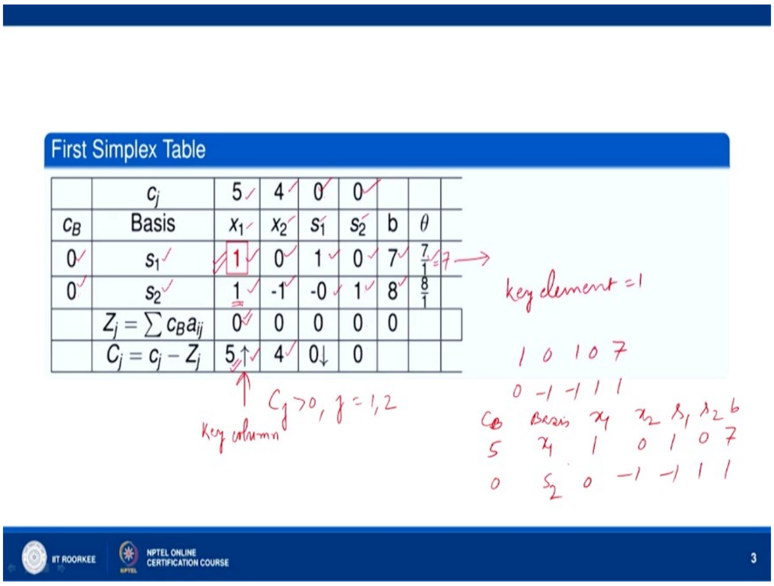

Now, let us write the corresponding 1st Simplex table, so we will have the Simplex table as C_j row contains the coefficients of the variables in the objective function so 5400 they are the coefficients of x_1, x_2, s_1, s_2. The basis elements are s_1, s_2, basic variables are s_1, s_2 their coefficients in the object are $0, 0$ and the body matrix is 10 because the coefficient of $x_1 = 1$ $x_2 = 0$, so 10 and then 0 minus 1, the coefficient of $x_1 = 1$ here, coefficient of $x_1 - 1$, so body matrix is $100 - 1$ and the other matrix is the unit matrix.

So 10 okay 01. Under b column we have the constants which occur on the right side of the equations they are 7 and 8, so we get 7 and 8 here. Now let us find the Z_j value $Z_j = \sum C_B a_{ij}$

$Z_j = \sum C_B a_{ij}$, so Z_1 for the column $Z_j = j = 1$ okay we have 0 into one plus 0 into one that is 0, so we get 0 here and then C_j capital $C_j = 5$, $C_j = c_j - Z_j$ so $5 - 0 = 5$ okay then similarly $z.0.0. = 0$, $0.(-1) = 0$, so we get $C_j = 4 - 0 = 4$ here and then $0.s_j = 0, 0.1 = 0, 0.0 = 0$, so we get 0 here $0 - 0 = 0$ and then we get $0.1 = 0$ so we get 0 here $0 - 0 = 0$ here.

Now we can see C_j is positive for $j = 1, 2$ okay C_j is greater than 0 when $j = 1, 2$ and the maximum value of $C_j = 4$ okay, so this is our key column okay. Now, let us find key row so we divide the elements of b column by the corresponding element in the key column, so $\frac{7}{1} = 1$, $7 y_1 = 1$ and $8 y_1 = 1$. Now we consider the minimum positive ratio, so this is 7 this is 8 minimum positive ratio is 7 okay, so this is our key row okay so at the intersection of key row and key column we have one here this is key element okay since it is already 1 we do not have to make it into one, so what we will do we will then make this element 0 in the other row okay in the key column we make with the help of the key element after it has been converted into duality all the other elements in the key column 0.

So there is one we have to make it 0, so we subtract this row key row from the 2nd row and then what we will get, we will get 10 this key row is 10107 okay and here it will become after subtracting 2nd row will become 0 and then we will get minus 1 then we get $0 - 1$, so we get minus 1, $1 - 0 = 1$ okay and here we get $8 - 7 = 1$ okay and this variable s_1 will be outgoing variable x_1 will be incoming variable.

So in the new Simplex table we will have C_B okay basis so basis now will contain variable x_1 s_2 the coefficient of x_1 is 5 coefficient of $s_2 = 0$ okay we have x_1, x_2 and then s_1, s_2 and b okay key row is 10107 okay and we have subtracted key row from the next row okay which became 0 minus 1 then we had minus 1 then we had 1 then we had $8 - 7 = 1$ so this is our new table, let us see the new table.

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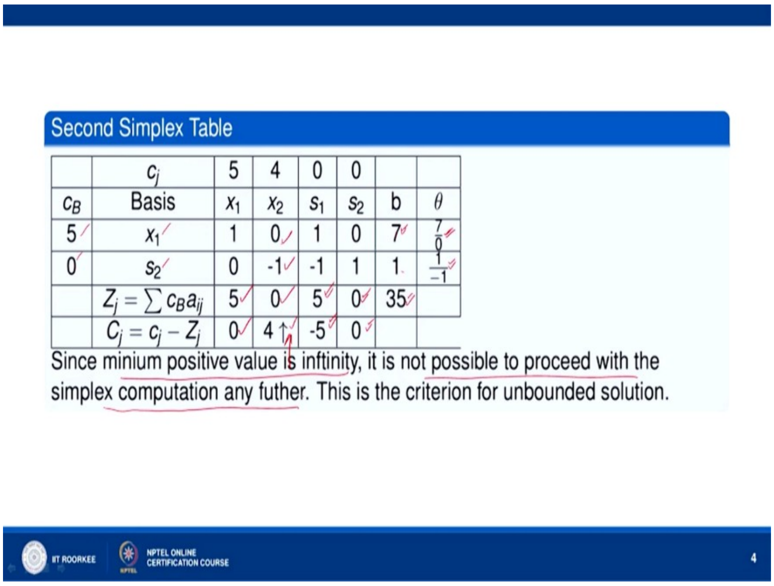

We have C_B column contains 5 0 $x_1\,s_2$ 1 0 1 0 7 0 minus 1 minus 1 1 1 okay now let us find $Z_j=0$, so x_1 $x_1 5.1=5$, 0 to 00 so we get 5 here $5-5-¿0$ and then $5.0=0$, $0.(-1)=0$ so we get 0 here $4-0=4$ and then $x_1 5.1.=5$, so we get 5 here $0-5=-5$ then $5.0=0$, $0.0=0$ here $0-0=0$. Now we can see $x_1 5.7=35$ this is 0, so this is $35\,C_B.\,b$ okay.

Now we have to see C_J is positive in the column $j=2$ okay, so this is our key column again this is a key column and so the elements of the key column are $0-1$ we will divide the corresponding elements and the key column okay, so $7/0$ we get here divided by 0 then 1 divided by minus 1 we get this. Now we have to consider only positive values in the theta column to determining the minimum positive theta okay, so minimum positive value is only 1 that is 7 by 0 okay so minimum positive value is infinity and therefore, it is not possible to proceed with the Simplex computation any further and so we get an unbounded solution here.

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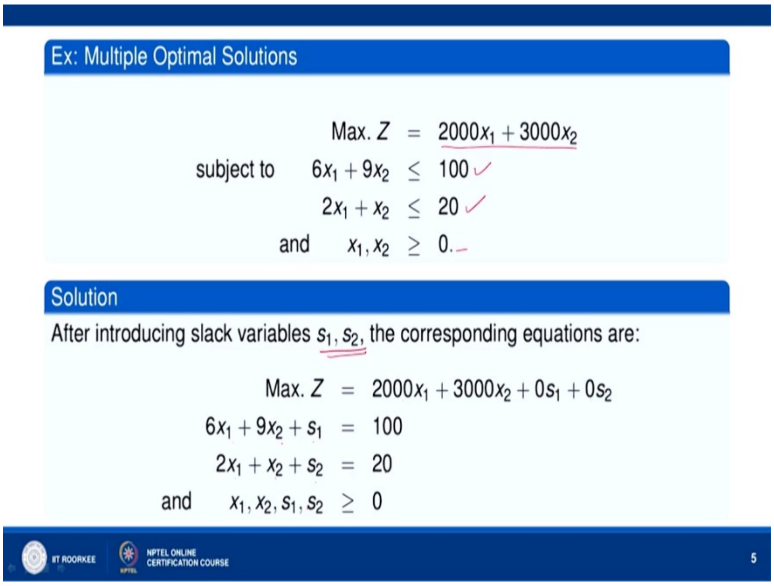

Okay, now let us consider 2nd problem we have the function objective function $Z=2000x_1+3000x_2$ and we are given to constants $6x_1+9x_2\leq 100$, $2x_1+x_2\leq 20, \wedge x_1, x_2\geq 0$ okay. We introduce slack variables x_1, s_2 to convert constants into equations, so $6x_1+9x_2+s_1=100, 2x_1+x_2+s_2=20$, $x_1, x_2, s_1, s_2\geq 0$ and in the objective function we take the coefficients of $s_1, s_2=0$, so maximise

$Z=2000x_1+3000x_2+0s_1+0s_2, \wedge x_1, x_2, s_1, s_2\geq 0$

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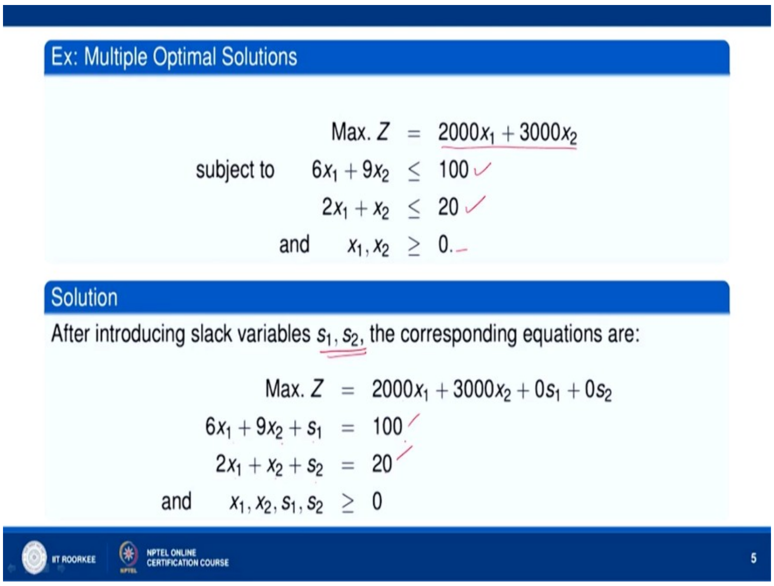

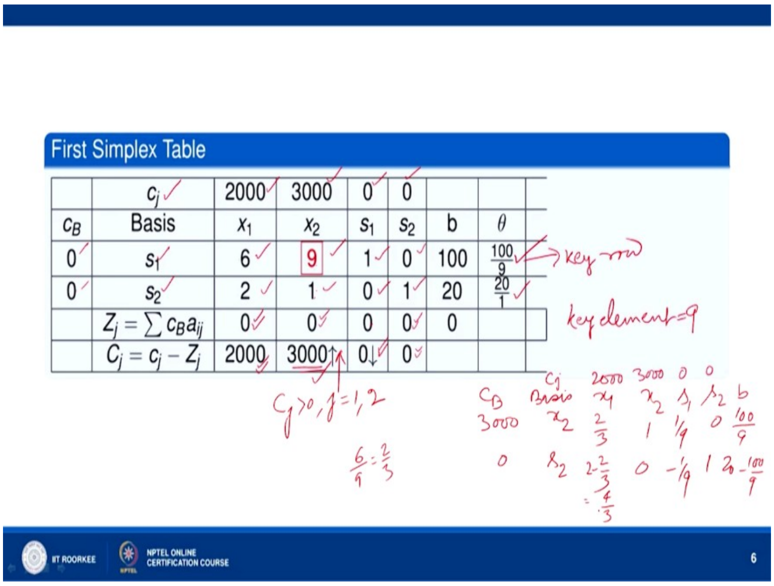

	c_j	2000	3000	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	6	9	1	0	100	$\frac{100}{9}$
0	s_2	2	1	0	1	20	$\frac{20}{1}$
	$Z_j = \sum c_B a_{ij}$	0	0	0	0	0	
	$C_j = c_j - Z_j$	2000	3000	0	0		

Now, let us write the 1st Simplex table, so C_j column contains C_j row contains 2000, 3000, 0, 0 which are the coefficients of x_1, x_2, s_1, s_2 basis variables are s_1, s_2 their coefficients in the objective function are 00 okay. Body matrix is 6921 that is body matrix 6921 and unit matrix is 1001 okay. Now we have to find the Z_j value theta column is the sorry b column is containing 100 and 20 these are the values 100 and 20 which constitute the b column okay. Now, let us find a Z_j. $Z_j=1$ we take so $0.6, 0.2$, so we get $2000-0=2000$ okay.

Similarly, $0.9, 0.1=0$ so we get 0 here as Z_2 value is $Z_2, j=2$ gives Z_2, $Z_2=0$,

$3000-0=3000$ then $Z_2 0.1=0, 0.0=0$, so we get 0 here $0-0=0$ and then $0.10=0, 1.0=0$ we get 0 here, $0-0=0$. Now so C_j is greater than 0 for $j=1,2$ the maximum value is 3000, so this is our key column okay. Elements of b column are divided by the corresponding elements in the key column, so 100 over 9 we get and 20 by 1 we get okay. We will take the minimum positive ratio so 100 by 9 is minimum ratio here, so therefore this is our key row. Okay at the intersection of key row and key column we have 9, so key element is 9 okay.

So this means what? This means that as 1 is outgoing variable okay x_2 is incoming variable and therefore the new Simplex table will be CB in place of 0 now we will have s_1 is outgoing x_1 is coming in so coefficient of $x_2=3000$, so 3000 okay basis will contain x_2 and s_2 okay and here we will have complement of $s_2=0$ in the objective function we have x_1, x_2, s_1, s_2, b so this is 2000 this is 3000 this is 0 this is 0 this is c_j of a small cj and then we divided key row by the key element, so we divided key row by 9 because 9 is the key element and we get here

$\frac{6}{9}, \frac{6}{9}-\frac{2}{3}$ okay so this is $\frac{2}{3}$ this is 1 here this is $\frac{1}{9}$ this is 0 okay they are dividing by 9 and then we get $\frac{100}{9}$ okay. Now, once we have made this key element 1 we subtract suitable multiples of it from the other rows to make the other elements in the key column as zeros, so this 1 okay and this is also 1, so we subtract just we subtract this row from the 2nd row, so we get $2-\frac{2}{3}$ and we get $1-10$ and then we have here $0-\frac{1}{9}=\frac{-1}{9}$ and here we get $1-0=1$, so we get one here and then $20-\frac{100}{9}$, so how much is this? $\frac{-4}{3}$ okay.

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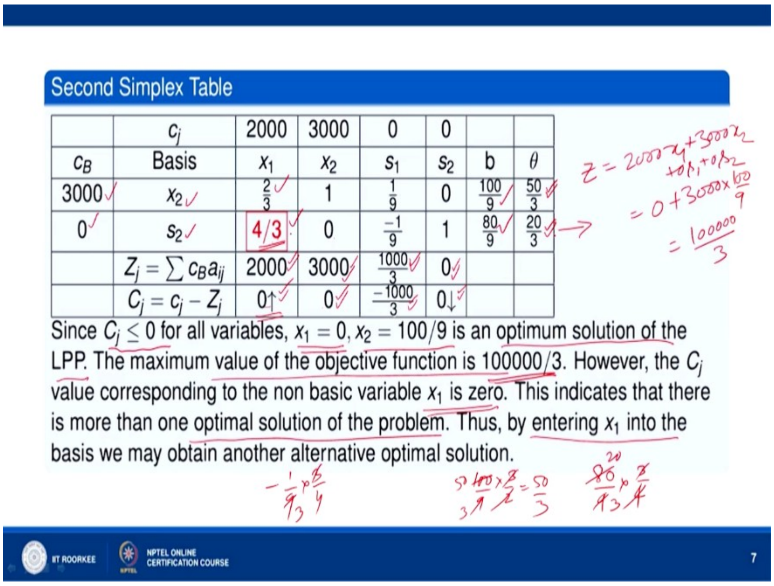

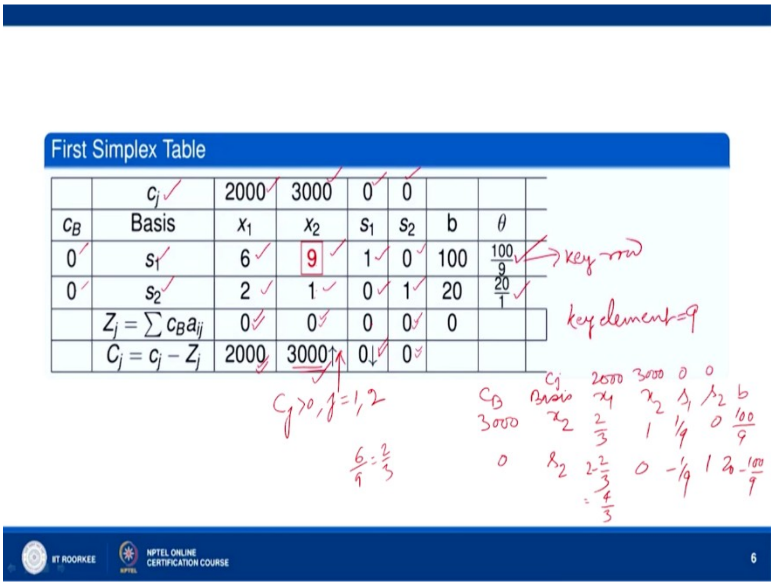

So what we get in the next table let us see, so 3000 0 x_2, s_2 okay and 2/ 3, 4 /3 that is 1st column 2 /3 , 4 /3 the 2nd column is 10 okay 3rd column 1 by 9 -1 / 9, 1/9- 1/ 9 and then 01 okay, so we have 01, 01 then b column becomes 100 / 9 and 80/ 9 okay, so 80/ 9 okay so here you can see 100 / 9 and 80 / 9 okay.

Now, let us find Z_j value Z_j is equal to 3000 .2 / 3 that is 2000 this is 0, so we get 2000 here, 2000 - 2000 = 0 and here we get 3000 . 1 = 3000, 0 . 0 =0, so we get 3000, 3000 -3000 =0 then we get 30001 /9, so we get 3000 to 1/ 9 that is 1000 by the and this is 0, so we get 1000/3 0 - 1000 / 3 = -1000 /3 okay so this is minus -1000/ 3 and then we get 0 . 3000 =0, 0 . 1=0, so we get 0 here 0-0=0 so we get 0 here.

Now we see C_j is 0- 3/ 1000 by 90 so C_j is less than or equal to 0 for all variables okay. Basic variables are x_2 and s_2 okay so $x_2=\frac{100}{2}$, $s_2=\frac{80}{9}$, $x_1=0$, so $x_2=0$, $x_2=\frac{100}{2}$ is an optimal solution of the LPP. The maximum value of the objective function is you can see

$$Z=2000x_1+3000x_2+0s_1+0s_2$$

$s_1=0, x_2=100/9$, so 0 x minus 0, so 3000 . 100 / 9, so what we get this 1,00,000 divided by 3 okay, so 1,00,000 by 3.

However C_j value corresponding to the non-basic variable now x_1 is a non-basic variable okay and C_j value corresponding the non-basic variable is 0 okay, C_j value corresponding the non-basic variable $x_1=0$ this indicates that there is more than one optimal solution of the problem, so what do we do?

By entering x_1 into the basis and may obtain another optimal solution okay. So if you take this as key column okay and this minimum positive ratio here you see 1000 / 9 okay we have this we take as key column if we take key column then we have to find key row so we divide by 4 by 3, so 80 / 9 .3 /4, so we get 20 / 3 okay and then 2 /3 when 100 / 9 is divided by 2 / 3 we get 3 / 2, so 50 / 3 okay.

Now we have to take minimum values, so 50 /3 this more than 20 / 3, so this is key row at the interception of key row and key column we get 4 / 3 so this is key element okay, so we divide key by 4 / 3 okay now what will happen? x_2 will be incoming variable s_2 will be outgoing variable and we will have the following table.

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Second Simplex Table

	c_j	2000	3000	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
3000	x_2	$\frac{2}{3}$	1	$\frac{1}{9}$	0	$\frac{100}{9}$	$\frac{50}{3}$
0	s_2	$4/3$	0	$\frac{-1}{9}$	1	$\frac{80}{9}$	$\frac{20}{3}$
	$Z_j = \sum c_B a_{ij}$	2000	3000	$\frac{1000}{3}$	0		
	$C_j = c_j - Z_j$	0↑	0	$\frac{-1000}{3}$	0↓		

Since $C_j \leq 0$ for all variables, $x_1 = 0, x_2 = 100/9$ is an optimum solution of the LPP. The maximum value of the objective function is $100000/3$. However, the C_j value corresponding to the non basic variable x_1 is zero. This indicates that there is more than one optimal solution of the problem. Thus, by entering x_1 into the basis we may obtain another alternative optimal solution.

Third Simplex Table

	c_j	2000	3000	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
3000	x_2	0	1	$\frac{1}{6}$	$\frac{-1}{2}$	$\frac{20}{3}$	
2000	x_1	1	0	$\frac{-1}{12}$	$\frac{3}{4}$	$\frac{20}{3}$	
	$Z_j = \sum c_B a_{ij}$	2000	3000	$\frac{1000}{3}$	0		
	$C_j = c_j - Z_j$	0	0	0	$\frac{-1000}{3}$		

However, the value of the objective function will not be improved, although the present solution $x_1 = 0, x_2 = 100/9$ is shifted to $x_1 = 20/3$ and $x_2 = 20/3$.

So 3000 2000 x_1, x_2 okay and we divide this by 4 / 3 so this will become 10 and - 1/ 9 into 3 / 4, so - 1 / 12 we are dividing by 4 by 3, so minus 1 / 12 we have and then 1 / 4 /3, so 3 / 4 we have so this is 3 / 4 okay and then 80 / 9 divided by 4/ 3 so we got 20 / 3, so this is 20 / 3 here okay. Now this element which has become 1 okay this element which has become 1 with the help of this we make the other entry in the x_1 column 0 okay so this has become 1 this we have to make 0, so we multiply it by 2 /3 and subtract from this row.

So this will become 0 okay we are multiplying the new row, new key row by 2/ 3 and subtracting from this row okay, so we have 2 /3 multiply here so and multiply here and subtract here this becomes 0, there is already 0 so it will not be change this is one and -1/ 12, minus 1 by 12 we are multiplying by 2/ 3 okay, so 2 /3 so minus 1 / 18 we are multiplying by

2 /3 and subtracting from the 1st row okay, so what is that 1 / 9, 1 / 9 + 1 / 18 this gives us 1 / 6 okay. Similarly this becomes minus half and this becomes 20 / 3, how there is 20 / 3? We have 100/ 9, 100 / 9 we have okay and this is 20 / 3, so -20 /3 into 2 /3 so this is 40 / 9 we get so 60 /9 we have okay which is 20 /3, so this is 20 / 3 okay now we find Z_j. Z_j is 0 . 3000, 1 . 2000, so 2000 here, here it is 3000 here it is 1000 / 3, 2000-2000 = 0 , 3000- 3000 = 0, 0 - 0 = 0 this minus 1000/ 3.

Now you can see value of the objective function will be how much, so $x_2=¿$ 3000, x_2 we have okay, so 3000 . 20 / 3 okay Z will be equal to 3000 into 20 by 3 plus 2000 into 20 by 3, so this is 20 . 3000 so 60,000 + 40,000 so we will get 100,000, so we get the same objective value of the… same value of the objective function okay but the present solution is now $x_1=20/3$ okay $x_1=20/3$ and $x_2=\frac{20}{3}$, so the solution has of course has change from $x_1=0$, x2 equals $x_1=\frac{100}{9}$ to $x_2=\frac{20}{3}$ but the value of the objective function remains the same, so we have multiple optimal solutions but the object functions value is the same okay.

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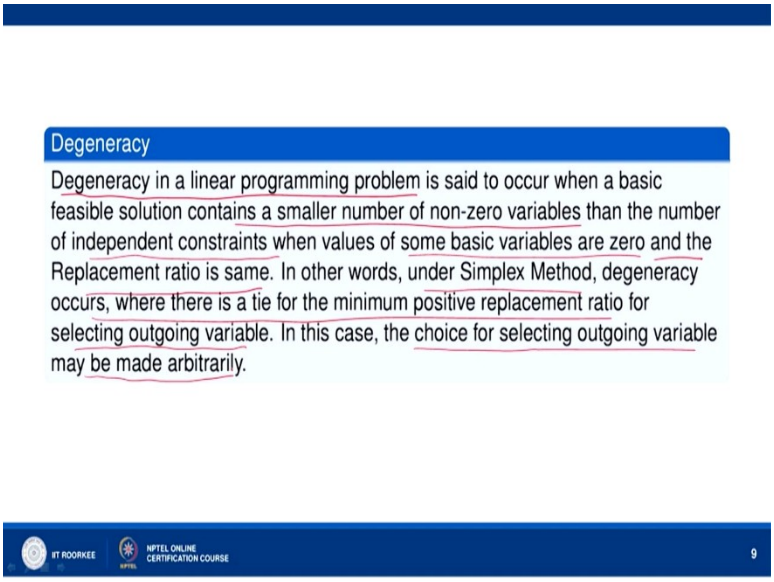

So now let us go to degeneracy, degeneracy is a linear programming problem is said to occur when a basic feasible solution contains a smaller number of non-zero variables than the number of independent constraints values of some basic variables are 0 okay, so and the replacement ratio theta is the same. In other words under simplex method, degeneracy occurs when there is a tie for the minimum positive replacement ratio theta, so when the theta is the same for 2… when there are 2 values which are same in the theta column okay then how to choose the outgoing variable in this case the choice for selecting outgoing variable can be made arbitrarily. So then in the theta column we have 2 values which are same than how to determining the minimum positive replacement ratio, so we can select any one as the minimum positive replacement ratio if there is a tie, so the choice for selecting outgoing variable may be made arbitrarily.

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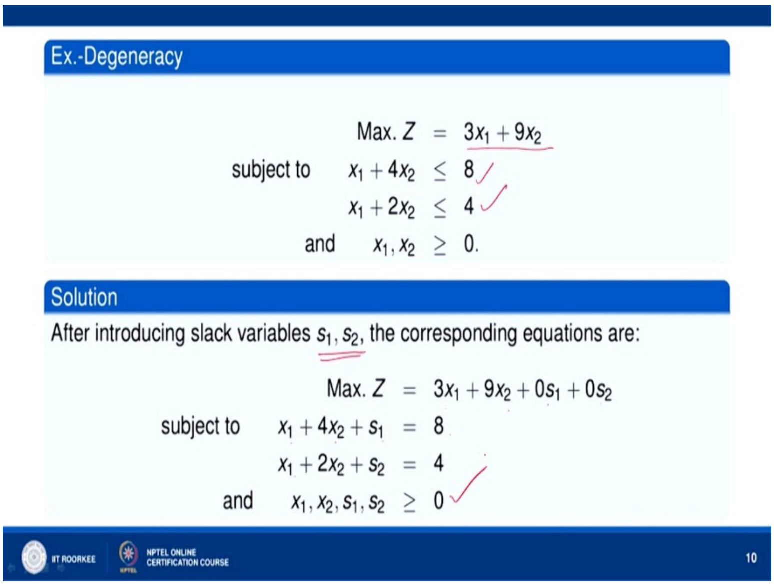

Let us take an example on this suppose we have maximum of $Z=3x_1+9x_2$

$x_1+4x_9\leq 8, x_1+2x_2\leq 4 \wedge x_1, x_2\geq 0$ to select variable as well as 2 and when the LPP into standard form so $3x_1+9x_2+0s_1+0s_2$, $x_1+4x_9+0s_1=8, x_1+2x_2+0s_2=4$, and $x_1, x_2, s_1, s_2\geq 0$.

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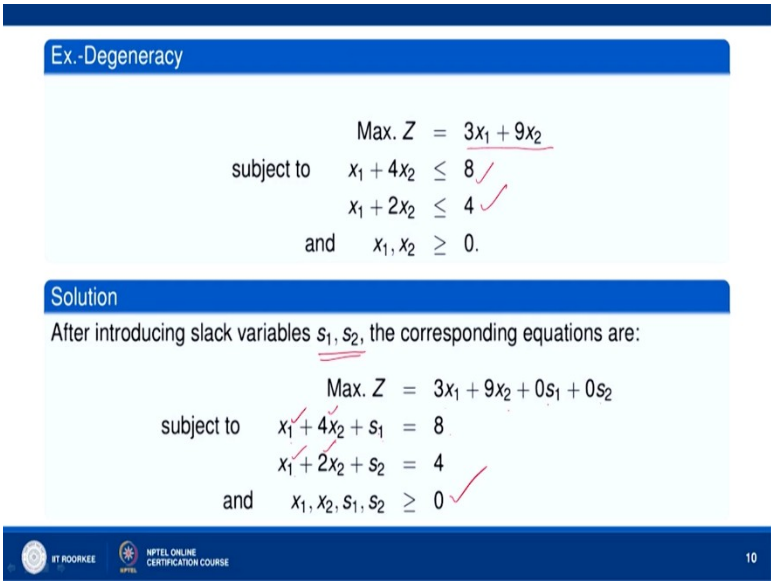

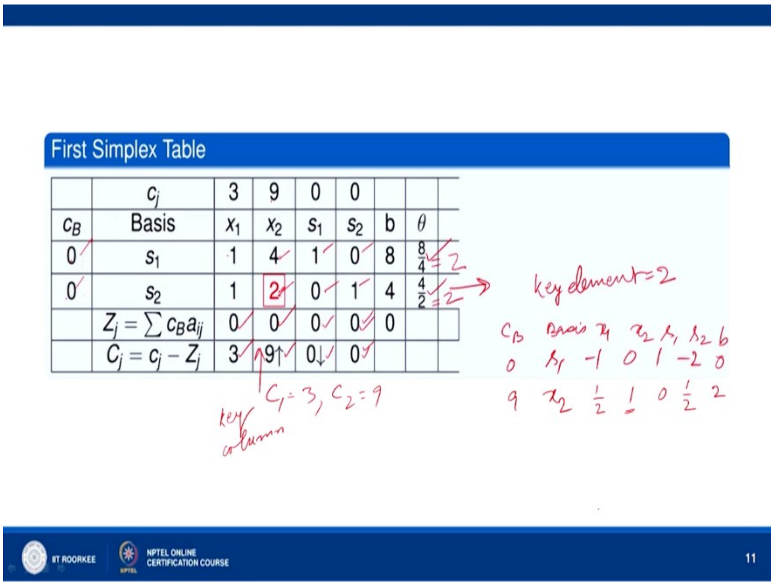

Now we can write simplex table so coefficients of x_1, x_2 are 3 , 9 coefficient of s_1, s_2 are 0, 0 basic variables are s_1, s_2 their coefficients in the objective functions are 0, 0 body matrixes this 1 4 then 2 okay let us see. 1412 okay and the unit matrixes 10 01 okay this is 10 01 then we can find Z_j. Z_j we can find 0. 1, 0. 1=0, so 3- 0 = 3 and then 0 . 4 = 0, 0 . 2 = 0, 9- 0= 9, 0.1 = 0, 0 . 0 = 0, so we get 0-0= 0 then 0 . 0 = 0, 0 . 1 = 0 we get 0 -0 = 0 . Now there are 2 values Z_j which are positive then j is 1 we get $C_1=3$ and $C_2=9$ okay, so we can take the maximum of these 2 so 9 is obviously the maximum, so key column is this one this is key column okay.

Now the elements in the key column are 4 and 2 divided b column values by these key column values, so 8 / 4 okay and then 4 / 2. Now you can see this is 2 this is 2 both the values of theta are positive and they are equal, so how to find the minimum positive theta, minimum positive theta is 2 okay so there is a tie and therefore it is how to select the outgoing variable, so we can choose any row as the key row. Let us choose this row as the key row okay then at the intersection of key row and key column we have 2, so this 2 is key element okay.

So we divide this row by 2 okay so now what will happen s_2 will be going out okay and x_2 will be coming in, so you simplex table will be C_B 0 basis we have s1 and here we will get in place of x_2, x_2 the coefficient of $x_2=9$ okay and we will have here this row will now be divided by 2 okay so we will have 1 by 2 1 0 1 by 2 okay 4 is divided by 2 so we get 2 and we have x_1 here x_2 here s_2 here s_2 here b here okay.

Now this new row okay in the new row this key element has become 1 the multiply suitably this row and subtract from the other row to make the corresponding element in the key column 0, so here it is 4 so we multiply it by 4 and subtract from here so 4 into half is 2, 1-2 =-1 then 4 - 4 =0 then we get 1 minus 0 is 0 sorry 1 minus 0 is 1, so we get one here and then here we have 0 and here we have half okay, so we multiply by 4 so 4 into half is 2 we are subtracting 2 from 0 so we get minus 2 here and here we are multiplying by 4 okay so 8, 8 we are subtracting from 8 so we get 0 okay.

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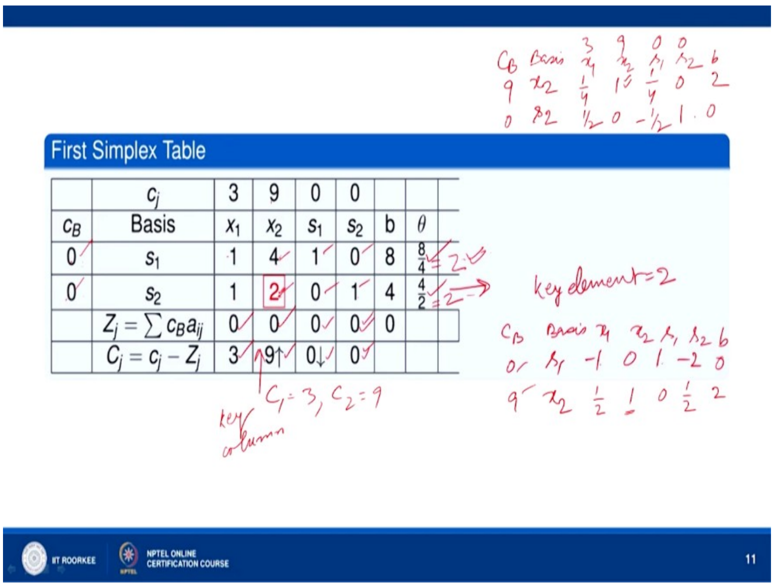

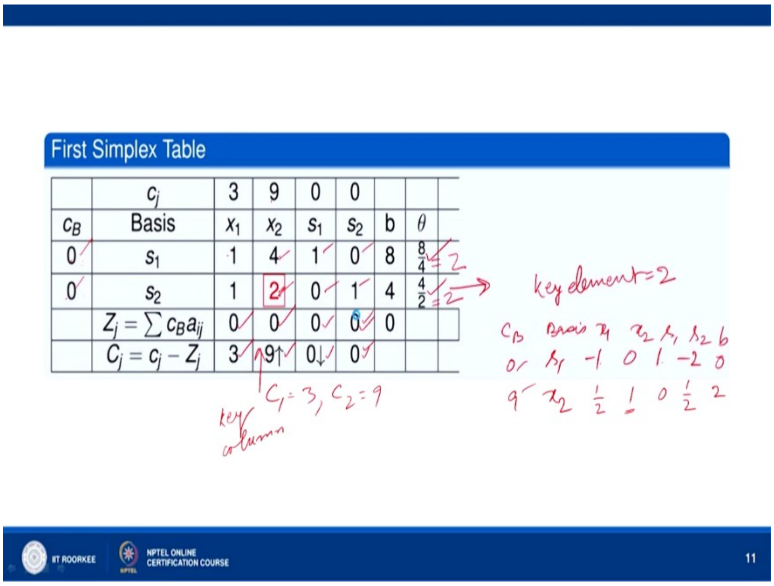

Now so this is our new table let us see, so let us go to the next table C_B okay we have then x_1, s_2 we have we have minus 1 -0 -1- 2, - 1 0 1 minus 2 okay, minus 1 0 1- 2 0 we have so let us see yeah this is okay then half 1 0 half 2, half 1 0 half 2 okay. Now we find Z_j here so

0 .(- 1) 9 into half okay so we get 9/ 2 okay 0 . (-1) .0 into half is 9 by 2 so it should be 9 by 2 so 3 - 9 / 2 will give us Cj so there is 6 - 9 / 2 will become - 3 / 2 okay and then here what will happen 0 . 0= 0 , 9 .1= 9 so this should be 9 okay so 9 - 9 = 0 then 0 . 1 = 0, 9 .0 =0.

So we get 0 here 0 . (- 2), 9 into half, so we get 9 / 2 so this is 0 .(-9) / 2 = - 9/ 2 okay and then 0 . 0= 0, 9 . 2= 18. So this should be 18 okay, this should be 18 now what Cj values are minus 3 / 2= 0- 0- 9, so $C_j \le 0$ so optimal solution is $x_1 = 0$, $x_2 = 2$ okay $x_2 = 2$ and $Z = 2$ okay. So here what we have got maximum value of $Z = 18$, $x_1 = 0$, $x_2 = 2$ now this we got while we considered this row as the key row. Let us see if I select this row as the key row then what happens? So if we do that what will happen?

Yes so if I take this as the key row okay suppose I take this as the key row then what will happen? $x_1$will be going out and $x_2$will be coming in okay so the new row will be $C_B$we will have this element, this element will be our key element now instead of 2 we will have 4 now as key element, so x_2 will be coming in in place of s1 the coefficient of $x_2 = 9$, so we have 9 0 basis will be having x_2 here x_2 will be coming in place of s_1 and we have s_2 here okay yes and we will have here 3 9 0 0 x_1, x_2, s_1, s_2 okay. After we have replaced this s1 by x_2 we divide this row by 4 okay, so we get 1 / 4 we get 1 here we get here 1 / 4 and we get here 0 and we get here to okay.

Now with the help of this element 1 okay this element 1 we make this element 0, so we multiply by 2 and subtract from here, so this 2 / 4 we subtract from 1 so we get 1 minus half, so we get half here then this is 2- 20 then there is 0 and here it is 1 /4 . 2 so we get 9 as half here and we multiply it by 2 right and subtracting from this okay so this will remain 1 okay and we are multiplying by 2 so this one is 4 4 we subtract from 4 we get 0 here okay, so let us see what we have there.

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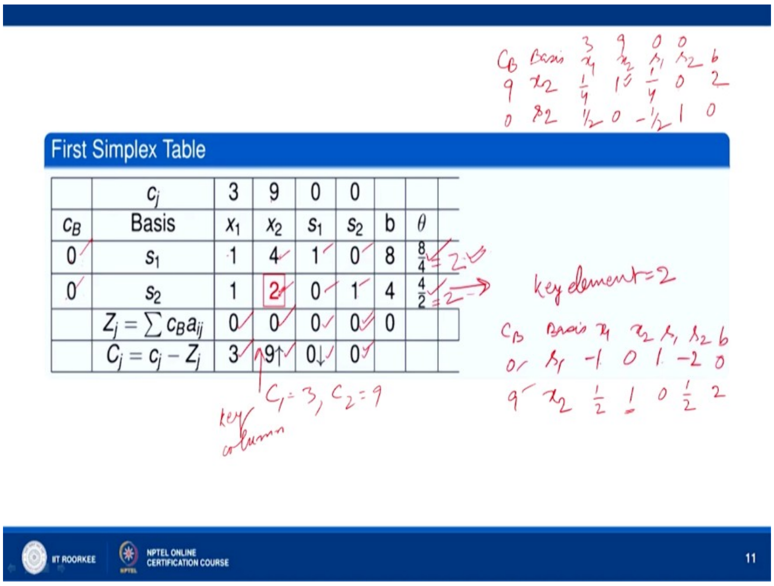

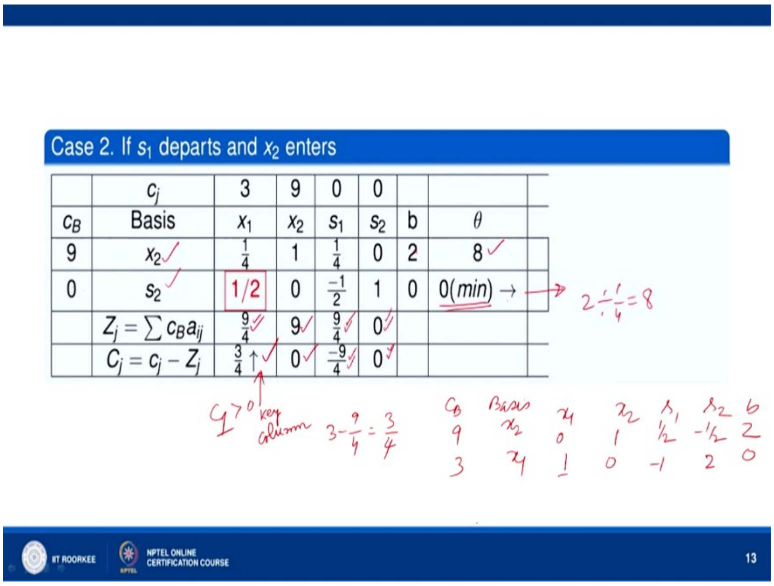

You see we have $x_2 . s_2$ okay and we have 9 0 here the elements of 1 by 4 1 1 by 4 02 1 by 41 1 by 402 and we have half 0 minus half half 0 minus half and we have 10 okay. So now we go there and we find the Z_j, Zj = 0.9 . 1 /4 , 9 /4 0 into half this is 9/ 4, 3- 9/ 4, 3 - 9 /4 gives you 3 / 4 so we get 3 /4 here and then 9 .1 = 9, 0 . 0 = 0 so we get 9 here 9 – 9= 0 and then 9 /

4, 9 . 1 /4 = 9/ 4 this 0 is 9 /4 , 0 -9 /4 = - 9 / 4, 9 .0=0, 0 so we get 0-0=0 0 okay, so C_j, j=1, $C_j \geq 0$ so this is key column okay.

Let us find the Theta value? So 1 / 4 we divide 2, 2 / 1 by 4 we get 8 and here what we have 0 divided by half so we get 0 okay, so minimum of the values is this 0 okay, so this is key row okay we divide key row by half okay so what we will get this will become 1. This s_2 will be replaced by what? Yeah s_1 okay so basis will be our s_2 x_1 okay C_B will be 9 and 3 okay. So this row will become after dividing by half it will become 1, 0 minus 1 2 okay and we are dividing by half so this will give 0 okay. Now with the help of this 1 we make this 0 so we multiply 1 / 4 subtract from here so 0 1 / 4 we multiply to this and subtract from here so we get 1, 1 / 4 we multiply here so minus 1 / 4 we subtract from there so we get 1 / 2 and then 1 / 4 we multiply here 2 / 4 is half, half we subtract from 0 so we get minus half and we multiply it by 1 /4 and subtract from there so this will remain 2 okay.

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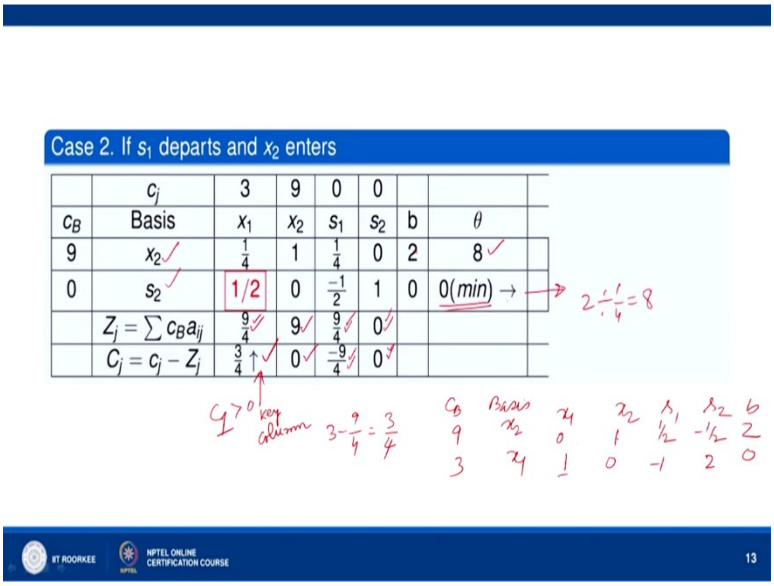
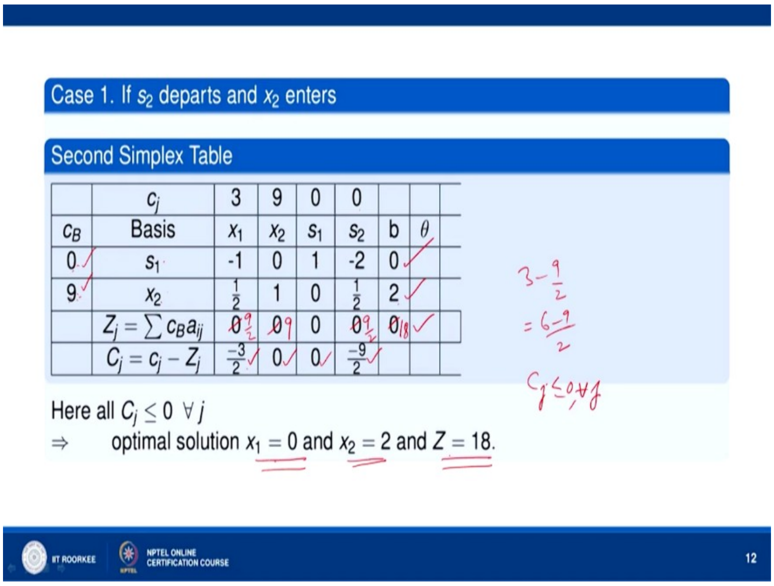
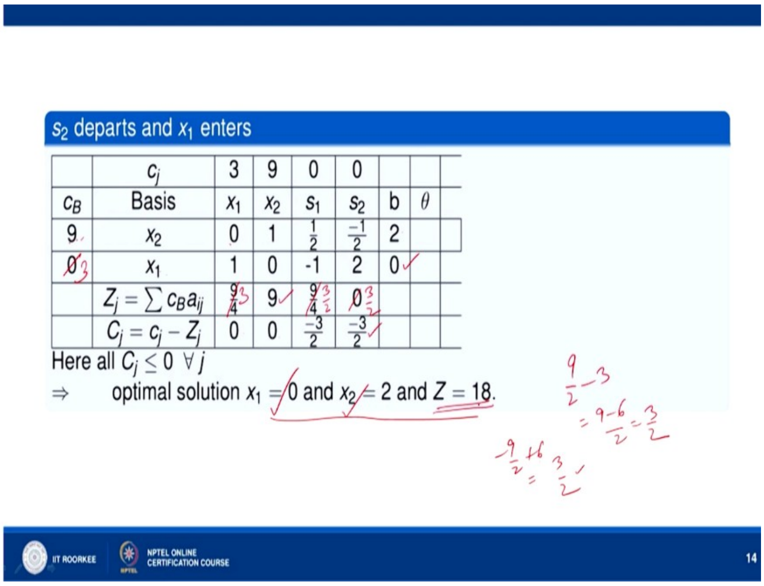




	c_j	3	9	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	-1	0	1	-2	0	
9	x_2	$\frac{1}{2}$	1	0	$\frac{1}{2}$	2	
	$Z_j = \sum c_B a_{ij}$	0	0	0	0	0	
	$C_j = c_j - Z_j$	$\frac{-3}{2}$	0	0	$\frac{-9}{2}$		



	c_j	3	9	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
9	x_2	0	1	$\frac{1}{2}$	$\frac{-1}{2}$	2	
0	x_1	1	0	-1	2	0	
	$Z_j = \sum c_B a_{ij}$	$\frac{9}{4}$	9	$\frac{9}{4}$	0		
	$C_j = c_j - Z_j$	0	0	$\frac{-3}{2}$	$\frac{-3}{2}$		

Yes so let us see this new table so we get $9x_1$ this should be 3 okay we get 0 one half minus half 2, 1 0 minus 1 2 1 0 minus 1 2 0 okay yes. Now let us find Z_j so 9. 0 0 3 into one is 3 9 into 0 0 9 3 into 1 is 3 this should be 3 okay 3 - 3 = 0 then 9 .1 =9 this is 0 so we get 9 and 9-9=0 is 0 then we get 9 .1/ 2, 9/ 2 and this is minus 3, 9 / 2 minus 3.

So we get 3 by 2, 3 / 2 and then 9 into minus half minus 9 / 2 3 into 2 6 so minus 9 by 2 plus 6 so minus 3 by 2 sorry this is 12 minus 9 so plus 3 / 2, so 3 by 2 you will have here okay 9 minus 9/2+ 6 so 3 by 2 here, so 0- 3 / 2 =-3 / 2 okay now $C_j \leq 0$ for all j so what do we have okay $x_2 = 2$, so $x_1 = 0$ $x_2 = 2$ and 9 . 2 = 18 and 3 . 0 = 0, so $Z = 18$, so we get the same value of Z okay. In the previous case we had $x_1 = 0$ $x_2 = 2$, $Z = 18$ and after the solution it was

$x_1 = 0$ $x_2 = 2$

$Z = 18$ and while making the other choice again we get the same optimal solution.

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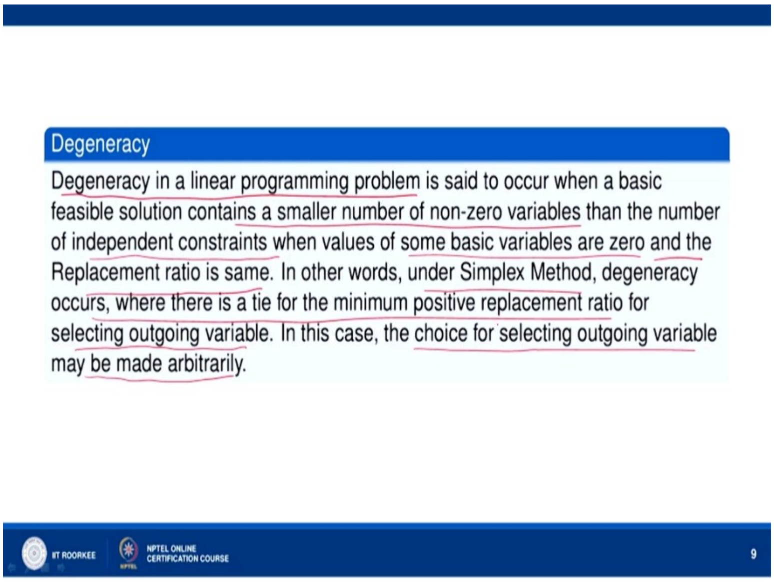

So we can choose when there is a tie in that it column okay when there is a tie for minimum positive replacement ratio for selecting outgoing variable the choice for selecting outgoing variable can be made arbitrarily this we can see from here okay we get the same solution okay. So we have discussed various cases, the cases where we found an unbounded solution, the case where we found more than one optimal solution and also the case of degeneracy where we discussed the problem where the value in the theta column were same.

So how we were not able to decide which one to take as the minimum positive ratio because because of a tie, so the selection of outgoing variable how it should be made we have discussed and we saw that choice can be made arbitrarily in both the cases we get the same optimal solution. So in case of a tie in the theta column the choice of selecting outgoing variable may be made arbitrarily, so with that I would like to end my lecture thank you very much for your attention.