Higher Engineering Mathematics
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Simplex Method - I
Mod09_Lec44

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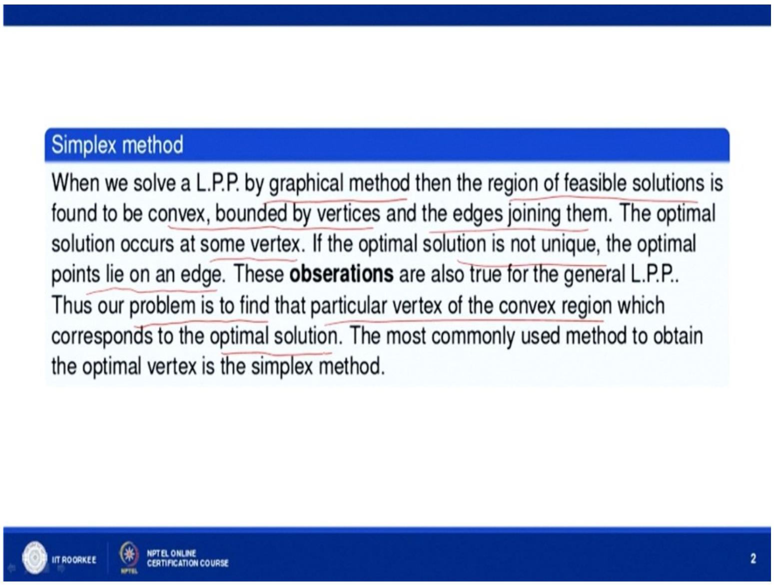

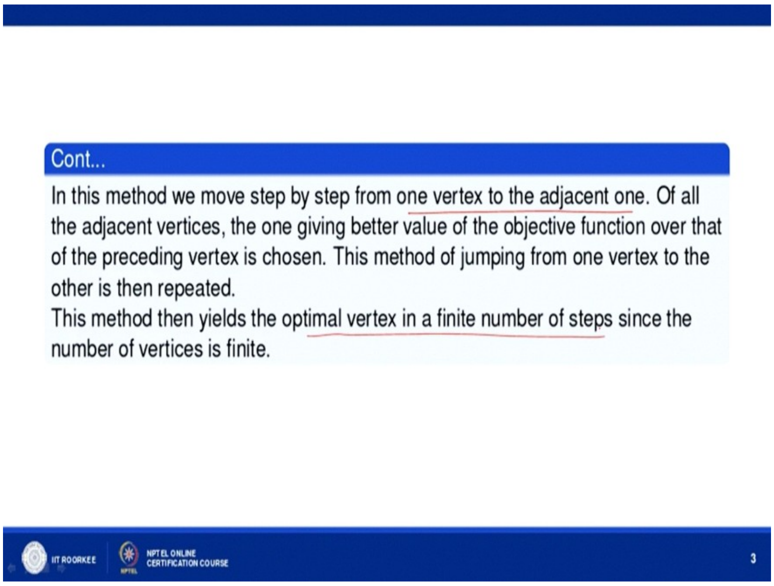

Hello friends, welcome to my lecture on Simplex Method. The first lecture on simplex method. When we solve a linear programming problem by graphical method, we notice that the reason of feasible solutions is a convex region, it is bounded by vertices and the edges joining them. The optimal solution occurs at some vertex. If the optimal solution is not unique, the optimal points lie on an edge. These observations are also true for the general linear programming problem.

Thus our problem is to find that particular vertex of the convex region which corresponds to the optimal solution. The most commonly used method to obtain the optimal vertex is the Simplex method. In this method, we move step-by-step from one vertex to the adjacent one, of all the adjacent vertices, the one giving better value of the objective function over that of the preceding vertex is chosen. This method of jumping from one vertex to the other is then repeated. This method then yields the optimal vertex in a finite number of steps since the number of vertices is finite.

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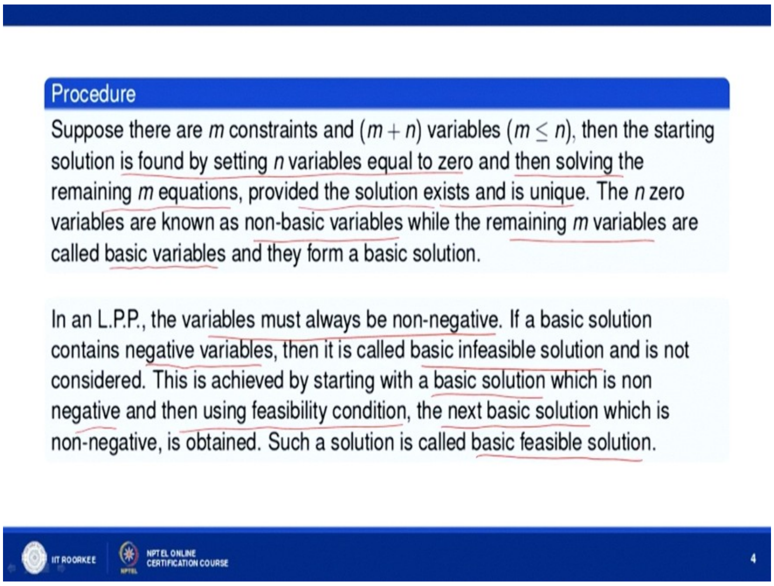

Suppose there are M constraints and M plus N variables, M less than or equal to N, then the starting solution is found by setting N variables equal to zero, and then solving the remaining M equations, providing the, provided that the solution exists and is unique. The N zero variables are called non-basic variables, while the remaining M variable are called basic variables and they form a basic solution.

So in an linear programming problem, the variables must always be non-negative, this is very important, in a linear programming problem the variables must always be non-negative, if a basic solution contains negative variables, then it is called a basic infeasible solution and it is not considered. To achieve this we start with a basic solution which is nonnegative and then use feasibility condition to obtain the next basic solution, such a solution is called basic feasible solution.

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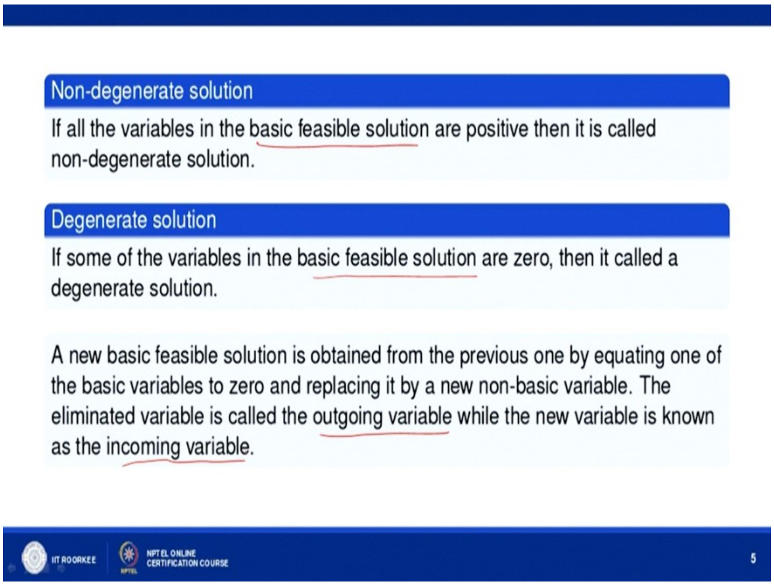

If all the labels in the basic feasible solution are positive then it is called non-degenerate solution. If some of the variables in the basic feasible solution are zero, we will call it as a degenerate solution. A new basic feasible solution is obtained from the previous one by equating one of the basic variables to zero and replacing it by a new non-basic variable, the eliminated variable is called as the outgoing variable while the new variable is called as the incoming variable.

Okay, so to obtain a new basic feasible solution we equate one of the basic variables to zero, replace it by a new non-basic variable and then the eliminated variable is called as the outgoing variable, the new variable that comes in is called as the incoming variable.

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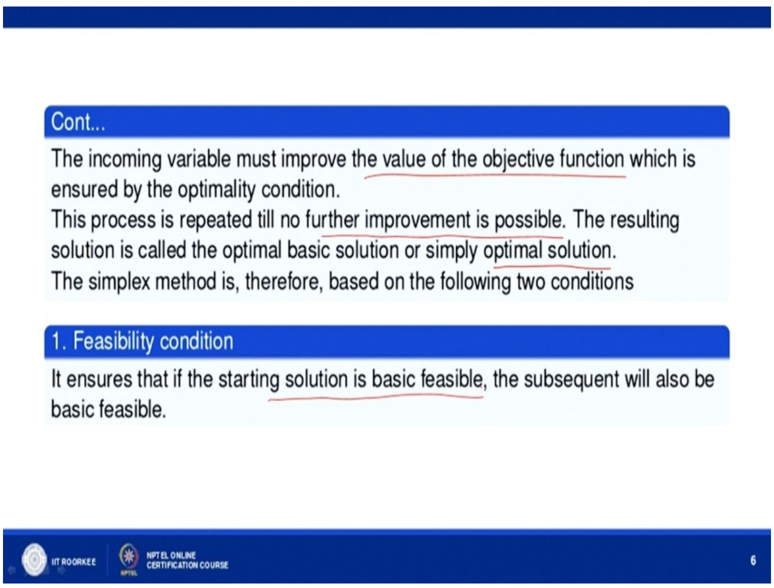

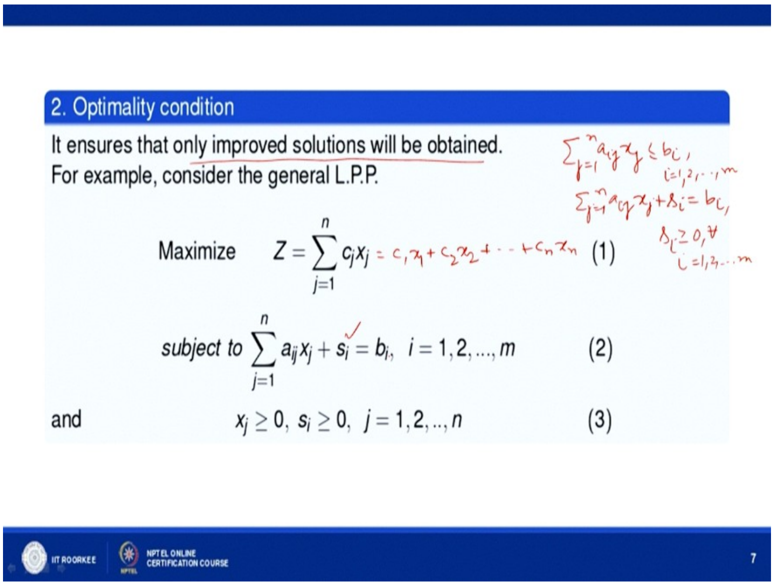

Now, the incoming variable must improve the value of the objective function, which is ensured by the optimality condition, the process is repeated till no further improvement is possible, the resulting solution is called the optimal basic solution or simply optimal solution. The simplex method is therefore, based on the two conditions, feasibility condition, feasibility condition ensures that the starting solution is basic feasible, the subsequent solution will also be basic feasible.

The optimality condition ensures that only improved solutions will be obtained, now say for example, let us consider the general linear programming problem, maximize

$Z=\sum_{j=1}^{n} c_j x_j$ i.e. $c_1x_1+c_2x_2+\dots+c_n$ subject to the constants, the question is

$\sum_{j=1}^{n} a_j x_j + s_i = b_i, i=1,2,\dots,m$ and $x_j \ge 0, s_j \ge 0, j=1,2,\dots,n$ and $i=1,2,\dots,m$.

So here you can see the constants $\sum_{j=1}^{n} a_j x_j \le b_i$ has been converted into equations by using the slack variables s_j, because when we have $\sum_{j=1}^{n} a_j x_j \le b_i, i=1,2,\dots,m$, we can convert them into equations by adding slack variables, so $\sum_{j=1}^{n} a_j x_j + s_i = b_i$ where $s_i \ge 0$, for all i, so these are slack variables, which are used to convert the M constants in to the equations.

Now, so this is our general linear programming problem, maximize

$Z = \sum_{j=1}^{n} c_j x_j$ i.e. $c_1 x_1 + c_2 x_2 + \ldots + c_n$, m equations, where x is, $x_j \geq 0$ for all j

and $s_i \geq 0$ for all i.

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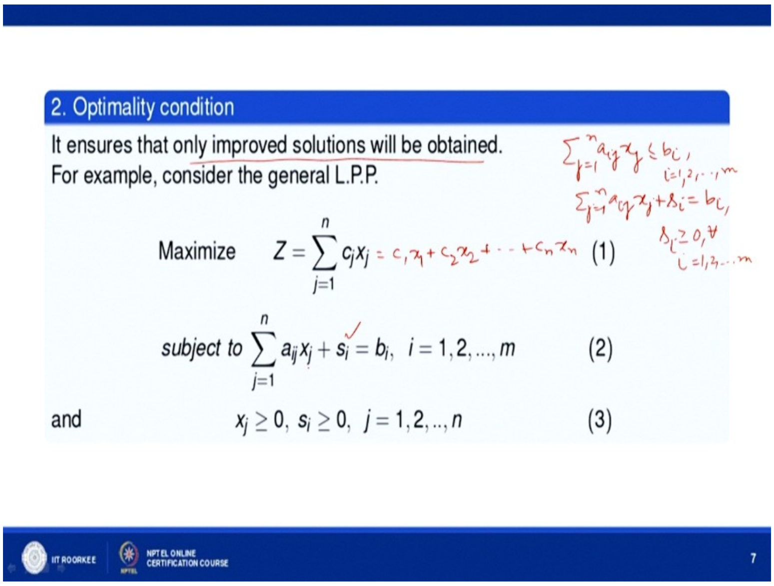

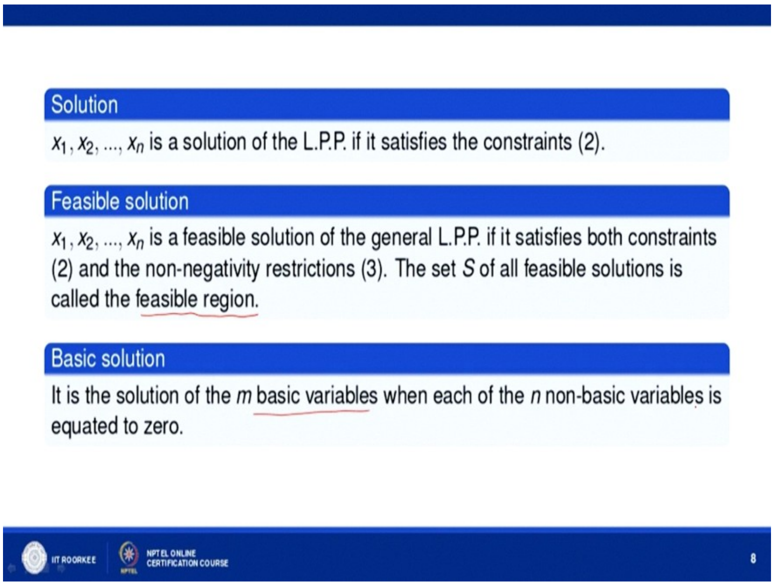

Now, $x_1, x_2, \ldots, x_n$ a solution of the linear programming problem if it satisfies the constrains, so $x_1, x_2, \ldots, x_n$ will be called as a solution of the linear programming problem if it satisfies this constraints, $\sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, \ldots, m$. Now, this solution will be called a feasible solution of the general L.P.P. if it satisfies both the constraints and the non-negativity restrictions, so $x_1, x_2, \ldots, x_n$ satisfies the constraints these m equations together with $x_j \geq 0$ for all j, $j=1,2,\ldots,n$ that is the non-negative and they satisfy is n constraints, a constraints equations which will say that $x_1, x_2, \ldots, x_n$ give us a feasible solution. Okay.

So $x_1, x_2, \ldots, x_n$ will be called a feasible solution for the general L.P.P., if it satisfies both the constraints and the non-negativity restrictions, now the set S of all feasible solutions is called as the feasible region. Okay, now it is the basic solution, it is the solution of M basic variables when each of the end basic variables are equal to, non-basic variables are equated 0.

So if you take the non-basic variables $x_1, x_2, \ldots, x_n$ to 0, then we have m equations involving m variables $s_1, s_2, \ldots, s_m$ okay, so we will be able to get the solution, the values of $s_1, s_2, \ldots, s_m$ from these m equations, then $x_1, x_2, \ldots, x_n$ and $s_1, s_2, \ldots, s_m$ will give us a basic solution of the linear programming problem, so it is the solution of m basic variables when each of the n non-basic variables is equated to 0.

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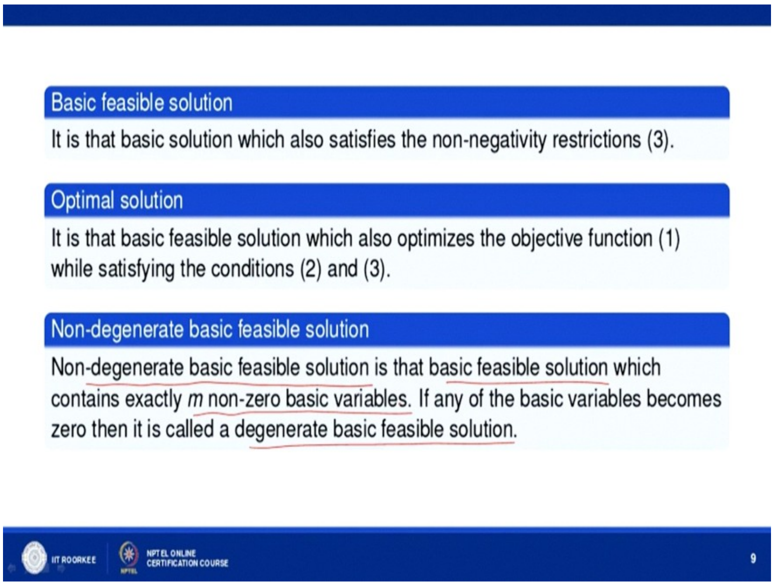

Now basic feasible solution, it is that basic solution which also satisfies the non-negativity restrictions 3, so basic solution will be called basic feasible solution if $x_1, x_2, \ldots, x_n, s_1, s_2, \ldots, s_m \geq 0$, now if optimal solution, it is that basic feasible solution which also optimises the objective function, so among all the basic feasible solution the 1 which is optimises the objective function while satisfying the conditions 2 and 3 is called as the optimal solution, there would be several basic feasible solution. Okay, so all those basic feasible solution the one which optimises the objective function. Okay, will be called as the optimal solution.

Now non-degenerate basic feasible solution, non-degenerate basic feasible solution is that basic feasible solution, which contains exactly M non-zero basic variables okay, so if any of the basic variables become zero than it is called a degenerate basic feasible solution, let say $x_1, x_2, \ldots, x_n$ we are taking a zeros, then $x_1, x_2, \ldots, x_n$ are basic variables, non-basic variables and $s_1, s_2, \ldots, s_m$ are called basic variables okay, so $s_1, s_2, \ldots, s_m$ are called basic variables, if any of the basic variable $s_1, s_2, \ldots, s_m$ becomes 0, then the solution will be call as a degenerate basic feasible to solution. Okay.

In case of non-degenerate basic feasible solution $s_1, s_2, \ldots, s_m$,will be known zeros okay, so it is that basic feasible solution which contains exactly M non-zero basic variables $s_1, s_2, \ldots, s_m$, must be non-zero, while taking $x_1, x_2, \ldots, x_n, x_n = 0$,the values of $s_1, s_2, \ldots, s_m$,if they turn out to be non-zero, then the basic feasible solution will be called as non-degenerate, if any, $s_1, s_2, \ldots, s_m$comes out to be 0 than it will be call as a degenerate basic feasible solution.

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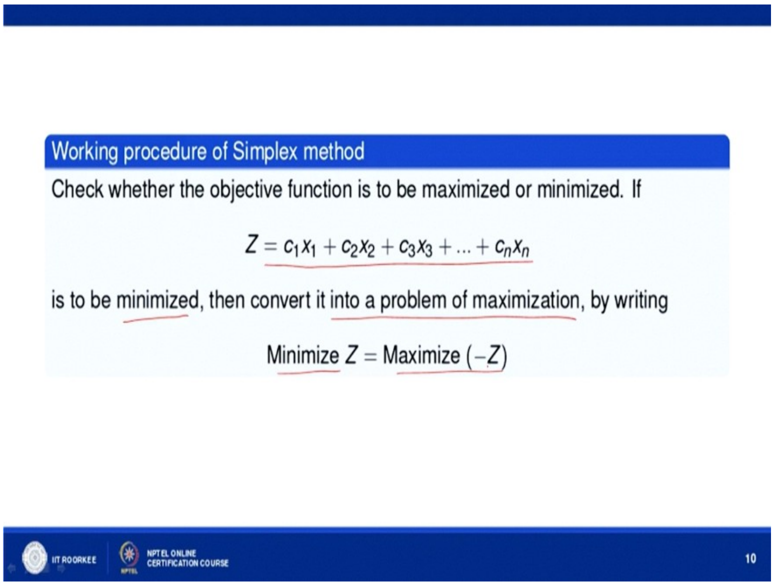

Now working procedure of simplex method, check whether the objective function, now, first of all, when you start to solving a problem by using a simplex method, we first check that whether the function that, objective function is to be maximized or minimized, if it is to be maximized then it is okay, if it is to be minimized then we will convert to a maximization problem.

So in the simplex method, first of all the objective function has to be maximized. Okay, so if it is already given as a maximized Z then we start as such, if it is given as a minimization of

Z, find minimum value of Z then we will convert it to a maximization problem, so suppose Z equal to $c_1 x_1 + c_2 x_2 + \ldots + c_n x_n$ is to be minimized, then we convert it into a problem of maximization by considering minimize Z equal to maximize minus Z, so minimize $Z = Min.Z$ means $Max.(-Z)$ Okay.

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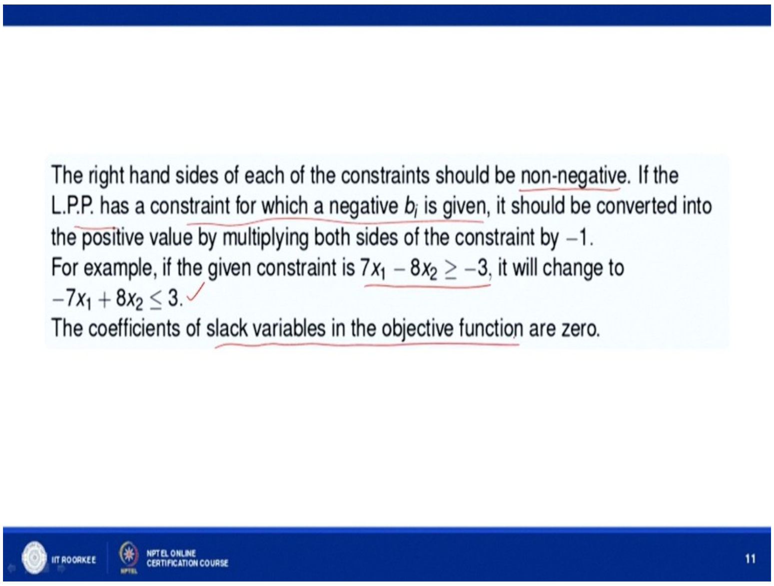

The right-hand side of each of the constraints, suppose there are m constraints than each right hand side each of the constraint should be made nonnegative, if the linear programming problem had a constraint for which a negative b_i is given. Okay, then we will multiply the constraint by minus 1 to make the right inside positive okay, so it should be converted into the positive value by multiplying both sides of the constraint by minus 1.

For example, suppose the constraint is given like this $7x_1 - 8x_2 \geq -3$, than it will be converted into $-7x_1 + 8x_2 \leq 3$, so we have to make this -3, 3 and that we can do by multiplying by -1. The coefficients of slack variables in the objective function are taken as zeros.

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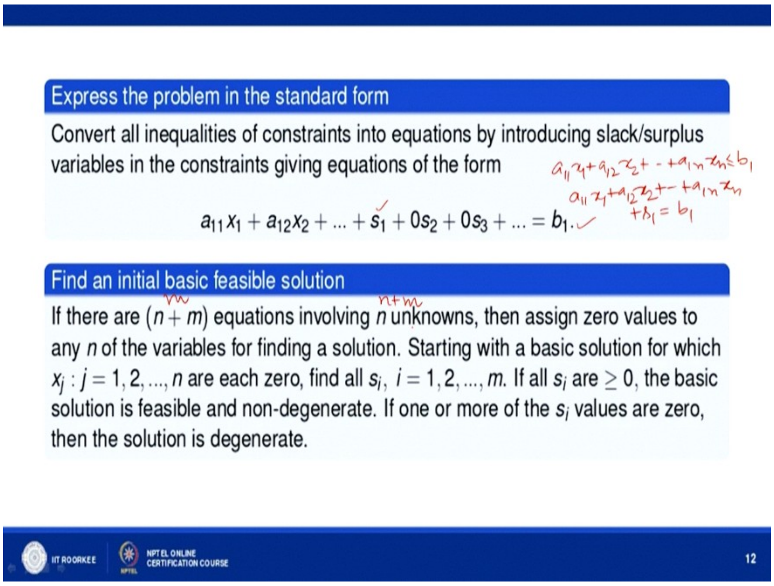

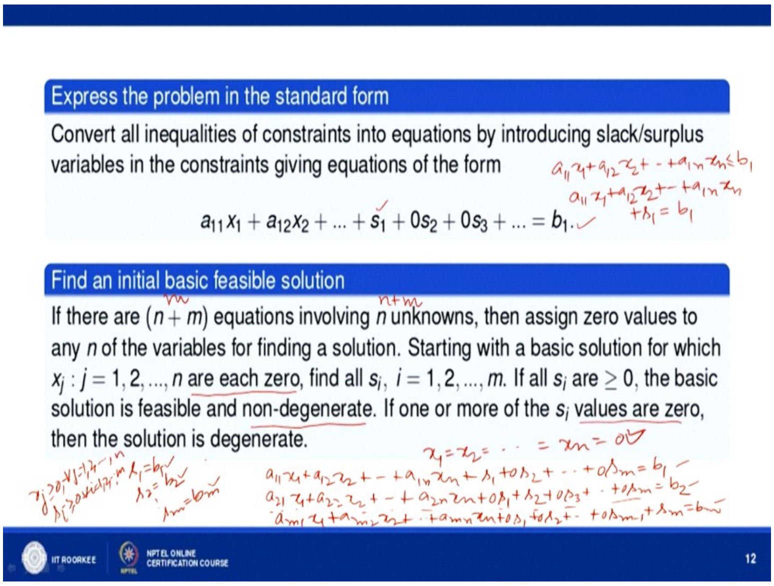

Convert all inequalities of constraints into equations by introducing slack oblique surplus variables in the constraints giving equations of the form this, suppose let us take the first equation, in the first equation we are using a slack variable S1 okay, then the question will become $a_{11}x_1 + a_{12}x_2 + \ldots + 1.s_1 + 0.s_2 + 0.s_3 + 0.s_m$ suppose there are m constraints, so $0.s_m = 0.b_1$.

So we will use slack variables s_1, s_2, s_3 to convert the given equation into, given constraint into equations. We might have to use a surplus variables also that we come later, so right now we are talking about the slack variable here, so suppose the constraint is like this $a_{11}x_1, a_{12}x_2, \ldots$ and so on a_1 and $x_n \leq b_1$, then it can be converted into an equation by using slack variable s_1, $a_{11}x_1, a_{12}x_2, a_1$ and $x_n + s_1 = b_1$ okay, we can write it as $a_{11}x_1 + a_{12}x_2 + \ldots$ and so on, a_1 and x_n, $1.s_1 + 0.s_2 + \ldots + ¿ b_1$.

Now find an initial basic feasible solution, after the constraints have been converted into equations we start to find a initial basic feasible solution, there are m equations, not $n+m$, suppose there m equations involving $n+m$ unknowns, what we will do? We will assign zero values to any n variables okay, so that we have m equations involving m unknowns and then we start with basic solution for which okay.

So those m equations in N unknowns can then be solved okay, if solution exists there, then we will find it, so if suppose there are m equations involving $n+m$ unknowns then assign zero values to any n of the variables for finding a solution, so when you put n variables as zeros then we will have m equations involving m unknowns okay, so we will then find a solution.

Now, starting with a basic solution, so what we will do? We will choose n variables $x_1, x_2, \ldots, x_n = 0$. Okay, we will take to be equal to zeros and then find all $s_1, i = 1, 2, \ldots, m$ so suppose the equations are like this,

$a_{11}x_1, a_{12}x_2, \ldots, a_{1n}x_n + s_1 + 0.s_2 + 0.s_3 + \ldots + 0s_m = b_1$, this is first equation, second equation $a_{21}x_1, a_{22}x_2, \ldots, a_{2n}x_n + 0.s_1 + 1.s_2 + 0.s_3 + \ldots + 0s_m = b_2$, and the last mth equation is Mth equation $a_{m1}x_1, a_{m2}x_2, \ldots, a_{mn}x_n + s_1 + 0.s_2 + 0.s_3 + \ldots + 0s_m = b_m$.

Now, you can see there are n equations, there are n equations involving n plus m unknowns okay, what we will do to obtain basic feasible solution, we will take $x_1, x_2, \ldots, x_n = 0$. Okay, so we will take $x_1, x_2, \ldots, x_n = 0$, then we will have m equations involving m unknowns $s_1, s_2, s_3, \ldots, s_m$ and we can find the values of these m unknowns $s_1 = b_1, s_2 = b_2$, and so on $s_m = b_m$.

Now, since we have already made right hand sides of these constraints $b_1, b_2, \ldots, b_m$ positive okay, the values of $s_1, s_2, \ldots, s_m$ will always be greater than or equal to 0, so we will get $x_1, x_2, \ldots, x_n = 0, s_1, s_2, \ldots, s_m$. $s_1 = b_1, s_2 = b_2$, and so on $s_m = b_m$, so we have $x_j \geq 0$ for all J and $x_i \geq 0$ for all I, so then the solution, $x_1, x_2, \ldots, x_n, s_1, s_2, \ldots, s_m$ okay satisfies these m equations and also satisfies the non-negativity restrictions, therefore, $x_1, x_2, \ldots, x_n = 0$ and

$s_1 = b_1, s_2 = b_2$, and so on $s_m = b_m$. okay will give us a basic feasible solution.

So if all SI are greater than or equal to 0, the basic solution is feasible and non-degenerate okay, if one or more of the SI values are zero, then the solution will be called degenerate solution, so if any $b_i = 0$ is zero then the solution will be called degenerate, if the $b_i = 0$, then $s_1, s_2, \ldots, s_m$ will be all positive and the solution will be called a basic feasible solution, so this is how we find the initial basic feasible solution for the given LPP.

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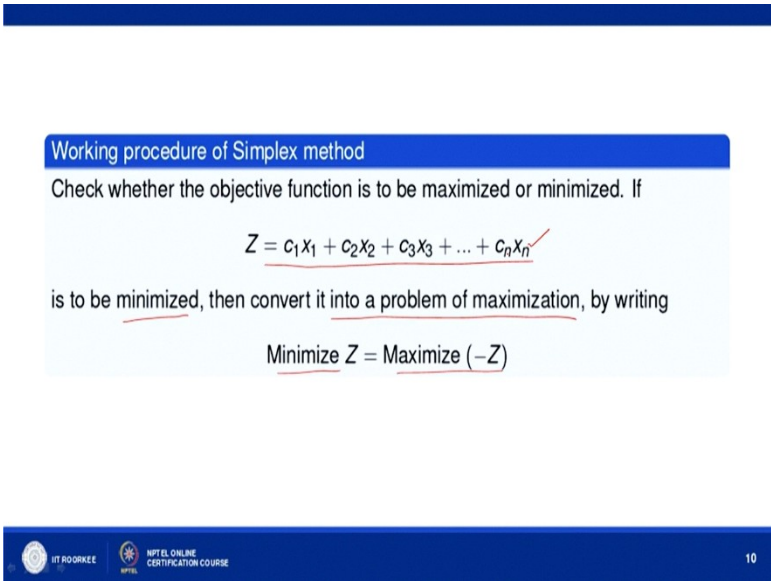

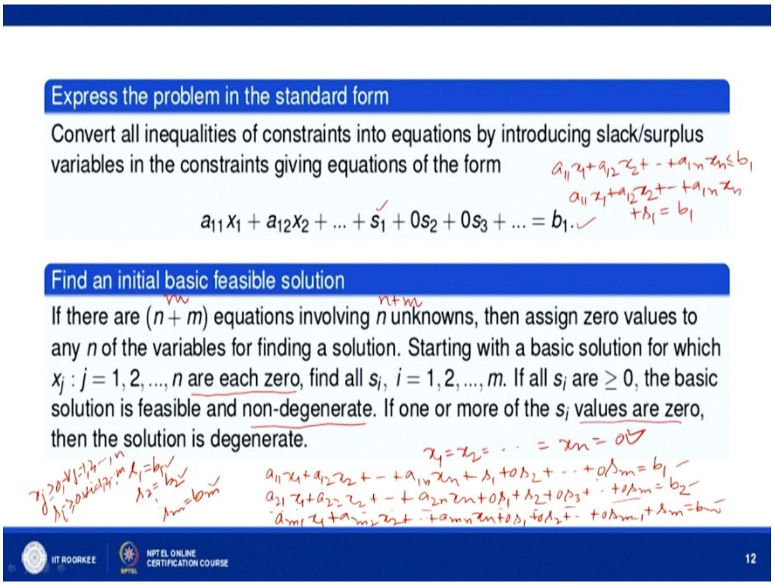

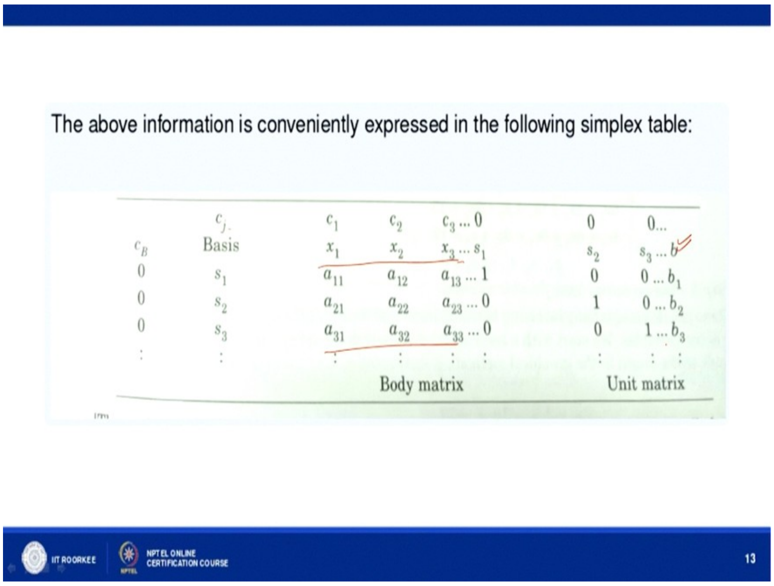

The above information is conveniently expressed in the following simplex table:

	c_j	c_1	c_2	$c_3 \dots 0$	0	$0 \dots$
c_B	Basis	x_1	x_2	$x_3 \dots s_1$	s_2	$s_3 \dots b$
0	s_1	a_{11}	a_{12}	$a_{13} \dots 1$	0	$0 \dots b_1$
0	s_2	a_{21}	a_{22}	$a_{23} \dots 0$	1	$0 \dots b_2$
0	s_3	a_{31}	a_{32}	$a_{33} \dots 0$	0	$1 \dots b_3$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots \quad \vdots$	$\vdots$	$\vdots \quad \vdots$
			Body matrix		Unit matrix	

Now, let us write this information in the form of this table. Okay, so here, what is c_j row, c_j row consist of $c_1, c_2, c_3$and so on this. Okay, so c_1, c_2, c_3 are the coefficients of A11, c_1, c_2, c_3 are coefficients of $x_1, x_2, \dots, x_n$ in the objective function. Okay, objective function is this, these our objective function, so $c_1, c_2, \dots, c_n$ are coefficients of $x_1, x_2, \dots, x_n$ the decision variables in the objective functions.

So, coefficient of x_1 is c_1 coefficient of x_1 c_1, coefficient of $x_3 c_3$ and the coefficients of $s_1, s_2, \dots, s_n$ okay are taken as 0 here. Okay, so we consider Z equal to, suppose the problem is minimization, then we will consider maximization of minus Z okay, so we will have

$-c_1 x_1 - c_2 x_2 - c_n x_n$ okay and then $0 s_1, 0 s_2, \dots, 0 s_n$, so coefficients of slack variables are taken as zeros okay and coefficients of $x_1, x_2, \dots, x_n$ are taken as $-c_1, -c_2, \dots, -c_n$, if the problem given is of minimization, if the problem given is of maximization then the coefficient of $x_1, x_2, \dots, x_n$ will be $c_1, c_2, \dots, c_n$ and coefficient of $s_1, s_2, \dots, s_n$ will be zeros in the objective function.

So here we are taking the coefficient of $x_1, x_2, \dots, x_n$ as $c_1, c_2, \dots, c_n$ that means we are assuming that the problem is of maximization and then $s_1, s_2, \dots, s_n$, the coefficient of $s_1, s_2, \dots, s_n$ are taken as zeros okay and then the variables $x_1, x_2, \dots, x_n$ are the non-basic variables $s_1, s_2, \dots, s_n$ are the basic variables, so under the basis we write the variables $s_1, s_2, \dots, s_n$ and their coefficients in the objective function are zeros, so we write 0, 0, 0 and so on. Okay, this is body matrix, this is body matrix okay, a_{11}, a_{12}, a_{13} and so on

$a_{1n}, a_{22}, a_{23}, \ldots, a_{2n}, a_{31}, a_{32}, \ldots, a_{3n}$, so that $n \times n$ matrix is the, that $m \times n$, $m \times n$ matrix not, because we will have here and we are having m equations. Okay.

So we will have M equations we have, so a_{11}, a_{12}, a_{13} and so on $a_{1n}, a_{21}, a_{22}, a_{23}, \ldots, a_{2n}, a_{31}, a_{32}, a_{33}, \ldots, a_{3n}$, these are the coefficients of $x_1, x_2, \ldots, x_n$ okay, they form the body matrix and the coefficients of s_1, s_2 in the first equation, coefficient of $s_1 = 1$ coefficient of $s_2 = 0$ coefficient of $s_m = 0$, so we write 1, 0, 0, 0 okay and then in the second equation coefficient of $s_1 = 1$, coefficient of $s_1 = 1$ so we get 0, 1, 0, 0 and so on, so we get 0 1, you see 0, 1, 0, 0 and so on, then in the third equation the coefficient of $s_1 = 1$, coefficient of $s_2 = 0$ coefficient of $s_3 = 1$ so we get 0, 0, 1 and so on. Okay, so and then this is B column, in the B column, we write b_1, b_2, b_3 and so on. Okay, the right-hand sides of the constraints, so this is how we find the first simplex table.

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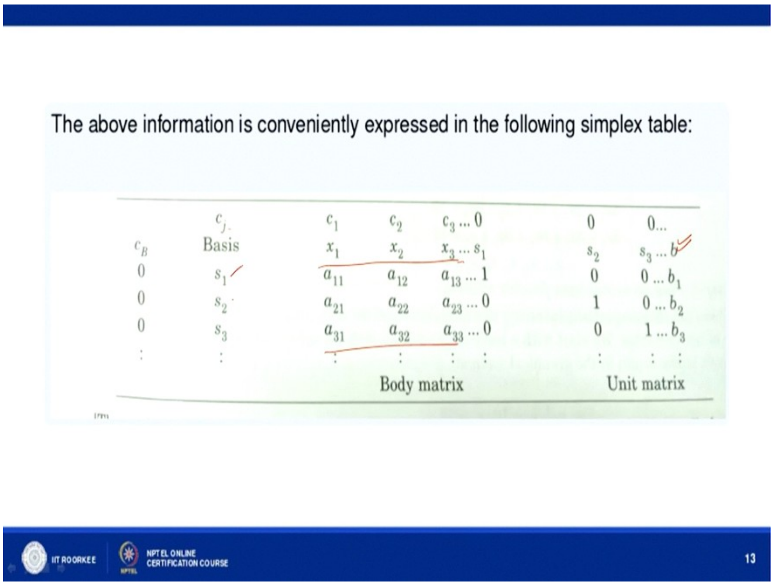

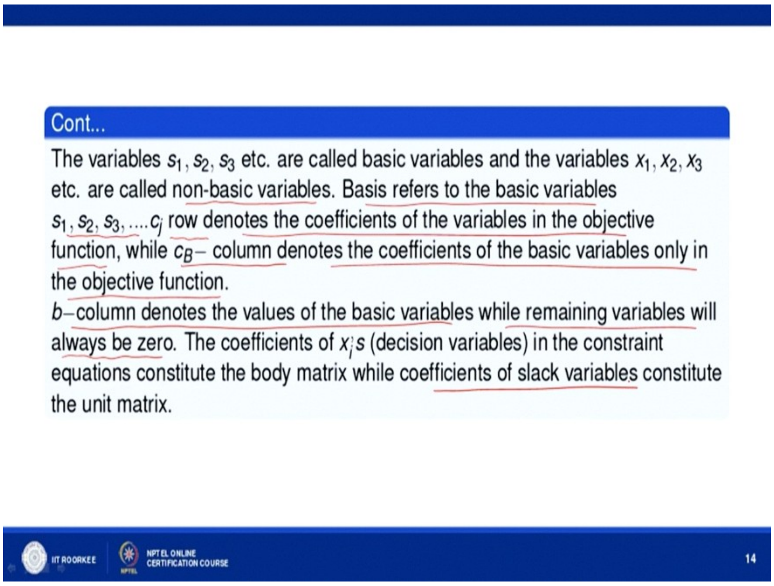

Now, the variables s_1, s_2, s_3 here, the variable s_1, s_2, s_3 are called as the basic variables and the variables x_1, x_2, x_3 are call as the non-basic variables, basis refers to the basic variables, s_1, s_2, s_3 and so on CJ row denotes the coefficients of the variables in the objective function, CB column denotes the coefficients of the basic variables only in the objective function. Okay.

So in the CB column with denotes, we write the coefficients of the basic variables only, in the objective functions, the B column denotes the values of the basic variables while the remaining variables will be always zero. Okay, so in the B column we get the values of the basic variables while the remaining variables are always zero, the coefficients of XIs that is decision variables in the constraint equations constitute the body matrix, so you see here, this is body matrix the coefficient of XIs, this is decision variables x_1, x_2, x_3 they are the decision variables, their coefficient constitute the body matrix, the coefficient of s_1, s_2, s_3 and so on. Okay, they constitute the unit matrix 1, 0, 0, 0, 0, 1, 0, 0 and so on, you see here, 0, 0, 0, 1 okay this M by m matrix because we have m rows and we have m columns here, s_1, s_2, s_3 so this is $m \times m$ matrix.

While this body matrix is $m \times n$ matrix, so body matrix has got m rows and n columns, A11,

$a_{12}, a_{1n}, \ldots, a_{21}, a_{22}, a_{2n}$ and we have $a_{m1}, a_{m2}, \ldots, a_{mn}$, so this is body matrix and this is unit matrix okay, so slack variables, coefficients of slack variables constitute the unit matrix.

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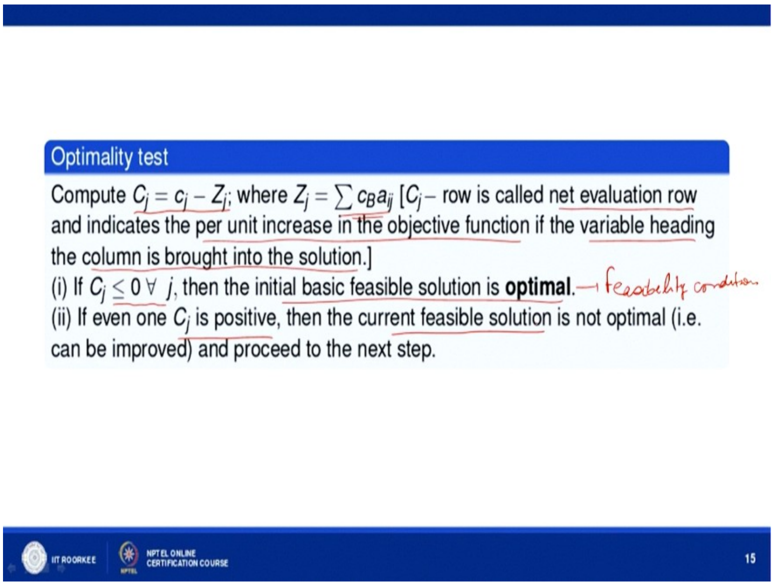

Now, optimality test. Okay, so they will compute C_j, $C_j = c_j - Z_j$ what is Z_j? $Z_j = \sum C_B A_{ij}$, C_j row is called the net evaluation row and indicates the per unit increase in the objective function if the variable heading the column is brought into the solution. Okay, now if C_j is less than or equal to 0 for all J. Okay, then the initial basic feasible solution is optimal, so this is feasibility condition, this is optimality condition, if C_j is less than or equal to J, if C_j is less than or equal to 0 for all J, this is called feasibility condition, so if C_j is less than or equal to 0 for all J than the initial basic feasible solution is optimal, if even one C_j is positive, then the current feasible solution is not optimal and we will proceed to the next step, it has to be improved.

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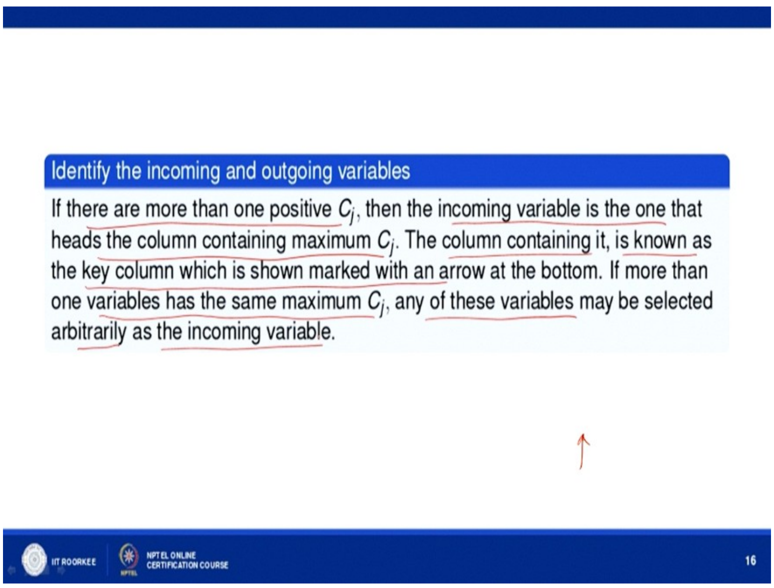

So now if there are more than one positive C_j, okay if there are more than one positive C_j then the incoming variable is the one that heads the column containing maximum C_j okay, if there is more than one C_j which is positive, then we will take that, consider that value, which is the maximum and in that particular column the variable which is heading that column that will be incoming, while.

So the column containing it is known as the key column, which is shown marked with an arrow, we will mark it by an arrow like this. Okay, so marked with an arrow at the bottom, if more than one variable has the same maximum C_j, any of this variables may be selected arbitrarily as the incoming variable.

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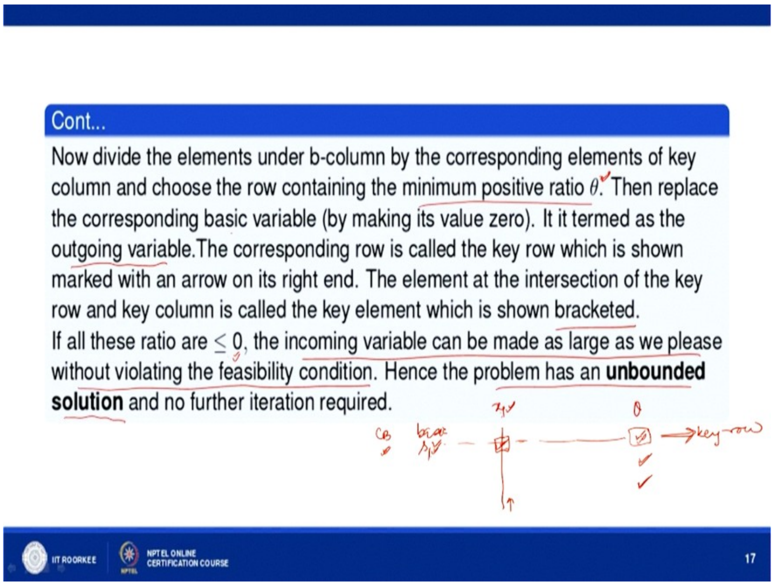

Now, divide the elements under B column by the corresponding elements of key column and choose the row containing the minimum positive ratio theta. Then replace the corresponding basic variable by making its variable value 0, if it termed as the outgoing variable. Okay, so will consider, we will have the key column, the values of B column are then divided by the corresponding values in the key column. Okay and that gives us the values of theta, the ratios will give us the values of theta, will find the minimum positive ratio theta, among all the positive ratios will take that one which is the minimum positive ratio.

So that, then the replace the corresponding, so suppose this is your theta column. Okay, we have here values in the theta column. Okay, suppose this is the minimum positive ratio, then this row. Okay, this row will be called as key row, this row will be called as key row and we replace the basic variables here, the basic variables which occurs in this row. Okay, this basic variable will be given value zero and it will be replace by the incoming variable, which contains the.

Suppose the incoming variable is X1 okay than this is your key column, so this element okay this element will be, which element which occurs at the intersection of key column and key row, that will be key element, this basic variables, support this is S1 okay, S1, then be replace by X1 in the new table and in the CB column we will write the value of the coefficient of X1 in the objective function, before this process, the coefficient of S1 will be here in the CB column, but after we have done this, we have got the key row and key column and then this will be incoming variable, this will be outgoing variable.

s_1 will be replaced by x_1 and the coefficient of x_1 will be put here in place of the coefficient of s_1, so that will become the new simplex table, so this variable will be called as the outgoing variable, this s_1 will be called as the outgoing variable, its value is made zero, now the corresponding row, this row is called the key row which is shown by an arrow on its right hand, so we will be showing it by arrow in its right hand.

The elements at the intersection of the key row and key column has key element, this element will be called key element which is shown bracketed, now if all these ratios are less than or equal to 0, suppose that all the ratios here. Okay, under theta column are less than or equal to 0, the incoming variable can be made as large as we please without violating the feasibility condition and therefore the problem will have a an unbounded solution and no further iteration is required, so if all the values in the theta column turn out to be is less than or equal to 0, then the problem will have an unbounded solution and we do not need to proceed further.

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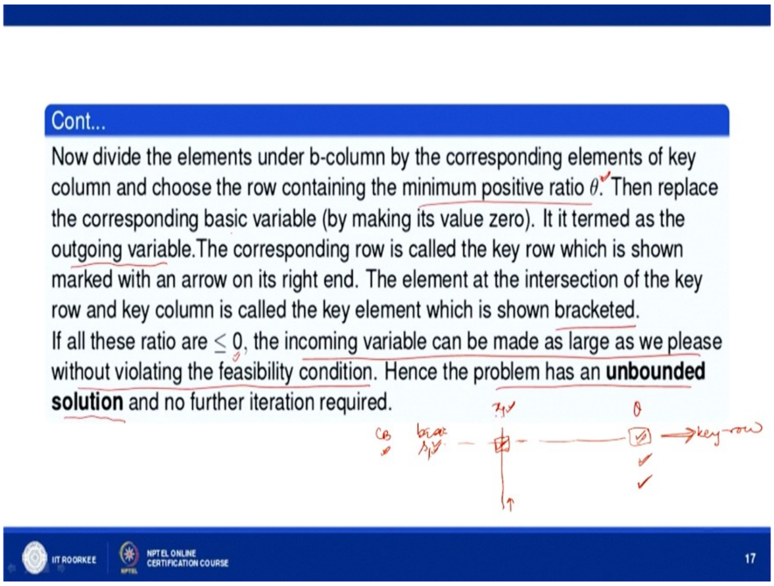

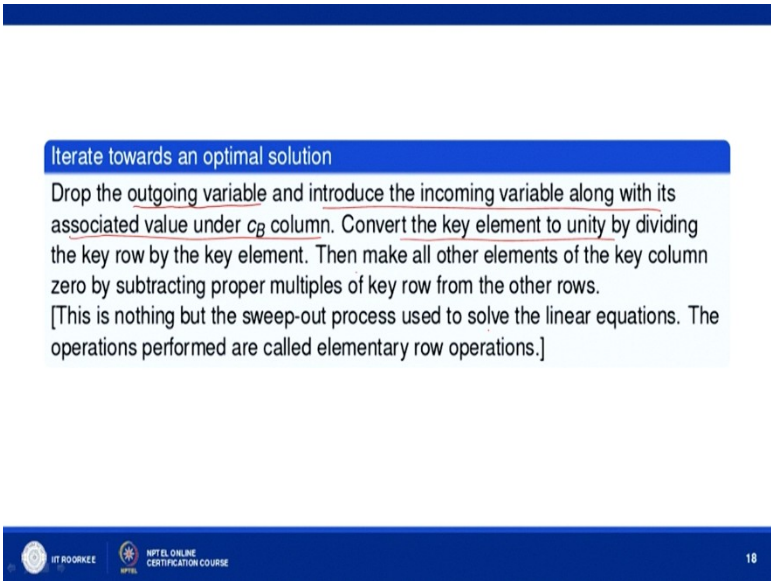

Now drop the outgoing variable as I said, we drop the outgoing variable and introduce the incoming variable along with its associated value under C_B column, convert the key element to unity, what we will do? We will, after we have replace s_1 by x_1 and here we put the coefficient of x_1 here in the C_B column, we divide the key row by the key element to make it unity okay and with the help of this unity here we make the elements, in the other elements in the key column zeros using elementary row operations.

So convert the key element to unity by dividing the key row by the key element, then make all other elements the key column zero by subtracting proper multiples of key row from the rows, this is nothing but sweep out process used to solve the linear equations, the operations performed are called elementary row operations.

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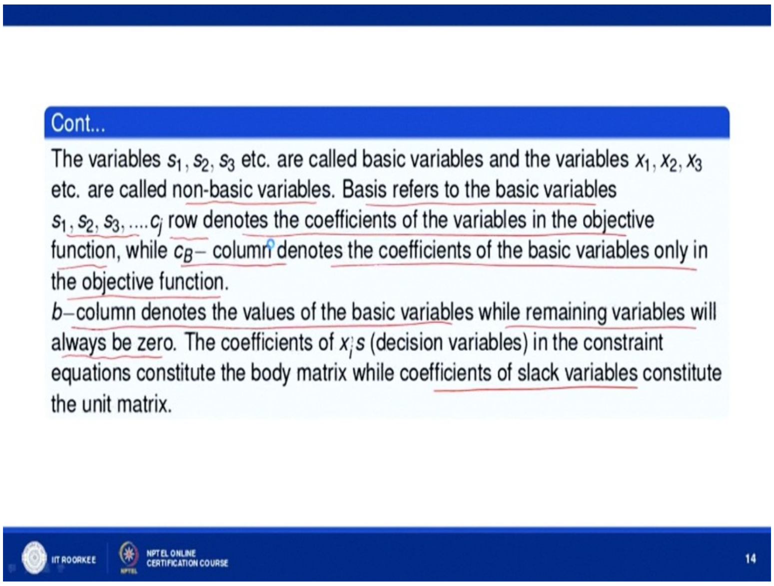

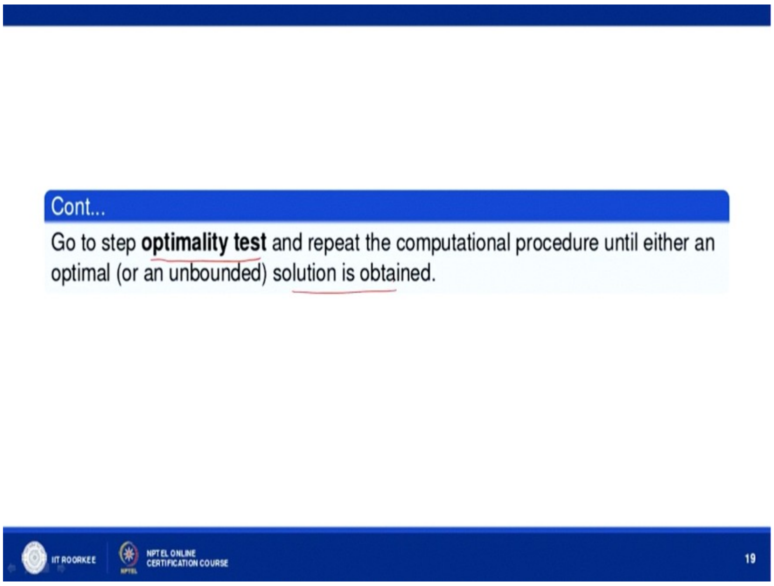

And then we apply optimality test. Okay, that is, we find capital c_j, so this one capital c_j which is equal to $c_j - Z_j$ said Jay is Sigma $c_{CB} A_{ij}$ and then check whether c_j is less than or equal to 0 for all J, if it is, $c_j \leq 0$ for all j, then the solution that we have got is the optimal solution, if even one c_j is positive, then this current feasible solution is not optimal and we proceed to next step, so go to step optimality test and repeat computational procedure until either an optimal or an unbounded solution is obtained okay.

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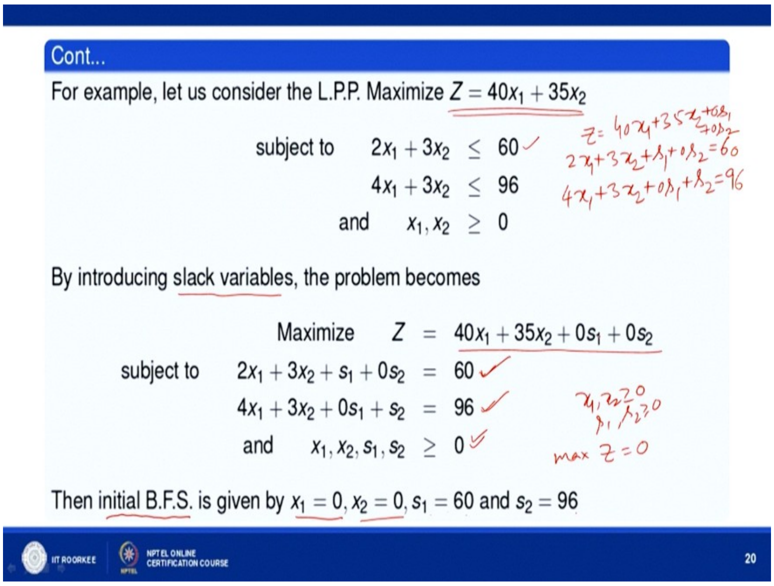

Now let us see how we can solve a given LPP using the simplex method, so let us consider we are given $Z=40x_1+35x_2$ and we have to maximize this objective function, so here we do not have to convert this problem into maximization because it is already a maximization problem, if it is minimization of

$Z=40x_1+35x_2$then we will convert into a maximization problem by considering maximum

of -Z that is maximum of $-40x_1-35x_2$ that we would have done okay.

But here it is already the solution problem, so we keep it as such maximization of

$Z=40x_1+35x_2,$ now we are given to constraints equations

$2x_1+3x_2\leq 60, 4x_1+3x_2\leq 96,$ so we convert this constraints into equations by using slack variables because there are two equations, so we need two select variables s_1 and s_2, so by introducing slack variables, the problem now becomes, now $2x_1+3x_2$, $2x_1+3x_2+s_1=60,$

okay and the second one we will write as $4x_1+3x_2+0s_1+1.s_2=96.$

Objective function will be $Z=40x_1+35x_2$ and 0 s_1 coefficient of slack variables in the objective functions are taken zeros, 0 s_1, 0 s_2 okay, so the problem will become maximize $Z=40x_1+35x_2+0s_1+0s_2$ subject to the two equations $2x_1+3x_2+s_1+0s_2=60, 4x_1+3x_2+0s_1+1.s_2=96$ and $x_1, x_2, \ldots, x_n, s_1, s_2 \geq 0.$

Now we finally initial BFS okay initial BFS we find, so for that we take, now there are two equations here and we have four unknowns okay, $x_1, x_2, \ldots, x_n, s_1, s_2$, so we can find a unique solution, if we take any two variables equal to 0, so we take x_1 and x_2 equal to 0, $x_1=0, x_2=0$, then we get $s_1=60, s_2=96, x_1=0, x_2=0$, S2 so $x_1=0, x_2=0, s_1=60, s_2=96$,is a solution of this L.P.P.

It is the basic feasible solution of this LPP because $x_1, x_2, \ldots, x_n, s_1, s_2 \geq 0$, a solution becomes a basic solution if it satisfies the non-negativity restrictions okay, so $x_1=0, x_2=0, s_1=60, s_2=96$, they are all, $x_1, x_2, \ldots, x_n, s_1, s_2 \geq 0$ Okay and therefore we have found the initial basic feasible solution. Okay and $Z=40x_1+35x_2=0$, maximum of $Z=0$ because

40 $x_1=0, x_2=0, s_1=60, s_2=96$so $Z=0$

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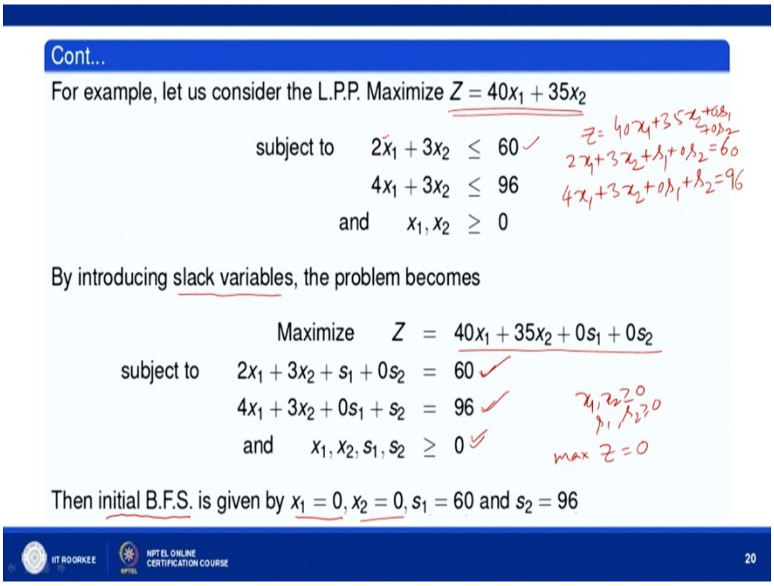

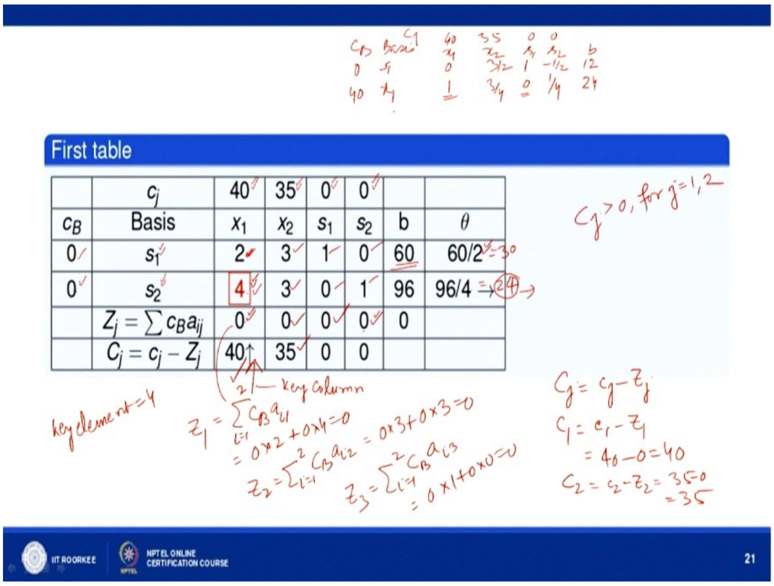

So now let us find the, write the first table. Okay, c_j row contains the coefficient of the variables x_1, x_2, s_1, s_2 in the objective function, so coefficient of x_1 is in the objective function is 40, coefficient of X2 in the objective function is 35, coefficient of the S1 in the object function is 0, coefficient of S2 in the objective function is 0 and x_1, x_2 are your non-basic variables s_1, s_2 are basic variables, so we write s_1, s_2 under the column basis, in the column basis we write s_1, s_2.

The coefficient of s_1 and s_2 the objective function are zeros, so we write them in the CB column, CB column contains the coefficients of the basic variables in the objective function and this is body matrix, 2, 3, 4, 3 okay, this is the body matrix, $a_{11}x_1 + a_{12}x_2$, A less than or equal to 60 given to us. Okay, this was the question, so 2, 3, 4, 3 okay, 2, 3, 4, 3 this is the body matrix and 1, 0, 0, 0, 0, 1 okay, it is unit matrix, so we have this 1,0, 0, 1, this is unit matrix and this is body matrix 2, 3, 4, 3.

Now what we will do and this is B column, B column contains the cost sense b_1, b_2. Okay, this b_1, b_2, this one, this one. Okay, now what will do, so we will find Z_j first, Z_j is Sigma $\sum C_B Z_j$, means the coefficients of, the values in the CB column are multiplied by the corresponding values in the x_1, x_2, s_1, s_2, columns, so C_B is, so first we multiply by 0, 0, these the values on the CB column to the coefficients in the X1 columns, so 0 into 2 is 0, 0 into 4 is 0, so totally is 0. Okay, so we get this, so this is Sigma $C_B a_{ij}$ means, we have Sigma C_B, this is obtained from Sigma C_B into AI1 okay, this is the first column okay and I takes values from 1 to 2. Okay.

So this is Z_j, Z_j means Z_1 okay, first column okay, so this is Z_1, $Z_1 = 0.2 + 0.4 = 0$, say this is Z_1, now Z_2, Z_2 equal to Sigma, $i = 1, 2$I equal to 1 to 2, $C_B a_{i1}$. Okay, so we have 0 into a_{12} 0 into a_{12} means, 0 into 3 plus 0 into 3, so we get 0, so this is Z_2. Okay, then Z_3, Z_3 means Sigma I equal to 1 to 2, C_B into this one, a_{11}, a_{12}, a_{13} you can say this a_{13}, a_{13}third column okay, so that is this a_{13}, that a_{13}, this is a_{13} okay, so we get 0 into 1 plus 0 into 0 equal to 0, so we get this, similarly we get 0 into 0, 0 into 1 equal to 0. Okay.

Now then find C_j, capital $C_j = c_j - Z_j$ okay, so that means suppose in the first column C_1, the first column J equal to 1, $C_1 = c_1 - Z_1$ will be equal to and $C_j = 40$, so $40 - Z_j = 0$, so we get 40 here and similarly $C_1 = c_2 - Z_2$, so 35 minus 0, we get 35, so this is 35 and then here, 0 minus 0 is 0, 0 minus 0 0, now we can see C_j is greater than or equal to 0 for C_j is strictly positive for J equal to 1, and 2. Okay, for J equal to 1 it is 40, for J equal to 2 it is 35, so will take the maximum value. Okay, maximum value is 40 okay, so this column is our key column we show it by an arrow pointing upwards. Okay, so this is our key column.

Now what we will do in the key column the elements 2, 4 okay, we divide the elements in the B column by the corresponding elements in the key column, so 60 divided by 2 gives you the value of theta for this first row. Okay, in the second we get 96 divided by 4 okay, so this is 30 and this is 24 okay, now both are positive, both value of theta are positive, so we can find the minimum value of, minimum positive value of theta, this is the minimum positive value of theta and therefore this is our key row okay, so this is key row and this is key column okay and the intersection of key row and key column we have 4, so these 4 we have bracketed, this 4 is in the key element, key element is 4.

Now what will happen? This s_2 will be outgoing variable and x_1 will be incoming variable, so in the next simplex table s_2 will be replaced by x_1 okay and the coefficient of x_1, which is 40 will replace the 0 here, 0 was the coefficient of S2 in the adjective function, now in the objective function the coefficient of $x_1=40$, so 40 will come here, x_1 will come here, after that we divide the key row by 4 okay, to make this element which is key element to make it unity we divide the elements of key row by 4 okay.

So in the next simplex table will be C_B and in the C_B 0, here we will get 40, we will get basis S1 and here we will get in place of C_2 will get x_1 okay and then C_j, C_j will be 40 as such. Okay, 35 and will get 0, 0 alright and here we will have x_1, x_2, s_1, s_2 okay and we divided by 4 so this row will become 1, 3 by 4 okay 1, 3 by 4, 0, 1 by 4 and here we will get 96 by 4 that means 24, this should B column okay, B column okay.

Now with the help of this 1 okay, we will make the entries, other entries in the key column zeros, so this entry is 2, we will have to make it 0, so we multiplied it by 2 and subtract from this row, so will get 0 here okay any multiplied by 2, so this become 3 by 2, 3 by 2 we subtract from 3, so we get 3 by 2 here and we multiply 0, we divide 0 by, ohh, we multiply this by 2 and subtract from 1, so we get 1 okay and here what we get here? We multiply it by 2, so we get 1 by 2, 1 by 2 we subtract from 0, so minus half we get here okay and here what we will have? 24 is multiplied it by 2 we get 48, 40 it is subtracted from 60 we get 12 here okay, so this is our due table you can see, this one. Okay.

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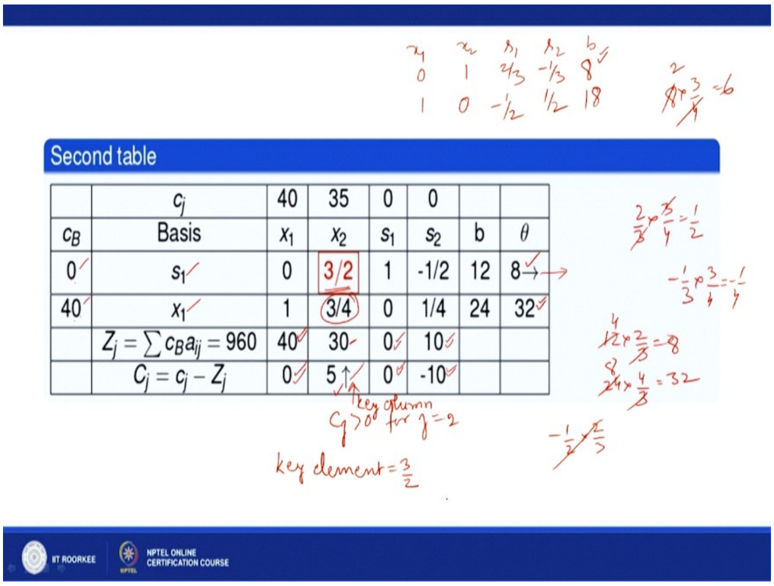

We have $s_1, x_1, 0, 40$ then 0, 3 by 2, 1 minus half, 12 and then 1, 3 by 4, 0, 1 by 4, 24 okay, now we find again Z_j, so ZJ$Z_J = \sum c_B a_{ij}$, so 0 into 0 0, 0 into 40 into 1 is 40 total is 40, so we get 40 here and then 40 - 40 =0 and then we get 0 .3 / 2 0, 40 . 3/ 4, that means is 30, so 30 is here. Okay total is 30, 30 is here, 35 - 30 =5 and then 0 into 1 is 0, 40. 0 =0, so we get total 0, 0 minus 0 is 0. Okay and then 0 into minus half is 0, 40. 1 / 4 = 10 total is 10, so we get 10 here, 0=- 10= - 10 okay, so we got the values of Z_j, we got the values of capital C_j.

Now we can see C_j is positive. Okay for J equal to 2 in the second column, C_j is positive. Okay, therefore we still have not got the optimal solutions. Okay, so this is our key column, there is only one positive value. Okay, so this is our key column, we divide the elements of under B column by the corresponding values in the key columns, so 12 divided by 3 by 2, so 12 into 2 by 3, so we get 8 okay, we get 8 here and then 24 divided by 3 by 4, 24 divided by 3 by 4 means 4 by 3, so we get 32. Okay, now both are positive values in the theta column, so minimum value will be considered, so 8 is minimum, so this is our key row okay.

At the intersection of key row in key column we get the elements 3 by 2, so this bracketed element is the key element, now what we will do? We divide this row okay, key row by the key element okay, what we will get? x_1, x_2, s_1, s_2, b okay, it will, after dividing 3 by 2 it will get 0, 1, 2 by 3 okay and we divide by 3 by 2, so we get minus 1 by 3 okay, minus 1 by 2 into 2 by 3, so we get minus 1 by 3 and then 12 divided by 3 by 2, so 12 divided by 3 by 2 means 12 into 2 by 3, so we get 8 here okay.

After we have converted the key element to unity okay, we make the element 3 by 4, this element we have to make 0, so we multiply it by 3 by 4 and subtract from this row okay, so what will get? We multiply it by 3 by 4 okay, 3 by so we multiply here and subtract from this, so this will remain 1 this will become 0 and this is multiplied by 3 by 4, so 2 by 3 .3 / 4, so we get half okay, so we multiplied by 3 by 4 and subtract from 0, so 0-1 / 2, so we get minus half okay and we multiply it by 3 / 4 okay, 3 by 4 we multiply, so minus 1/ 3 . 3 / 4, we get -1/4.

So we subtract after multiplying 3 /4 to this row, we subtract it from the second row, so 1 / 4 plus 1 / 4, 1 by 2 we will get here and here what will happen? We multiply by 3 / 4, so 8 into 3 by 4 means 6, so 6 we subtract from 24 and we get 18 okay and moreover that S1 will be replace because this is key row and this is key column okay, so S1 will be replace by x_2 here, so we will get here 35, here 40, here x_2, x_1 okay.

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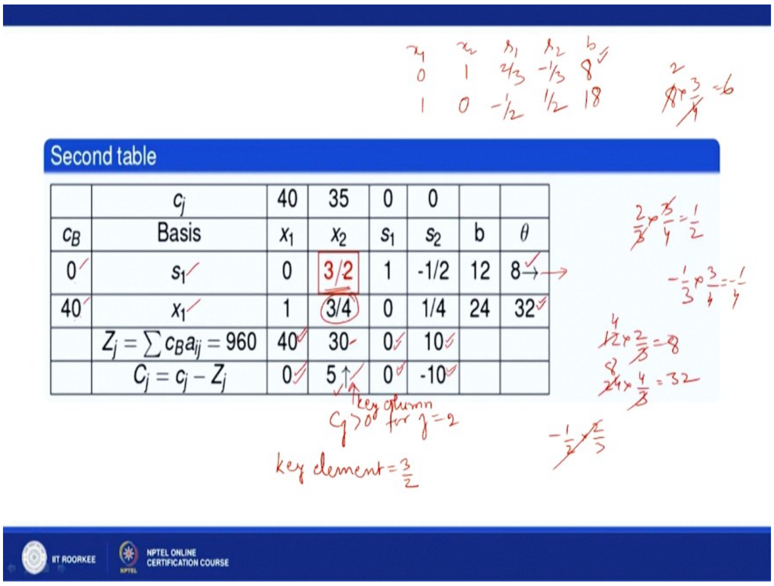

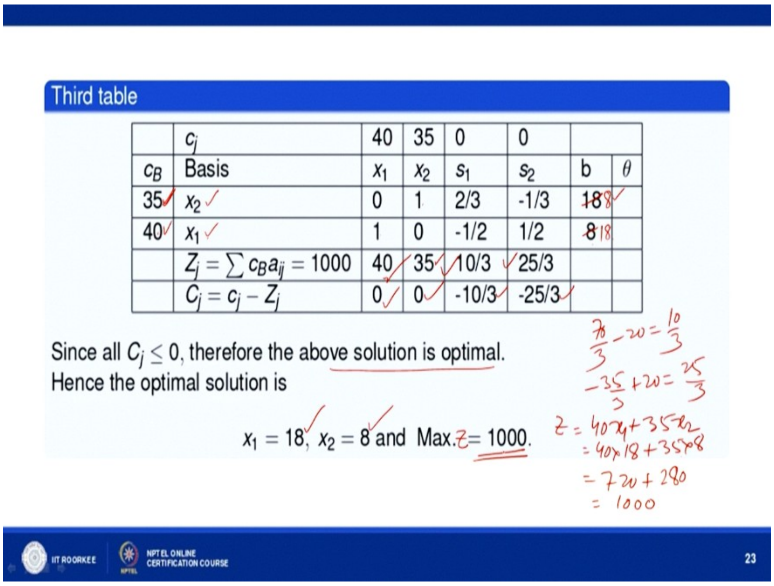

	c_j	40	35	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
35	x_2	0	1	2/3	-1/3	~~18~~ 8	
40	x_1	1	0	-1/2	1/2	~~8~~ 18	
	$Z_j = \sum c_B a_{ij} = 1000$	40	35	10/3	25/3		
	$C_j = c_j - Z_j$	0	0	-10/3	-25/3		

So let us see 35, 40, x_2, x_1 and 0, 1, 2 by 3, 0, 1, 2 by 3 okay minus 1 by 3 and here we will have 8 and then 1, 0, minus half, 1, 0, minus half, half and here we will get 18 okay, so this is 18, this is 8 okay, this is 8 and this should be 18 okay and what we will get then yes, so now we can see, we can find $Z_j, Z_j = \sum c_B a_{ij}$, so 35 . 0 = 0, 40 .1 = 40, we get 40 here, 35 . 1 = 35, 40 . 0 = 0, so we get 35 here, 35 / 2 / 3, so we get 70 / 3 and then we get 40 into minus half, so minus 20, so we get 10 by 3, so we get 10 by 3 here and then 35 into minus 1 by 3, so minus 35 by 3 and then we get 40 into half, so we get 20, so we get 60 minus 35, so 25 by 3, so we get 25 by 3 here.

And then 40-40 = 0, 35-35 = 0, 0- 10 / 3 = -10 by 3, 0 - 25 / 3 = -25 / 3. Now $C_j \le 0$ for all J and therefore the above solution is optimal, so what we get? We get $x_1 = 18$ and $x_2 = 8$ and the value of $Z = 35.8$ okay, so we find C_B into yes, we have $Z = x_1$, 40 into $x_1 + 35. x_2$ that is Z okay, $x_1 = 18$, so 40.18 okay, 40 to 18 and 35 into 8 that is, we multiply C_B columns okay, by B column.

So 35 . 8 okay and then 40 to 18, so this is 720 plus, so 720 + 280, so that is 1000, so

$Z = Max. Z = 1000$ okay, so that is how we solve the given linear programming problem. In our next lecture we will discuss a more examples on simplex method where we will find unbounded solution, here we found optimal solution, in those problems we will find unbounded solution, so that we will do in our next lecture. So thank you very much for your attention.