Higher Engineering Mathematics
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Lecture 37
Coloring of Graphs - I

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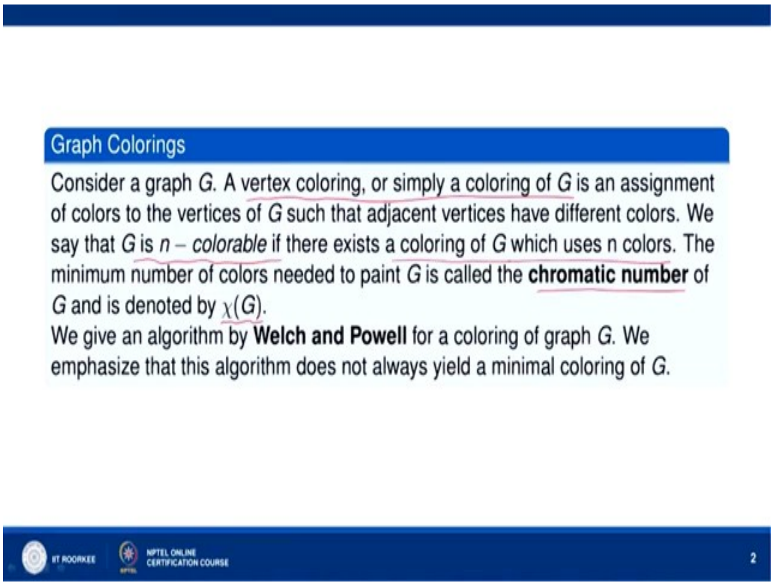

Hello friends, welcome to my lecture on Coloring of Graphs. Let us consider a graph G vertex coloring are simply a coloring of G is an assignment to the vertices of G such that adjacent vertices have different colors. We say that G is N colorable if there exist coloring of G which uses N colors. The minimum number of colors needed to paint a graph G is called the chromatic number of the graph G and is denoted by $\chi(G)$. Now we shall discuss an algorithm by Welch and Powell for coloring of graph G. We will decide that this algorithm does not always yield minimal coloring of G.

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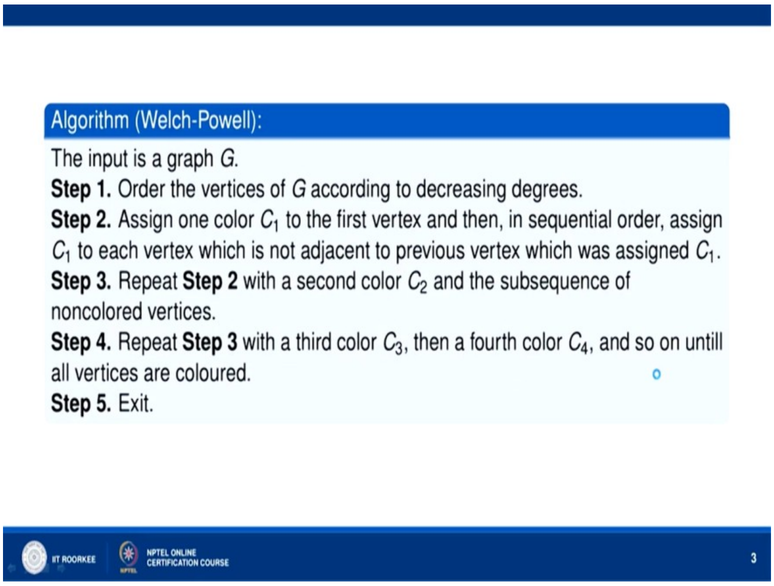

Now, let us see what is the Welch Powell algorithm? The input is the graph G, the 1st step is order the vertices of G according to the decreasing degrees okay so we will find the decrease of all the vertices of the graph and then arrange them in the order of decreasing degrees. Now, step 2 is assign one color C_1 to the 1st vertex okay and then 1st vertex means the vertex with the highest degree, and then in sequential order assign C_1 color to each vertex which is not adjacent to the previous vertex that is vertex with the highest degree and which was assigned C_1 okay. Repeat step 2 with 2nd color C_2 and the subsequence of non-colored vertices, so in the step 3 we shall start repeating the step 2 with 2nd color. Now in the step 4 we repeat step 3 with the 3rd color C_3 and 4th color C_4 and so on until all vertices are colored and then we exit.

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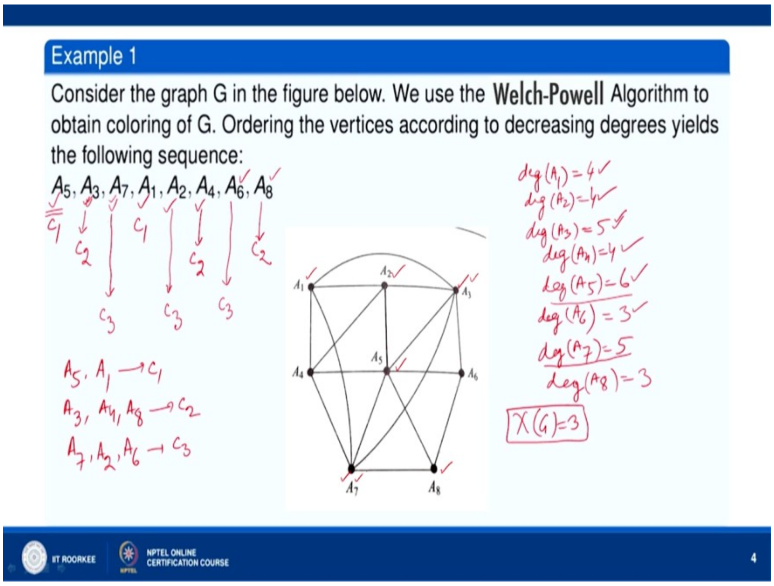

So, let us see how we do this, let us consider this graph okay this graph let us consider. In this graph you see there are 8 vertices okay, we have to find degrees of each vertex and then arrange them in the order of decreasing degrees. So degree of A_1, let us 1st find the degree of A_1, degree of A_1 is 1, 2, 3, 4, degree of A_2 is 1, 2, 3, 4, degree of A_3 is 1, 2, 3, 4, 5, degree of A_4 is 1, 2, 3, 4, Degree of is 5, A_5 has degree you see 1, 2, 3, 4, 5, 6 and then degree of A_6 so your 1, 2, 3, degree of A_7 okay degree of A_7 is 1, 2, 3, 4, 5, degree of A_8 okay 1, 2, 3, now let us see which one has the highest degree. You see we have degree of A_7 is equal to 5, degree of A_5 is equal to 6 okay so A_5 has the highest degree, so we have put A_5 in the in the 1st place in the sequence.

Next comes A_3 okay, A_3 has degree 5 okay 1 lower than degree of A_5 so A_3 becomes next and then A_7, A_7 also has the same degree we can write A_3, at the 2nd place we can write A_3 or we can write A_7 because A_3 and A_7 have equal degrees. After A_7, A_1 has degree 4 okay so we have A_1 then A_2, A_2 has degree 4 so we have A_2 then A_4, A_4 has degree 4 so we have A_4 then A_6, A_6 has degree 3 and we have A_8, A_8 has degree 3 ok. Now, let us see how we shall find the colors to these vertices okay. So we have arranged the vertices in the order of decreasing degrees then we start with the 1st vertex A_5.

We assign color C_1 to A_5 ok so C_1 color you find to A_5 then A_5 is adjacent to A_3 okay so then we leave it then A_5 is adjacent to A_7 so this also is left and then A_1.A_1 is here, A_5 is not adjacent to A_1 so we assign C_1 color, now then let us say whether A_2 is adjacent to A_5 or not?

A_5 is adjacent to A_2 okay so we leave A_2 then A_4, A_4 is adjacent to A_5 so we leave it, then A_6, A_6 is adjacent to A_5 so we also leave it and then A_8, A_8 is also adjacent to A_5 so C_1 color is given to A_5 and A_1 ok. Now we have completed one round that is from A_5 starting with A_5 to which we assign color C_1, we have checked all the vertices in the sequence okay up to A_8.

Now we come back and start with A_3 ok, A_3 we assign color C_2 ok then A_3 you see let us see A_7 is adjacent to it, so A_3 is here A_7 is here, A_3 is adjacent to A_7 ok they are joined okay so A_7 is not adjacent to A_3 and therefore, we leave A_7 and then A_1 has already been assigned the color A_2. Yeah A_2 is adjacent to A_3 so we leave it, A_4, A_4 is not adjacent to A_3 so A_4 is assigned color C_2. And then A_6, A_6 is adjacent to A_3 so we leave it, A_8, A_8 is not adjacent to A_3 so we give it color C_2 so A_8 is also given color C_2. Now we have computed the color C_2, we go back and start with A_7 .

A_7 is given color C_3 okay, A_7 is here okay A_7 is here. Now we have to see this one, this one, this one, this one, this one, this one, yeah A_2, where is A_2? A_2 is not adjacent to A_7 so A_2 is given color C_3 ok. And then what about A_6? A_6 is here, it is not adjacent to A_7 so it is also given color C_3 ok. So A_5 and your A_1 they have color C_1, they are colored with color C_1, A_3 and then A_4 and then A_8, they are colored with color C_2, and then A_7, A_2 and then we haveA_6 , they are colored with color C_3. So 3 colors are used to color all the vertices of this graph and we see that let us now find the minimum number of colors to paint this graph ok.

We see that A_1, A_2, A_3 okay, A_1, A_2, A_3 are adjacent ok, A_1 is adjacent to A_2, A_2 is adjacent to A_3 ok so they are to be painted with different color. So A_1, A_2, A_3 are to be painted with different colors so there we will need 3 colors to paint A_1, A_2, A_3 consider A_5,A_7, A_8 ok A_5, A_7,A_8 they are adjacent vertices ok A_5 is adjacent to A_7, A_5 is adjacent to A_8 so they will have to be painted with different colors so 3 colors will be needed to paint A_5, A_7, A_8 with different colors. So minimum here we have used 3 colors to paint the entire graph so minimum number of colors needed to color this graph okay are 3, so that means chromatic number is chromatic number of G is 3 which is the minimum number of colors to be used to color the given graph, so 3 is the committing number here.

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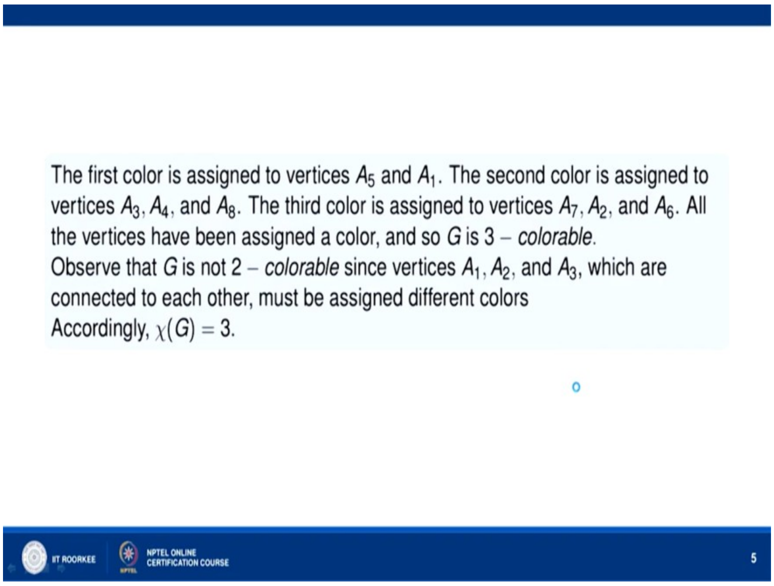

So this is the so thus we can say that this graph G is 3 color. If 3 colors are to be used to color this graph then we say that it is free colorable. Now it is not two colorable as we have seen it is not 2 colorable because A_1, A_2, A_3 they are adjacent, A_1 is adjacent to A_2, A_2 is adjacent to A_3 so that means they are to be painted with different colors.

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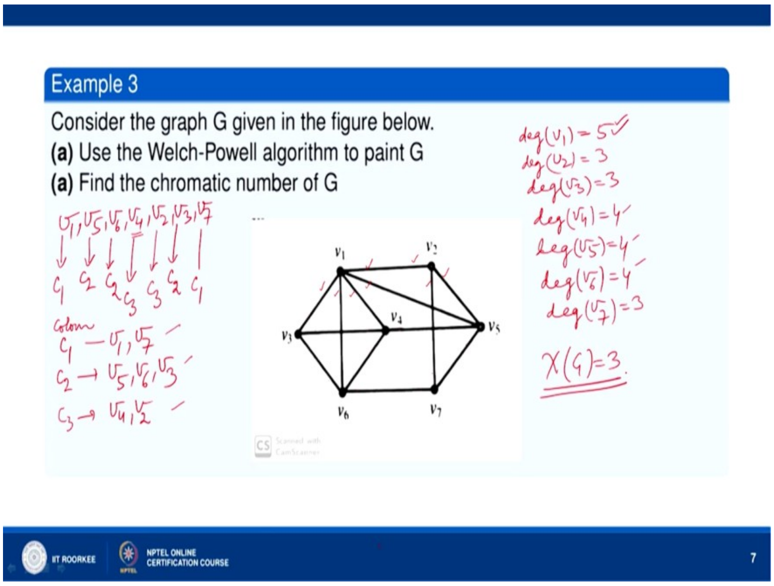

Now we have to consider the complete graphic A_6 okay, K_{10}, and in general we can take K_n. K_n is complete graph, in the case of complete graph every vertex is adjacent to every other vertex ok. So if vertices are V_1, V_2, V_3, V_4, V_5, V_6 ok then each vertex is adjacent to every other vertex that means they will all have to be painted with different colors okay so 6 colors will be needed, 6 colors are required for painting K_6 ok. And similarly K_{10}, in the case of K_{10}, 10 colors will be required, 10 colors for coloring K_{10}. In general n colors are required and this is the minimum number of colors okay X_n equal to n here because it is a complete graph, you cannot color this graph with less than n colors.

Now let us consider this graph okay, so here we are again use Welch Powell algorithm to paint this graph and we shall find the chromatic number of G. So there are how many vertices? 7 vertices so let us find the degree of V_1, you see 1, 2, 3, 4, 5 so it is 5, degree of V_2 is 1, 2, 3. Degree of V_3, degree of V_3 is 1, 2, 3, degree of V_4, V_4 has got degree 1, 2, 3, 4, degree of V_5 is 1 2 3 4, degree of V_6 is 1, 2, 3, 4, degree of V_7 is 1, 2, 3 ok.

Now let us write the vertices according to that degrees okay, so we have maximum degree is 5 okay so V_1 has the maximum degree okay so we have V_1 in order of decreasing degrees. So then V_4, V_5, V_6 they all have degree 4 ok so V_4,V_5, V_6, we can write in any order,V_4,V_5, V_6 because we all have same degrees ok and then we V_2, V_3, V_7, they all have degree 3 okay.

V_1 has degree by 5,V_4, V_5, V_6 have degree for each and then V_2,V_3, V_7 have degree 3 ok. Now so we color V_1 with C_1 color ok and let us see then which vertices are adjacent to V_1, so V_1 is here V_1 is adjacent to V_5 ok V_1 is adjacent to V_5 so we leave V_5.V_6,V_1 is adjacent to V_6 so we will leave V_6.V_4,V_1 is adjacent to V_4 ok, V_1 is adjacent to V_2 yes, V_1 is adjacent to V_3 yes, V_1 is adjacent to V_7 no, so V_7 is also colored with C_1 ok.

Now we go to V_5 okay V_5 we color with C_2 now V_5 is here okay so V_5, V_6, where is V_6? V_6 is here soV_5 is not adjacent to V_6 so we give color C_2 to V_6. And then V_4, V_4 is adjacent to V_5 okay so we leave V_4 .V_2, V_2 is adjacent to V_5 so we leave V_2. Then V_3, V_3 is here ok V_3 is not adjacent to V_5 soV_3 is also given color C_2 okay.

Then we come to V_4 okay, now we will give color C_3 to V_4 ok. V_4 is colored with C_3 and V_2 and very V_2? V_2 is here so V_4 is not adjacent to V_2 so V_2 is also colored with C_3 ok. So thus, C_1 color is used for V_1 vertex, V_7 vertex, and C_2 color is used for coloring V_5, V_6 and V3, and C_3 color is used to color V_4 and V_2 ok so 3 colors are required to paint this graph okay.

Now you see, V_1 is adjacent to V_4, V_4 is adjacent to V_6, V_1 is adjacent to V_4 and V_4 is adjacent to V_6 so V_1,V_4 , V_6 have to be painted with different colors and therefore 3 colors will be required to paint the vertices of V_1, V_4 and V_6 so at least 3 colors have to be used but here we have used 3 colors to paint the entire graph therefore, the chromatic number which is the minimum number of colors to be used to paint the graph is $\chi(G)$ is equal to 3 okay, so the chromatic number here is $\chi(G)$ is equal to 3.

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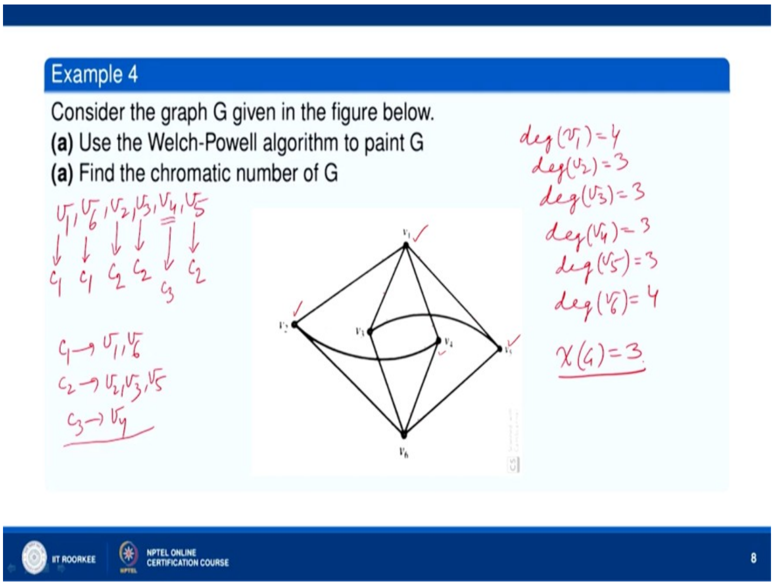

Now let us consider this graph okay, so here let us see what is the degree of V_1? We have V_1, V_2, V_3, V_4, V_5, V_6 ok degree of V_1so 1, 2, 3, 4. Degree of V_2, degree of V_2 1, 2, 3. Degree of V_3 so 1, 2, 3. Degree of V_4 is 1, 2, 3. Degree of V_5 so 1, 2, 3 and degree of V_6 is 1, 2, 3, 4 ok. So there are 2 vertices V_1 and V_6 with the degree 4 each, we can write anyone at the 1st of the sequence okay so we can write V_1, let us write V_1 then next is V_6, they both have equal degrees then all others have 3 degrees each ok so we have V_2,V_3 ,V_4 , V_5.

Okay now we use color C_1 to paint V_1 vertex so for this we use C_1 color. Now V_6 is here, V_6 is not adjacent to V_1 so C_1 is used to paint V_6 okay. Then V_2, V_2 is adjacent to V_1 ok so we leave this,V_3 is also adjacent to V_1 so leave this, V_4 is adjacent to V_1 so leave it, V_5 is adjacent to V_1 so leave it ok so V_1, V_6 are painted with C_1 color.

Now, we go to V_2 and paint it with C_2 color and then V_2, V_2 is here, V_2 and V_3 they are not adjacent okay so C_2 color is used for V_3 also ok. And then V_2 is adjacent to V_4, V_2 is not adjacent to V_5, no V_2 is not adjacent to V_5, V_5 is here, so this is also painted with C_2 color ok, and then what is left? Then we are left with V_4 ok, V_4 is painted with color C_3 ok. So C_1

color is used for V_1 , V_6, C_2 color is used for V_2 , V_3 and V_5 and C_3 color is used for V_4 ok, so 3 colors are used to paint the vertices here.

Now, let us see what is the chromatic number, you see we have V_1, V_2, V_1 is adjacent to V_2, V_1 is adjacent to V_5 okay so 3 colors will be needed to paint V_1,V_2, V_5 okay and here we have used 3 colors to paint the entire graph. Chromatic number means minimum number of colors to be used to paint the graph so $\chi(G)$ is equal to 3, we cannot paint the graph with colors less than 3 because V_1, and V_5, V_1 is adjacent to V_2 and V_1 is adjacent to V_5.

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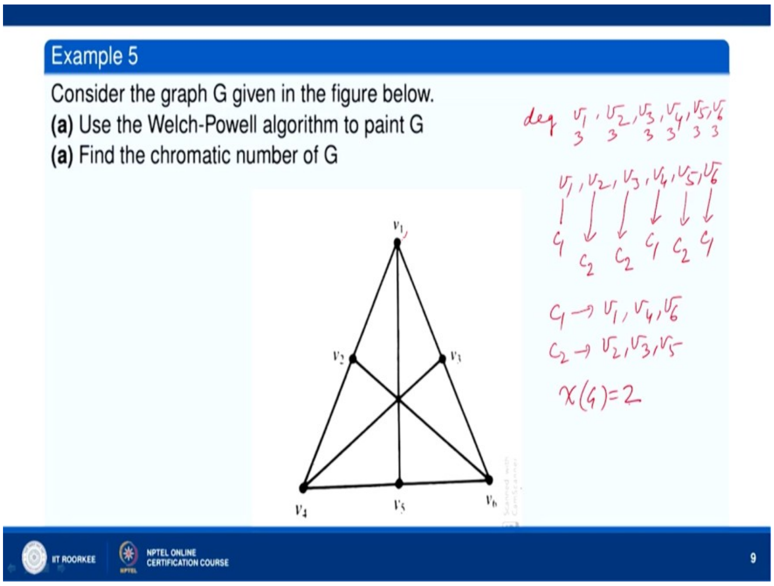

Now, let us consider this graph, ok so use the Welsh Powell algorithm here, so degree of V_1, what is degree of V_1? 1, 2, 3 degree of V_1 is 3. Degree of V_2 okay, degree of V_2 is how much; 1, 2, 3. Degree of V_3 is 1, 2, 3. Degree of V_4 1, 2, 3 and degree of V_5 1, 2, 3 and degree of V_6 1, 2, 3 ok so they all have degrees 3 each okay soV_1, V_2,V_3,V_4, V_5, V_6. Here, let us start with V_1, we can color it with C_1 color, now V_1 is adjacent to V_2 so we have to leave V_2, then V_1 is adjacent to V_3 so leave V_3, V_1 is adjacent to V_4, no so V_4 can be colored with C_1. V_1 is adjacent to V_5, yes, V_1 is adjacent to V_5 so we cannot color V_5 with C_1. V_1 is not adjacent to V_6 so we can color it with C_1 color okay.

Then we go to V_2, V_2 can be colored with C_2 then V_2 is adjacent to V_3 no, so it can also be colored withC_2 . And then V_2, is it adjacent to V_5? V_2 is not adjacent to V_5 so it can be colored with C_2 ok. So, C_1 color is used to paint V_1, V_4 and V_6 okay, and C_2 color can be used for V_2, V_3 and V_5 so 2 colors are required. And you see we have V_1 and V_2, they are adjacent to each other so we will have to use different colors to paint V_1 and V_2 so 2 colors are needed for painting V_1 and V_2 and here we are using 2 colors to paint the entire graph so minimum number of colors used to paint this crap that is chromatic number is 2. So that is the end of this, thank you very much for your attention.