Higher Engineering Mathematics
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Lecture 27
Various Types of Graphs-II

Hello friends, welcome to my lecture on Various Types of Graphs, it is second lecture on Various Types of Graphs. Let us first define union of two graphs.

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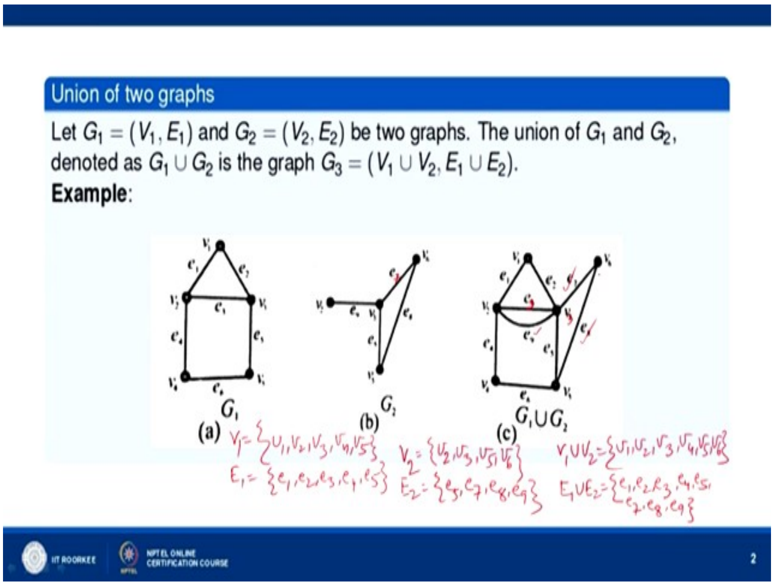

Now, let us first define union of two graphs. Let $G_1 =$ (V_1, E_1¿. $G_2 = (V_2, E_2)$, then the union of G_1 and G_2 denoted by$G_1 \cup G_2$ is the graph G_3, okay which is the union of the, which is the union of vertices of V_1,V_2 and the union of edges of E_1 and E_2. So, the vertices of G_3 are (the union of) union of the vertices of V_1 , V_2 and the edges of (G_1) G_3 are the union of the edges ofE_1 and E_2.

Now, we can see that in G_1 , V_1 is equal toV_1 is equal to e_1 vertices, vertices are v_1, v_2,v_3 , ,v_4, v_5 okay and E_1 is equal to e_1 , e_2, e_3,e_4, ande_5 and G_2 consists of vertices v_2 v_3 , the vertices are the vertices are given by V_2 , V_2 equal to V_2 consists of vertices v_2 ,v_3 , v_5, v_6, okay and E_2 consists of e_5 ,e_7, e_8 and e_9. , okay this is e_7 this is e_7, okay. So when we consider $V_1 \cup V_2$, $V_1 \cup V_2$ when V is the union of the set of vertices of V_1 and V_2 , so v_2, v_3, v_5, v_6 so v_6 will add because v_2,v_3, v_5 are already there, okay. So $V_1 \cup V_2$ will be the set v_1, v_2,v_3, v_4, v_5 ,v_6 and $E_1 \cup$ E_2 w*Type equation here*.ill consist of e_1 , e_2, e_3,e_4, e_5 after e_5 we have e_7 ,e_8 and e_9, okay.

So, now let us see we have between v_2, andv_3 , okay we have edge e_3 here, okay here we have the edge between v_2, andv_3 as the edge e_9, okay so when we take the union, okay between v_2, andv_3 we also consider e_9 apart from e_3, so this is e_3 , this ise_3 and this one is e_9, so there are two edges e_3 and e_9 between v_2, andv_3 so they are parallel edges and then at v_3 we have the edge between v_3 and v_6 , okay so we add this this is v_3, so we add this edge at v_3, okay v_3, v_6, v_8 this e_7, okay and from v_5¿v_6 we have the edge e_8, remaining figure is the same, so we have added when we have to consider the union of G_1 and G_2 between v_2 andv_3 we add the edge e_9, okay so that means there are parallel edges e_3 and e_9 and at v_3 vertex and at this $v_5$we add the edge e_7, this is e_7 and e_8, okay. Sov_3, v_6 is connect by e_7 and v_5, v_6 are connected by e_8, so that is the union of G_1 and G_2.

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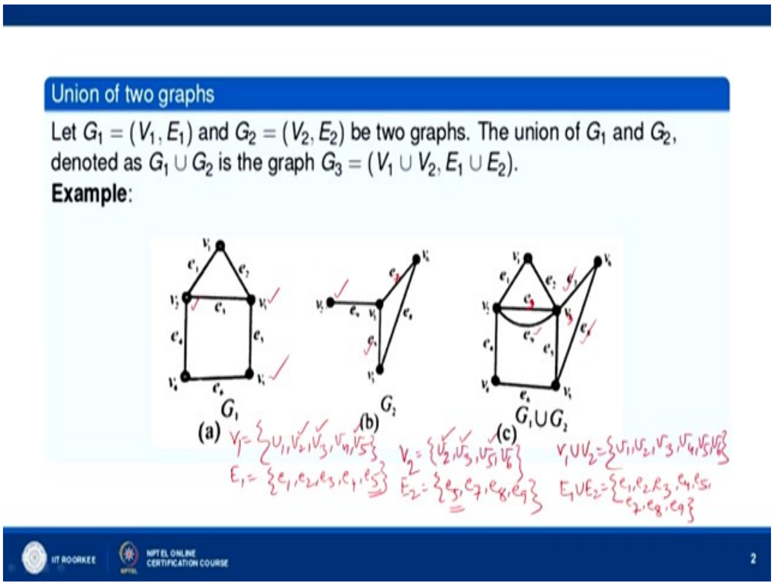

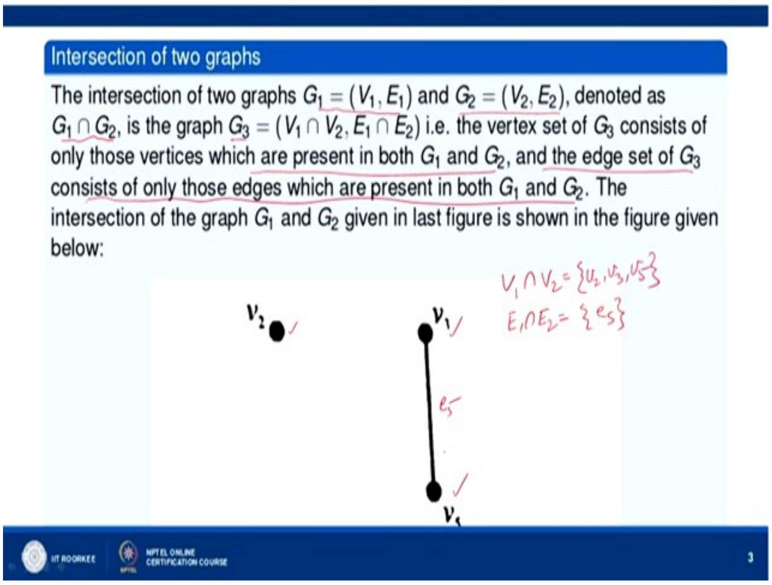

Now, the intersection of two graphs, the intersection of two graphs G_1 be equal to V_1, E_1 ,G_2 equal to V_2, E_2 denoted by G_1 intersection G_2 is the graph G_3, where we take the intersection of V_1 andV_2, okay to get the set of vertices of G_3, so the set of vertices of G_3, is obtained by taking intersection of $V_1$and V_2 and the set of edges of G_3 is obtained by taking intersection of E_1 and E_2. The vertex set of G_3, consists of only those vertices which are present in both G_1 intersection G_2 and the edge set of G_3 consists of only those edges which are present in both G_1 and G_2.

So, let us take the intersection of the graphs we have considered earlier G_1 , G_2 so V_1 intersectionV_2 if you take here then$V_1 \cap V_2$ turns out to be see what is common v_1, v_2,v_3, so v_2,v_3, are there and then v_4,v_5, okay,v_5, , so $V_1 \cap V_2$ is equal to the set consisting of v_2,v_3, v_5. v_2,v_3, v_5 and $E_1 \cap E_2$ gives us the set of edges of $G_1 \cap G_2$, so $E_1 \cap E_2$ will consist of e_5 here and e_5 here, so that is the only edge, so e_5, okay.

Now, you can see here v_3 and v_5 are joined by e_5, , okay and here also v_3 and v_5 are joined by e_5, so when you take the intersection of G_1 and G_2 you get v_3 and v_5 are joined by this e_5 , okay and v_2 which is here okay and here okay (it is) it occurs as an isolated vertex okay so v_2 is here and v_3, v_5 so this is the graph of $G_1 \cap G_2$ where v_2 is an isolated vertex and v_3, v_5 are joined by the edge e_5.

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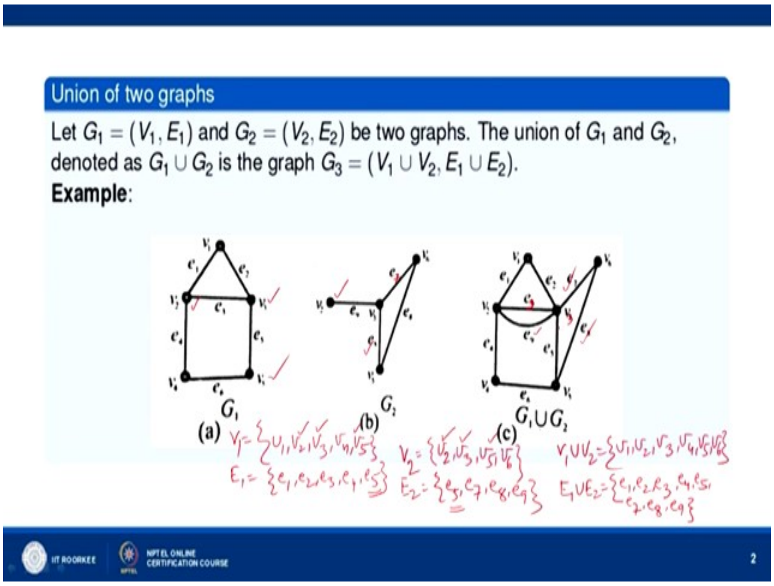

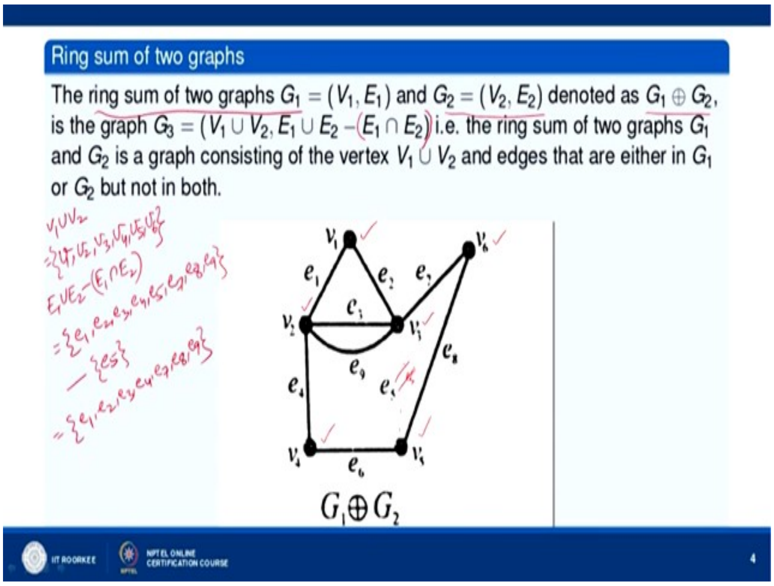

Now, ring sum of two graphs, the ring sum of two graphs $G_1 = (V_1, E_1$¿$, G_2 = (V_2, E_2)$ denoted by G_1 ring sum G_2 is the graph $G_3 = (V_1 \cup V_2 , E_1 \cup E_2 - E_1 \cap E_2)$, so the set of vertices of G_3 is the union of the set of vertices of V_1 andV_2 , but the set of edges of G_3 is obtained by removing the edges that are common to E_1 and E_2 from $E_1 \cup E_2$ so that gives us the ring sum of two graphs G_1 and G_2.

So, let us see here $V_1 \cup V_2$ we had as in this example $V_1 \cup V_2$ was v_1, v_2,v_3, v_4,v_5, v_6, okay so (V_1) $V_1 \cup V_2$ $V_1 \cup V_2$ is equal to v_1, v_2,v_3, v_4,v_5, v_6, okay and what is $E_1 \cup E_2 - E_1 \cap E_2$, okay so this equal to $E_1 \cup E_2$ is this one e_1 , e_2, e_3,e_4, e_5, e_7 , e_8, e_9 minus E_1 intersection E_2 it is $E_1 \cap E_2$ is e_5 , so we get e_1 , e_2, e_3,e_4, e_7 , e_8, e_9 okay so we have all vertices v_1, v_2,v_3, v_4, v_1, here,v_2, here, v_3, here, v_4,v_5, v_6 all this all of them are there, we have to from E_1 union

E_2 we just have to remove e_5, okay so in this is E_1 union E_2 okay this is $E_1 \cup E_2$ from there we have to removee_5 , if you remove e_5 this edge, this edge will be removed, okay and we will have this figure, this figure okay this was e_5 which we have removed, okay so this is e_5 is removed and we get the figure for $(E_1 \cup G_1)$ the ring sum of G_1 and G_2, okay.

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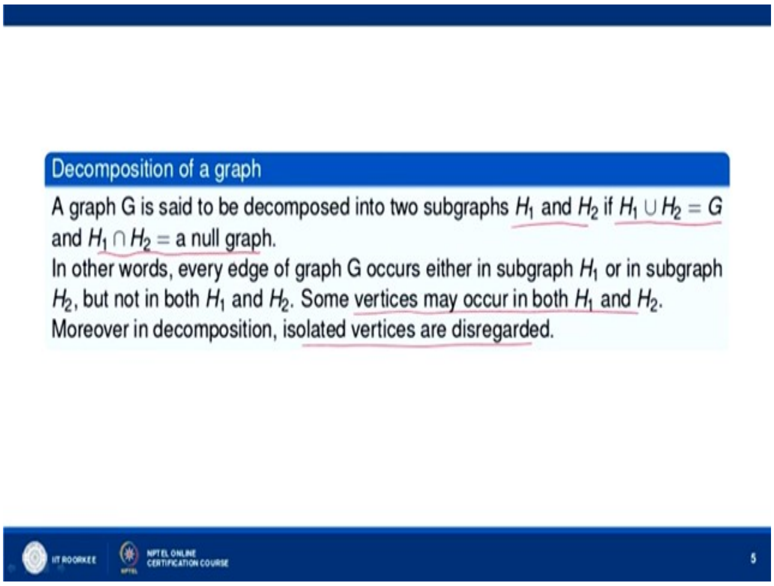

Now, decomposition of a graph. A graph G is said to be decomposed into two sub graphs H_1 and H_2 if $H_1 \cup H_2$ is equal to G and $H_1 \cap H_2$ is equal to a null graph. In other words, every edge of graph G occurs either in subgraph H_1 or in sub graph H_2 , but not in both H_1 and H_2. Some vertices may occur both in H_1 and H_2. In decomposition, isolated vertices are disregarded. So, some vertices which are isolated may occur there in the case of H_1 and H_2. Okay, so when you take the intersection $H_1 \cap H_2$ when we say it is a null graph there it might happen that some vertices occur there but they are isolated so we do not count them and we call $H_1 \cap H_2$ to be a null graph.

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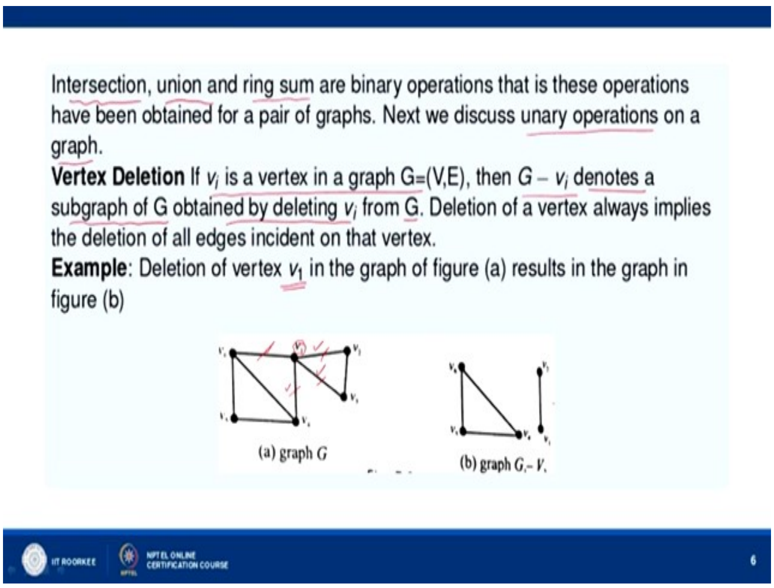

Now, let us say intersection, union and ring some are binary operations that is these operations have been obtained for a pair of graph. When we have taken two graphs G_1 and G_2 G we have defined their union, their intersection and their ring some so they are binary operations, okay in the set of graphs. Now, we discuss unary operations, okay on a graph. So, by vertex deletion, if v_i is a vertex in a graph G then G minus v_i denotes a subgraph of G obtained by deleting v_i from G, okay. Deletion of a vertex always implies the deletion of all edges incident on that vertex.

Say in the figure here, if we delete the vertex v_1, if we delete the vertex v_1 then this edge, this edge, this edge and this edge four edges are incident on v_1 they will all be removed, okay to obtain G - v_1, so the graph for G - v_1 will be like this, so this is $v_6, v_5, v_4,$ okay we have removed this edge, this edge has been removed, this edge has been removed, this one and this one and we have this, okay so $v_2 \wedge v_3$ this one, okay. So when v_1 vertex is removed the graph of G - v_1 is like this.

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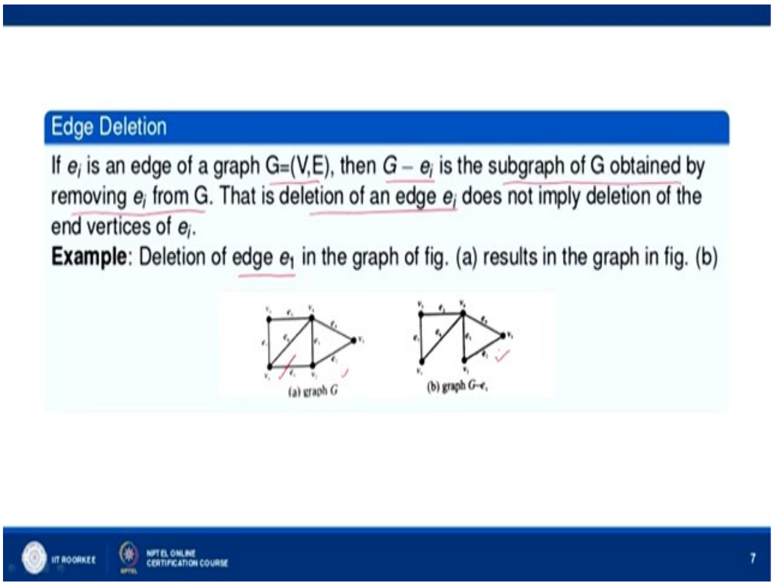

Now, edge deletion, if e_i is an edge of a graph G = (V, E), then G minus e_i is the subgraph of G obtained by removing e_i from G, okay that is deletion of an edge e_i does not imply deletion of and vertices of e_i. So, suppose we have this graph, okay and we delete e_1 from here edge e_1 okay so edge e_1 is this one, okay edge e_1 is this, this is edge e_1, so when you delete edge e_1, okay only edge is removed but this is not removed okay so we have this edge has been removed from here, okay and we get this figure this is the graph of G - e_1 .

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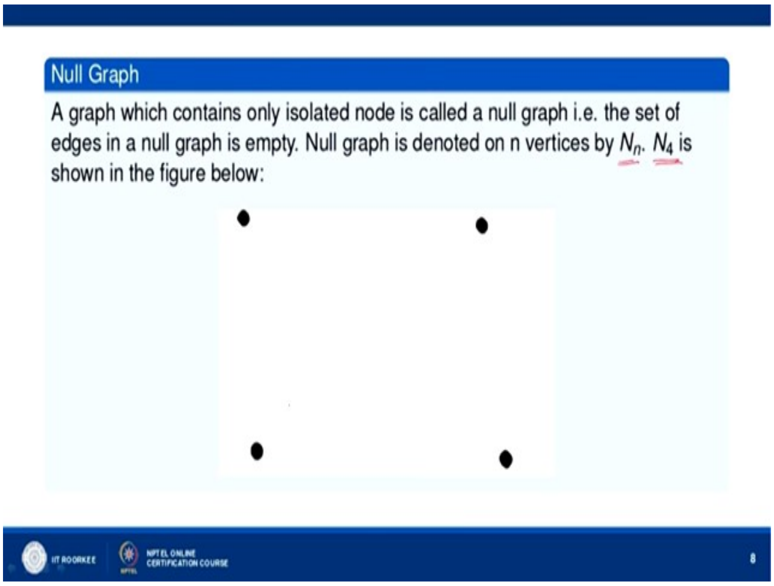

Now, a graph which contains only isolated node is called a null graph that is the set of edges in a null graph is empty like here you can see null graph N_4 okay null graph is denoted on n vertices byN_n, so you can see here this graph for N_4 there are four dots in this, no edge.

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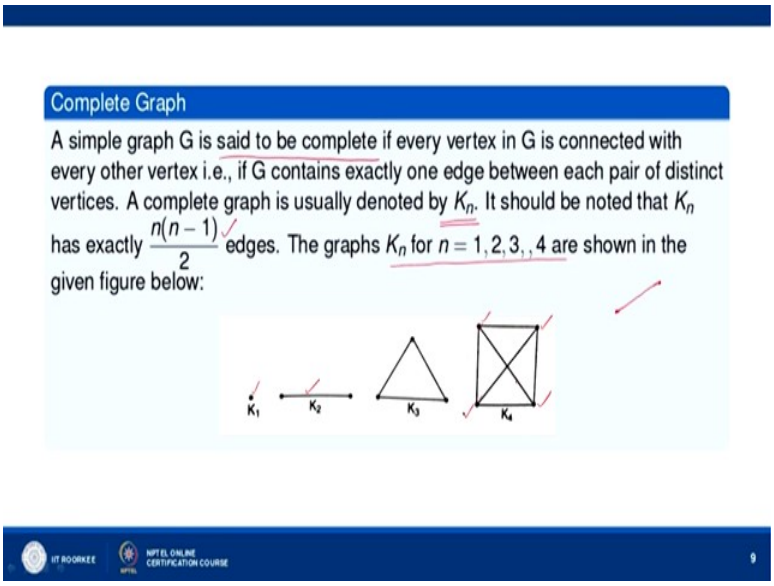

Now, complete graph. A simple graph G is called complete if every vertex in G is connected with every other vertex, that is if G contains exactly one edge between each pair of distinct vertices, you take any two vertices there is an edge connecting them, okay. A complete graph is usually denoted by K_n , okay it should be noted that K_n has exactly $\frac{n(n-1)}{2}$ edges because each vertex suppose in K_n there are n vertices each vertex is connected to every other vertex so every vertex will have n - 1 edges, so total number of edges will be $\frac{n(n-1)}{2}$ because an edge is counted twice, okay if suppose we have a vertex this, this is the another vertex so this is joint to this, so it is counted once for this vertex and when we come to this vertex this is again counted so an edge is counted twice and therefore the total number of edges will be $\frac{n(n-1)}{2}$ in the case of a complete graph.

Now, the graph K_n, graphs of K_n for n = 1, 2, 3, 4 are so on here, this is K_1 , K_1 means only one vertex, okay and K_2 means two vertices the two vertices are joint by this edge, K_3 means three vertices, every vertex is joint to every other vertex so there are these three vertices joined by these edges and in the case of K_4, okay you can take there are four vertices because there are four vertices, every vertex is joint to every other vertex, okay this is joint to this, this is joint to this and this is joint to this and similarly the other vertices so this is the graph for K_4.

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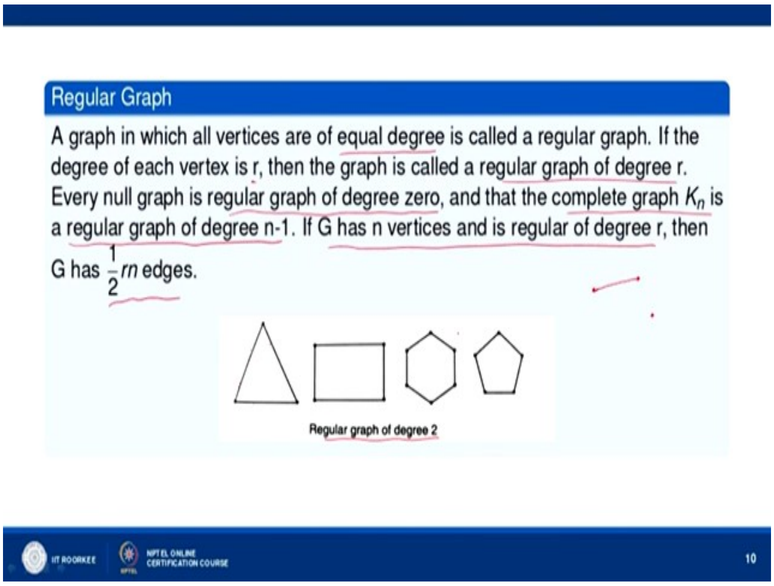

Now, regular graph. A graph in which all vertices are of equal degree, okay is called a regular graph. If the degree of each vertex is r, so that means there are r edges at each vertex which are incident on it, the degree of each vertex is given to be r so if there are r edges incident on each vertex, then the graph is called a regular graph of degree r, okay. So, when each vertex of a graph has equal degree say degree is r then it is called a regular graph of degree r and every null graph, null graph is a regular graph of degree zero. The complete graph K_n is a regular graph of degree n - 1 because in the case of a complete graph every vertex is joined to every other vertex, so every vertex has degree n - 1, okay so every complete graph is a regular graph, complete graph K_n is a regular graph of degree n - 1.

Now, if G has n vertices and is regular of degree r, okay so then the number of edges will be r $\frac{n}{2}$, okay so you have vertices, okay say 1, 2, these vertices then if this vertex each vertex here has degree r then and there are n vertices then $\frac{nr}{2}$, $\frac{nr}{2}$ will be total number of edges because the (())(17:14) second theorem where we say that every vertex this edge which joints this vertex to this vertex is counted for degree here as well as for degree here, okay.

So, when it is of degree r, okay this edge is counted towards degree r for this vertex as well as towards degree for this vertex and therefore total number of edges will be $\frac{m}{2}$. So in the case of see this is regular graph of degree 2, you can see every vertex has two edges incident on it, this one, this one, here for this vertex this one, this one, for this vertex this one, this one, so each vertex here has degree 2, here again each vertex has degree 2, here also each vertex has degree 2, and here also each vertex has degree 2, so they are regular graphs of degree 2.

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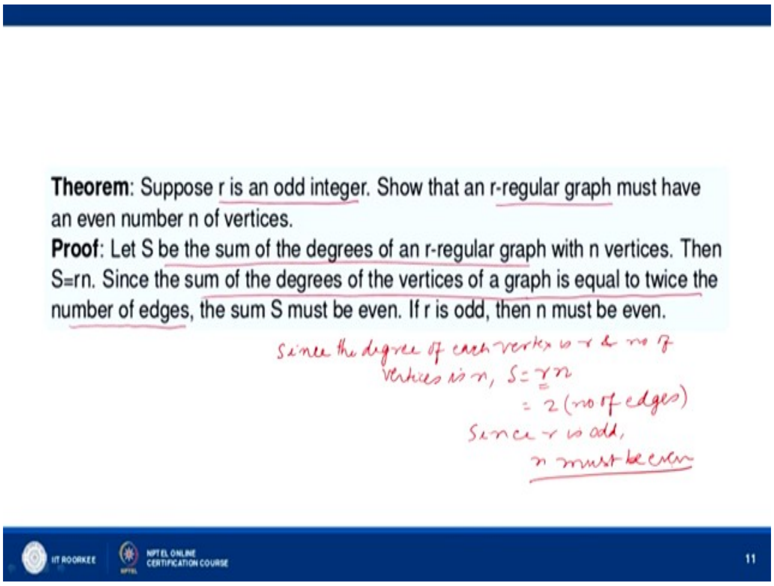

Now, suppose r is an odd integer, okay let us say r be an odd integer. Let us show that an r-regular graph must have an even number n of vertices. Suppose S be the degree sum of degrees of an r-regular graph, okay r-regular graph means each vertex has degree r, okay. So, if S is the sum of the degrees of an r-regular graph with n vertices then S = rn because the degree of since the degree of each vertex is r and number of vertices is n, S = rn , okay.

Now, further the sum we know that the sum of the degrees of the vertices of a graph is equal to twice the number of edges, okay so twice the number of edges and therefore this is twice the number of edges, sum of the degrees of the vertices of a graph is equal to twice the number of edges, so S is an even number, now r is an odd number given, okay. Therefore, n must be even, okay since r is odd, n must be even. So, if r is an odd integer an r-regular graph will have an even number of vertices.

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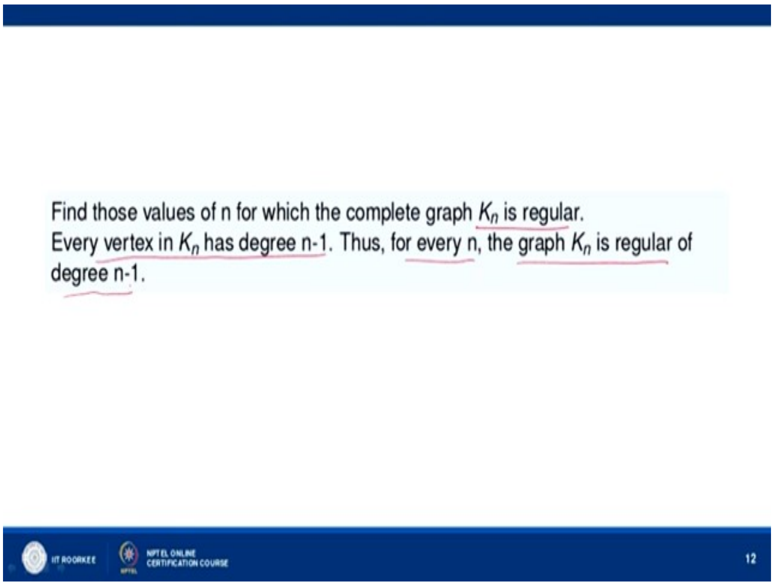

Find those values of n for which the complete graph K_n is regular. Now, every vertex in K_n has degree n - 1 because every vertex is joined to every other vertex so if there are n vertices every vertex will have degree n - 1. Therefore, for every n, the graph K_n is regular of degree n - 1, okay because each vertex has degree n - 1.

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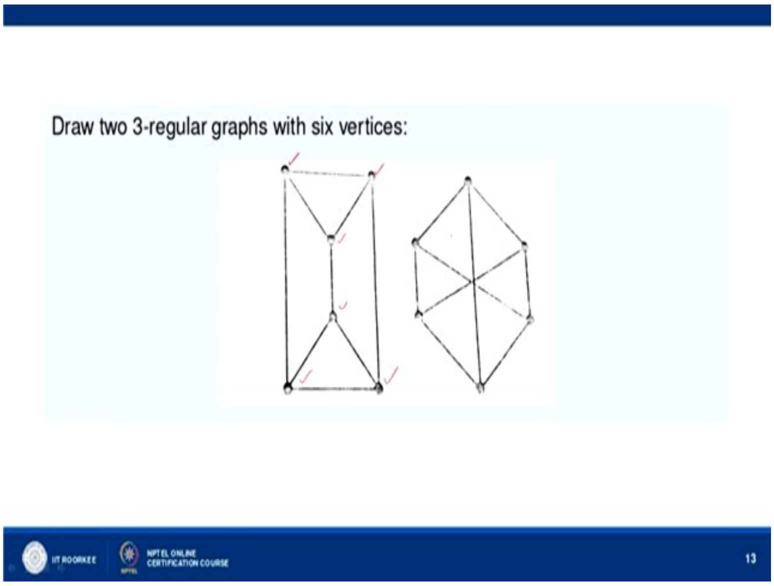

Now, there are two 3-regular graphs with six vertices, 3-regular graph means the degree of each vertex must be 3, okay so you can see and six vertices must be there so 1, 2, 3, 4, 5, 6 so this is the regular graph 3-regular graph with six vertices you can see at each vertex there the degree is 3, okay. Here also there are six vertices and the degree of each vertex is 3.

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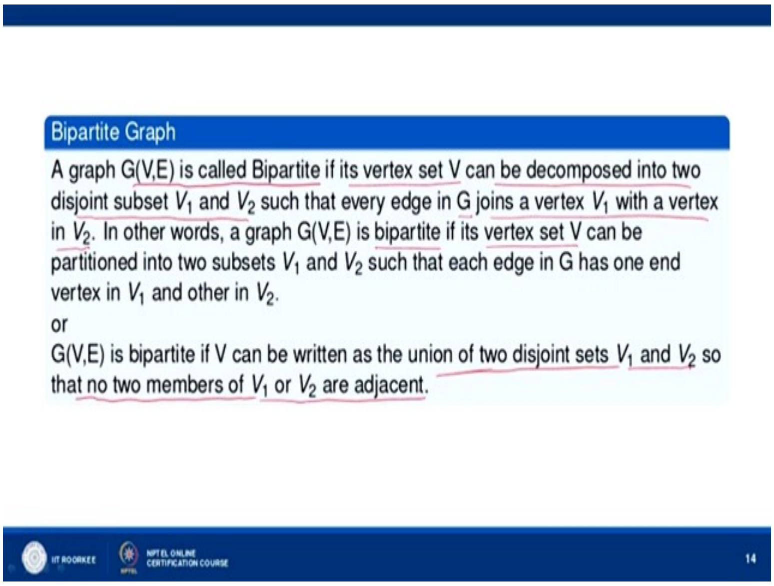

Now, a bipartite graph. A graph G(V, E) is called bipartite if its vertex set V can be decomposed into two joint subset V_1 and V_2 such that every edge in G joins a vertex V_1 with a vertex in V_2 , okay every edge in G joins a vertex of V_1 joins a vertex of V_1 with a vertex in V_2 . In other words, a graph is bipartite if its vertex set V can be partitioned into two subsets V_1 and V_2 such that each edge in G has one end vertex in V_1 and the other in V_2 or

G(V, E) is bipartite if V can be written as the union of two disjoint sets V_1 and V_2 so that no two members of V_1 and V_2 are adjacent.

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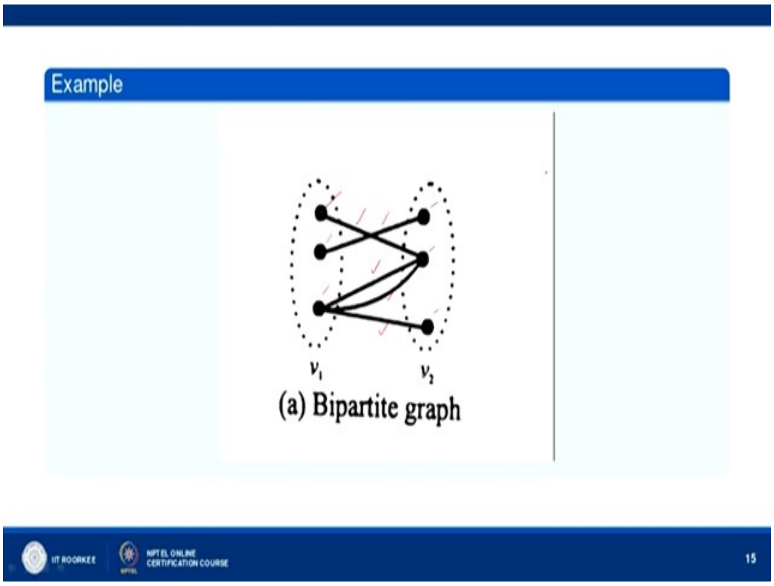

Now, let us see for example this graph, okay. You see here there are six vertices 1, 2, 3, 4, 5, 6 these six, set of six vertices has been (())(22:04) as two sets of 3 vertices each, one is this one, one set of 3 vertices, another one is this one a set of 3 vertices, and every edge of G, okay joints a vertex of (V_1) vertex of this set to vertex of this set, okay these are edges of G, this is edge of G, this is edge of G, okay and this is edge of G, this is edge of G, this is edge of G, every edge of the graph G joints a vertex of V_1 to a vertex of V_2so it is a bipartite graph.

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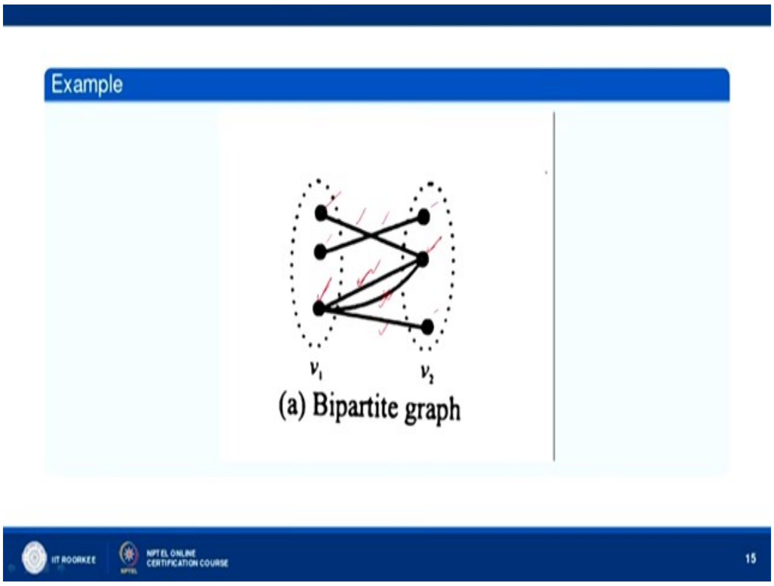

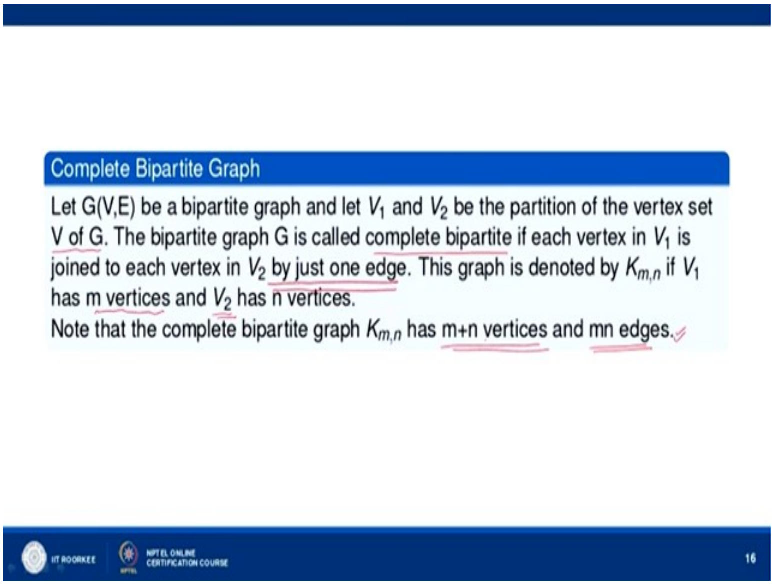

Complete bipartite graph. Let G(V, E) be a bipartite graph and V_1 , V_2 be the partition of the vertex set V of G, okay like in the previous figure we have seen V has been partitioned into V_1 and V_2 , here if G(V, E) is a bipartite graph and V_1 , V_2 form the partition of the vertex set V of G, the bipartite graph will be called complete bipartite if each vertex in V_1 is joined to each vertex in V_2 by just one edge.

Now, here you can see this vertex this vertex is joined to this vertex by two edges this edge, okay and this edge okay they are parallel edges. So this is not a complete bipartite graph, each vertex in V_1 is joined to each vertex in V_2 by just one edge, so then we will say that the graph is bipartite graph is complete. This graph is denoted by $K_{m,n}$ if this graph is denoted by $K_{m,n}$ if V_1 has m vertices and V_2 has n vertices.

Note that the complete bipartite graph $K_{m,n}$ has m + n vertices, complete bipartite graph has m plus n vertices and mn edges, okay m vertices are joined to n edges, okay so m vertices here are there m vertices are joined to n vertices. So, we have every vertex is joined to every other vertex so, if you take any vertex here okay (it can) it will be joined to n vertices here and there are m vertices which are to be joined to n vertices, okay each vertex is joined to every other vertex so total number of edges will be mn, okay total number of edges will be mn and total number vertices will be m + n.

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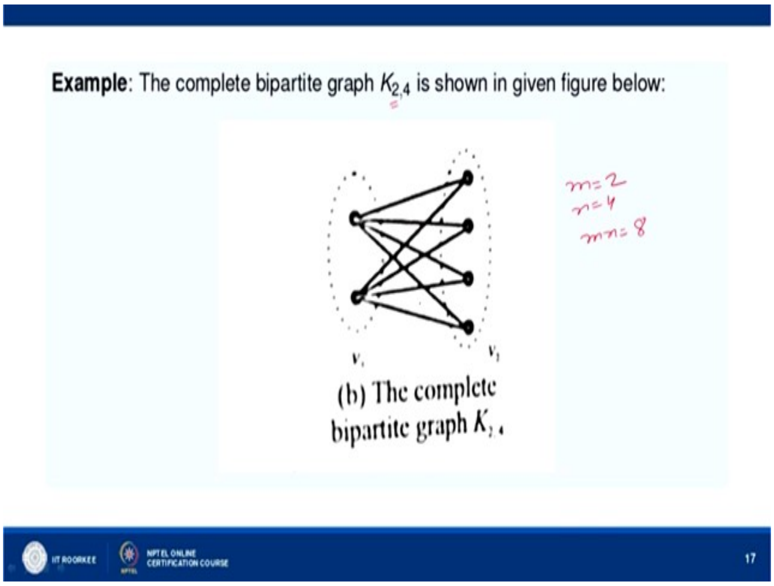

Now, complete bipartite graph. For example here, so you can see there are six vertices, two vertices in V_1, four vertices in V_2 and every vertex here of V_1 is joined to every other vertex of V_2, okay every other vertex of V_2, so total number of vertices are m + n that is 2 + 4, so 2 + 4 this is m, m = 2, n = 4 and total number of edges = mn = 8 so there are 1, 2, 3, 4, 1, 2, 3, 4 total number of edges are 8, so it is a complete bipartite graph.

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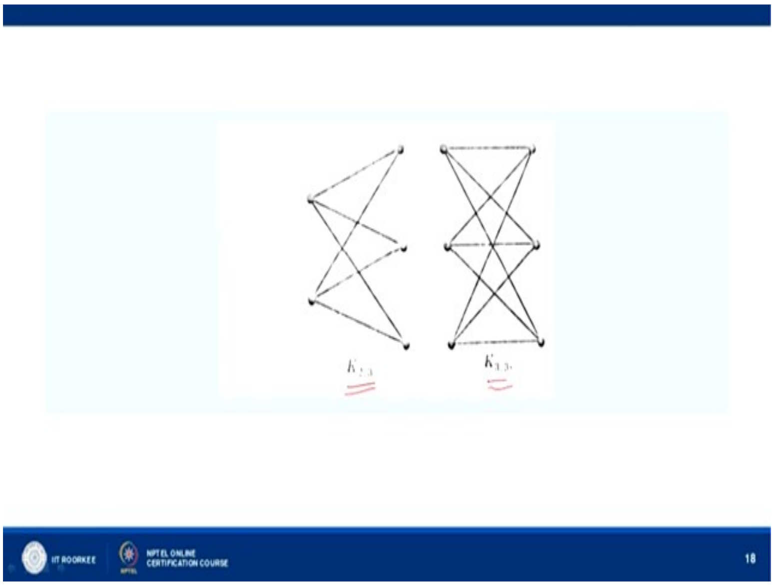

Now, this is example of $K_{2,3}$ there are two vertices here and three vertices here and every vertex here is joined to every other vertex here, okay. So total number vertices are 5 and the total number of edges are six and here we have three vertices here, three vertices here, every vertex here in V_1 is joined to every other vertex of V_2, so this is the example of $K_{3,3}$, thank you very much. So that is the end of my lecture, thank you very much for your attention.