Higher Engineering Mathematics
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Lecture 40: Trees-2

Hello friends, welcome to my second lecture on Trees. Let us recall the definition of a tree. A tree is a connected graph without circuits or cycles. Now, what is a spanning tree? A sub graph T of a graph G is called a Spanning tree of G if T is a tree and T includes all the vertices of G.

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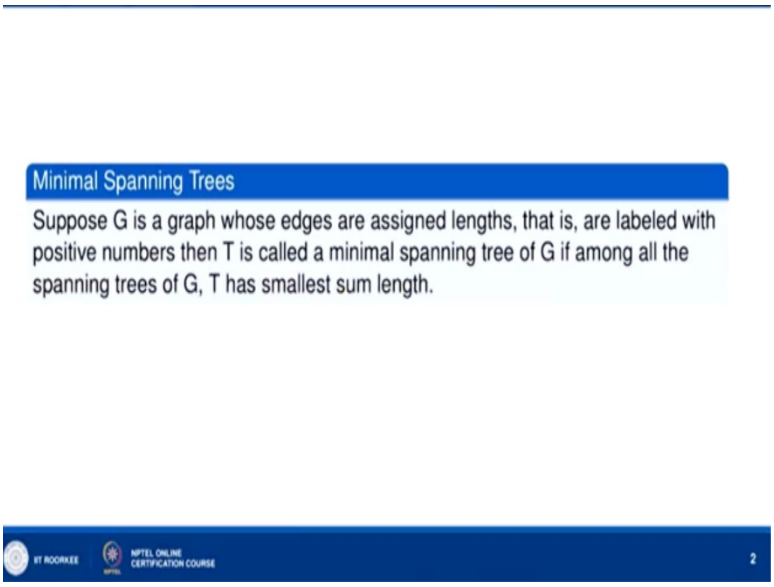

Now, among the spanning trees of a graph G the one which has the shortest length is called is as the minimal spanning tree. So, in this lecture we shall be finding, we shall be discussing the algorithms to determine to find the minimal spanning tree of a given graph G okay.

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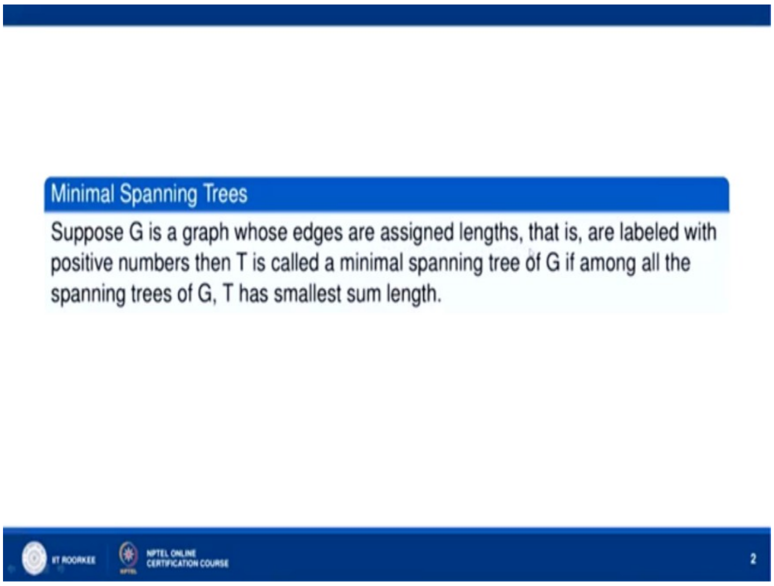

Now suppose G is a graph okay whose edges are assigned lengths, that is, are labeled with positive numbers then T is called a minimal spanning tree of G if among all the spanning trees of G, T has smallest sum length or length sum okay the sum of lengths of the edges of T is the smallest. So we are going to discuss two algorithms by which we can determine the minimal spanning tree.

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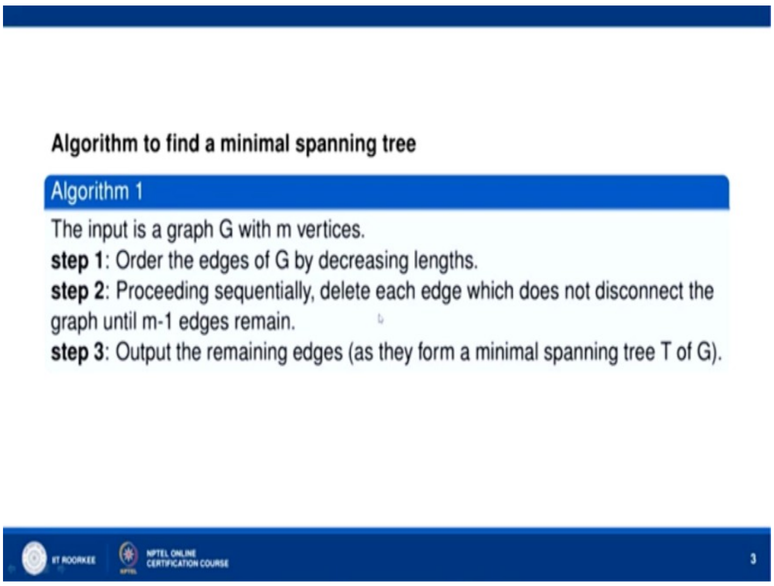

Now, let us discuss first algorithm. This algorithm says that the input is a graph G with m vertices we are given a graph G with m vertices, let us order the edges of G by decreasing lengths because the edges of G are labeled by positive numbers so we can order the edges of G by decreasing lengths. Now, proceeding sequentially delete each edge which does not disconnect the graph until m minus edges remains, this is because if a tree is there with n vertices then it has got n - 1 edges okay.

So minimal spanning tree will have m-1 edges because the graph G has m vertices. So the minimal spanning tree will also have m vertices and will therefore, have m - 1 edges. Now, step 3 is output the remaining edges as they form a minimal spanning tree T of G. Now let us see how we can apply this algorithm.

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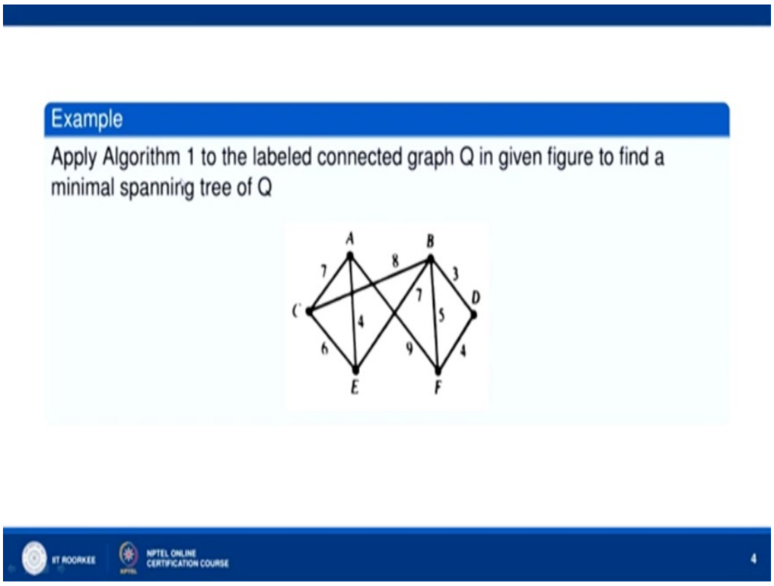

Let us discuss a problem here, so apply algorithm 1 to the labeled connected graph, we are given this labeled connected graph, the edges of this graph are labeled by positive numbers, positive integers, so apply algorithm 1 to the labeled connected graph Q in the given figure to find a minimal spanning tree of Q. So we are going to find minimal spanning tree of Q.

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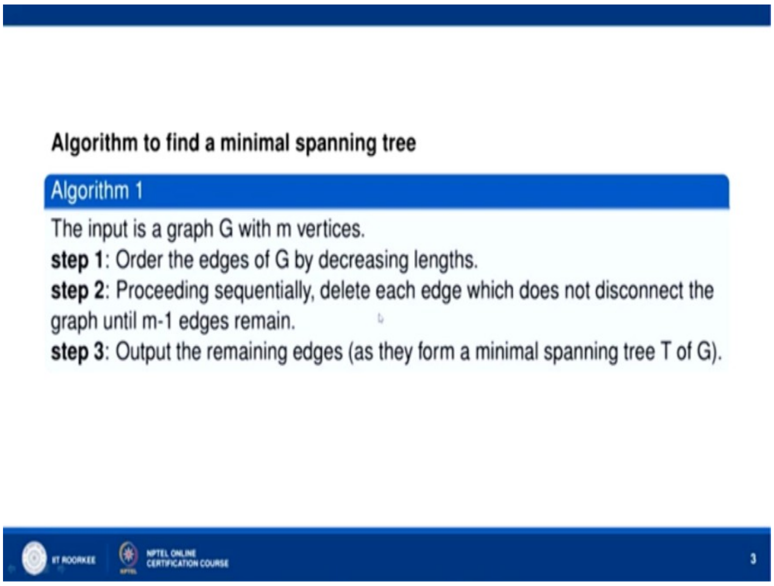

Let us follow step one, Step 1 says that order the edges of G by decreasing length okay.

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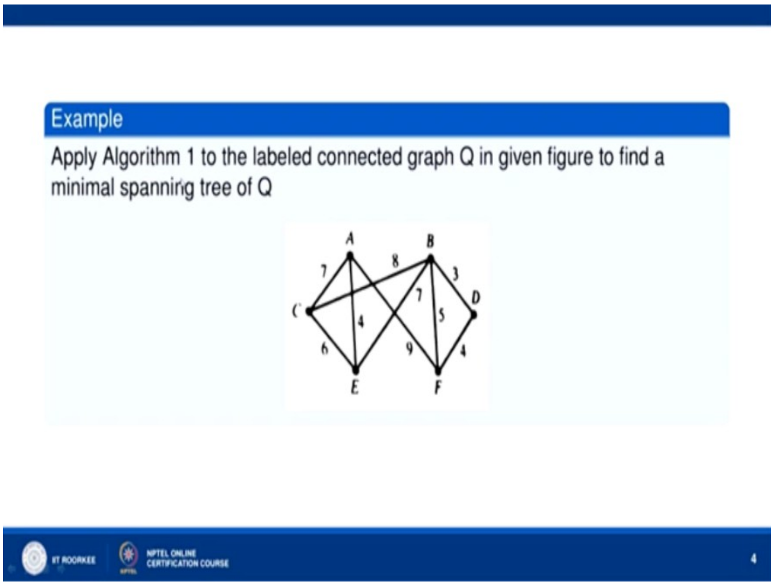

So we can see which edge has got the maximum length, you see we have edge AF okay, edge AF has got length 9 okay, then BC has length 8 okay, next highest okay, and then AC has length 7, then we have BE has also length 7, and then we have edge CE which has got length 6 okay, then BF has got length 5, then we have AE and FD which have got length 4 each, and then we have BD which has got length 3, and then yeah so we will arrange them in the decreasing order of their lengths. So, let us see how we can do that.

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Sol. Graph Q has 6 vertices, hence any spanning tree of Q will have 5 edges. By Algorithm 1, the edges are ordered by decreasing lengths and are successively deleted (without disconnecting Q) until five edges remain. This yields the following data:

Edges	AF	BC	AC	BE	CE	BF	AE	DF	BD
Length	9	8	7	7	6	5	4	4	3
Delete	Y	Y	Y	N	N	Y	N	N	N

See here, we see that graph Q has 6 vertices okay, here we have 6 vertices A, B, C, D, E, F okay and therefore, minimal spanning tree of this graph will have 5 edges okay, so by algorithm 1 the edges are ordered by decreasing lengths, and are successively deleted without disconnecting the Q, graph Q okay, until 5 edges remain okay. So, now AF has got the length 9, BC has got length 8, then AC has got length 7, BE has got length 7, CE 6, BF 5, AE 4, DF 4, BD 3, so we have arranged the edges in order of their decreasing lengths

Now, let us see, we will start deleting edges okay, without disconnecting the graph until only 5 edges remain okay. So, how we do that? So, we start with the edge which has hot the maximum length from here we start, so deleting AF okay, if we delete AF whether the graph gets disconnected okay, so if we delete AF okay, the graph is not disconnected okay, so we delete AF, so yes, then we delete, then should we delete BC, if we delete BC okay, B and C, if we delete BC the graph does not become disconnected so we can delete BC okay so okay.

Then AC is, where is AC, this is AC if we delete AC the graph is not disconnected and therefore, we delete AC okay, and then BE let us see if you delete BE okay, what will happen, because we have already deleted AF, BC, AC okay AF this is deleted, AF and then we have deleted AF, we have deleted BC, AC BC, we have deleted and then we have deleted AC okay. So we have deleted BF we deleted BC we have deleted AC okay. Now if we delete BE what happens okay, so B and then E okay, if we delete B, what will happen okay. See this is AC means we have this figure after deleting AC.

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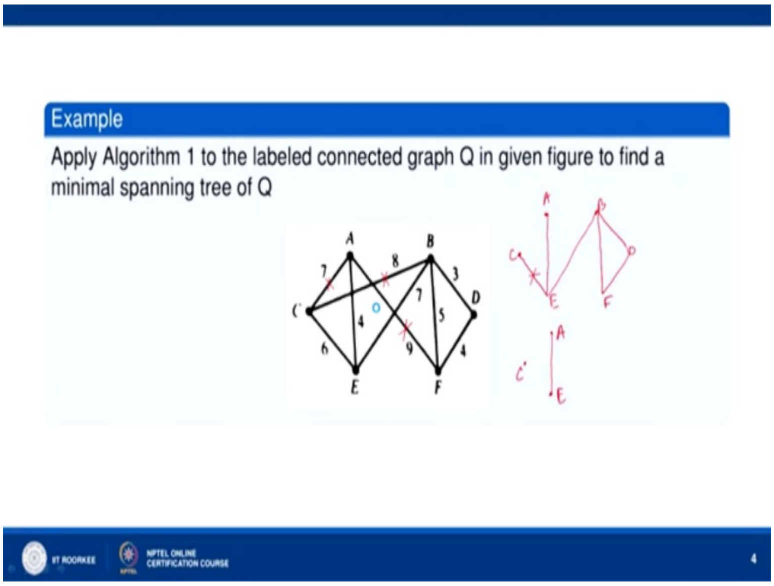

A this is C E okay then we have AE we have deleted AC okay and we have deleted BC also and then we have B here, okay we have deleted A, okay we have deleted AF also now BD and we have BD DF, we have this okay. So what we are doing, we are deleting, see we have deleted AC, this is AC, we have deleted AF, we have deleted BC, we have deleted BC here.

Now, if we delete BE, okay if we delete BE this one, then the graph becomes disconnected, so we cannot delete BE okay so this BE remains there so it is not to be deleted and then CE okay if we delete CE okay what will happen if you delete CE here then C will remain and we will have like this because deleting an edge does not delete the vertex. So C vertex will remain there okay, so we will have C that will become an isolated vertex. So CE and then will have this situation so it becomes disconnected okay and therefore we cannot delete CE cannot be deleted.

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Sol. Graph Q has 6 vertices, hence any spanning tree of Q will have 5 edges. By Algorithm 1, the edges are ordered by decreasing lengths and are successively deleted (without disconnecting Q) until five edges remain. This yields the following data:

Edges	AF	BC	AC	BE	CE	BF	AE	DF	BD
Length	9	8	7	7	6	5	4	4	3
Delete	Y	Y	Y	N	N	Y	N	N	N

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Okay then we have in all 1, 2, 3, 4, 5, 6, 7, 8, 9. We have 9 edges we have to delete 4 edges so that we get minimal spanning graph with minimal spanning tree with 5 edges. Okay so we have already deleted 3 edges we have to delete 2 more edges. Now let us can we delete BF okay so BF yes, we can delete BF right okay we can delete it because it because tree is in the tree there should not be any circuital cycle so we can delete BF.

Okay so we have deleted 4 edges now one more has to be deleted, now we have deleted 4 so now how many are remaining 1, 2, 3, 4, 5 are remaining, so we have deleted, with the deletion of this edge BF we have in all deleted 4 edges and therefore the remaining 5 edges give us the minimal spanning tree and that is the following.

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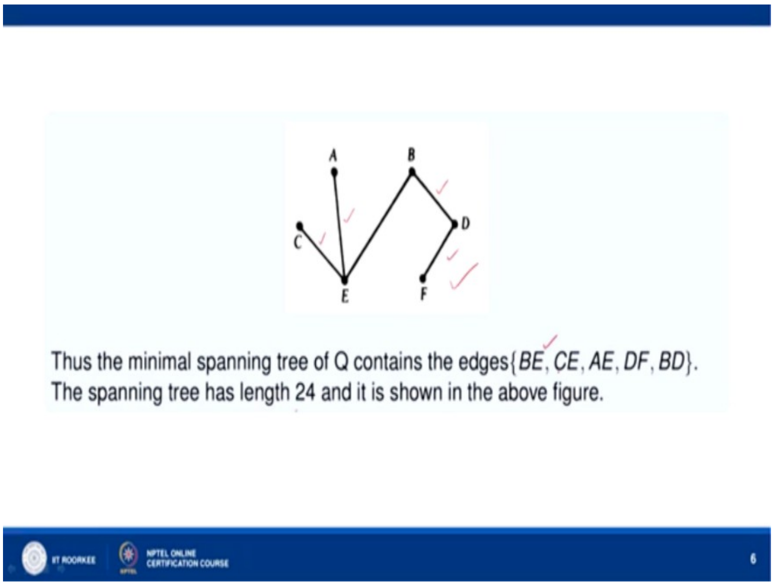

This graph okay where we have deleted 4 edges and the reaming edges are BE okay CE this CE, and then AE, and then we have DF, and we have BD okay so it the minimal spanning tree it is a tree with 5 edges and it the minimal spanning tree and the sum of lengths here, lengths sum is equal to 24 because. We can see that what are the values assigned to BC and AE, DF, BD.

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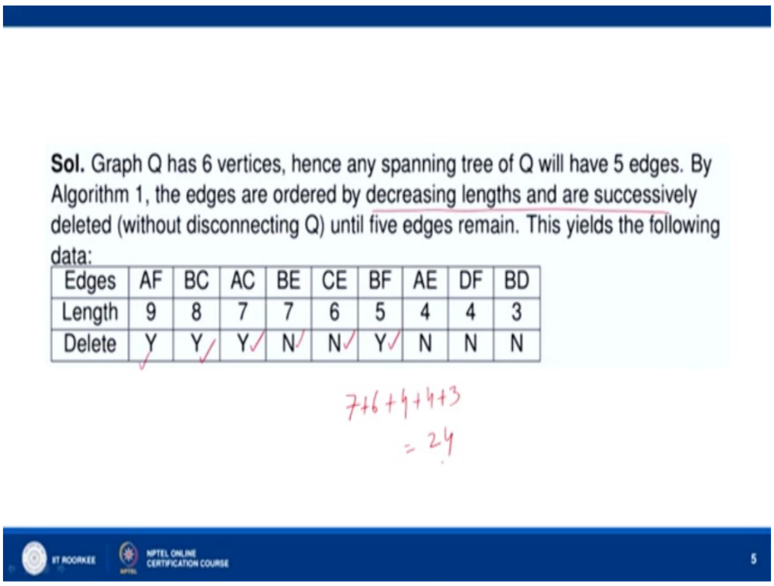

So that we can see from here so BE has length 7 BE is given 7, and then CE is 6 okay and then we are given AE with us 4 okay, DF with 4 and BD with 3. So this is 7 + 6 = 13, 13 + 8 = 21 + 3 so 24.

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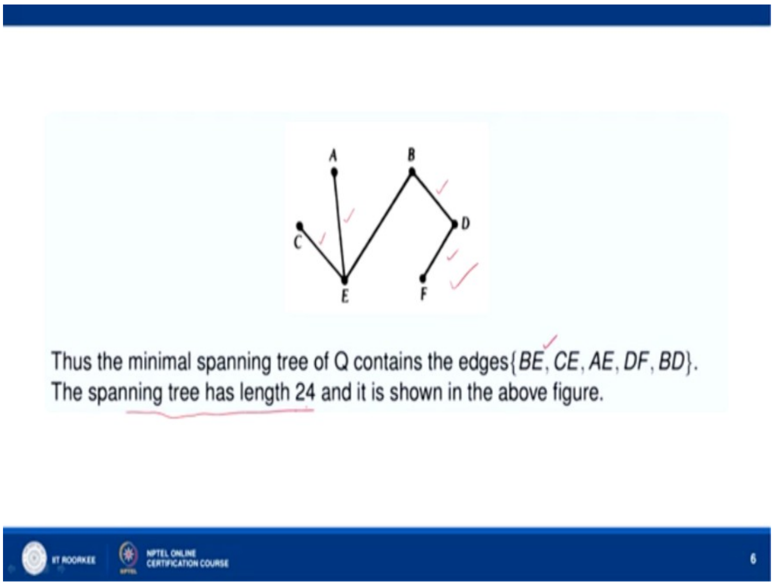

So the spanning tree minimal spanning tree has length 24 and it is this shown in this figure okay.

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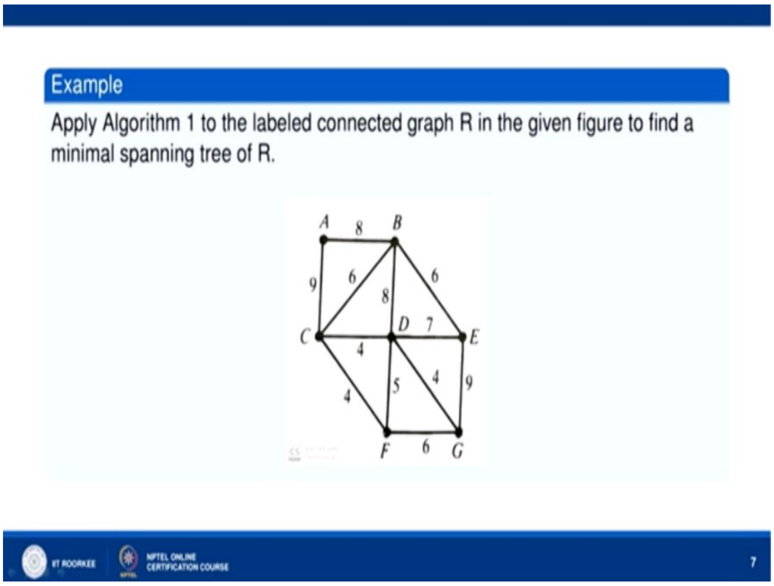

Now, let us apply algorithm 1 to this graph another example, let us take and we can find a minimal spanning tree of R okay. So you can see here the 9 is the highest, maximum value okay so we are going to arrange them in the decreasing order, lengths in the decreasing order so EG will come and after that EG will have BD because BD has 8 lengths okay, so AC also has 9, AC then EG then BD and after that DE because it has 7 length, and then DE we have BC 6 which has 66, and then BE which has 6, then DF which has 5 and so on, so we have this table okay.

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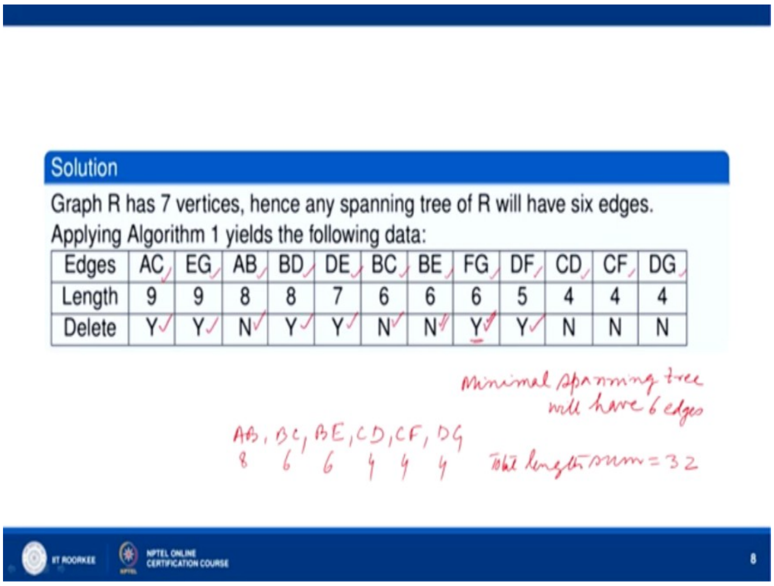

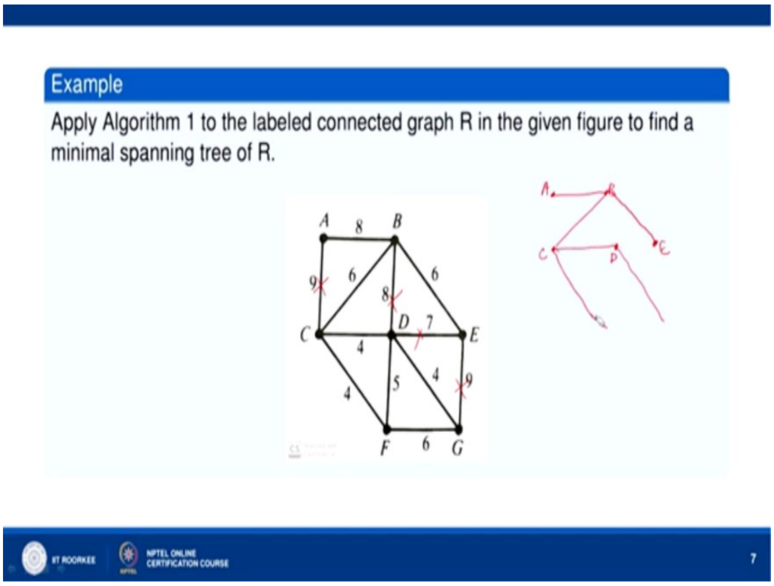

So, we will have this table AC with 9, EG 9, AB 8, BD 8, ED 7, BC 6, EF 6, DF 5, CD 4, CF 4, DG 4 okay. So, and now we are given the graph R with 7 vertices A, B, C, D, E, F, G, 7 vertices are there. Okay and therefore for the minimal spanning tree there will be 7 - 1 that is 6 edges okay, so minimal spanning tree will have 6 edges. Now, how many edges are there 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, (it total) twelve edges are there so we have to remove 6 edges. Okay by applying algorithm 1 now let us see how we delete the edges here so let us follow the algorithm okay so we will go on deleting edges in such way that the graph does not become disconnected.

So AC okay we can delete AC okay let us see we can delete AC, yes, then EG can we delete EG, yes, we can delete EG, the graph does not become disconnected and then can we delete AB? See AB okay, if you delete AB we have already deleted AC, if you delete AB, then vertex A will become an isolated vertex so the graph will become disconnected and therefore we cannot delete AB. So no then BD can we delete BD, B here and D here we can delete BD the graph does not become disconnected.

So yes and then DE can we delete DE yes we can delete DE the graph is not disconnected. Okay so yes now BC if we delete BC we have already done deleted AC right so we are having this situation if we delete BC BD we have deleted. BD we have deleted perhaps yes BD we have deleted right so what will happen .So this is BD so this is A, B but BD we have deleted already. So this will not be there okay so this is A, B we have deleted AC we have deleted BD okay so and now the question is whether can we delete BC this your BC.

If we delete BC we will have AC here and then what will've have we have already we have deleted AC we have deleted EG we have deleted AC okay. Let us see AC we have deleted EG we have deleted. EG and then we have deleted. AB and no AB we have not deleted BD we have deleted BD so BD we have deleted. And then we have deleted DE okay so DE we have deleted okay yeah so then what now BC if we delete what happens okay so BC if we delete we will have this situation okay BC if we delete we will have suppose we delete this okay we will have this situation and we will have BE okay BE we have not deleted BE we have not deleted so we will have this and then okay CF is there.

CF is there and also DF we will delete later on okay DF we will delete later on, the question was whether BC can be deleted, if we delete yeah that is the situation actually okay see, if we delete BC the 2 graphs becomes disconnected because D has already been deleted okay, there is no, D is not there so graph is actually like this now. The graph which is remaining is like this, so if we delete BC if this BC is deleted the graph becomes disconnected and therefore, BC is not to be deleted

Okay so BC can't be deleted and BE can be deleted if you delete BE the again the graph will become disconnected okay so because BC is there right if we delete BE then the graph will become disconnected because E will become an isolated vertex. If I delete this B I will have this situation so again this is disconnected BC that cannot be done we cannot delete BE okay so this is also correct.

Then FG can be deleted yeah FG can be deleted, yeah FG can be deleted okay, and then DF okay DF now the situation is like this okay we have deleted BE alright and then we have deleted this one yeah so you can delete DF is this you can delete DF and when you delete DF what we get is the following. So, let us see so DF can be deleted FG is already been deleted yes so this is not there okay so we have this and we can see that this graph is not disconnected okay.

So we have all together deleted 1, 2, 3, 4, 5, 6 edges and so 6 edges are remaining and therefore the graph that we are getting after deleing the 6 edges is nothing but the minimal spanning tree. Okay and minimal spamming tree has edges AB, then we have BC, we have BE, we have CD, and we have CF, and we have CF okay so these are the edges and then DG, and there edges are labeled with the numbers AB is equal with 8, BC is with 6, BE with 6 again, CD with 4, CF with 4, and DG with 4 okay so 8, 6 that is 14 + 6 that is 20. 20 + 12 so this is total length sum = 32   8 + 6 + 6 + 4 + 4 + 4 so minimal spanning tree has these 6 edges and its total lengths sum is 32 and the minimal spanning tree is following this figure.

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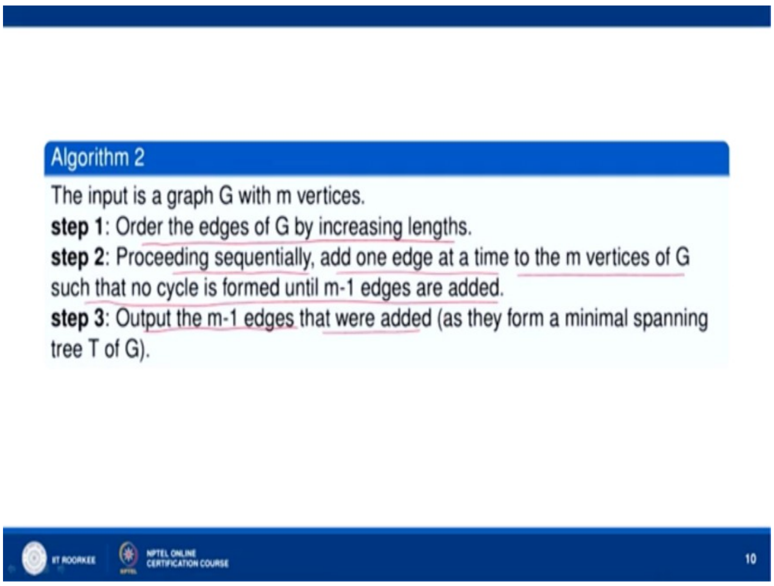

Now, let us discuss algorithm 2 okay this algorithm is based on arranging the edges by increasing lengths, in the algorithm 1 we have arranged them in the by decreasing lengths here we arrange them by increasing lengths, so this is the step 1, we arrange the edges of G by increasing lengths and then proceeding sequentially we add 1 edge at a time to the m vertices of Group such that no cycle is formed until m - 1 edges are aided. As soon as we have an m-1 edges and ensured that no cycle is formed okay. The output will be the m-1 edges that were added and they will form a minimal spanning tree of G.

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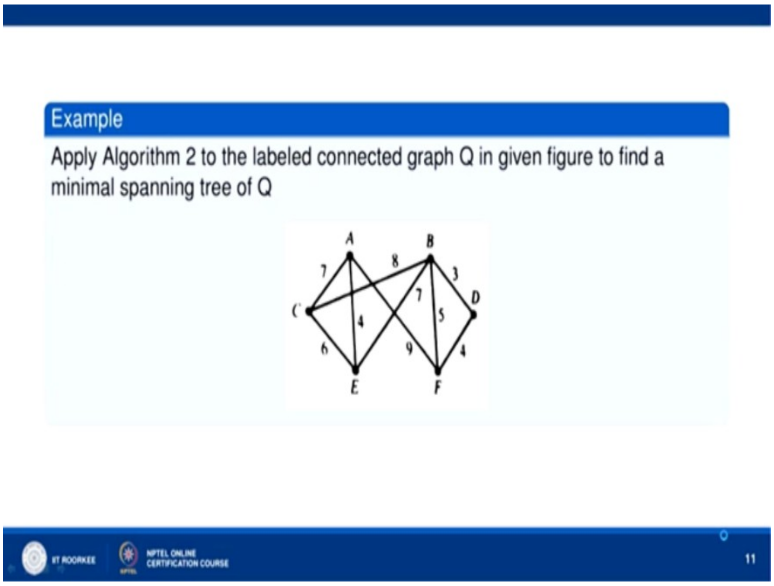

So, let us see, we are going to apply this algorithm to the labeled connected graph Q in this figure this we have discussed this graph we discussed earlier and we applied algorithm 1 to this graph to obtain minimal spanning tree. Now let us apply algorithm 2 to this graph and find the minimal spanning tree okay. Let us again, now this time we arrange the edges in order of their increasing lengths okay.

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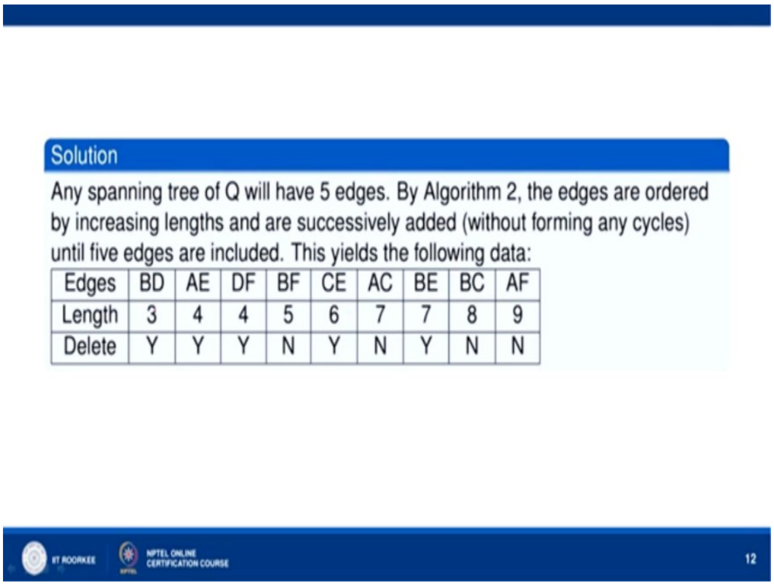

So we have this so BD, AE, DF, BF .CE, AC, BE, BC, AF they are in order of the increasing lengths 3, 4, 5, 6, 7, 7, 8, 9 okay. Now, we have in all here edges are 6 okay sorry these vertices are 6.

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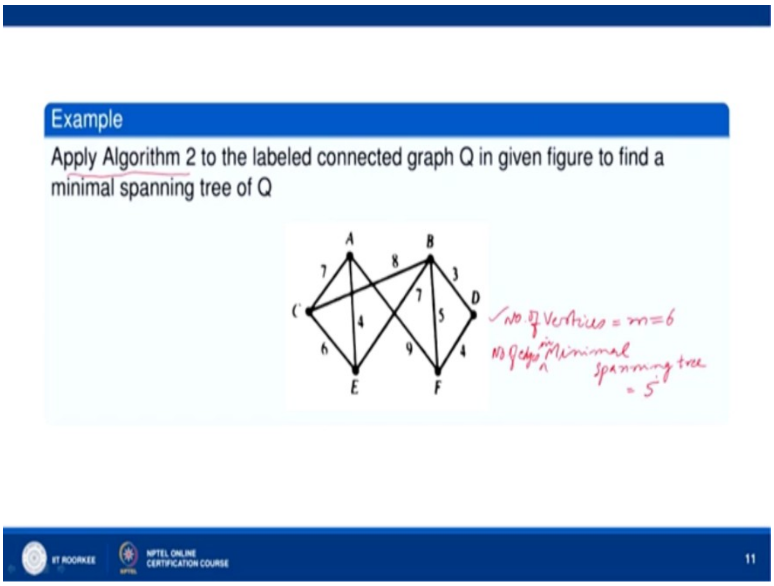

But still number of vertices = m = 6 so minimal spanning tree will have 5 edges. Number of edges in minimal spanning edges will be m - 1 that is 5 okay. So, what we do we go on adding edges till 5 edges are added m - 1 and it is ensured that no cycle or circuit is formed okay in that process, so we see we have arranged them in the increasing order now what we will do okay.

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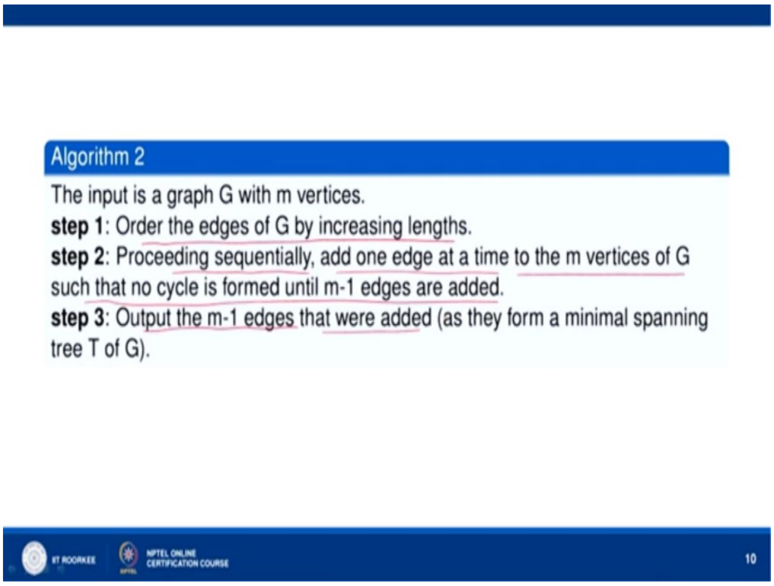

So see proceeding add 1 edge at a time to the m vertices of G such that no cycle is formed until m - 1 edges are added okay. Yeah so, let us see where is B? You see, we have B here okay.

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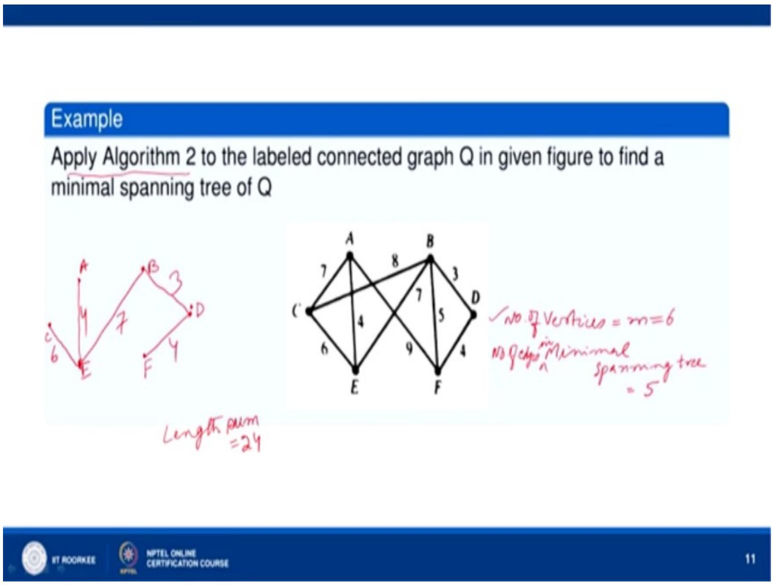

B and we have D here okay, so first step is that BD is added, we are adding okay BD is added and then add 1 at a time to the m vertices okay. So we are adding 1 edge BD here and then the second the case is what second case is AE okay so AE this is A here at the same level A here and E here so we add AE okay so we have added 2 edges we have to add 3 more okay 3 more right so then yes and then DF let us add DF okay D and then F if we add DF so this is

DF okay DF yes and then now shall we add BF okay so let us see if you add BF a circuit is formed so we should not add BF okay, so no.

Then CE, shall we add CE, C is here okay C, so we can add CE okay, alright, now we have 4 edges, 1 more is to be added and so CE, yes, then AC if we add AC, what happens if you add AC okay it becomes a circuit so we will not add AC, and then BE, shall we add BE okay we can add BE, okay now you can see we have 5 edges 1, 2, 3, 4, 5 okay and the graph is a tree and without any circuits.

So it is a spanning tree and by the algorithm which is the minimal spanning tree because we have added edges in such a way that no cycle is formed until 5 edges are formed okay. So this is the minimal spanning tree in this case and you can see that CE is labeled with 6, AE is labeled with 4 okay, BE is labeled with 7 alright, and then BD is labeled with 3, and DF is labeled with 4 okay. So total length is 6 + 4 = 10 , 6 + 4 + 7 that is 17 + 3 = 20 + 4 = 24 so minimal spanning tree is formed from this 5 edges and the length sum is 24. So this is how we reach the minimal spanning tree by applying algorithm 2 to this problem so there's the figure.

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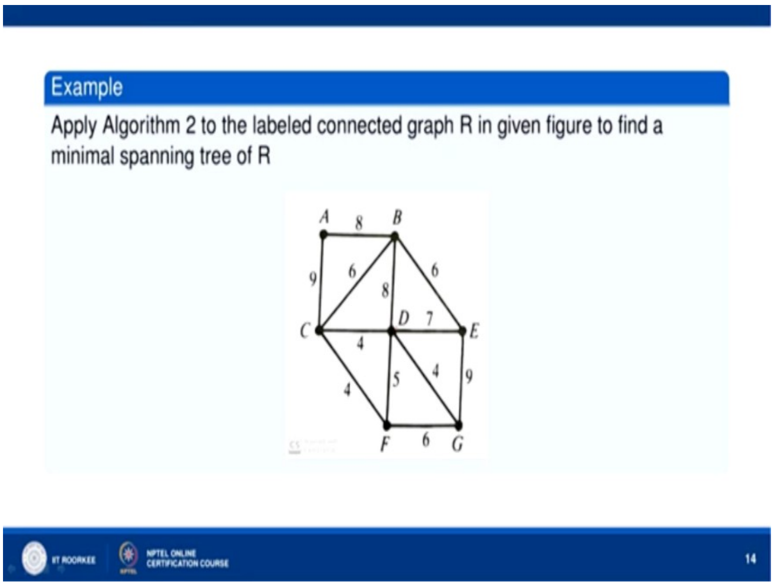

Now let us do this other example also this we have discussed earlier there we applied algorithm 1 now let us supply algorithm 2 to this problem okay. So let us arrange them in the increasing length in the order of increasing lengths.

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Solution

Any spanning tree of R will contain 6 edges. Applying Algorithm 2, this yields the following data:

Edges	CD	CF	DG	DF	BC	BE	FG	DE	AB	BD	AC
Length	4	4	4	5	6	6	6	7	8	8	9
Delete	Y	Y	Y	N	Y	Y	N	N	Y	N	N

Edges to be arrange in the increasing lengths so CD, CF, BG, DF they are then BC, BE, FG and then DE then AB then BD then AC they are arranged in the order of increasing lengths.

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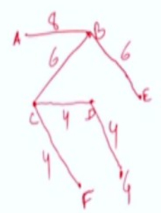
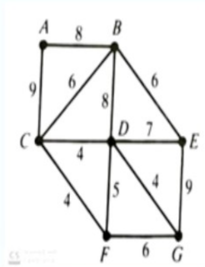
Example

Apply Algorithm 2 to the labeled connected graph R in given figure to find a minimal spanning tree of R





No. of vertices = m = 7

No. of edges in the minimal spanning tree = 6

Length sum = 32

Solution

Any spanning tree of R will contain 6 edges. Applying Algorithm 2, this yields the following data:

Edges	CD	CF	DG	DF	BC	BE	FG	DE	AB	BD	AC
Length	4	4	4	5	6	6	6	7	8	8	9
Delete	Y	Y	Y	N	Y	Y	N	N	Y	N	N

Now, what we do is there are total how many vertices are there 1, 2, 3, 4, 5, 6, 7 vertices number of vertices = m = 7. So number of edges in the spanning tree, minimal spanning tree will be in the will be 6 okay. Yeah, so now let us add edges to the vertices of this graph G and ensure that no circuit cycles are formed until we have got 6 edges okay in the graph, so okay, so let us see yes, so the first thing that we to see is CD okay, shall we add CD to the graph okay, so C is here, D is here okay so this CD okay and then after CD CF can CF be added CF okay CF is here right so CF can be added okay

CF can be added yes and then DG where is DG yeah DG can be added okay there is no circuit cycle has been formed so far and then we have DF now if we add D to F the we are seeing that CDF okay becomes a cycle so we cannot add D to F DF cannot be added and then we so this is no and then then BC okay so BC this is BC we can BAB can be added okay now 1, 2, 3, 4 edges we have drawn 2 more are to be added, now we okay, so we have added BC, then BE okay shall we add BE this is BE

Okay we can add BE now okay so yes now so how many we have added 1, 2, 3, 4, 5 one more is to be added to this graph. Now then we have FG, can FG be added FG, if we add FG, sorry if we add FG you see what happens is a cycle is formed so we cannot add FG okay so what we will do is FG cannot be added okay and then DE, can DE be added, if you add DE okay we add DE okay, then it becomes a cycle so we cannot add DE.

So it cannot be added okay now what we have okay so DE cannot be added now we have AB okay so AB is this yes we can add AB and we can see now we have in all 6 edges 1, 2, 3 ,4 , 5 ,6 okay and the no circuit is formed okay and therefore the we stop here okay we have got the minimal spanning tree and the length of this spanning tree is 8 here then we have BC is 6 okay BE is 6 and then DG is 4 and we have CF is 4 and we have CD is 4 ,so 8 + 6 = 14, 14 + 6 = 20, 20 + 4 = 24 + 4 =28 + 4 = 32 so length sum = 32 okay so we can draw the minimal spanning tree by applying the algorithm to here, in this case so we have got the sum length equal to 32 and we have this figure of the minimal spanning tree.

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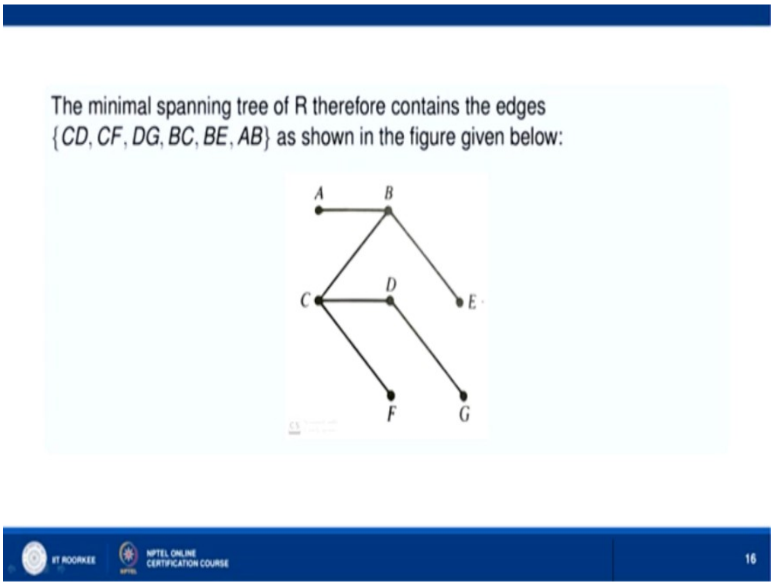

So that is all in this lecture thank you very much for your attention.