Higher Engineering Mathematics
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Karnaugh Map - II
Mod05_Lec25

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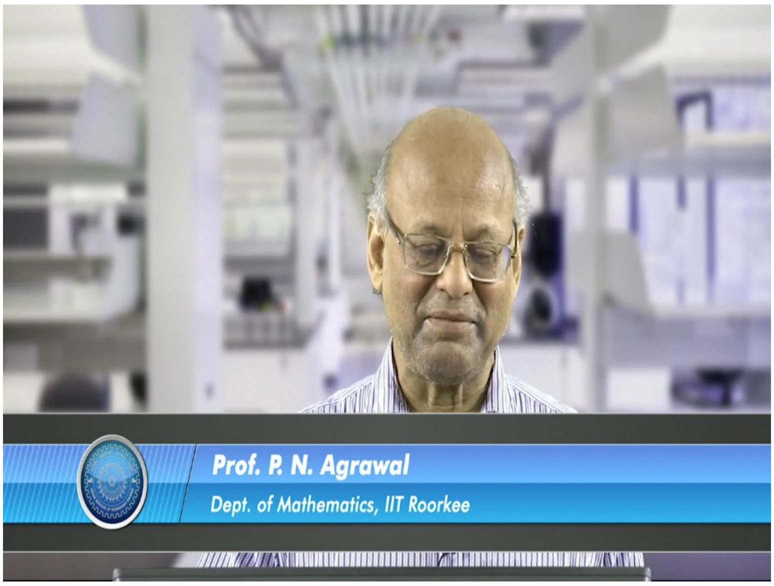

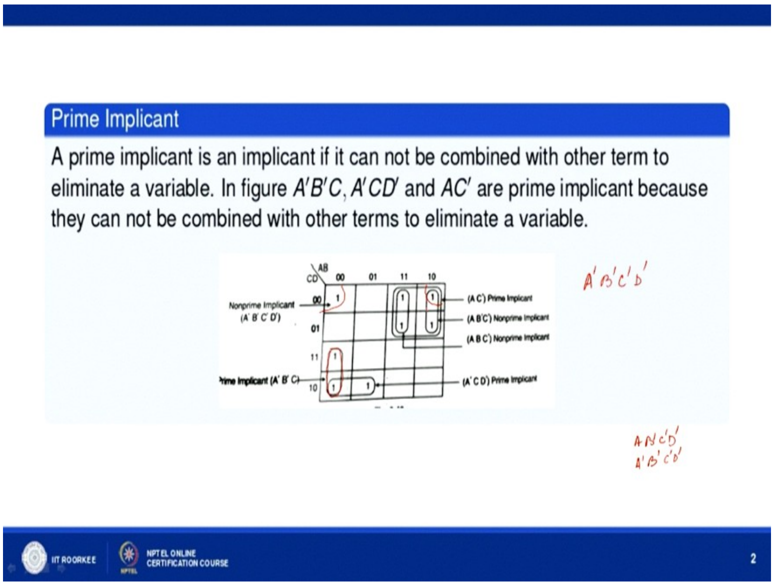

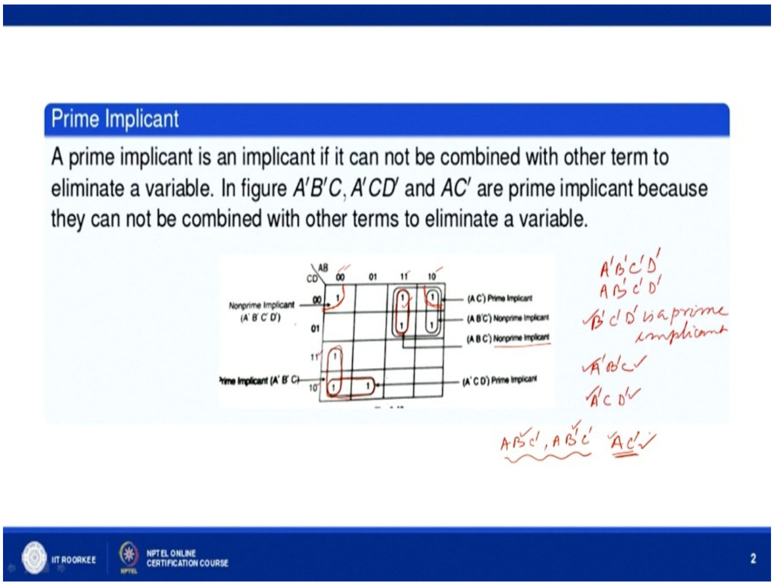

Hello friends, welcome to my lecture on, second lecture on Karnaugh maps, let us first define a prime implicant, a prime implicant is an implicant if it cannot be combined with other term to eliminate a variable, now let us look in this figure, in this figure, we see that this is square, they have 1, it represents $A^{'}B^{'}C^{'}D^{'}$ okay, $A^{'}B^{'}C^{'}D^{'}$ now here we again have 1, so if we can combine this $A^{'}B^{'}C^{'}D^{'}$ with this one and when we do that, we will have $A^{'}B^{'}C^{'}D^{'}$ and for this one, we can combine this one with this one and this one is $AB^{'}C^{'}D^{'}$. So we see that when you combine this one with this one, this square with this square what we get is A occurs in both complemented and uncomplemented form, so $B^{'}C^{'}D^{'}$ is a prime implicant, other prime implicants are, let us consider this loop. Okay, so in this loop we see that this is D, D occurs in both complemented as well as uncomplemented form and therefore we have $A^{'}B^{'}$ okay, this is 00, so $A^{'}B^{'}$ and C okay, so this loop, this loop gives us the prime implicant $A^{'}B^{'}C$ okay and when we consider this loop, so here in this loop we see that A occurs in the uncomplemented form, while B occurs both in the complemented as well as uncomplemented form, so B will eliminated and B will have $A^{'}CD^{'}$ okay, $A^{'}CD^{'}$.

So this is the prime implicant, this is the prime implicant, now when we consider this square, this consisting of four squares okay, so this 11 when we consider, in this 11, we notice that 01, 01 means D occurs both in the complemented as well as uncomplemented form, so this loop we will have minterm corresponding this A B and then we will have D will be moved and we will have $ABD^{'}$so this loop will minimise, will eliminate one variable that is D and we will have $ABC^{'}$and here, if you look at the other one, other one we have 10 and we have okay, 10 here we have 00, 01, so this will also eliminate one variable and we will have AB' okay, A B dash and we will have C dash okay.

Now, when we consider the entire D square consisting of this four squares, we notice that this B and B' they occur both in the complemented as well as uncomplemented form, so this will reduce to AC' okay, so when you consider this square, consisting of four squares two variables are eliminated, the variable D is eliminated because it, the variable B is eliminated because it occurs in the both complemented as well as uncomplemented form and the variable D is eliminated because it occurs both in the complemented uncomplemented form, so this square which consist of four squares reduces to AC', so there are prime implicants, $B'C'D'$ okay, $B'C'D'$, $A'B'C$ and then we have $A'CD'$ and AC' okay, they are the prime implicants.

The nonprime implicants are, because this ABC' this is nonprime implicants because it can be reduce further by combining with this one and similarly this is not the prime implicant because it can be combine with this to reduce further, this two can be reduced further to get AC' so this is a prime implicant.

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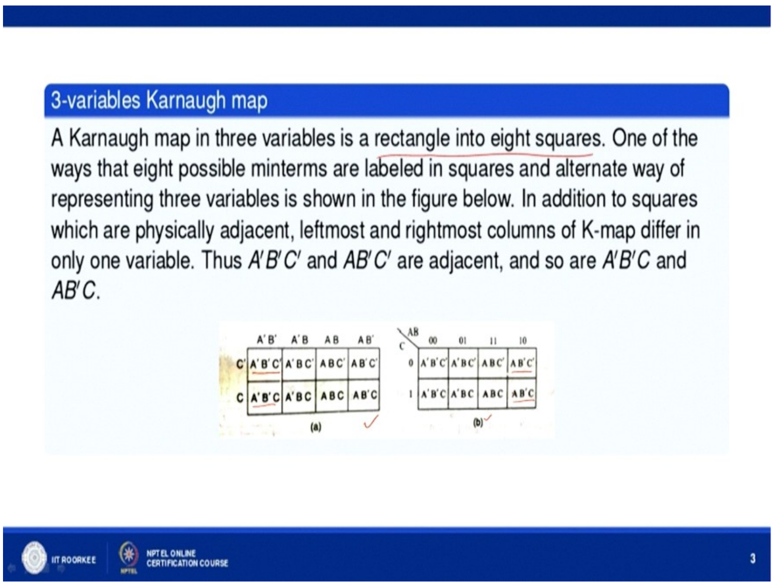

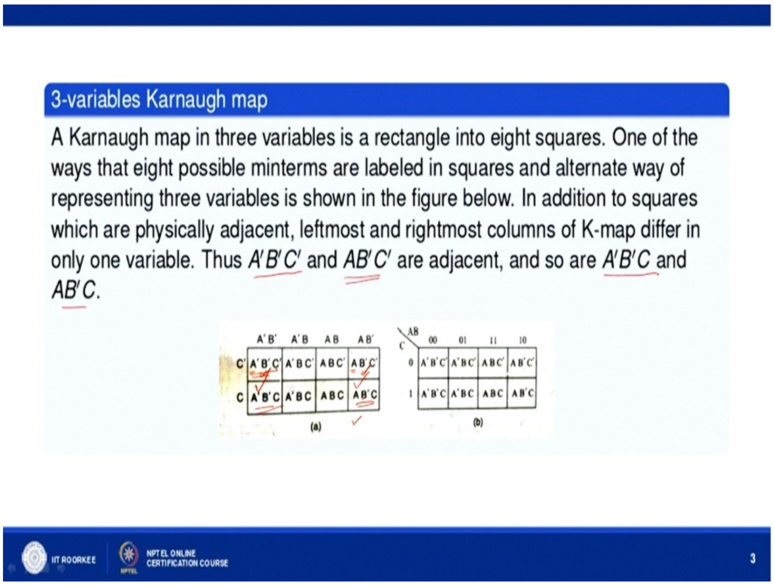

Now, let us go to the three variables Karnaugh map, a Karnaugh map in three variables is a rectangle into eight squares. One of the ways that eight possible minterms are labelled in squares and alternate way of representing three variables is shown in this figures okay, this figure and this figure okay, here we have, this is $A^{'}B^{'}$, $A^{'}B$, AB, $AB^{'}$ and we have $C^{'}C$, so these are eight squares and here $A^{'}B^{'}$ correspond to 00, then we have 01, $A^{'}B=01$, $AB=11, AB^{'}=10$, 10, $C^{'}=0$ and $C=1$, so these $A^{'}B^{'}C^{'}$ can, is that this one.

So $A^{'}B^{'}C^{'}$ corresponds to 000 okay and $A^{'}B^{'}$, $A^{'}B$, $C^{'}$ corresponds to 010, so in addition to squares which are physically adjacent leftmost and rightmost columns of K-map differ in only one variable, you can see here, let say leftmost and rightmost okay, this is a leftmost okay, this is leftmost, so here we have $A^{'}B^{'}C^{'}$, $A^{'}B^{'}C$ okay and here what do we notice is this is rightmost no, sorry, wait.

So, let us consider one of the two representations, I let us consider this one, so leftmost column is this one. Okay, this is leftmost column and this is rightmost column, now here if you notice we have $A^{'}B^{'}C^{'}$ okay and here we have $AB^{'}C^{'}$, so they differ in one variable that is A which occurs both in the complemented and uncomplemented form, so this can be combined with this to eliminate A and we can $B^{'}C^{'}$.

Similarly, here we notice that $AB^{'}C$ here we have, here we have $A^{'}B^{'}C$, so this rightmost column can be combine with the leftmost column okay and we can eliminate the variable A which occurs both in the complemented as well as uncomplemented form, so $A^{'}B^{'}C^{'}$ this one and $AB^{'}C^{'}$ are adjacent this one and this one, they are adjacent and $A^{'}B^{'}C$ this one and

$AB'C$ this one, they are complement, they are adjacent and they can be combined to remove one variable that is A.

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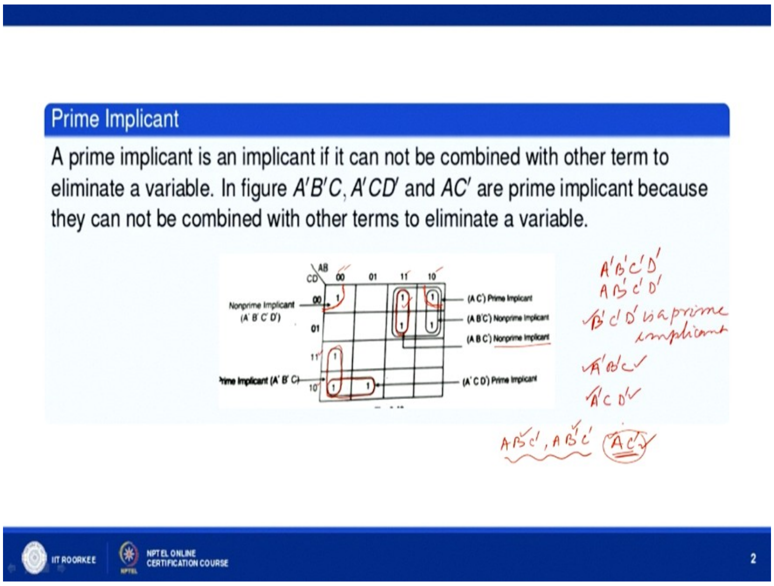

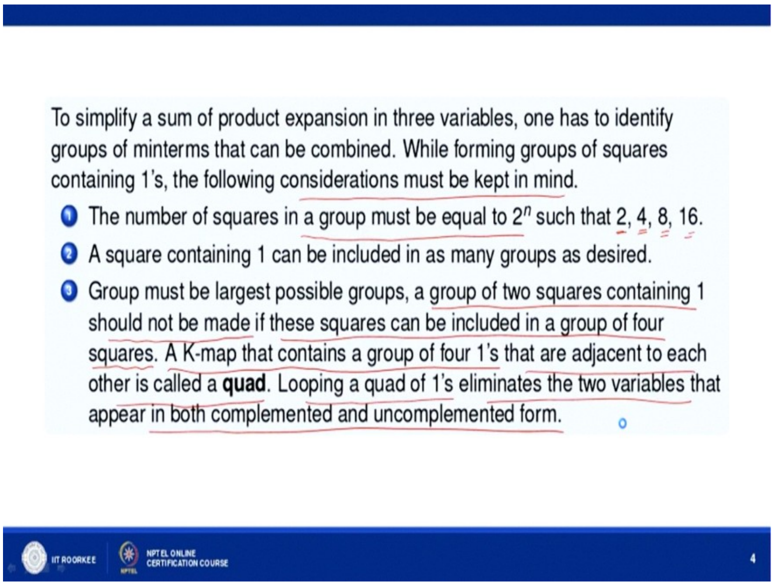

Now, to simplify the sum of product expansion in three variables, one has to identify groups of minterms that can be combined. While forming groups of squares containing 1s, now let us see what considerations we have to kept in mind, now we have to keep in mind, the number of squares in the group must be equal to 2 to the power N, so if it is a 2 variables case we will have 4, if it is have 4 variables case we will have 16 squares and so on.

Now, will consider those squares which contain 1, so such squares will be either 2 or 4 or 8 or 16, which can be combine containing 1s, a group square containing 1 can be included in as many groups as desired, a group must be largest possible group, a group of two squares containing 1 should not be made, so this we have to keep in mind, a group of two squares containing one should not be made in this squares can be included in a group of four squares. A K-map that contains a group of four 1s. Okay, a K-map that contains the group of four 1s that are adjacent to each other is called a quad. Okay.

So, let us go to previous example where we had considered a square containing four, this one, so this is a quad we can see, this one here we have combine four squares, so this is a quad okay, so in the case of a quad two variables are eliminated, so looping a quad of 1s eliminates two variables that appear in both complemented and uncomplemented form, like there we had seen that two variables are eliminated and they are B okay, B because here it is 1, here it is 0 B it is eliminated and then D is also eliminated because D occurs both in complemented as well as uncomplemented form and we have obtained prime implicant AC'.

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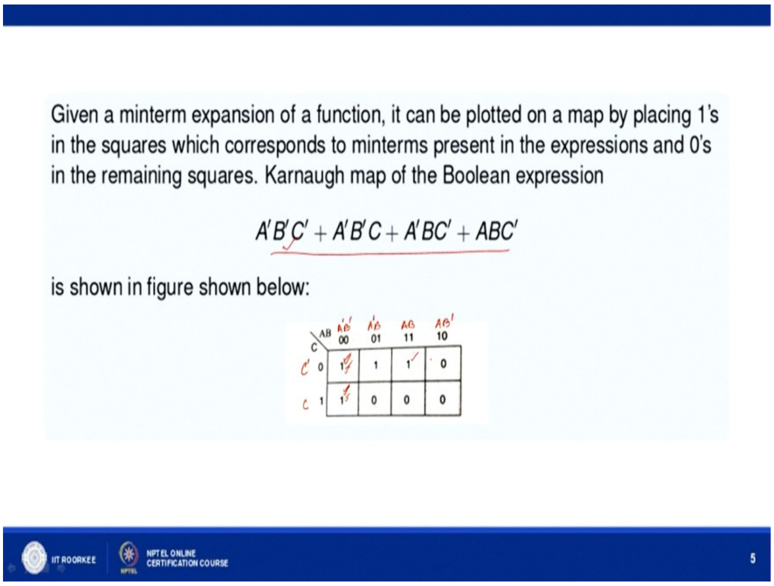

Now, given a minterm expansion of a function, we can plot it on a map by placing 1s in the squares which corresponds to minterms present in the expression and 0s in the remaining squares, say for example, we have this the Boolean expression. Okay, we have AB here, C down here, then this is $A'B'$ this is $A'B$ and this is AB, so this is $A'B'$, this is $A'B'$, this is AB, this represents AB and this one is AB' and we here here the C' and we have C okay.

So, $A'B'C'$, this is $A'B'C'$, $A'B'C$ so $A'B'C$ okay, so this is $A'B'C$ so $A'B'C'$ is corresponding to minterm $A'B'C'$ this is the square. Okay, so here we place 1 and corresponding to $A'B'C$ this is the square. Okay, so in that we have put 1 okay and then $A'BC'$, $A'BC'$ is this square, so in that we have 1 because those minterms which occur in the given Boolean expression, they are represented by 1 in the Karnaugh map, others are represented by 0, now then here ABC', so this is ABC', so here we have put 1, others are all given 0, now, so this is Karnaugh map for this Boolean expression.

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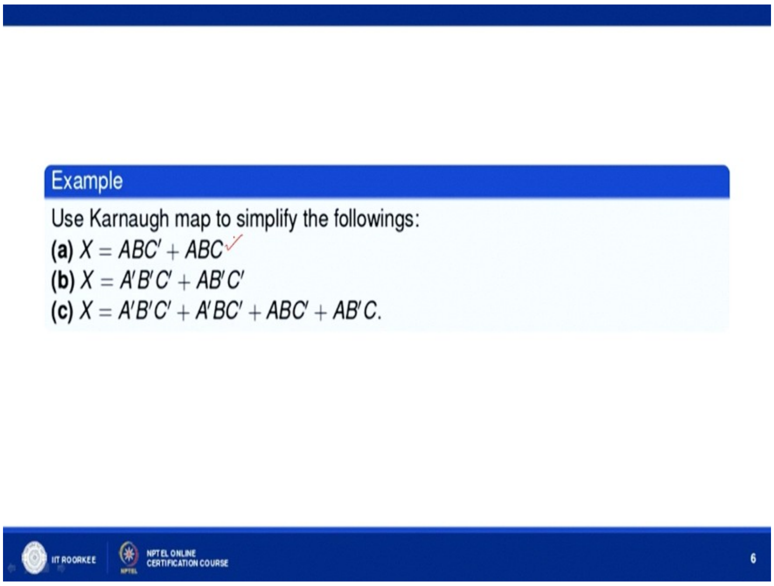

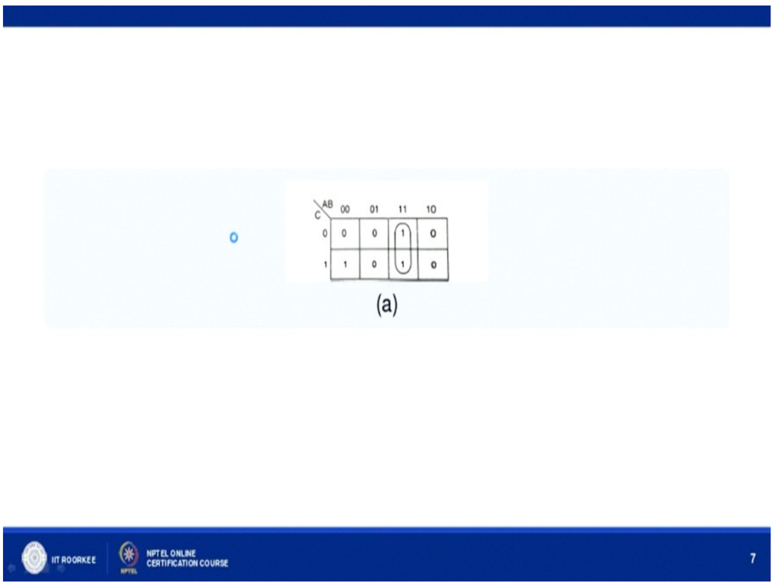

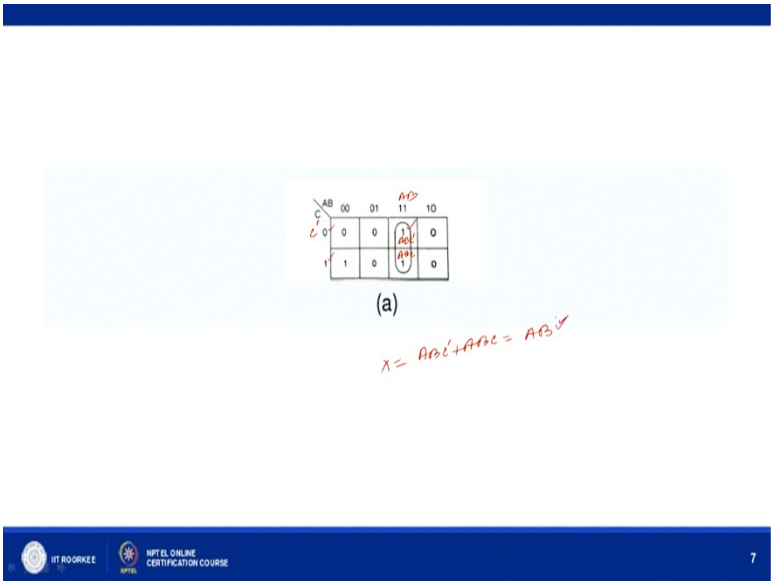

Now, let us use the Karnaugh map to simplify the following, so let us consider $X = ABC' + ABC$, let us plot it Karnaugh map and then find the prime implicant of I mean this, find the minimal form of X. So this is Karnaugh map, let see we have the terms of ABC', ABC okay, so this is AB, this is C' okay, so ABC' is here and then be other term that we have is ABC okay, so we have this is ABC, so this is ABC and this is ABC'okay.

Now when we consider this loop okay, it will eliminate one variable and the variable that will be eliminated is C because C occurs both in the complemented and uncomplemented form and so $ABC' + ABC = AB$, C will be eliminated, so $X = ABC' + ABC = AB$, so this is the minimal form of X.

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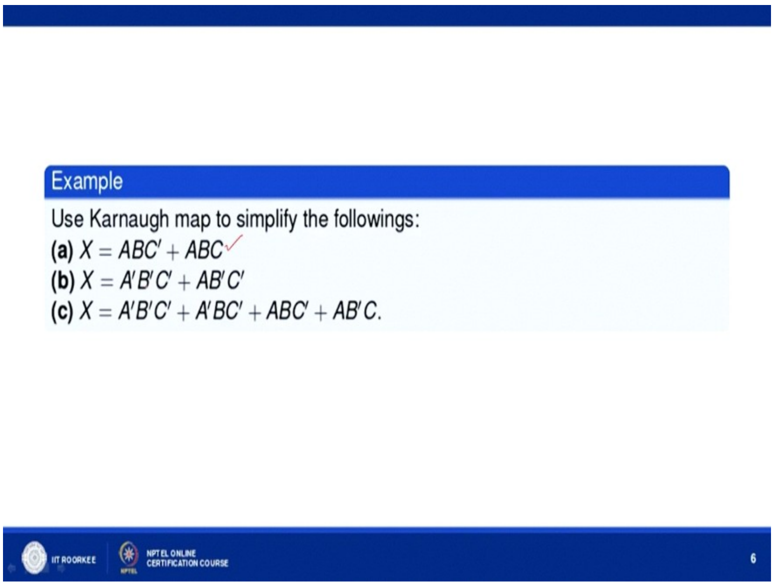

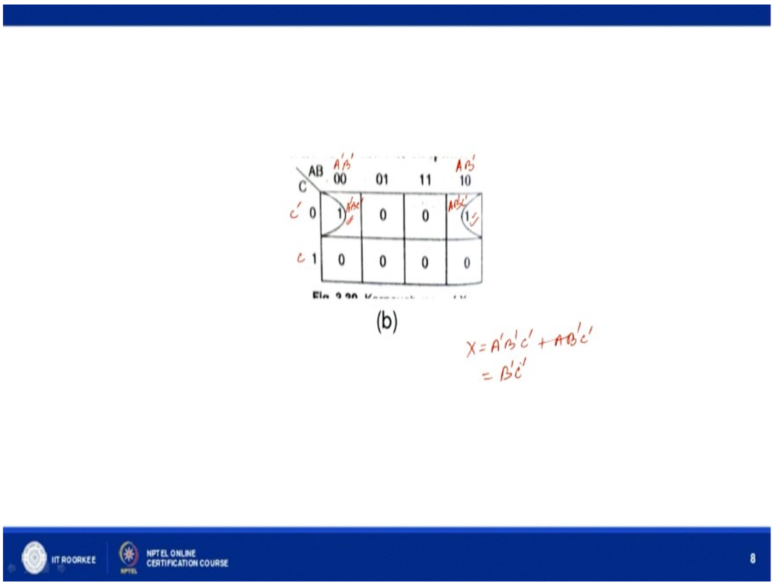

Now, let us go to a second part, in the second part we have $A^{'}B^{'}C^{'}$ okay, so $A^{'}B^{'}C^{'}$we have here $X=A^{'}B^{'}C^{'}$ and $AB^{'}C^{'}$ okay, so this is $C^{'}$, this is C, this is $AB^{'}$ okay and here we have $A^{'}B^{'}$, so this square represents $A^{'}B^{'}C^{'}$, okay and this square represents $AB^{'}C^{'}$, so we have put 1 in the squares which represents $A^{'}BC^{'}$ and $AB^{'}C^{'}$.

Now let us combine the rightmost, leftmost and the rightmost column okay, so here in this one, this one and this one can be combined and we see that one variable will be eliminated and the variable will be, because here we have $AB^{'}$ so here if it is 10, here we have 01 okay, so when we eliminate, the variable A will be eliminated because it occurs both in the complemented as well as uncomplemented form and we shall have $B^{'}C^{'}$, so this will be equal to $B^{'}C^{'}$.

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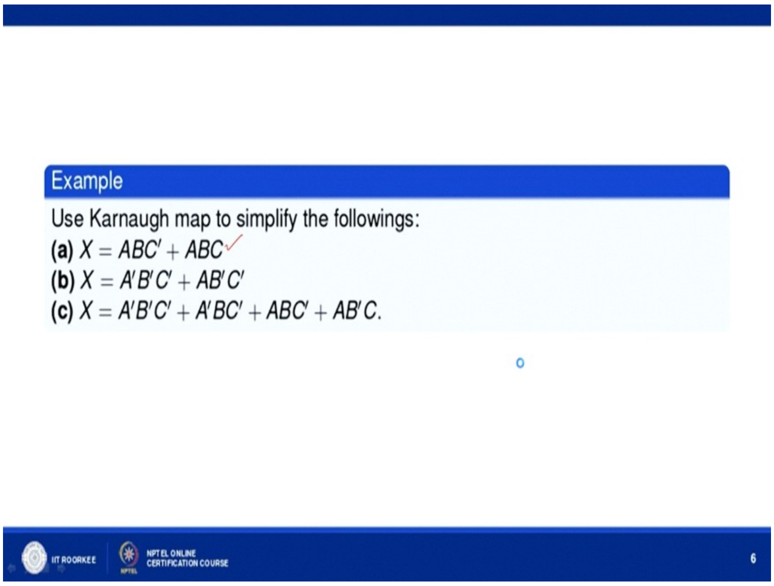

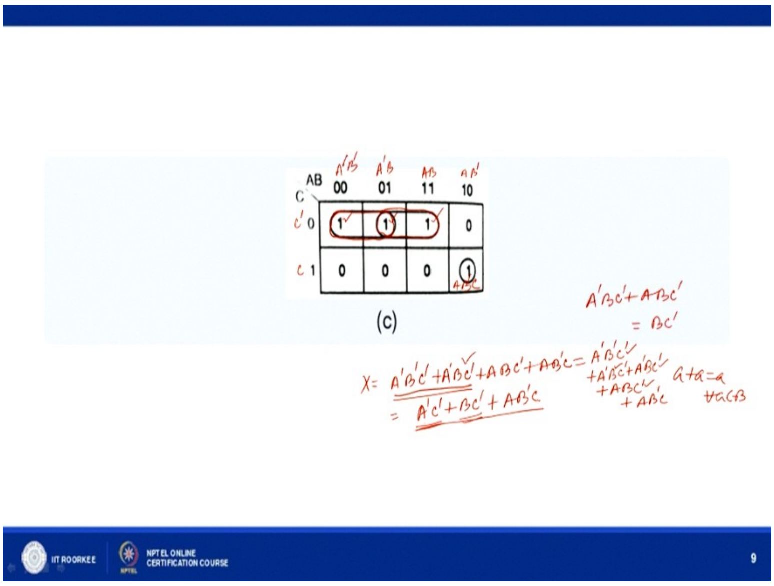

Then let us consider the third case, third part and, third part is we have $X = A^{'}B^{'}C^{'} + A^{'}BC^{'}$, we have $ABC^{'}$, $AB^{'}C$. Now corresponding to the square $A^{'}B^{'}C^{'}$, so this square is $A^{'}B^{'}C^{'}$ because this is $A^{'}B^{'}$ and this is $C^{'}$ so this square represents $A^{'}B^{'}C^{'}$,so we have put 1 here. Okay, then $A^{'}BC^{'}$ okay, $A^{'}BC^{'}$, so this is $A^{'}BC^{'}$ we have put 1 here, then $ABC^{'}$, so we have $ABC^{'}$, so this we have put 1 here and then A, $B^{'}C$ so $AB^{'}$ and we have here $AB^{'}C$ so this is $ABC^{'}$. Okay, so we have put 1 here.

Now, we can consider first this loop, we can consider this loop, if you consider this loop, what do you notice is that B occurs both in the complemented as well as uncomplemented form, so $A^{'}B^{'}C^{'} + A^{'}BC^{'}$okay, this plus this. Okay, if you consider this loop okay, then B will be eliminated and $A'B'C'+A'BC'$ it will reduce to $A'C'$ and when you consider this. Okay, $A'B$, $A'BC'$ this one. Okay, $A'BC'$ and ABC' okay, we notice that A occurs both in the complemented as well as uncomplemented form, so $A'BC'+ABC'=BC'$ okay.

Now, this is equal to $A'B'C'+A'B'C'$, $A'BC'=A'C'$, $A'B'C'$, we can write one more time because we know that $A'B'C'\ A+A=A, \forall A \in Boolean\ algebra\ B$, so I, we can write it one more time and then we, that one more time when you write $A'BC'$it will combine with ABC' and you get BC' and then we get ABC'.

Okay, so here what you do is you write it once more this term $A'BC'+A'BC'+A'BC'+ABC'$ and that the first two terms will give you $A'C'$ the next two terms will give us, one more term is that ABC', ABC' okay, so next two term, this term and this term give us $A'C'$,this term and this term give us BC' and we have $B'C$, so the minimal form of $X=A'C'+BC'+ABC'$.

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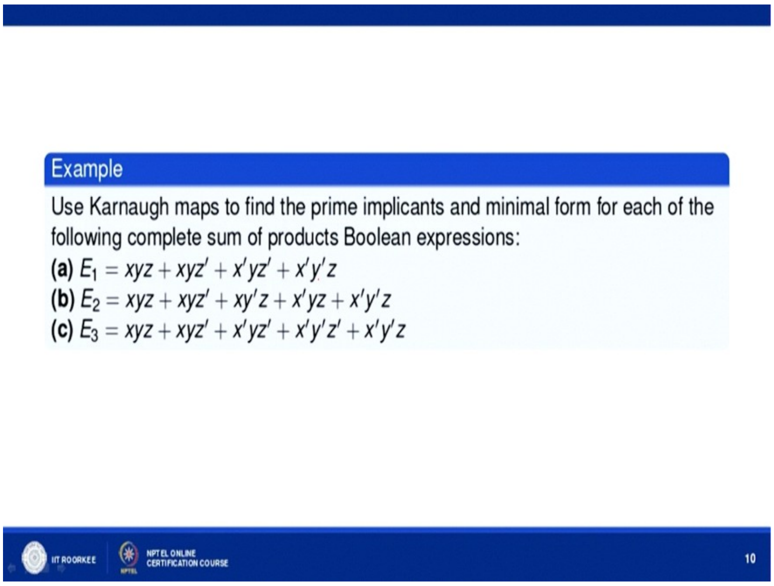

Now, let us use Karnaugh maps to find the prime implicants and minimal form for each of the following complete sum products Boolean expressions, so let us consider complete sum of products a Boolean expressions E_1, E_2, E_3 and determine the prime implicants and the minimal form for each of them.

So we have see the figure for E_1, this is the figure for E_1. Okay, we have the terms XYZ' so this is XYZ, this is XYZ' okay and then we have $X'YZ'$, this is $X'YZ'$ and then we have $X'Y'Z$, so this represent $X'Y'Z$, we have put a tic, this means that it is 1,1,1,1 else are 0s.

Okay, now we consider, we want to find the prime implicants and the minimal form of E_1 so let us consider this loop, when we consider this loop it will reduce the one variable that is Z and we will get the $E_1 = XY$, when we consider XYZ this square representing XYZ and this square representing XYZ'.

So, Z occurs both in the complemented and uncomplemented form, so we shall remove that is Z and we will get XY, when we consider the other loop this one which represents XYZ' and $X'YZ'$, then we notice that X occurs both in the complemented and uncomplemented form, so we shall get YZ' and then, this occurs isolated, it is isolated, so it is, it will come as such, so $X'Y'Z'$ it cannot be combined with any other term to eliminate a variable. Okay, so prime implicants are XY, YZ' and $X'Y'Z$ and this is the minimal form. Okay, so there are three prime implicants XY, YZ', $X'Y'Z$ and the minimal form of E_1 is $XY + YZ' + X'Y'Z$.

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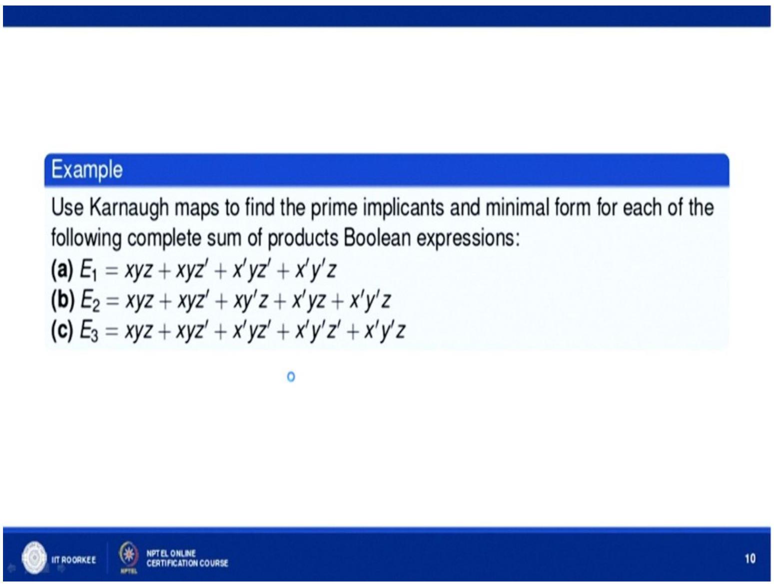

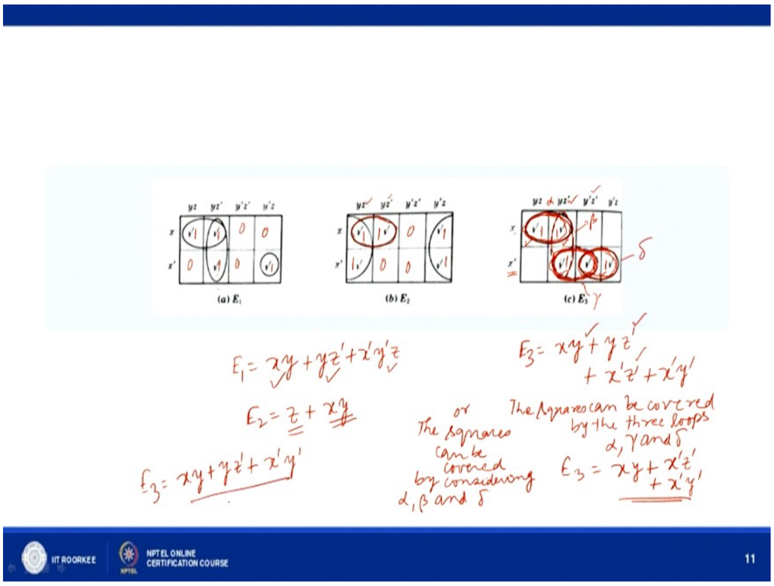

Now, let us look at the second part E_2, XYZ, so this is XYZ, okay this is XYZ and then we have XYZ', XYZ', this is XYZ', then we have $XY'Z$ so $XY'Z$ this is $XY'Z$ and then we have $X'YZ$, $X'YZ$. Okay and then we have $X'Y'Z$, $X'Y'Z'$ okay else are 0s. Now what we can do is? We can combine the leftmost column and the rightmost column okay and we can eliminate two variables because it will consist of four squares okay, when you combine the leftmost column with the rightmost column okay.

So combining the leftmost column with the rightmost column if you do that, then what will happen? Y occurs here in the both complemented as well as uncomplemented form, so Y will be removed and then X occurs both in the complemented as well as uncomplemented form, so X will be removed and the result will be Z. Okay, so E_2 will be equal to Z plus, when you consider this one, this loop okay, so this loop will reduce one eliminate, one variable which one is Z. Okay and will get XY okay.

So considering the leftmost and rightmost column eliminate two variables which are X and Y and when we consider this loop it eliminates one variable that is Z and we get XY, so XY and Z are prime implicants and $Z+XY$ is the minimal form of E_2 and then go to, let us go to E_3, so in E_3, we have XYZ, so this is XYZ and then we have XYZ', XYZ' here, then we have $X'YZ'$, $X'YZ'$, so this is $X'YZ'$ and then we have $X'Y'Z'$, $X'Y'Z'$and then we have $X'Y'Z$, so this is $X'Y'Z'$. Okay.

Now, let us consider this loop first, let us consider this loop first, so if we consider this loop, then Z occurs both in complemented as well as uncomplemented form, so Z will be elevated and we get $E_3 = XY$ okay and then we can consider, so we have taken this one, now we can consider suppose this one, we can consider this one. Okay, if we consider this loop, then this loop, because we have to find the minimal form this loop will give us or okay before that, let us consider this loop, let us consider this loop, this loop will eliminate the variable X because X occurs both in the complemented as well as uncomplemented form and we will get $Y Z'$.

Now, let us consider this loop, when we consider this loop, then Y will be eliminated because Y occurs both in the complemented as well as uncomplemented form and we will get $X' Y'$, $X' Z'$ okay, because X' is here, so this is $X' Y Z'$, this is $X' Y' Z'$, so Y will be eliminated and we will get $X' Z'$ and when we consider this loop, when we consider this loop okay, then Z will be eliminated and we will get $X' Y'$ okay, so we can E_3 equal to this, now let us notice that, see these square. Okay, this one, this one and this one, this one and this one can be covered if we consider this loop, this loop and this loop okay.

So, that will give us $XY + X' Z'$ okay, $XY + X' Z'$, so let me call it as loop number 1 okay, let me call them as okay, say, there are four loops okay, I can call alpha, this is say alpha, this one, this loop is beta, this loop is gamma and this loop is say delta okay, so there are four loops okay, now these squares, 5 squares can be covered by considering the loops alpha, gamma and delta okay, so the loops can be covered by the three loops alpha and gamma and delta okay, which give E_3 equal to, $E_3 = \alpha$ gives XY and this β gives your $X' Z'$ and this one delta gives us $X' Y'$ okay, we can also do.

Okay, we can also consider α, β and δ okay, the loops can be covered, the squares can be covered rather, I should say squares, squares can be covered by the three loops α, β, δ or the squares can be covered by considering α, β and δ okay, so the minimal form of E_3 will be α will give us XY, β will give us $Y Z'$and δ will give us $X' Y'$, so E_3 is either $XY + X' Z' + X' Y'$ or it is $XY + Y Z' + X' Y'$, so there are four implicants $XY + Y Z' + X' Z'$ and $X' Y'$, but the minimal form of E_3 is either this or this.

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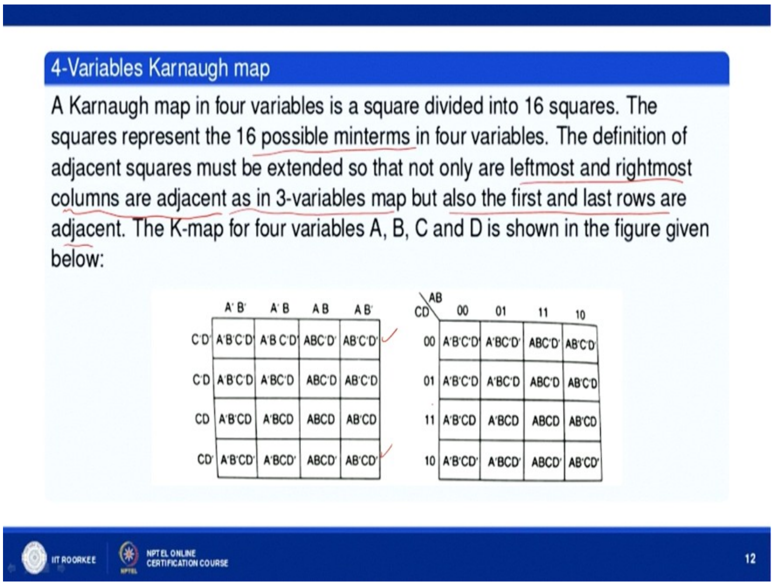

Now, let us consider four variables Karnaugh map, a Karnaugh map in four variables is a square divided into 16 squares, you can see there 16 squares here, the square represents the 16 possible minterms in four variables. The definition of adjacent squares must be extended so that not only are leftmost and rightmost columns are adjacent as in 3 variables map but also the first and last rows, this first row and last rows are adjacent, this first row also and last row are adjacent.

The K-map of four variables A, B, C, D is shown in the figure, will also you can see here, we have the $A'B', A'B, AB, AB'$ that we have $C'D'$, $C'D$, CD and CD' and these columns, these quads represent all possible, 16 possible minterms and when we give the binary values to A and B, 0 or 1, then we have the $A'B'$ becomes 00, $A'B$ become 01, AB becomes 11 and AB' becomes 10 and similarly we have $C'D'$, $C'D$, CD and $C'D$, so we have this Karnaugh map in the case of four variables.

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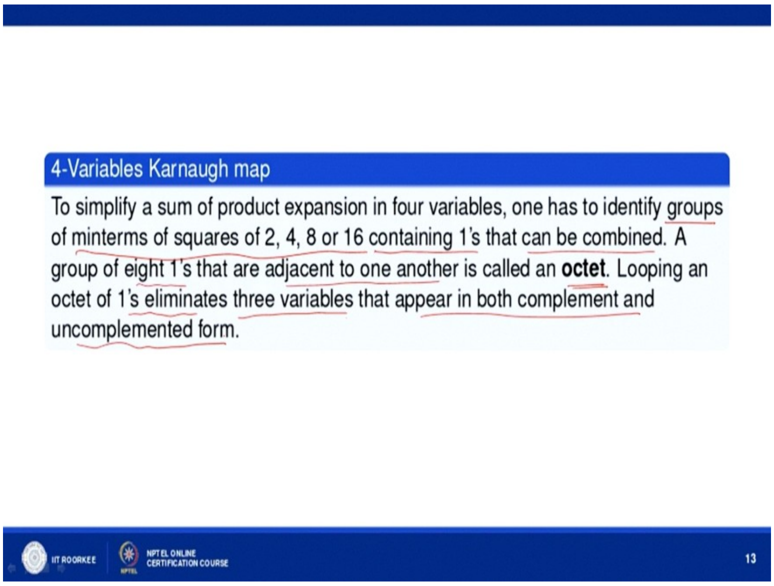

Now, to simplify sum of product in four variables, one has to identify groups of minterms of squares of 2, 4, 8 or 16 containing 1s that can be combined. A group of eight 1s that are adjacent to one another is called an octet okay, like in the case of three variables we had a quad okay, where we had a group of four squares, here we have a group of eight squares containing 1s.

So loop an octet of 1s eliminates three variables, there when we consider a quad it eliminated two variables, here when we considered a octet of 1s it eliminates three variables that appear both in the complemented and uncomplemented form.

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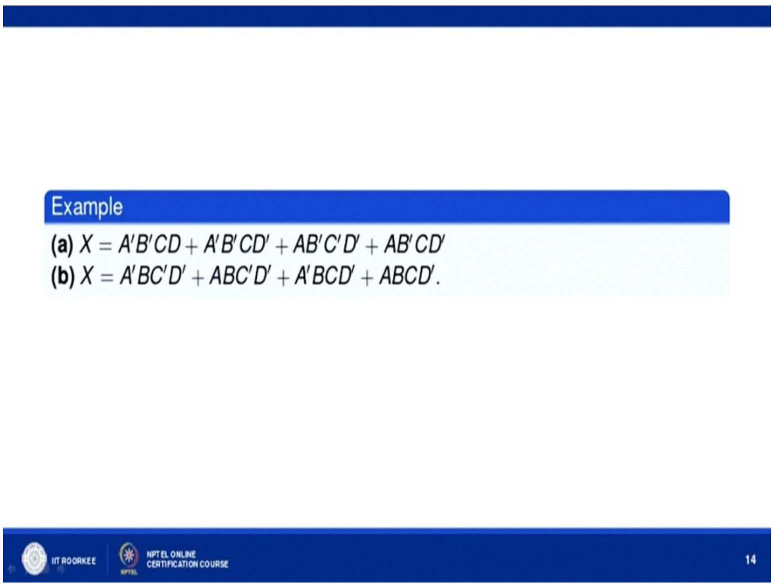

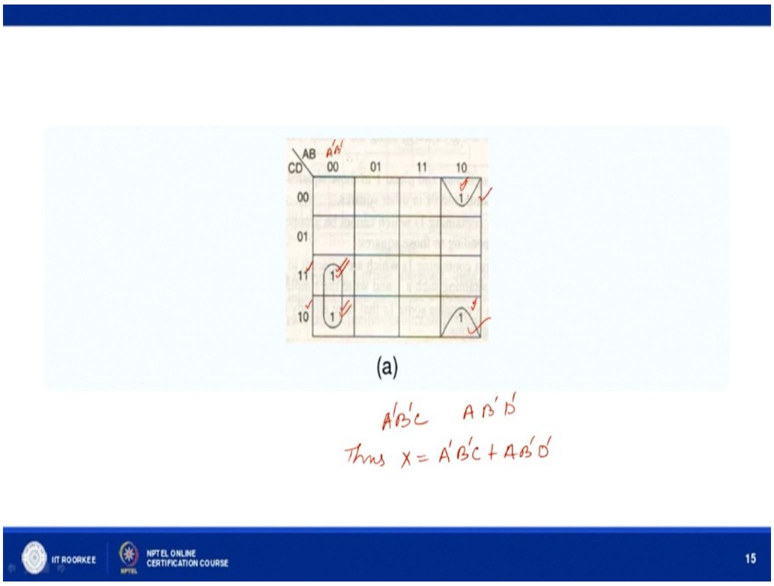

Okay, now let us consider this problem. Okay, so $X = A^{'} B^{'} CD + A^{'} B^{'} C D^{'} + A B^{'} C^{'} D^{'} + A B^{'} C D^{'}$ okay, let us draw Karnaugh map okay, so this the Karnaugh map, we have, see we have $A^{'} B^{'} CD$ okay, $A^{'} B^{'} CD$ so this is $A^{'} B^{'}$, $A^{'} B^{'} CD$ okay, $A^{'} B^{'} CD$ means this one. Okay, so $A^{'} B^{'} CD$ is represented by this square, then we have $A^{'} B^{'} C D^{'}$, so this is $A^{'} B^{'} C D^{'}$ okay and then we have $A B^{'} C^{'} D^{'}$ so that means it will be 1000 okay, yes, this one 1000.

So that is, that represent the third term and the fourth term is $A B^{'}$ so that means 1010. Okay, so this is 1010, this one. Okay, now we can see we have four 1s. Okay, corresponding to the four terms present in the Boolean expression, so we combine this 11 by this loop okay, it will remove one variable, okay, and what will be that variable, which occurs both in complemented as well as uncomplemented form, so you see here.

D occurs in both complemented as well as uncomplemented form, so we shall have the result of this looping, as a result of this looping we shall have $A^{'} B^{'} C$ okay, $A^{'} B^{'} C$, so and then as a result of this, this is a horizontal when we consider first row and last row. Okay, we can see that, we have here 00, here we have 10, so C occurs in the complemented as well as uncomplemented form. Okay, here it is C is 0, here C is 1.

So for these when we consider the first row and last row. Okay, when we combine them, then C will be eliminated and we shall have A and then B dash okay, because 10 represents A and B dash and then C is eliminated. Okay, C is eliminated and we shall have $D^{'}$, so as a result of this looping, we have X equal to, thus $X = A^{'} B^{'} C + A B^{'} D^{'}$.

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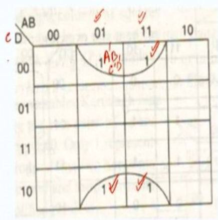
Example

(a) $X = A'B'CD + A'B'CD' + AB'C'D' + AB'CD'$

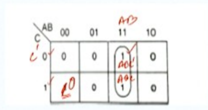
(b) $X = A'BC'D' + ABC'D' + A'BCD' + ABCD'$.



(b)

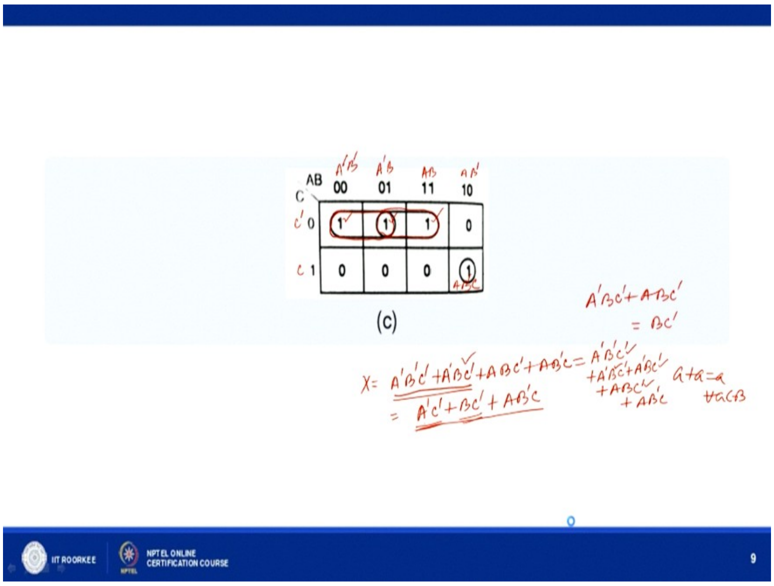
0110

$X = BD'$



(a)

$X = ABC' + ABC = AB$



Now, let us go to second case, in the second case we have $X=A^{'}BC^{'}D^{'},A^{'}BC^{'}D^{'}$, so $A^{'}$, this is $A^{'}BCD^{'},CD^{'}$, so this is $A^{'}BC^{'}$ okay, right, so this is $A^{'}BCD^{'}$ okay and then we have, oh it was $A^{'}BC^{'}D^{'}$, $A^{'}BC^{'}$, so this is $A^{'}BC^{'}D^{'}$ okay, that is first term and next term is $ABCD^{'}$ that means 11 00, so what is that 11 00, 11 00, so this is second term. Okay and then third term is 01 10, 01 10, 01 10, 01 and 10 okay, so this one is third term and then the fourth term is 11 10, 11 10, this is fourth term. Okay.

Now, we can see when we consider the first row and the last row, we can consider this loop and then this loop will eliminate as the result of this looping because we are now, when we consider the first row and the last row four squares are combined, four squares are combined, so what will get? Two variables will be eliminated, one variable that will be eliminated is this one because it occurs A, A occurs both in the complemented as well as uncomplemented form, so A will not be there, we will have B okay and as a result of this one, here you see in the first row and the last row when you compare D, C occurs in the complemented as well as uncomplemented form, C is 0 here, C is 1 here.

So, we will have $BD^{'}$okay, so we will have here, B here and then here we will have $D^{'}$, so $BD^{'}$ so $X=BD^{'}$ as a result of this looping and then we go to, now here let us notice that C is missing, so what we can write AB the first row represent AB, second row represents the variables CD okay and here in this part we have here 1. Okay, this 1 it means that $A^{'}B^{'}$ and C should be present, A dash B dash and C should be present, this is not here. Okay, there are two 1s only corresponding to the two terms in the part A, we notice that, there are three 1s, so this should be 0. Okay, so in the seventh slide this square should be having 0 instead of 1.

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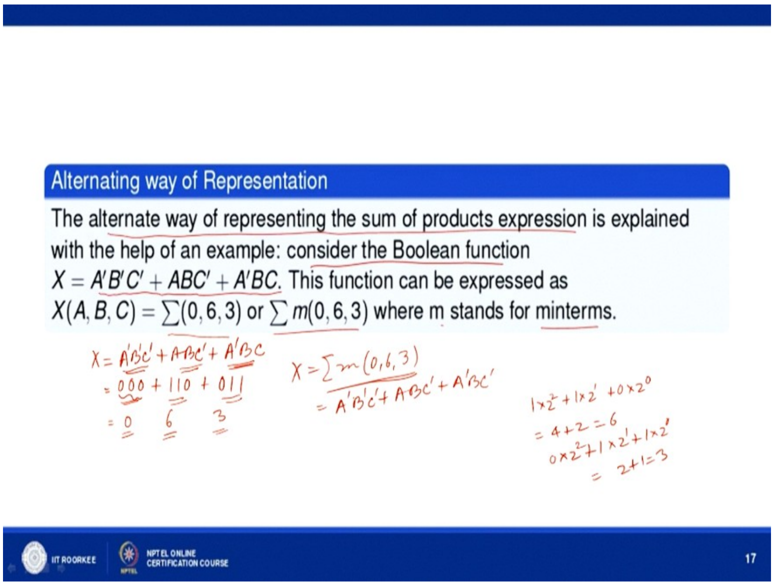

Okay, now let us consider alternating way of representation, the another way of representing the sum of products expression is explained with the help of an example, let us consider this Boolean function. Okay, $A^{'}B^{'}C^{'}$, $ABC^{'}$, $A^{'}BC$ okay, then this function can be expressed as Sigma 0, 6, 3 or Sigma m 0, 6, 3 where m stands for minterms, that means there are three minterms corresponding to 0, 6 and 3.

The first minterm has the decimal value 0, second minterm has the decimal value 6, third minterm has the decimal value 3 and that we can see also because

$X=A^{'}B^{'}C^{'}+ABC^{'}+A^{'}BC$ okay, this is in the Boolean form, Boolean representation, $A^{'}=$¿0, $B^{'}=0, B^{'}=1$ and this is 1, 1, 0 and we have 0, 1, 1 okay. So first term in decimal representation will have value 0, second term in decimal representation will have value 0 into 2 the power 0 plus 1 into 2 to the power 1 plus 1 into 2 to the power 2 okay.

So we have $4+2=6$. Okay, so this will have value 6 and the third one will have

$0.2^{2}+1.2^{1}+1.2^{0}=2+1=3$, so third term in the Boolean expression has decimal value 0, so these terms in the Boolean expression X, in the Boolean expression for X they have decimal values 0, 6, 3 and in the Boolean form, Boolean representation they have, they are represented by 000, 110, 011.

So X can also be written as Sigma m 0, 6, 3 okay and then we will have to write the corresponding terms for the Boolean function X, the minterm have been decimal value 0 will be $A^{'}B^{'}C^{'}$and then minterm having decimal values 6 will be $ABC^{'}$ and then minterm having the decimal value 3 will be $A^{'}BC^{'}$, so we can also write the Boolean expression in terms of this Sigma m 0, 6, 3, we can write the corresponding Boolean form by converting the decimal values 2 the variables to the Boolean form.

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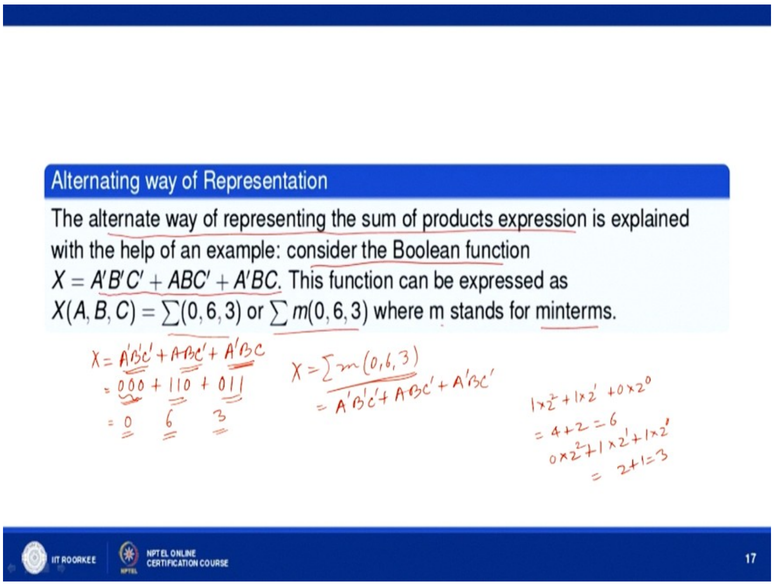

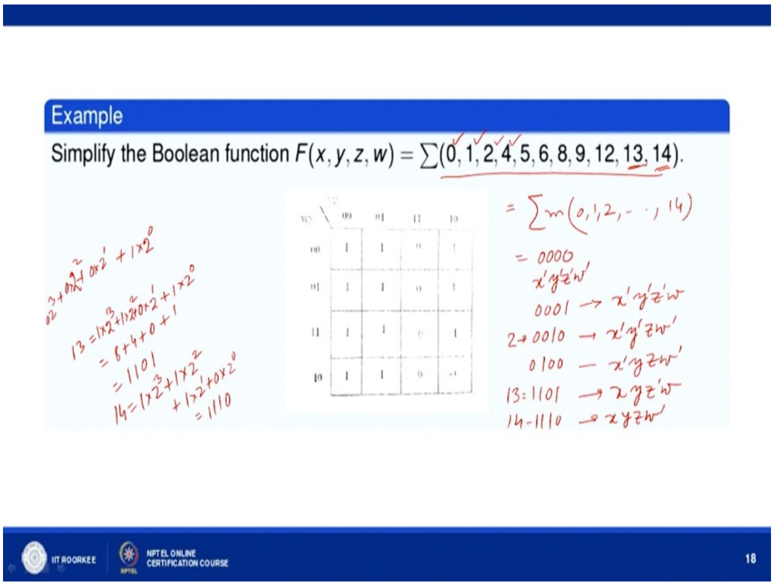

Now, let us say, consider this one, you see here we have $\sum(0,1,2,4,5,6,8,9,12,13,14)$, so this is nothing but 0, $\sum m(0,1,2)$ like we have said already here, we can write $\sum(0,6,3)$, or we can write $\sum m(0,6,3)$, Okay, so $\sum(0,1,2,4,5,6)$, means 0, 1, 2 and so on up to 14 okay, so first term has decimal value 0 means it is 000, so that means we are having X, Y, Z, W, so 0000, so that means we have $X^{'}Y^{'}Z^{'}W^{'}$ corresponding to this 0 okay, corresponding to 1.

Now decimal value is 1, so we shall write it in the Boolean forms, so that mean, 1 means, 1.2^{0}

then 0.2^{1} then $0.2^{2}$and then 0.2^{3} Okay, so we shall have 0001 for this one. Okay, that will mean that, it is, it will represent $X^{'}Y^{'}Z^{'}W$.Okay and when we take 2, 2 will be similarly written as 0010 okay 2, 2 will be represented by 0010 and this will be written as X dash Y dash ZW dash okay, like this.

So this way we can write here anyone okay, I have 2 here, 2 can be written like this and for will be 0100 okay, 0100, so this will be $X^{'}YZW^{'}$, so we can write when the corresponding Boolean expression for any decimal here, like supposed we want to write for 13 okay, so 13 can be written as 1.2^{0}, we have here 1.2^{3}, so $8+9$ and then we have 1.2^{1} and then we have 1.2^{2} okay.

So we have, no, $8+4+2$, oh, $8+4$ okay, that should be taken 0, so this is, if we can take it as 0 here, so 8, 4 and then this is $0+1$ so we get 13 okay, so this means that 11 and 0 and 1 okay, so 1101, 1101 that is 13 okay and this is $X^{'}Y^{'}$no, $XYZ^{'}W$, if you want to write 14 it will be fourteenth can be written as 2 to the power 3, so that means 14 will be want to write, 14 can be written as 1.2^{3}, so that will give us 8 and then 1.2^{2}, then we have $8+4,12+1.2^{1}+0.2^{0}$ and we will have then 1110 okay, so 1110, so this will be equal to XYZW dash.

So we can write this way okay and you can see there are 15 values right, there are 15 values and we have here. Okay, so minterms present are, there are, how many minterms there? We have 15 yes.

Student: 11.

Prof: why 11? Okay, oh, oh, sorry, sorry, okay, okay, see, so we have here 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 11 minterms are there and there are 11 1s here corresponding to the minterms present here, 11 1s here and remaining are 0s okay.

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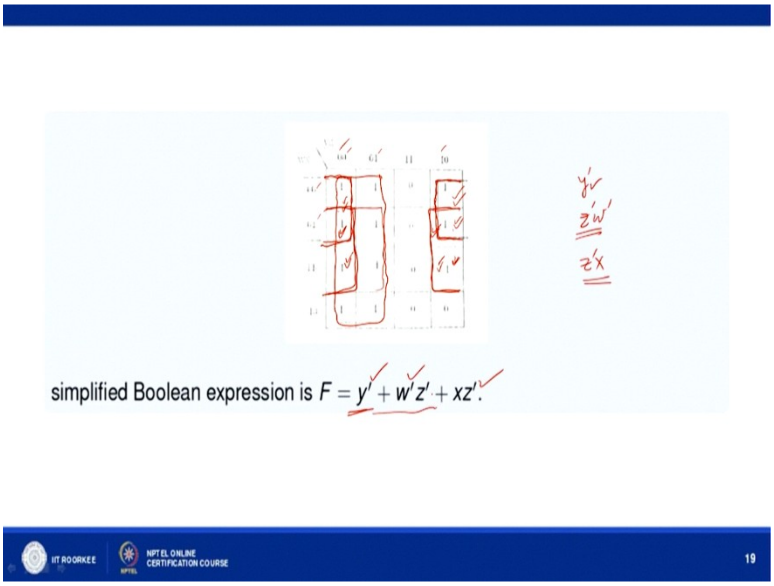

Now, let us see this Boolean expression, this Boolean expression, so we have to see how we can combine okay, so this octet we can consider, this octet will remove three variables okay and what will be those three variables, you see here 0, here we have 1, so Z will be removed okay, we will have Y dash okay and here you see 0, 1, so W will be removed okay and here we have 0 and here we have 1, so X will be removed, so as a result of looping of these eight squares, we get the result S Y dash okay.

Then we consider this looping, we consider this looping, so this will, they have four squares, four square means two variables will be removed, so here you see here we have 0, here we have 1, so that means Y will be removed okay and here we have 0, here we have 1, so X will be removed and we shall then have, this is Z okay, so Z' okay, Z' and here we shall have W. Okay, so we will have $Z'W$, as a result of looping of these four squares, $Z'W'$.

Okay, so as a result of looping of this four squares, we get $Z'W'$ okay, now we still have to consider these ones, this one is there, so we consider this loop, this loop, no we consider this loop okay, that will involve four squares okay, so this, this, this and this, we consider this four squares, so as a result of this looping what will happen? Here we have 10, okay and here we have, then we consider this will have 00, so 10, 10 means Y will be removed okay, 1 here and 0 here, why will be removed and a result of this we will see that W will be removed okay.

So we shall have $Z'$and we shall here X okay, so we get Y dash okay, $W'Z'$ and we get XZ', so as a result of this looping, first consider an octet which gives us Y' and then we consider a looping four squares this one. Okay, we had consider this one, this one, which gives us $Z'W'$

and that we consider this looping, this looping, that gives us $X Z'$, so this is the simplified Boolean expression, that is all in this lecture. Thank you very much for your attention.