Higher Engineering Mathematics
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Karnaugh Map - I
Mod05_Lec24

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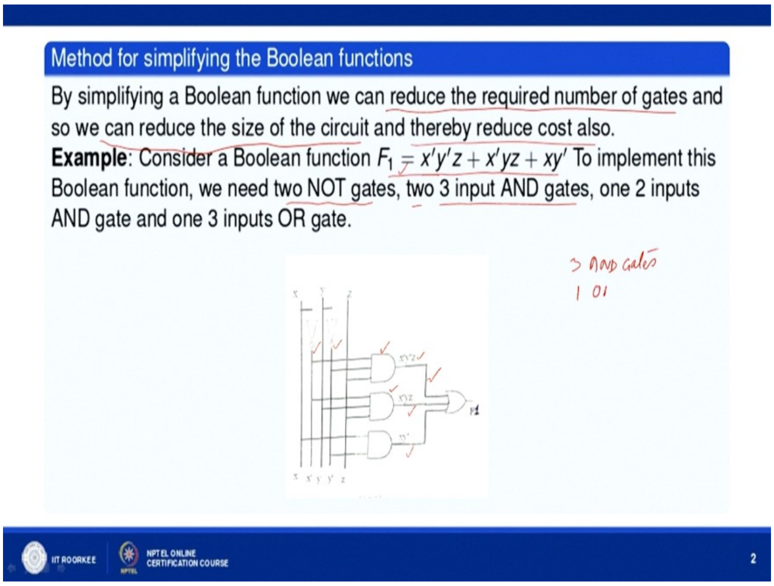

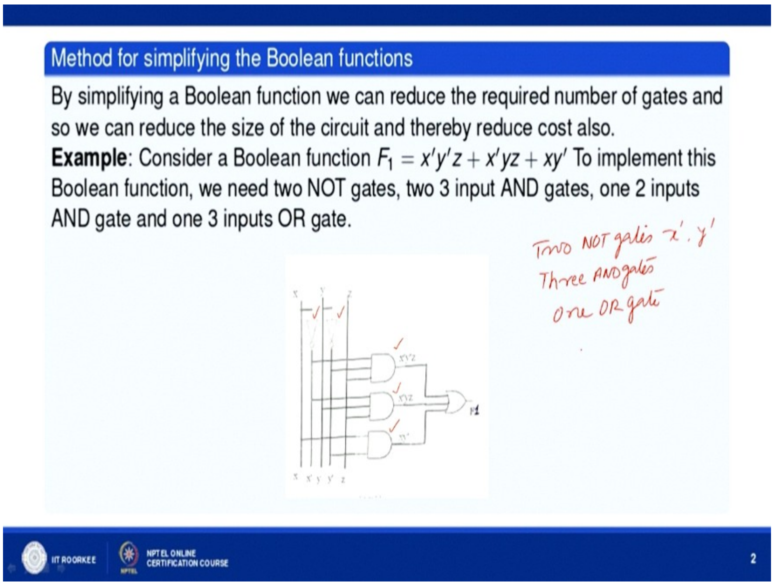

Hello friends, welcome to my lecture on Karnaugh maps. In this lecture we shall discuss the matter of simplifying Boolean functions. Why we simplify Boolean function? Okay, if we simplify a Boolean function, we can reduce the required number of the gates and so we can reduce the size of the circuit and thereby reduce the cost also. Say for example, consider this

Boolean function, $F_1 = X^{'} Y^{'} + X^{'} YZ + X Y^{'}$ Now this is the figure of this Boolean function, you see this is X, this is X conjugate, sorry this is X, this is $X^{'}$, this is Y, this is $Y^{'}$ and this is Z.

Okay, so then $X^{'} Y^{'} Z^{'}$,okay, $X^{'} Y^{'}$,so we have this input $X^{'}$, then we have this input $Y^{'}$and then we have this input Z. Okay and $X^{'} Y^{'} Z^{'}$, to implement $X^{'} Y^{'} Z^{'}$, we have consider this AND gate, okay. So when these are 3 inputs $X^{'} Y^{'} Z^{'}$ then to get the output $X^{'} Y^{'} Z^{'}$we have used this AND gate. Okay, then $X^{'} YZ$, so we have this $X^{'}$ input then Y and then Z. Okay, so 3 inputs are there, $X^{'} Y$ and Z and to get the output as $X^{'} YZ$ we accused again another AND gate, okay.

So, then the third term is $X Y^{'}$ so we have this input X okay and this input $Y^{'}$ okay, so 2 inputs are here X and $Y^{'}$and to get the output $X Y^{'}$ we use the AND gate. Okay, so in the first AND gate there are 3 inputs $X^{'} Y^{'} Z$.Okay, in the second AND gate there are 3 inputs again $X^{'} Y$and Z and in the case of third AND gate there are 2 inputs X and $Y^{'}$. So, there are 3 AND gates, first AND gate considers two, 3 inputs, okay, this AND gate has 3 inputs, this AND gate has 3 inputs again and this AND gate has 2 inputs and there are two NOT gates, this is one, okay, $X^{'}$ and $Y^{'}$,there are two NOT gates.

So, there are two NOT gates, okay, two, 3 input AND gates, there are 3 input AND gates okay, there are 3 input AND gates here and then to obtain F_1 okay, the output F_1 okay, we have to use this OR gate okay, so for OR gate there are 3 inputs $X^{'} Y^{'} Z$.Okay, $X^{'} YZ$and $X Y^{'}$ so there are 3 inputs and to get the output F_1 we use this OR gate okay, so there are 3 AND gates with one has 3 inputs, the one other has 3 inputs again, the third one has 2 inputs and 3 AND gates and one OR gate.

So there are two NOT gates okay, $X^{'}$and $Y^{'}$, to get $X^{'} Y^{'}$ okay and there are 3 AND gates okay, this, this and this 3 AND gates, two NOT gates this and this okay and there is 1 OR gate, okay, to get the desired output $F_1 = X^{'} Y^{'} Z + X^{'} YZ + XY^{'}$. Now that is simplify this Boolean expression. Okay, using the postulates and the theorems that we are earlier studied.

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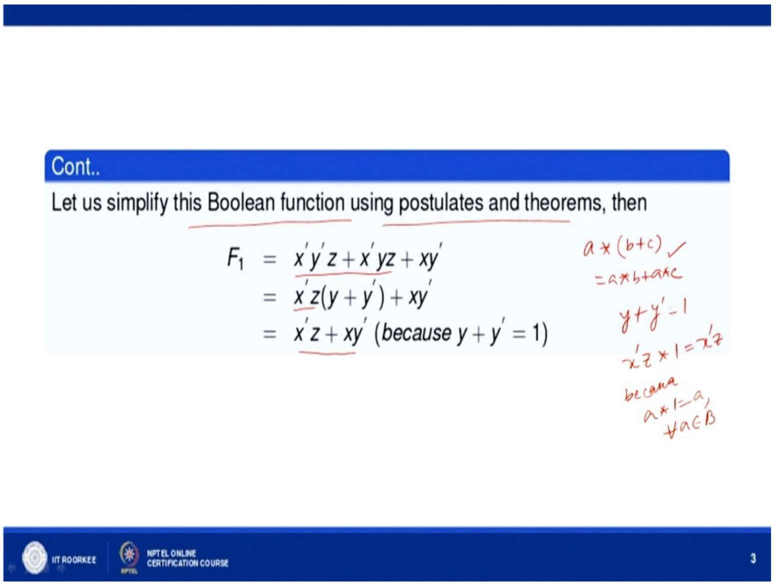

Okay, but we will have them, so let us simplify this Boolean function using postulates and the theorems $F_1 = X'Y'Z + X'YZ + XY'$, we can then use this distributive law and we say that

$A*(B+C) = A*B + A*C$ Okay, so $X'Y'Z + X'YZ + XY'$ can be written as $X'Z* + Y + Y'$ by using this distributive law, so $X'ZY + Y' + XY'$, now $Y + Y' = 1$ okay.

So, we have $X'*X'Z*1$ here. Okay and this is equal to $X'Z$because $A*1 = A \forall A \epsilon B$ okay, this we know for all $A \epsilon B$, $A*1 = A$, 1 is the unit element, so here $X'Z1 = X'Z$and therefore we will have $X'Z + XY'$ making use of $Y + Y' = 1$. Now, let us see how to get this desired this output $X'Z + XY'$, let us again draw the gates.

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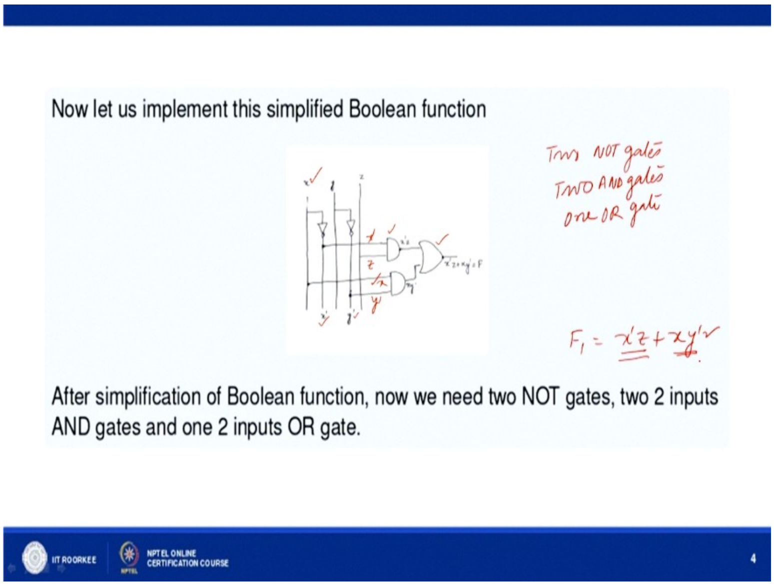

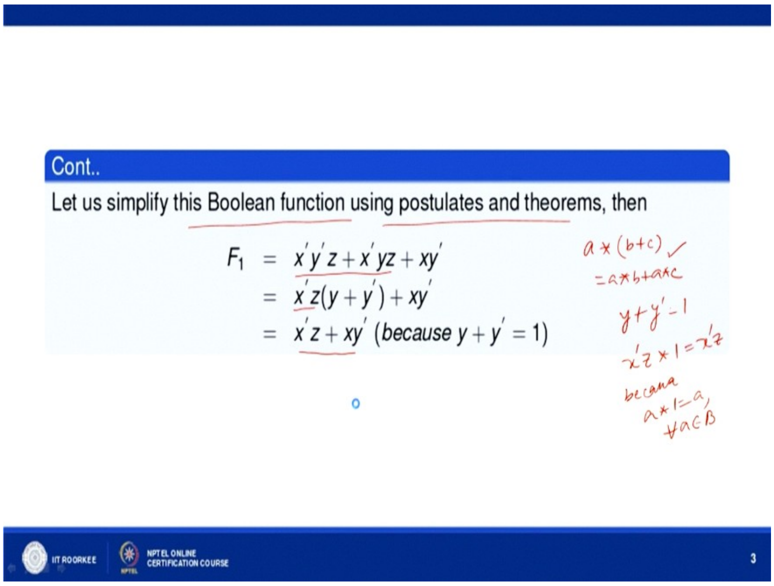

So, here we have this, this is your X, this is X' okay, the output, so we have one NOT gate and then for Y we have one NOT gate Y', so there are two NOT gates, okay, and since our output

$F_1 = X'Z + XY'$, how to get is $X'Z$? We take the input X', okay, this is input X',and this is input Z. Okay, so X', this is X', this is Z, to get $X'Z$ we used this AND gates, okay, so 1 AND gate we use to get the $X'Z$, so there are 2 inputs $X'Z$ and 1 AND gate to get the output $X'Z$ and then for the other term XY' we use again 2 inputs this is X and this is Y', okay, so two inputs X and Y' with an AND gate then gives us XY' okay.

So, and to get then $X^{'}Z+XY^{'}$ we use one OR gate okay, so this is one OR gate, so we have two NOT gates, two AND gates and one OR gate, okay, and we get $F_1=X^{'}Z+XY^{'}$. So you can see now the number of gates that we need are less, okay, in comparison to the previous case and number of inputs also are less, okay, there we had 3 inputs, okay, if we see this one, this figure we had two NOT gates okay, we also have two NOT gates there, we have three AND gates here, while we have one AND gate there, we have two AND gates there.

So, this three AND gates reduced to two AND gates and moreover the inputs like here in this two AND gates the inputs were three each. Okay, here, three and here also three and there we see that we have two AND gates with inputs 2 each. So, by using postulates and the theorem we can simplify given Boolean expression, so that many draw the figure okay, we use less number of AND or OR gates and NOT gates to get the desired output, so that actually reduces the cost of the output, to get the desired output, we have to spend much less okay, by using the theorems and postulates to simplify the given Boolean function.

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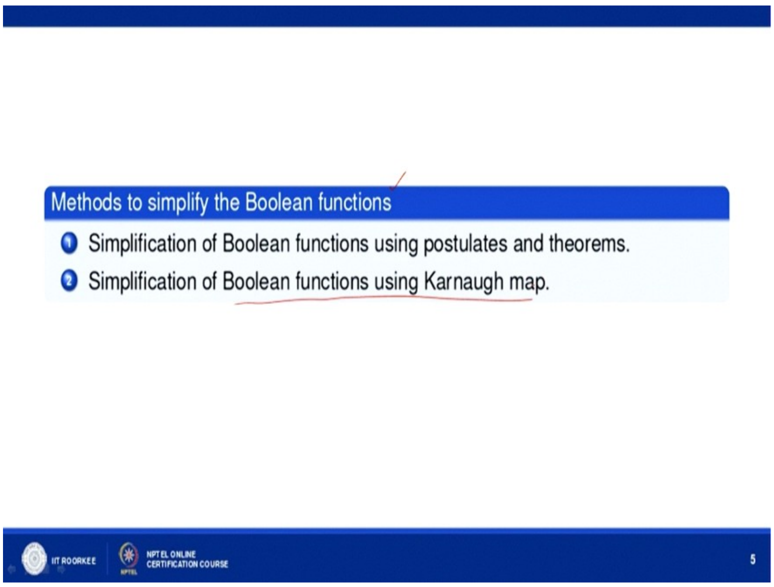

Now, let us consider methods to simplify the Boolean functions. Okay, one method that we have just now seen by using postulates and theorems, we can simplify the Boolean functions, the other method is using Karnaugh map okay, so let us see how we use Karnaugh map to simplify the given Boolean functions.

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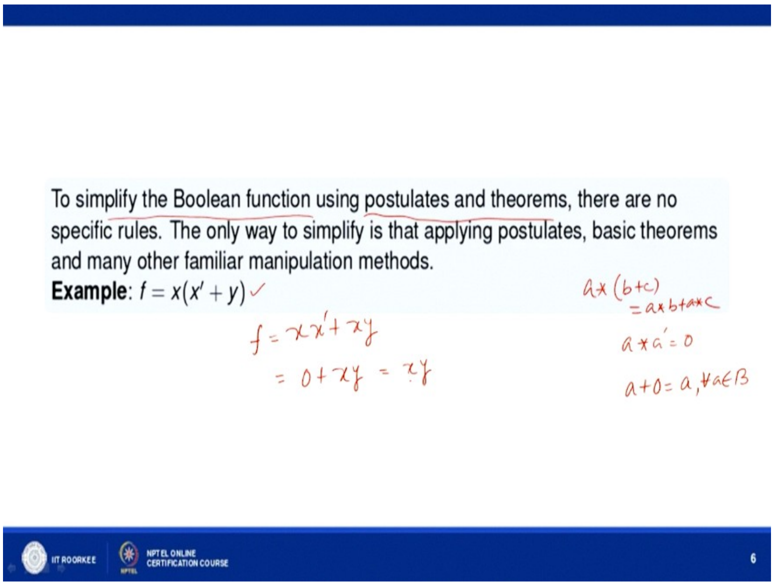

Now, let us say to simplify the Boolean function if you use postulates and theorems okay, then there are no specific rules. The only way to simplify is that the apply postulates and basic theorems and some other manipulation methods to arrive at the simplification of the given Boolean expression, let say for example, let us consider $F = X\,X' + Y$ okay, then we can simplify this, so F is equal to we use distributive law $a*(b+c) = a*b + a*c$ okay and this gives us $X\,X' + XY$. Now, we know that $a' * a = 0$. Okay, so this is equal to $0 + XY$,okay, and we know that by identity law $A + 0 = A\,\forall\,A \epsilon\,B$ okay, so $0 + XY = 0$So, we see that the expression $F = X\,X' + XY$ can be simplified to $F = XY$ by using the postulates and theorems.

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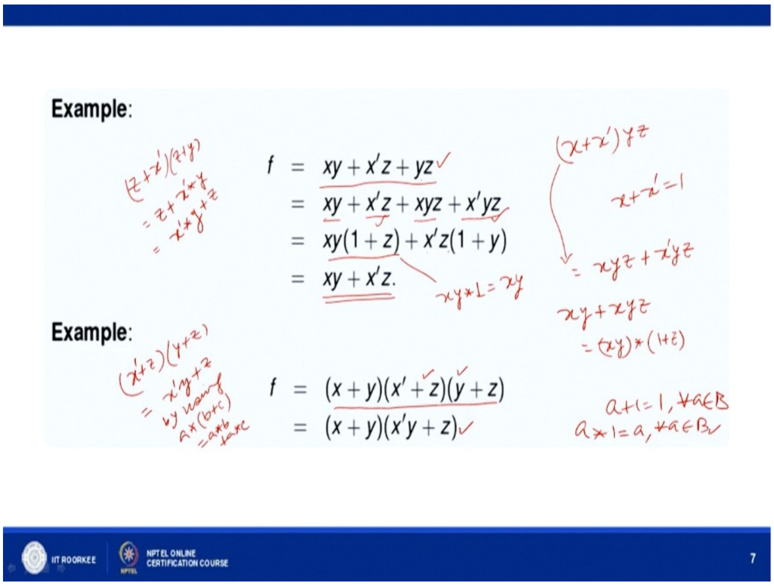

Now, let consider another example $XY+X^{'}Z+YZ$, let consider this Boolean function, we can write it as $XY+X^{'}Z$ and then YZ, YZ can be written as $X+X^{'}YZ$. Okay, because $X+X^{'}=1$ okay, so this YZ can be written as $X+XZ^{'}$ and then this is equal to by distributive law

$XYZ+X^{'}YZ$, so we have $XY+X^{'}Z+XYZ+X^{'}YZ$ here. Okay, now $X^{'}XY+XYZ$ can be combined and we can write $XY*1+Z$, Okay.

So, $XY+XYZ$ can be combined and we have $XY*1+Z$.. Okay, so we have this, now we know that $A+1$, $A+1=A, A\,\forall\, A\epsilon B$, so

$XY*1+Z$, $1+Z=1$ okay, so this will be $XY*1$ okay, this is equal to $XY*1$ and $XY*1=XY$ okay, because we know that $A*1=A, A\,\forall\, A\epsilon B$.Okay, B, so this gives us XY and similarly we combine $X^{'}Z$ and $X^{'}YZ$ so $X^{'}Z.1+Y$ we can write, $1+Y=1$, okay, so we have

$X^{'}Z.1, X^{'}Z.1$ we get X, $X^{'}Z$ by using this result, this theorem. So we have

$XY+X^{'}Z$. Okay, so this given expression which consisted of the 3 terms XY, $X^{'}Z$ and YZ is reduced to now 2 terms XY and $X^{'}Z$.

Okay, now let us consider this another Boolean function $F=ZX+YX^{'}+ZY+Z$. Okay, then what you notice is that if you take the, these 2 terms $X^{'}+Z$ then$X^{'}+ZY+Z$, we can write as

$X^{'}Y+Z$ by using $a*(b+c)=a*b+a*c$ okay, we can write it as X plus, $X^{'}+Z.y+z$ we can literally write as $Z+X^{'}Z+Y$, okay. So then it is equal to $Z+X^{'}*Y$, okay. So, we have or we can write it as $X' * Y + Z$. Okay, so $X' Y + Z$ we have here, okay, so the function F which consisted of 3, product of 3 terms, now has been reduced to product of 2 okay, $X + Y$ and $X' * Y + Z$

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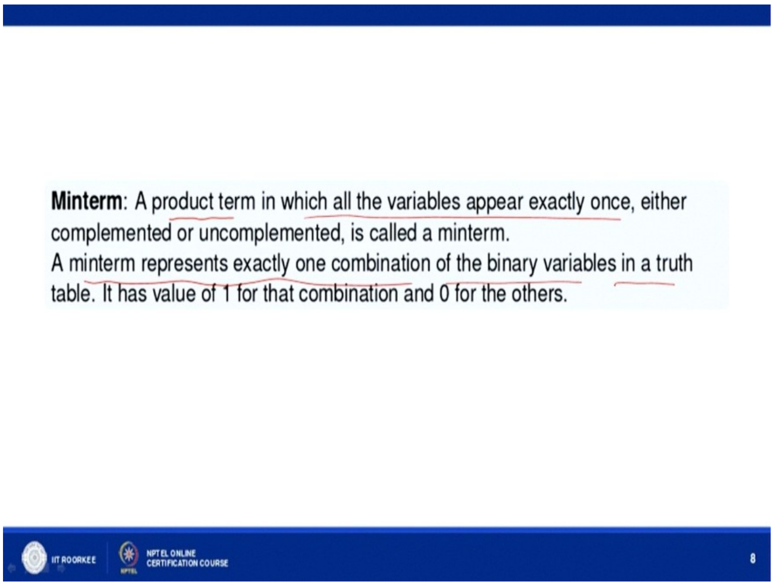

Now, let us define a Minterm, a product term in which all the variables appear exactly once. Okay, either complemented or uncomplemented is called a minterm, okay. So a product term in which all the variables appear exactly once, okay, either complemented or uncomplemented is called a minterm. This we define or in the previous lecture also. Okay, we have defined it again to emphasise it because we are using, going to use it when we discuss Karnaugh maps, so we have defined it earlier. Again here, a minterm represents exactly one combination of the binary variables in a truth table. Okay, it has the value 1 for that combination and 0 for the others and we shall see while discussing Karnaugh maps.

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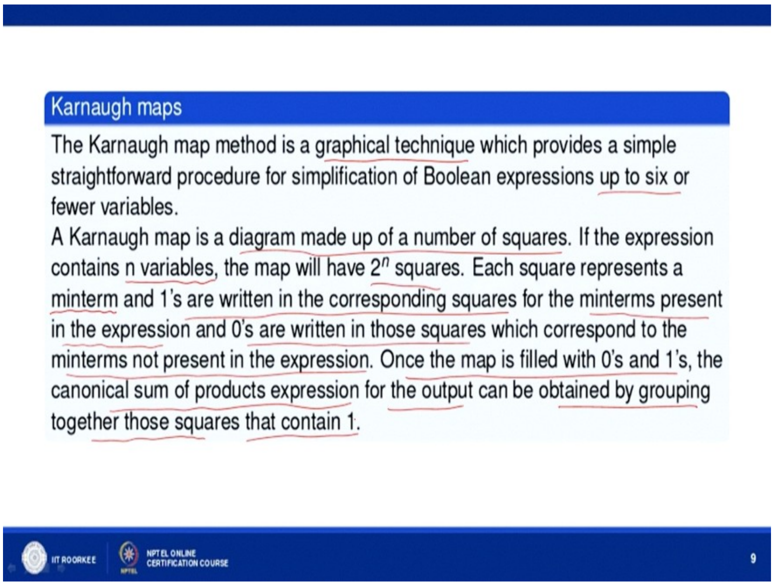

So, the Karnaugh map method is a graphical technique, it is a graphical technique which provides a simple procedure for simplification of Boolean expressions and it can be applied to six or fewer variables beyond that it is not easy to handle this situation, so Karnaugh maps can be used to Boolean expressions containing up to six variables, Karnaugh map it is a diagram made up of a number of squares, okay.

If the expression contains N variables, the map will have N squares, 2 to the power N squares, each square represents a minterm, okay, and 1s are written in the corresponding squares for the minterms present in the expression and 0s are written in those squares which correspond to the minterms not present in the expression okay, so those minterms which are present in the expression, in the corresponding boxes.

Okay, we shall right 1 and the minterm which are not present in the expression in those boxes, we shall in those squares, we shall put 0. Okay, now once the map is filled with 0s and 1s the canonical sum of products expression. Okay, for the output is obtained by grouping together those squares that contain 1, okay, so we shall then group those squares that contain 1 to get the desired output.

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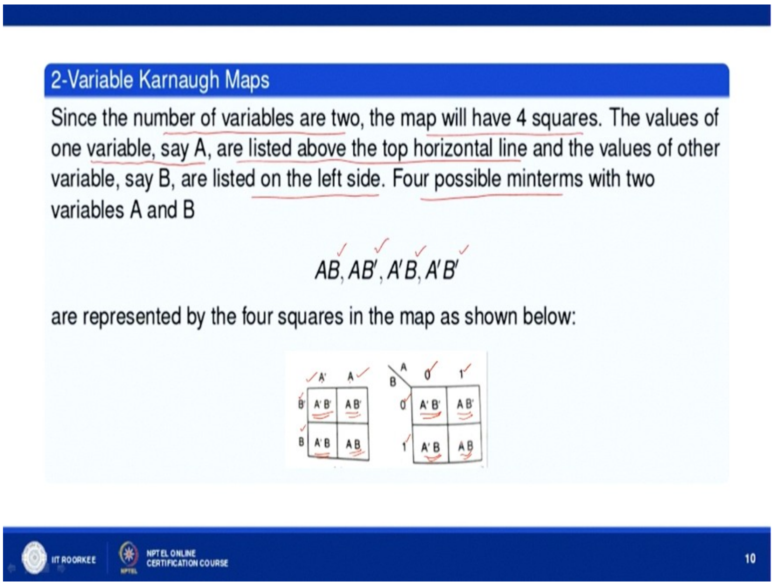

Now, let us consider two variable case for Karnaugh maps. So, if you consider a Boolean expression having with two variables then the number of, since the number of variables are two, the map will have, Karnaugh map we have 2 the power 2 that is 4 squares. The values of one variable okay, the values of one variable say A, are listed above the top horizontal line like here okay, the values of A are listed above the top horizontal line, this is top horizontal line. Okay, and the values of the other variables say B are listed on the left side.

So here they are listed, the 2 values of B, B and B'okay, four possible minterms okay, with variables A and B, what are the minterms? There are 4 minterms, AB, AB', $A'B$, $A'B'$ okay, so those 4 minterms are than represented by four squares, so you can see A', B' okay, this is $A'B'$, then AB', this is AB' and then $A'B$ okay, this is $A'B$ and then AB, this is AB.

Now if you use the value of $A=1$ then value of A dash will be 0, value of B if you take as 1 value of B'will be 0, so we can use the alternate Karnaugh map, this one also to find the desired output, here instead of A and B and A', B' we use their values. Okay, so A has value 1, A' has value 0, B has value 1, B' has value 0. Okay, then 0, 0, this isA', B' okay, A' has value 0, B' has value 0, so A', B' 0, 0 means A', B' 1, 0, 1 0 means AB' okay and then 0, 1 means, 0,1 means $A'B$, 1, 1 means AB okay, so this also can be use.

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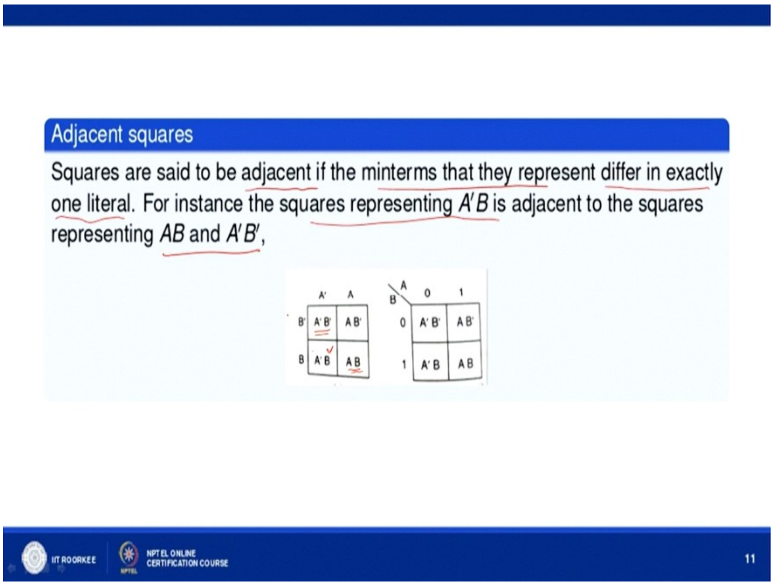

Now, let us define adjacent squares, which squares are called adjacent, okay, so squares are said to be adjacent if the minterms that they represent define in exactly one literal. Okay, two squares will be called adjacent, if the minterms that they represent. Okay, differ in exactly one literal. For instance, the squares representing $A^{'}B$ okay, you can see here $A^{'}B$ okay, representing $A^{'}B$ it is adjacent to this one AB because $AB^{'}$ differs with AB only in one literal, okay, and here $A^{'},B^{'}$ and $A^{'}B$ differs with $A^{'}B^{'}$ only in one literal that is B okay. So this square $A^{'}B$ this square is called adjacent to the square representing AB and the square representing $A^{'}B^{'}$, okay, so we can easily see which square is adjacent to which one using the definition of the adjacent squares, okay.

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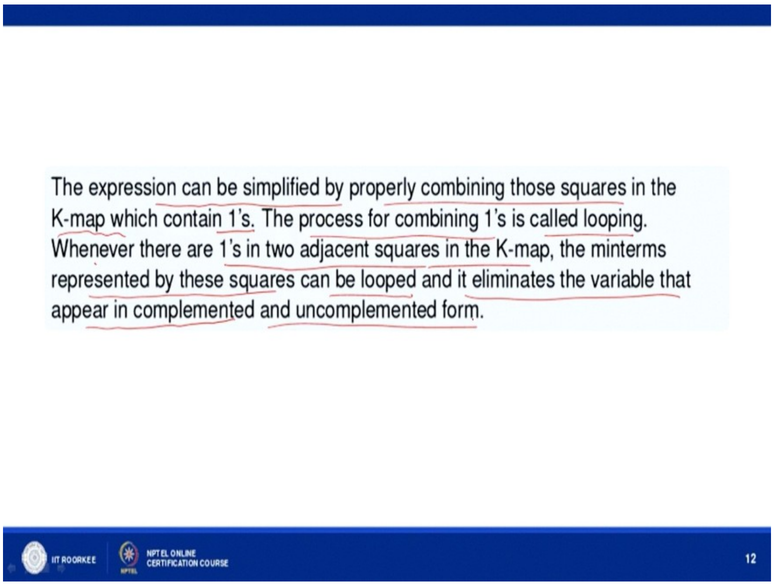

Now, let us go to how to simplify the Boolean expression using Karnaugh maps, so the expression can be simplified by properly combining those squares in the K-map which contain 1's. Okay, the process for combining, how will we combine? The process of combining 1's is called looping okay, wherever there are 1's in two adjacent squares in the K-map, the minterms represented by these squares can be looped and it eliminates the variable that appear in complemented and uncomplemented form.

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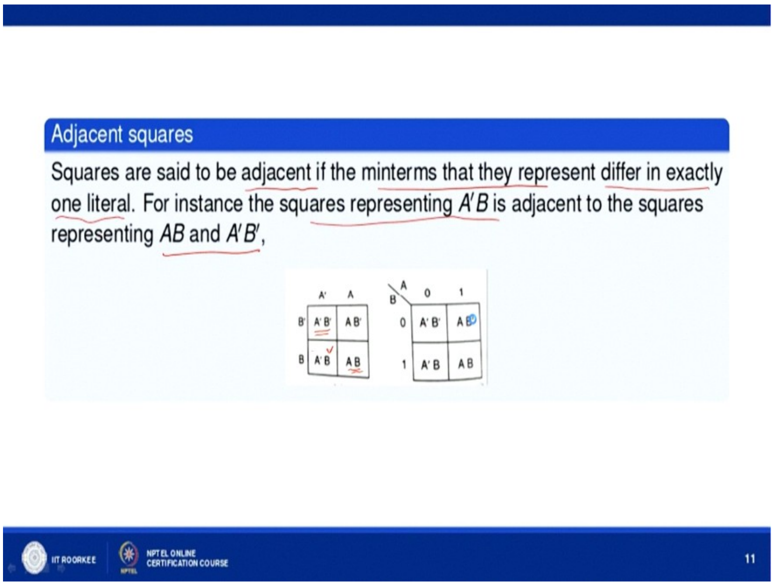

Say for example here. Suppose this value is 1 and this value 1, we will consider loop about that and then we will draft that variable, which appears in complemented and uncomplemented form in the adjacent squares. S1o let us see how we do this.

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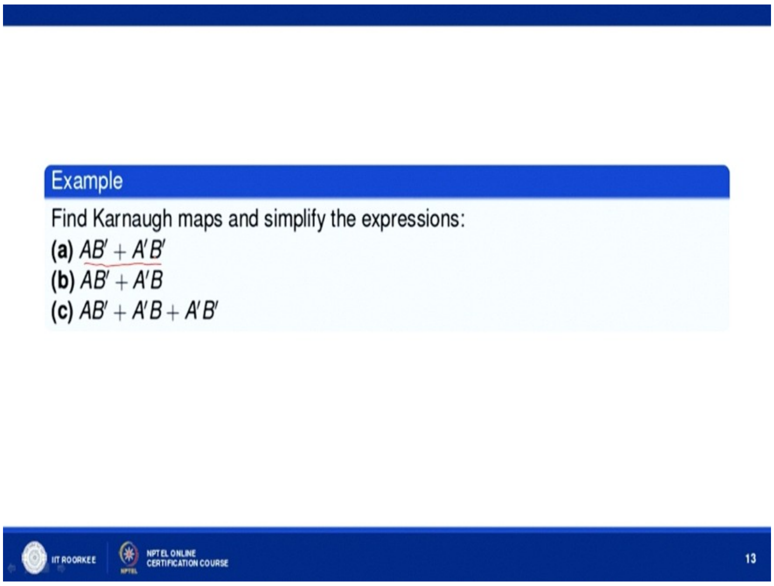

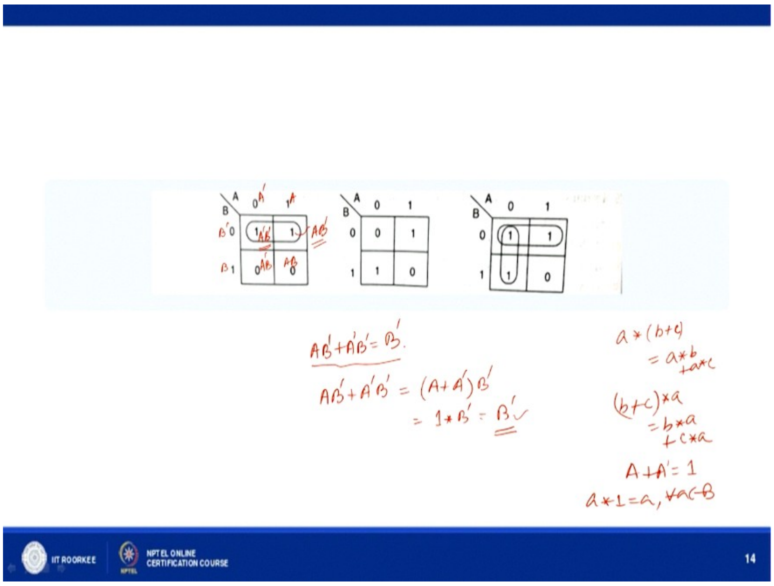

For example, find Karnaugh maps and simplify this expression, $AB' + A'B'$ okay, so let us see okay, if we have used this one. So AB' and $A'B'$ okay, we have this. Okay, so this is A', okay, this is A dash, this one is A, this one is B dash, this one is B, and so therefore, A dash B dash, this is $A'B'$. No, we have $A'B'$ okay, AB'means A will have value 0, B' will have value, this is AB' okay, this is AB' okay and this is $A'B'$ okay and this one is $A'B$, this one is AB.

Now, what is our first part $AB^{'}+A^{'}B^{'}$. $AB^{'}+A^{'}B^{'}$ okay. Now, these two squares representing $AB^{'}$ and $A^{'}B^{'}$ are adjacent because they differ only in one variable, that is B okay, that is A okay, so here we have A, here we have $A^{'}$ and $B^{'}$ is same, so they differ only in one variable that is A and therefore, they both have value 1, so we combine them, be it consider loop, this loop okay and then, $(AB)^{'}+A^{'}B^{'}$ will be equal to, now see, this A appears, this square it appears in the uncomplemented form, here it appears in the complemented form.

So, will consider $AB^{'}+A^{'}B^{'}=B^{'}$ okay by using this Karnaugh method. Karnaugh method says that eliminate the variable that appear in complemented in uncomplemented form after you have taken the loop okay. So here we are considered the loop okay and then we eliminate the variable, which appears in the complemented and uncomplemented form, so

$AB^{'}+A^{'}B^{'}=B^{'}$. Now you can also verify this by using the postulates and theorems that we have done studied so far.

So, $AB^{'}+A^{'}B^{'}=B^{'}$, if we use the postulates and theorems, we can write it as $A+A^{'}B^{'}$ if we use the distributive formula, $A*(B+C)=A*B+A*C$. Okay, if we use this formula and the commutative with formula of commutative law also. Okay, because we have here the situation, by using commutative law we can write it as $B+C*A, B+C*A=B*A+C*A$, okay.

So using this formula $AB^{'}+A^{'}B^{'}$ can be written as $A+A^{'}B^{'}$ and $A+A^{'}, A+A^{'}=1$, we know okay, so $1.B^{'}$ and we know that $A*1=A, \forall A \in B$, so $B^{'}*1=B^{'}$ okay, so by using postulate theorems $AB^{'}+A^{'}B^{'}$ turns out to be $B^{'}$ and by using Karnaugh map also $AB^{'}+A^{'}B^{'}$ comes out to be $B^{'}$.

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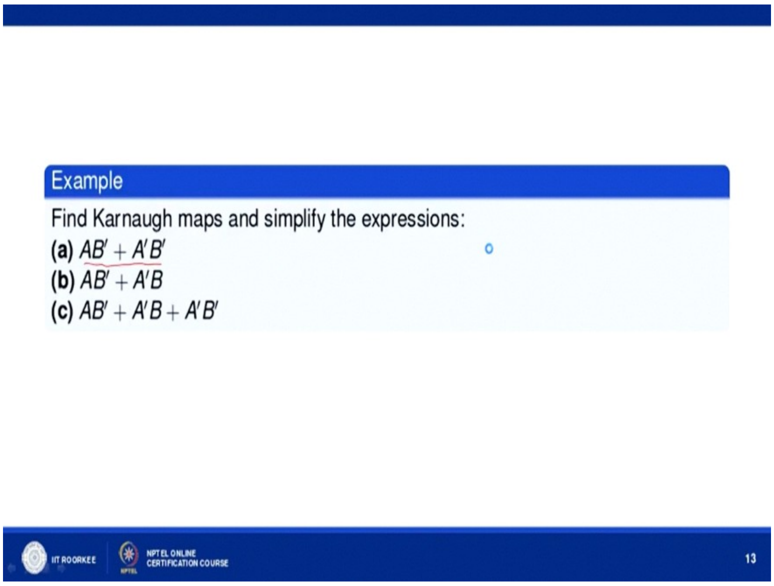

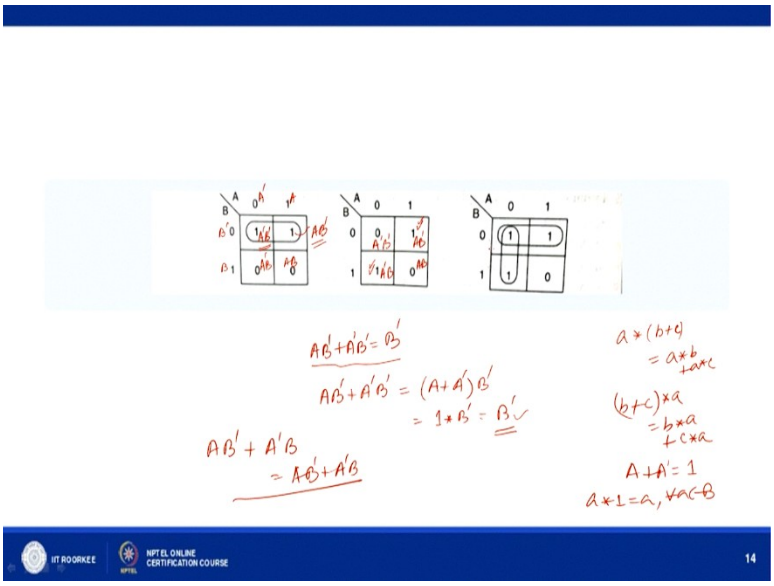

Now, let us go to the second part is $AB'+A'B$ okay, so $AB'+A'B'$ okay, now this is $A'B'$, this is $A'B$, we have $A'B$ here and we have AB here. Okay, so which squares represent $A'B$ and AB', is this, okay, and AB' is this. Now, so these two are not, these two squares are not adjacent okay, because that of the fact that AB' and $A'B$ okay, they do not differ in just one literal okay, in exactly one literal, so they are not adjacent squares and therefore we cannot make looping okay, so $AB'+A'B=AB'+A'B$ okay.

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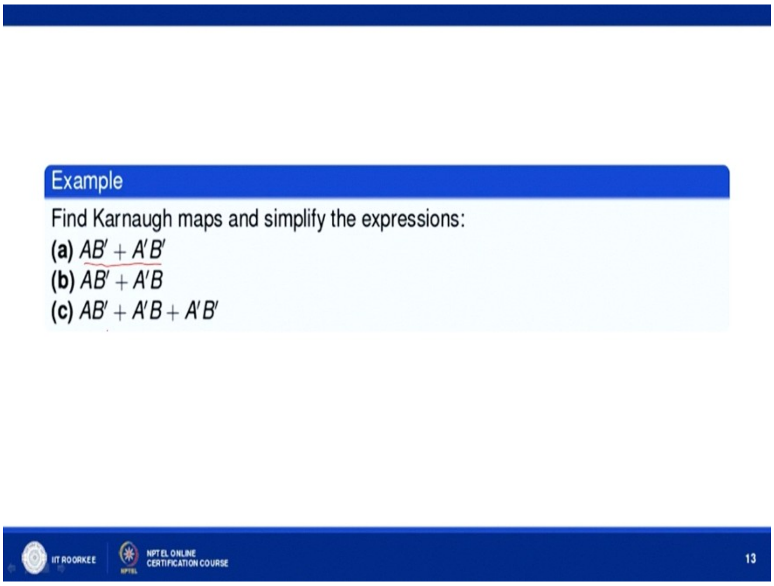

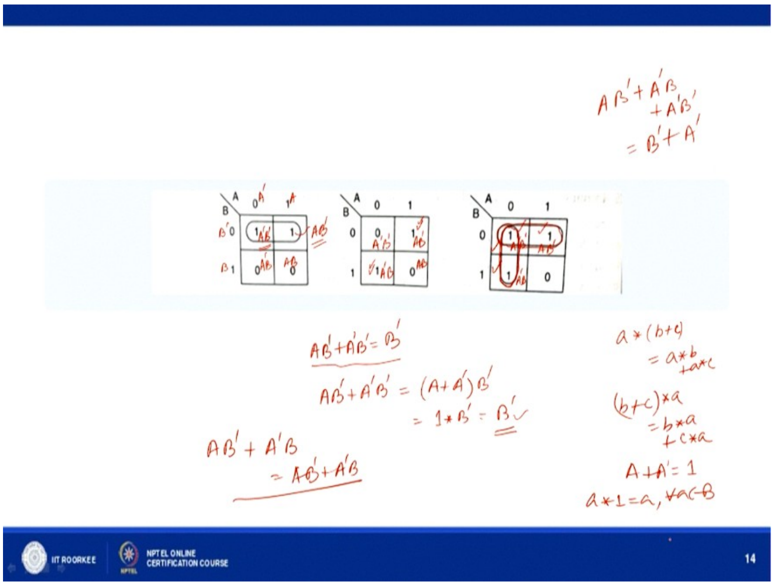

Now, let us consider the third part which is $AB^{'}+A^{'}B+A^{'}B^{'}$. So $AB^{'}+A^{'}B+A^{'}B^{'}$ okay, now let us see this is $A^{'}B^{'}$, this is square represents minterm $A^{'}B^{'}$, this is represents $AB^{'}$ and this represents $A^{'}B$ okay. So here $A^{'}B^{'}$ and $A^{'}B$, this square and this square, they differ in exactly one literal okay, that is B okay, so they are adjacent squares and we can have looped this 1 and 1 okay, so we have this looping okay and this looping then eliminates the variable which appears in, eliminate the variable which appears in complemented as well as uncomplemented form and which is B, here it appears as B, here it appears as $B^{'}$.

So, $A^{'}B+A^{'}B^{'}$ that will be equal to $A^{'}$ okay. Now let us consider this square, this square represents $A^{'}B^{'}$ and this represents $AB^{'}$ then here are we see that $B^{'}$ is same in the two minterms that they are representing, A is occurring here in the uncomplemented form, here it is occurring in the complemented form. So $A^{'}B^{'}$, we have considered this loop, $A^{'}B^{'}+AB^{'}, A^{'}B^{'}+AB^{'}$

when you find it will give you $B^{'}$ because A occurs in the complemented and uncomplemented form.

So, $AB^{'}+A^{'}B+A^{'}B^{'}$ is then the sum of this $B^{'}$ okay, which is the result of looping, this looping, it gives us $B^{'}$ okay and at the result of this looping, as the result of this looping we get $A^{'}$, so we get the output as $B^{'}+A^{'}$ or we can say $A^{'}+B^{'}$ in the third part.

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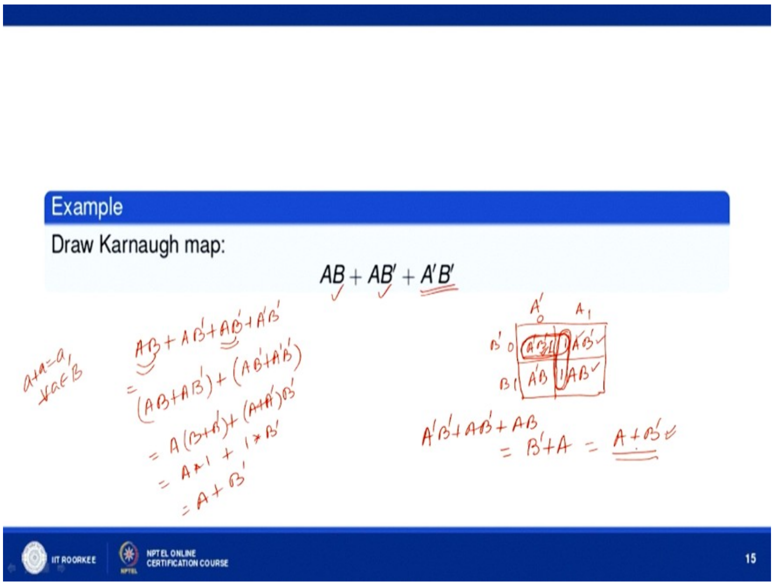

Now, let us draw Karnaugh map for $AB+AB'+A'B'$, so we have seen that this is A' okay, this let us say is A, this is B' this is B okay, so the value here is 0, here the value is 1, here the value is 0, here the value is 1 okay, then $A'B'$, this box, this square represents the minterm $A'B'$, this represents the minterm AB' and it represents the minterm $A'B$ and this square represents the minterm ABB, AB okay, now we see $A'B'\ AB+AB'$.

Let, us look at this square and this square okay, the two squares are adjacent because they differ in just one literal that is B okay, so this square and this square are adjacent and this is 1, 1, this is 1 okay, so we considered this loop because we give the value 1 to the minterm that is occurring here in the given Boolean expression, so AB' is occurring here, AB is occurring here, so there squares will be represented by the 1, they will have value 1 and then we have $A'B'$, so this is $A'B'$, okay, this has a value 1 and then $A'B'$ and AB', this square which represents $A'B'$and this square which represents AB' they are also adjacent because they differ in one literal that is A okay.

So we loop 1 here, 1 here. Okay. Now, because of this looping okay, if you consider this loop first, this loop first, then as the result of this looping the value will be B' because A and A' are occuring here, so A is occurring in the complemented form, as well as uncomplemented form, so as a result of this looping $A'B'+AB'=B'$ okay $A'B'+AB'=B'$, and as a result of this looping okay, as a result of this looping B is occurring in the complemented and uncomplemented form, so the result will be A okay, so $A'B'+AB'+AB$ will be equal to $B'+A$

okay or you can say $A+B^{'}$ so this is how we can use Karnaugh map to simplify this expression, this can be obtained, this output well can be obtained by using postulates also, a postulate and theorem, what we can do is.

Okay, now let us get this output $A+B^{'}$ by using postulates and theorems, so $AB+AB^{'}+A^{'}B^{'}$ I can also write as $AB+AB^{'}+AB^{'}+A^{'}B^{'}$ because

$A+A=A, \forall A \in B$ okay. Now what we can do? We can combine this and this. Okay, so

$AB+AB^{'}$ we can combine and we can combine $AB^{'}+A^{'}B^{'}$, then this equal to, now using distributive law this is $A*(B+B^{'}), A*(B+B^{'})$ and here we get $A+A^{'}*B^{'}=1$, so $A.1+1.B^{'}$ and this is equal to $A+B^{'}$, so that is how we can arrive at the output given by the Karnaugh map, that is all in this lecture. Thank you very much for your attention.