Higher Engineering Mathematics

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Lecture 29

Subgraphs and Traversable Multigraphs

Hello friends, welcome to my lecture on subgraphs and Traversable (graph) Multigraphs. Let, G be a graph, okay then H is a subgraph of G, if the set of vertices of H is a subset of set of vertices of G, okay that is the vertices of H are also vertices of G, and the set of edges of H is a subset of the set of edges of G, that is the edges of H are also edges of G. In other words, H ($V^{'}$, $E^{'}$) is a subgraph of G(V, E) if $V^{'} \subset$ V and $E^{'} \subset$ E.

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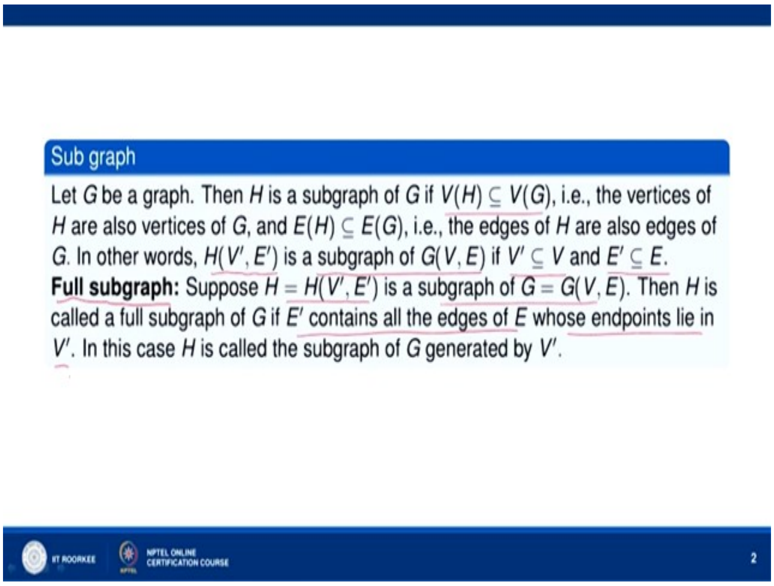

Now, what do we mean by a full subgraph? If H = H($V^{'}$, $E^{'}$) then it is called as full subgraph of G, if $E^{'}$ contains all the edges of E whose endpoints lie in $V^{'}$, that is, H is the subgraph of G generated by $V^{'}$. So, if H is the subgraph of G generated by $V^{'}$, then it is also called as a full subgraph of G.

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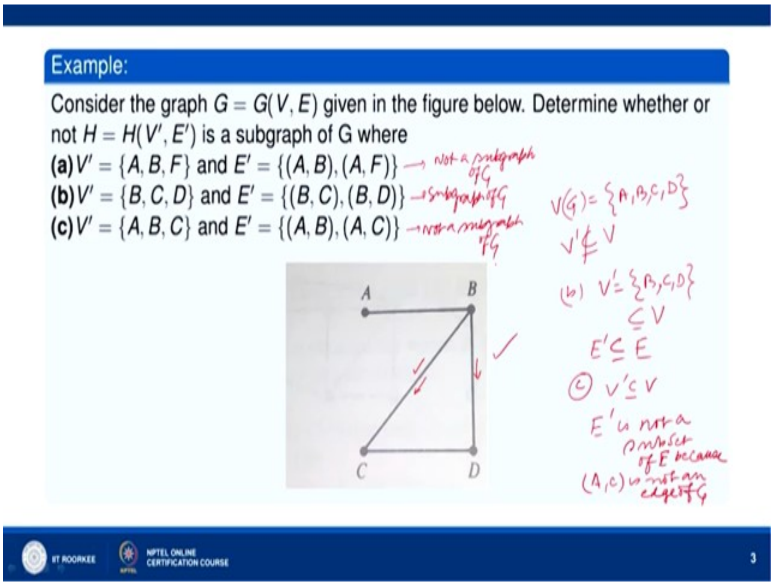

Now, let us consider the graph G = G (V, E) given in this figure, okay. Determine whether or not H = H(V', E') is a subgraph of G, where V' is {A, B, F}; E' is {(A, B), (A, F)} okay. Now, V' is {A, B, F} and the vertices, set of vertices of G is V (G), V (G) equal to {A, B, C, D}, okay.

So, since V' contains A, B, F and F is not an element of V (G), (V G) $V' \not\subset V$, okay so this $V' \not\subset V$. Therefore, H (V', E') is not a subgraph of G, so it is not a subgraph of.

Now, let us look at the second part, V'= {B, C, D} so clearly $V' \subseteq V$ in the part (b), okay V' ={B, C, D} it is a subset of V, okay and E', E' consists of B, C, this is B, C and B, D, okay so B, C, and B, D, okay B, C, and B, D, okay B, C, and B, D is so the set of B, C, and B, D is a contained in E, okay so this is $E' \subseteq E$. Therefore, H (V', E') is a subgraph of G, okay so this is a subgraph of G.

Now, then V' third part (c), so, if V = V' ={ A, B, C} then $V' \subseteq V$, okay E' is A, B, A, C, okay but A, C is not an edge so E' is not contained in, this $E' \not\subset E$ because A, C is not an edge of G, okay so it is not a subgraph of G.

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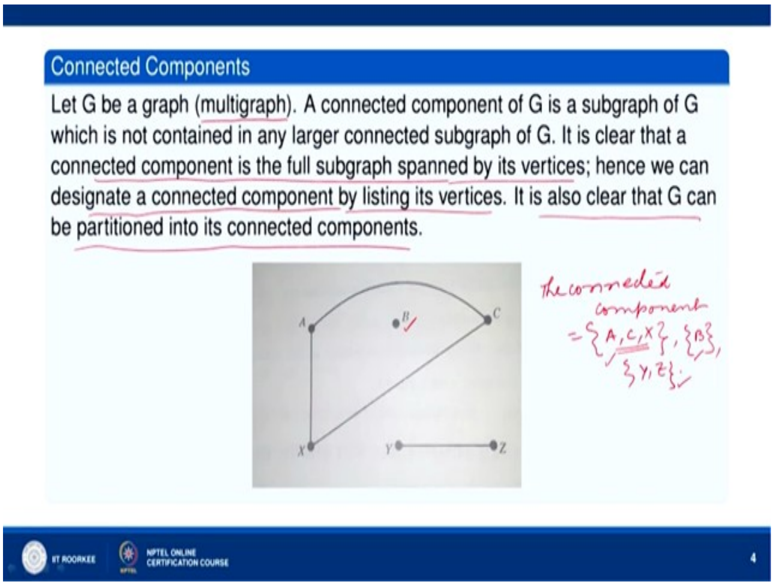

Now, let us go to connected components. Let G be a graph a multigraph, okay. A connected component of G is a subgraph of G which is not contained in any larger connected subgraph of G, okay. That means a connected component is the full subgraph of G spanned by its vertices, okay it is the full subgraph spanned by its vertices. Hence, we can designate a connected component by listing its vertices, okay. So it is also clear that G can be partitioned into its connected components.

Say for example in this case, the connected components are in this graph A, C, X we just list the vertices, okay of the connected components and then the other connected component is B, okay this one, okay which an isolated vertex and the other connected component is Y, Z. So to determine the connected component, you start at any vertex and then find all vertices that are connected to it, okay.

Say for example here, we started with A and found the other vertices C and A, X connected to it, so that determines the connected component A, C, X. Here, we started with say Y or we can start with Z and then find see that Y is connected to it, so Y, Z is a connected component. And here, V is an isolated vertex, so it is a connected component. So, you can see the graph G is partitioned into its connected components, the graph G consists of three connected components A, C, X, B, and Y, Z.

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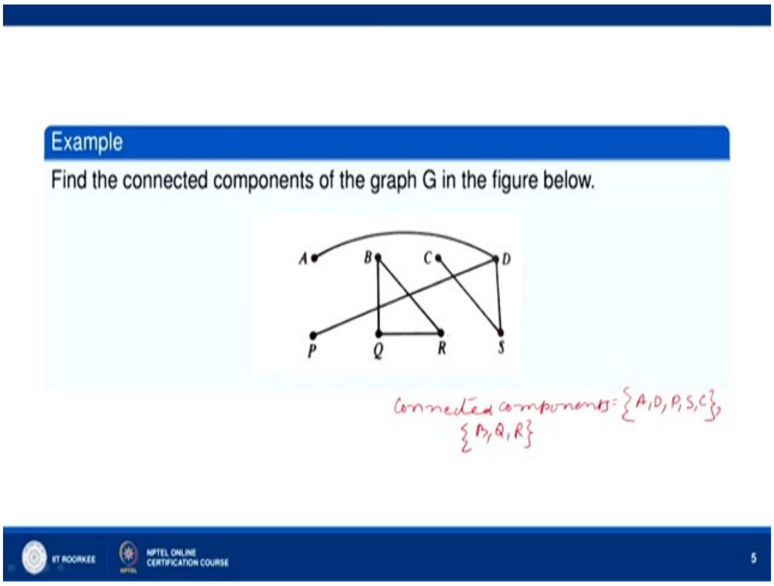

Now, let us find the connected components of this graph, okay. So, connected we start with the vertex A, okay. So, A, D, P, S, C, okay the vertices D, P, S, and C are connected to A, so connected component is, one connected component is A, D, P, S, C, other connected component we can say, we can start with B, B, Q, R components are this and then B, Q, R, and third connected component, so there are only two connected components A, D, P, S, C and B, Q, R.

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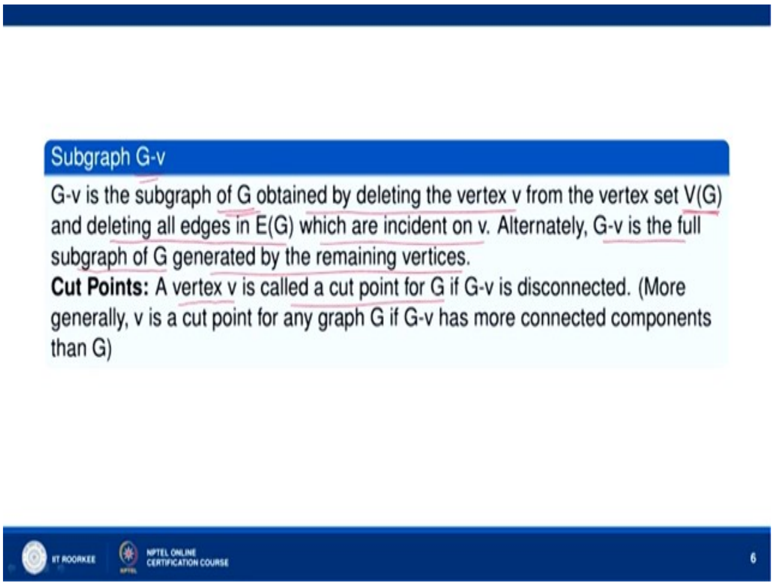

Now, let us go to subgraph G - v. So from the graph G if we remove, delete one vertex, okay we get G - v, okay so what is G - v? G - v is the subgraph of G, okay obtained by deleting the vertex v from the vertex set V (G), okay and deleting all edges in E (G) which are incident on v. Alternately, G - v is the full subgraph of G generated by the remaining vertices, okay.

So, when you find G - v from the set of edges E (G), delete all edges that where which are incident on v, okay and you will get G - v and what is the cut point? A vertex v is called a cut point for G, okay if G - v is disconnected, okay. That means v is a cut point for any graph G if G - v has more connected components than G.

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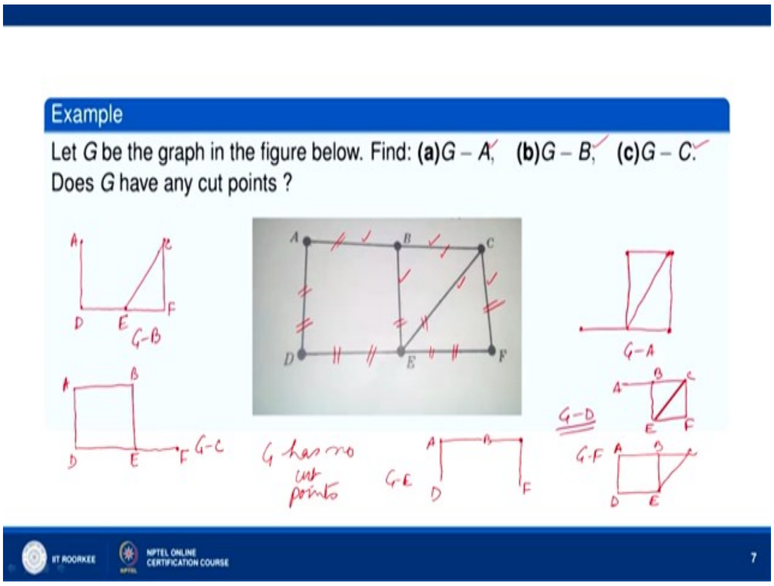

So, let us look at this graph. Let G be the in the figure below. G - A, let us remove the vertex A, when you remove the vertex A, you remove all edges that are incident on A. So you remove this edge, you remove this edge, okay and what is the graph then left? So we will have, okay so this is the graph of G - A, okay when we remove A, okay the two (edges) incident on A will be removed, okay.

And G - B, when we remove B, then this edge, this edge, this edge they are incident on B, so they will be removed and we will have, so A, D, E, okay and we will have C, okay and we have F, okay. So this edge, this edge and this edge are removed and we have this, so this is G - B.

And, then we have the third part G - C. So, let us remove C now. When we remove C, we delete C, then this edge, this edge and this edge, they will be removed and we will have (A, B, C, D sorry) A, B, E, D, okay C is removed so these three are, C deleted so these three are removed and we have this, okay. So this is the graph for G - C, does G have any cut points, okay.

Now, we can see, we have deleted A, we have deleted B, we have deleted C and we did not see that, there is any, A, B, C are not cut points, I mean the graph is not disconnected, okay when we delete A, B, or C. Let us see other cases. If I delete A, B, C we have deleted, let us delete D, okay. So G - D, what is the graph for G - D? So this edge, this edge, and this edge they will be removed and we will have A, B, C, E, F, we can see still the graph is connected, okay so D is not a cut point, okay.

And, then let us delete E, okay. So, G - E. When we delete E, this edge, this edge, this edge, this edge, four edges will not be there and we will have D, A, B, okay C, okay and F, okay this edge will not be there, this edge will not be there, this edge will not be there, this edge will not be there, so we have D, A, B, C, F and we can see this graph is connected, okay.

Now, let us see F, okay delete F. So, G - F. So if we delete F, this edge and this edge will be deleted and we shall have this graph A, B, D, E, and C, okay. So this graph is also connected. So G does not have any cut points, G has no cut points.

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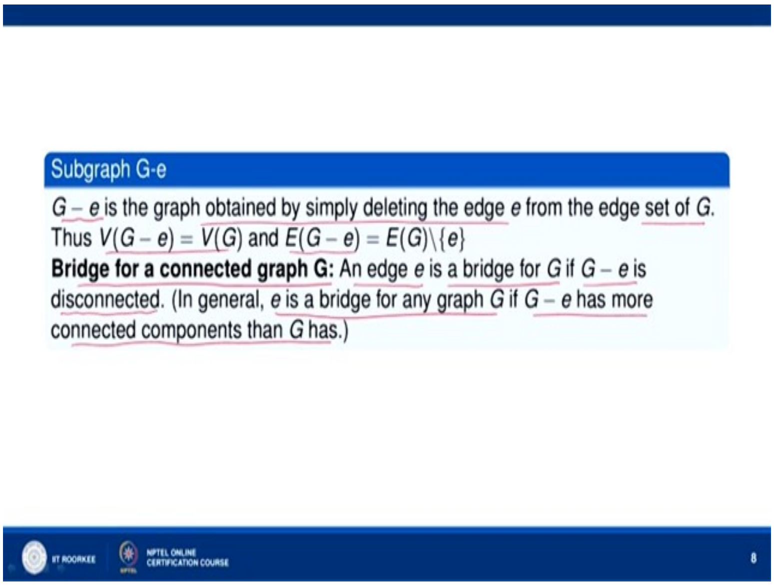

Now, let us say, let us suppose we delete edge, okay from the graph G, okay. Then, let us consider G - e. So, we are deleting an edge e from the graph of G, okay then the graph obtained by simply deleting the edge e from the edge set of G gives what? See, when you delete the edge, you do not delete the vertices joining I mean vertices of the end points of that edge, okay so only edge is deleted.

So, that means when you delete an edge from the edge set of G, okay then the vertices of B remain as it is, okay so vertices of G - e are same as vertices of G, E (G-e) will be E (G-e) , so only edge is deleted, vertices are not removed. So, V(G-e) is V (G) and E (G-e) is E (G-e).

Now, bridge for a connected graph. An edge e is a bridge for G if G - e is disconnected. So, after you delete the edge e, if the graph becomes bridge connected, then e is called a bridge. In general, e is a bridge for any graph G if G - e has more connected components than G has. So if G - e becomes disconnected, then G minus e will have more connected components than G.

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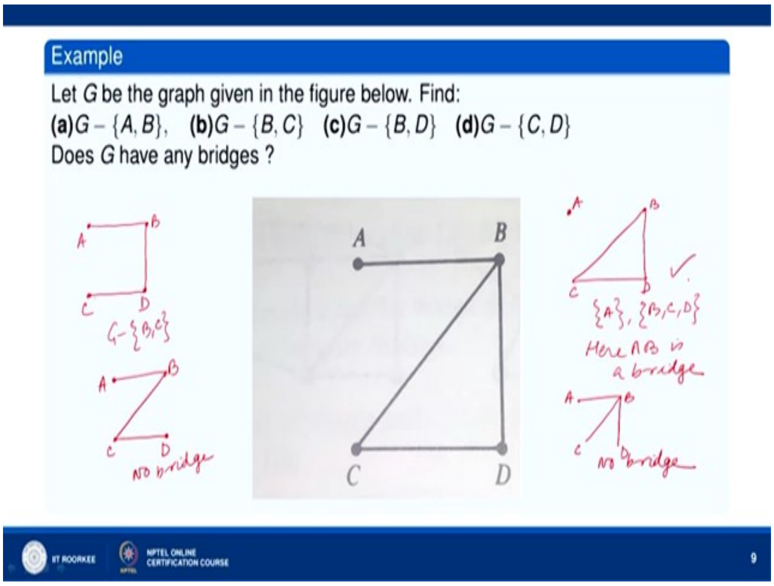

Say, for example this case let us consider, G be the graph given in the figure below. Let us delete A, B edge from here. So G – {A, B}. So when you delete A, B edge from here, what do we get? The vertex A will be there and we will have B, C, D. Now you can see, earlier the graph was connected, earlier the graph was connected, now it has become disconnected, it has two connected components, two connected components, one is this {A}, connected components are {A} and then {B, C, D}.

So after deleting the edge {A, B} we see that number of connected components are more than the connected components of G. Therefore, here {A, B} is a bridge.

Now, G- {B, C}. If you remove {B, C}, what we get? This is A, this is B, okay B, C is removed and we get this, so which is not disconnected. So, since this is not disconnected, this G – {B, C} it does not have any bridge.

Then, let us consider G – {B, D}, we delete {B, D}. So, we will have A, B, C, D after deleting B, D, it is again a connected graph, so there are no bridges, no bridges.

Then, G – {C, D}, when you remove {C, D}, what we have? A, B, C, D, it is again a connected graph, A is connected to B, A is connected to C, A is connected to D, so no bridge. So only the part A, in the part A when we delete A, B, we get two connected components A and B, C, D and therefore, A, B is a bridge.

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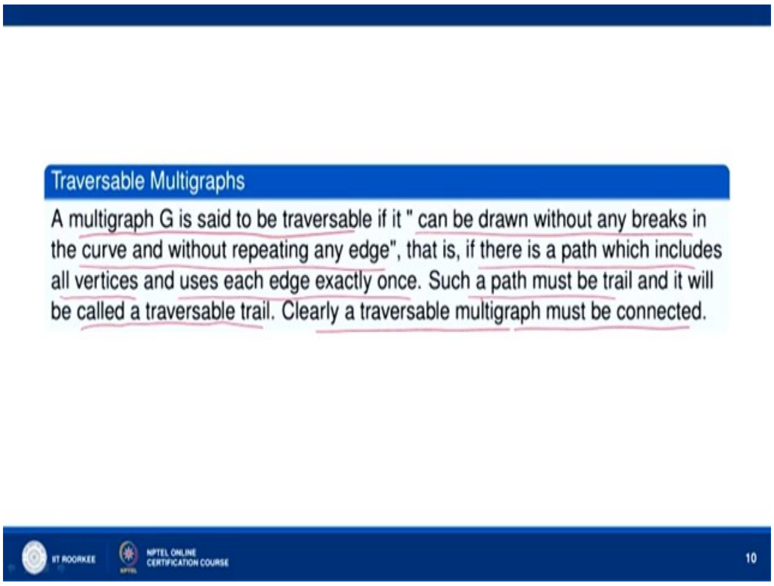

Now, let us consider traversable multigraphs. A multigraph G is called traversable if it “can be drawn without any breaks in the curve and without repeating any edge” that is, if there is a path which includes all vertices and uses each edge exactly once. Then such a path must be a trail and it will be called a traversable trail. Trail is a path where all the vertices distinct.

So, clearly when we are using each as exactly once and all vertices then such a path must be trail and we call it traversable trail. A traversable multigraph must be connected that is clear from the definition.

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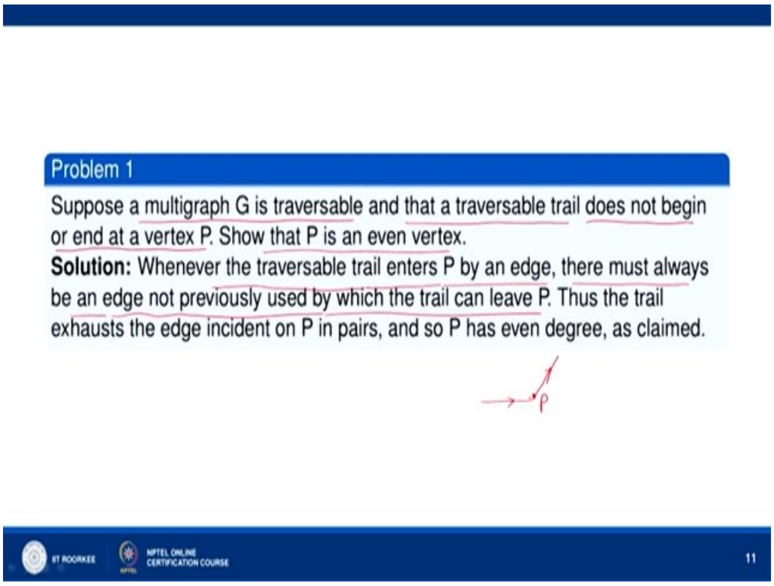

Now, suppose a multigraph G is traversable and that a traversable trail does not begin or end at a vertex P. So, let us see what we are assuming. A multigraph G is traversable and that a traversable trail does not begin or end at a vertex P. Show that P is an even vertex. So solution is that, since a traversable trail enters P by an edge, there must always be an edge, whenever the traversable trail enters P, the vertex P by an edge, there must always be an edge not previously used by which the trail can leave P.

So, suppose this is a P point, edge E an edge is entering here, then because of traversable trail there must be an edge not previously used by which the trail can leave. So, thus the trail exhausts the edge incident on P in pairs, so there will be one edge which enters P and there will be an edge which will be leaving P and therefore, the P has an even degree and so P has an even vertex.

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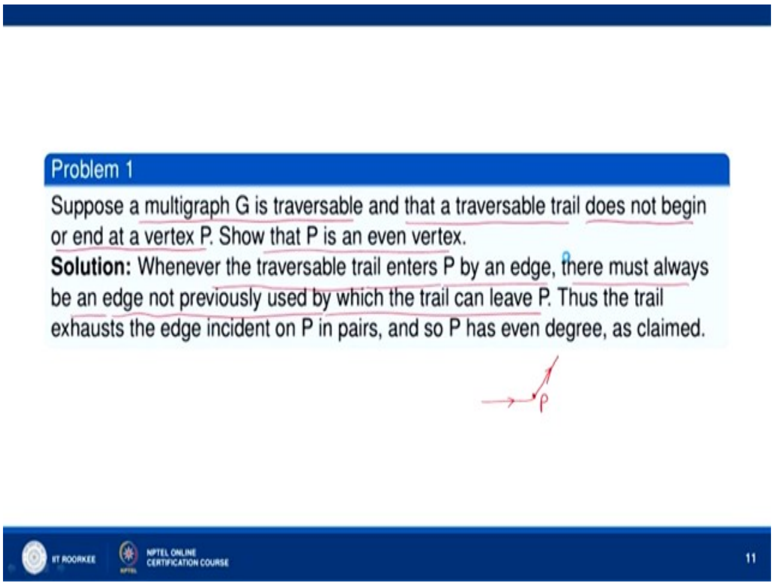

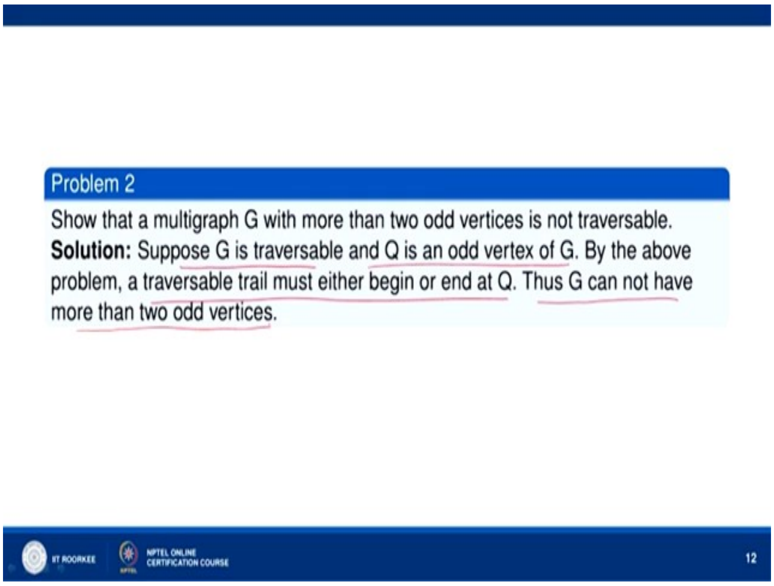

Now, show that a multigraph G with more than two odd vertices is not traversable. Suppose G is traversable and Q is an odd vertex of G, odd vertex of G means the degree of Q is odd. Then, by the above problem, this problem, this problem says that a traversable trail, a traversable trail must either begin or end at Q, because if it does not do that then the Q must be an even vertex.

So, if G is a traversable trail, Q is an odd vertex of G, then by the previous problem the traversable trail must either begin or end at Q, if it does not do that than by the previous problem Q must be an even vertex. So, thus G cannot have more than two odd vertices. So, that is all in this lecture. Thank you very much for your attention.