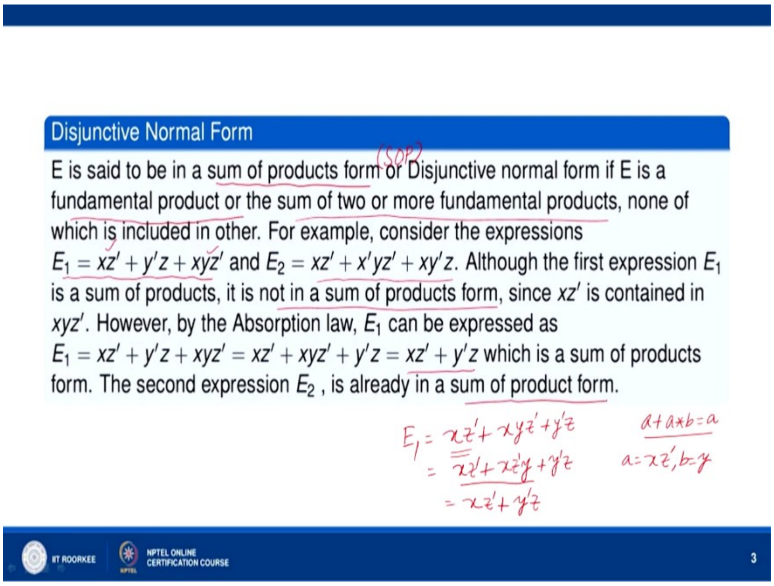
Higher Engineering Mathematics
Professor P. N. Agrawal
Department of Mathematics
Indian Institute of Technology Rookee
Lecture No 22
Boolean Algebra - IV

Hello friends welcome to my lecture on Boolean Algebra. A product term in which all the variables appear exactly once either complemented or uncomplemented is called a mean term. A mean term represents exactly one combination of the binary variables in a truth table it has a value one for that combination and 0 for the others.

(Refer Slide Time: 00:59)



Let us now define disjunctive and normal form or we also call it as sum of products forms. E is said to be in a sum of products from or disjunctive normal form if E is a fundamental product or the sum of two or more fundamental products, none of which is included in the other. For example consider the expressions $E_1 = x z' + y' z + xy z'$ and $E_1 = x z' + x' y z' + x y' z$.

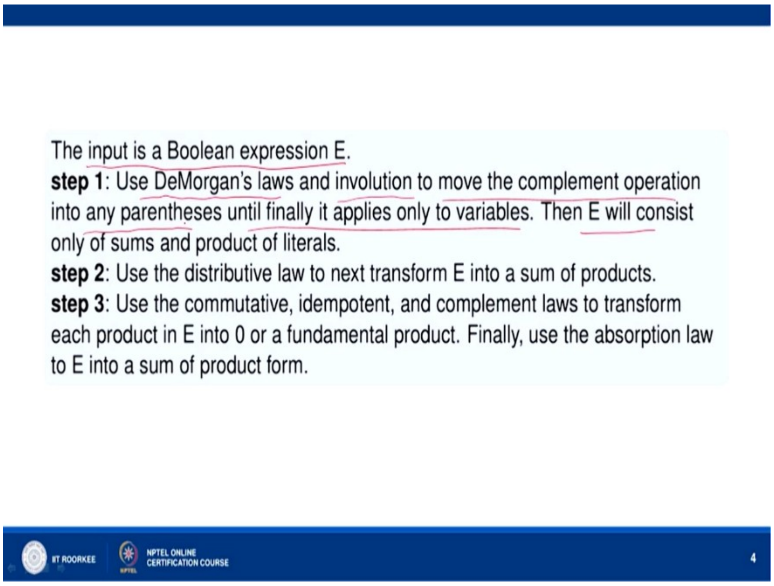
Now here in the first expression, the first and expression is the sum of products but it is not in sum of products forms. Why? Because $x z'$ is contained in $xy z'$ terms okay. Since $x z'$ is contained $x z'$ is contained in $xy z'$ therefore it is not in a sum of products forms. Now but we can apply the absorption law okay $E_1 = x z'$ plus we can write it as $xy z' + y' z$ okay by using a associative laws. Now let us use absorption law, absorption law is $a + (a*b)$ okay $a + (a*b) = a$, so here $x z'$ and this is $x yz'$ I can also consider it as $x z' + x z' y$, okay, so this is $x z' y + y' z$.

Now, let us apply absorption law $a+(a*b)=a$ take a equal to xz' and $b=y$, then

$xz'+xz'y=xz'$ and we get $xz'+y'z$ okay. So, using absorption law E_1 can be expressed as $xz'+y'z$ which is a $+$¿ of $\times$ from okay. Now the 2nd expression E_2,

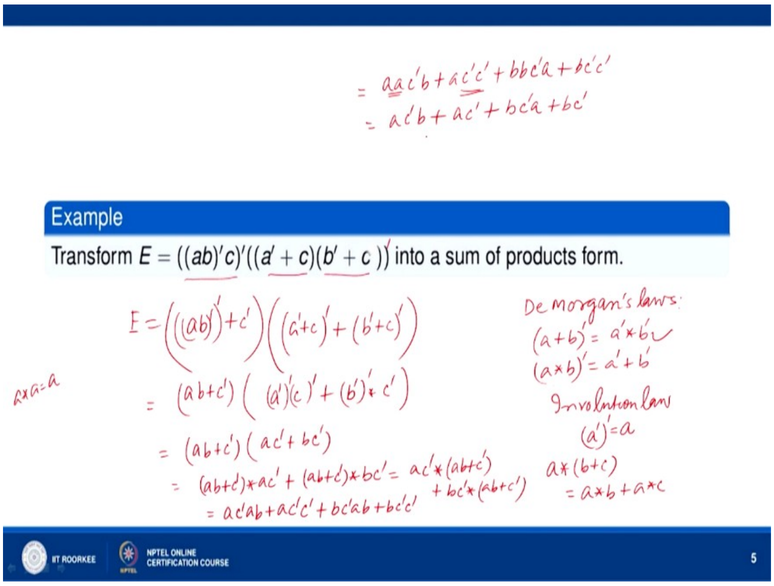
$E_1=xz'+x'yz'+xy'z$okay so that is in a sum of product form, some of product form because xz' is not included in $x'yz'$ or in $xy'z$ okay, so E_2 is a sum of two or more fundamental products and therefore none of which is included in other therefore it is disjunctive normal form or we also call it as sum of products form. Sum of product form is denoted by a SOP okay, so when E is a sum of product form we also call it as E is in SOP form.

(Refer Slide Time: 4:20)



Now, let us discuss how to bring given expression into SOP form okay, so let us say the input is a Boolean expression E then what we do is we use DeMorgan's laws and Involution law to move the complement operation into any parentheses okay until finally it applies only to variables. Then E will consist of only sums and products of literals. Use then the distributive law to next transform E into a sum of products and use the commutative, idempotent and complement laws to transform each product in E into 0 or a fundamental product until finally use the absorption law to E into a sum of product forms. So in order to convert a given Boolean expression E to SOP okay we have to follow these three steps okay.

(Refer Slide Time: 5:24)



Let, us say for example consider this problem $E=((ab)'c)'((a'+c)(b'+c))'$ okay, so we will use DeMorgan's laws and they are $(a+b)'=a'*b'$ and $(a*b)'=a'+b'$. So here we have $E=((ab)'c)'$ that means we will use the formula $(a*b)'$ okay. So a is $(ab)'$, so $(ab)'$ okay plus c' okay, this is first expression okay then we have this is $((a'+c)(b'+c))'$ here okay in the problem.

So, we have now again this is a, this is b and $(ab)'$, so we have $(a)'$, that is

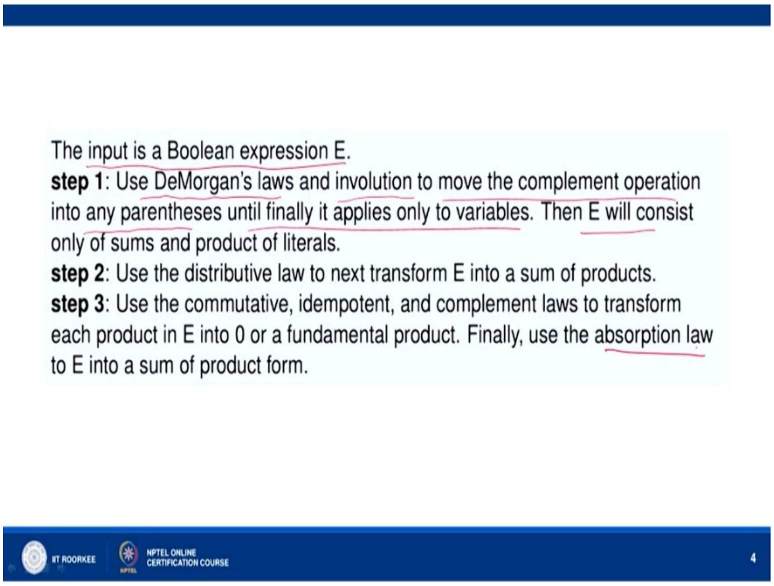
$(a'+c')+(b'+c)'$, let us recall Involution law which is that $(a')'=a$ and should have $((ab)')'+c'=ab+c'$ and here we have… Now we use again DeMorgan's laws this one $(a+b)'=a'*b'$, so we have here $(a')'c$, this is c not c', okay so we have $(a')'c'$ okay by this DeMorgan's laws first one and then here we have $(b')'c'$ okay so what do we get

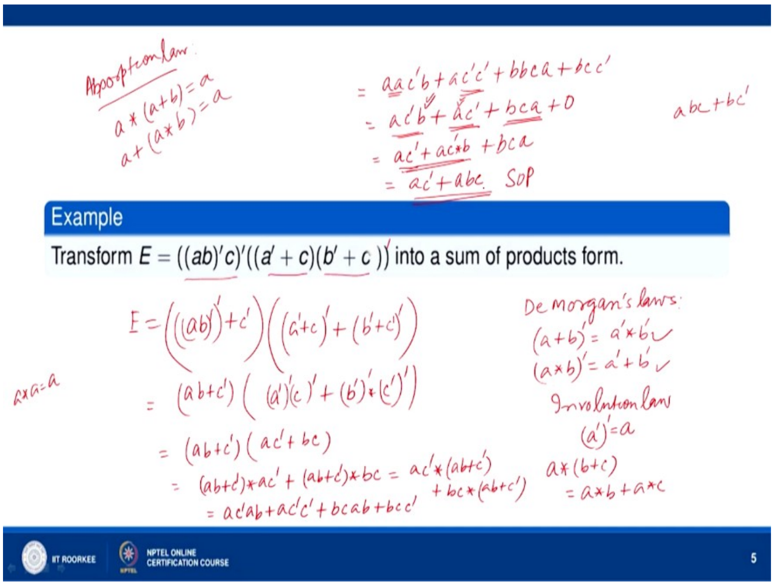
$(ab+c')$ and here $(a')'=a$, so $ac'+bc'$ okay.

Now, let us use the distributive law, so $a*(b+c)=a*b+a*c$ okay. So let us say this is a and this is $a+c$ so we get $(ab+c')*ac'+(ab+c')*bc'$ and what we have then again by commutative law we can write $(ab+c')*ac'$ as $(ab+c')*ac'$ and use this formula, so ab we have we can write it as $ac'*(ab+c')$ and similarly, here $bc'*(ab+c')$. Now, then $a*(b+c)=a*b+a*c$ so $ac'ab$ okay then $ac'c'$ then we have $bc'ab$ and we have $bc'c'$ okay.

Now, let us recall that $a*a=a$ okay, so $ac'ab$ okay $ac'ab=aac'b$ and here we have $ac'c'$ okay. Here we have, we can write it as $bbc'a$ using commutative (())(10:03) law and then $bc'c'$. So $c'c'=c'$ okay and $aa=a$ so $ac'b+ac'$ okay plus $bb=b, bc'a+bc'$ okay and lastly we use absorption law.

(Refer Slide Time: 10:34)





So let us use absorption law okay, so let us use absorption law, so we will use absorption law okay absorption law says that $a*(a+b)=a$ okay. Now here what do we have, R its dual is $a+(a*b)=a$ okay that is the dual statement, so here what we have let us notice this one and this one, okay, what we notice is, we can write it as we have $ac'+ac'*b$, so let us take this one okay these two.

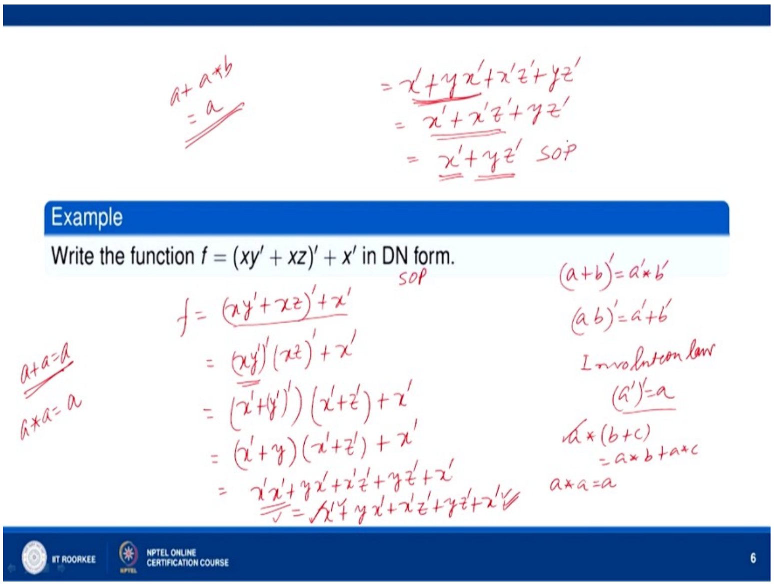
This one and this one okay so $ac'+ac'$ we have here and here we have ab' okay plus $bc'a$ okay, so $bc'a$ is $bc'*a$ okay, so this is $a+(a*b)$ type okay $a+(a*b)$, so $ac'+ac'*b$ will be ac' and $bc'+bc'=bc'$ and so $E=ac'+bc'$ that is in the form of sum of products, SOP. We are considering $(ab)'c'$ so that is by using this low $(ab)'+c'$ and $(a'+c)'(b+c)$ so when we use this law we have $(a'+c)'+(b'+c)'$.

Now, this is $(a')'(c)'+(b')'(c')'$ that this $(c')'$ okay. $(c')'$and that will give us $ac'+bc$ okay, so that will be bc not bc' okay $ac'+bc$. Now what we have here, so $(ab+c')*bc$okay and this will give us okay so ac, $ac\,ab\,ac'ab$ and then $ac'c'$ then $abbc$ okay so this is $bcab$ okay, $bcab$ and then we have… Okay so this is what we have here, so this is to be made correct $bcab$ okay $aac'b+ac'c'+cab$ okay we can write like that and then bc' this is bca okay, $bca$and we have here $ac'+(ac')*b$ okay this one $ac'b+ac'$ we have and then we have $abc+bc'$, so $bc'+abc$. $bc'+bca$

Now, $ac'+ac'*b$ by using absorption law becomes ac' okay and what we have here, we have abc this is $abc+bc'$ okay, so we have to use absorption law okay. So $abc+bc$, so it will be cc' okay $bbc'=0$, this is 0 okay and then what we will get here,

so $ac'+ac'*b+bca=0$ so this is not there, so we get bca, so now what do you get? We have $ac'+bc'*b$ is … so here we use the absorption law and we get ac' and here we get abc Okay so this is the SOP sum of products.

(Refer Slide Time: 16:18)



Okay, now, what we do is, let us write the function $f=(x y'+yz)'$, $(x y'+xz+x')'$ in destructive normal form, disjunctive normal form or you can see SOP form okay, so $f=(x y'+xz)'+x'$ okay and x' we have, so we can write it as, we can use the formula

$(a+b)'=a'*b'$ okay, so what it will be, this will be ¿¿ okay and then now you use $(a*b)'=a'+b'$ okay, so we have $x'+(y')'$ okay for this and then we have $(xz)'$ should we have $(x'+z')+x'$ okay.

Now, what we have, but let us use the formula Involution law, which says $(a')'=a$, so we have $x'+y$ here and then we have $(x'+z')+x'$ okay. Now we have to use Distributive law okay so $a*(b+c)=a*b+a*c$ okay so we have $x'x'+yx'+x'z'+yz'+x'$ okay and $a*a=a$, so $x'x'=x'$ then you have yx' then you have $x'z'$and then you have $yz'+x'$ okay.

Now, $a+a=a$, so $x'+x'$ okay that becomes x' and what we have is *B* is equal to

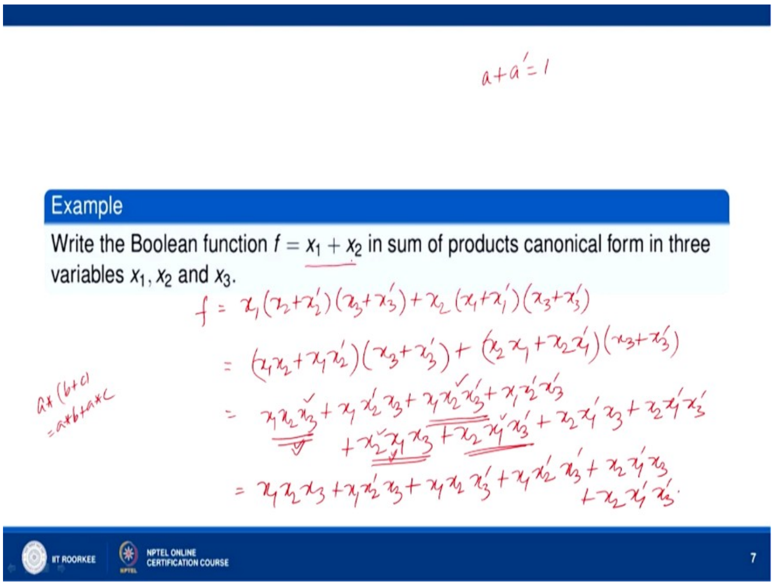
$x'+yx'+x'z'+yz'$ yes okay. So we have this $x'x'$ gives you x dash and thisx' this x' and this x' okay this x' and this x' we can combine and give x', so $+yx'$. Now this can be further simplified using absorption law $a+(a*b)$ equal to sorry $a+(a*b)=a$, so here we have $x'+(x'*y')$so by this formula this is equal to x' okay and here we have $(x$¿¿$'z'+yz')$¿okay, so $x'+x'z'$ will then be equal to x dash and we have x dash plus yz dash okay.

So, what we have done is we are given $f=(x\,y'+xz)'+x'$. Now, first we use the formula $(a+b)'=a'*b'$ and we get $(x\,y')'(xz)'+x'$, now we use the formula $(a*b)'=a'+b'$ so we get $x'+(y')'$ and then into $(x¿¿'+z')+x'¿$ is there $(y')'$ Involution law give y, so $(x'+y)(x'+z')+x'$ and then we use the Distributive law so

$a*(b+c)=a*b+a*c$that gives $x'\,x'$ then $y\,x'$ then $x'\,z'$ then $y\,z'$ then x'.

Now, $x'\,x'$ by this formula $a*a$ okay $a*a=a$ okay, so $x'\,x'$ gives x' okay, so this x' and this x' can be combined and we will get x' okay, so $x'+¿$ why $x'+y\,x'+x'\,z'+y\,z'+x'$. Now $x'+y\,x'$ can be combined and we get what? We get x dash because a plus a star b equal to a okay, so we get x' here and then we have $x'\,z'+x'$. $x'+x'\,z'$ can again be combined by this formula to get x' okay and then we have yz'. So this is the sum of products form SOP for this problem.

(Refer Slide Time: 22:37)



Now, let us write the Boolean function $f=x_1+x_2$in sum of products canonical form in three variables again, so we have to write it in three variables that means we have

$f=x_1(x_1+x_2')(x_3+x_3')+x_2(x_1+x_1')(x_3+x_3')$ Okay because we know that

$a+a'$, $a+a'=1$ okay, so $x_1$can be written as $x_1(x_2+x_2')(x_3+x_3')$ now what is this we can use $a*(b+c)=a*b+a*c$

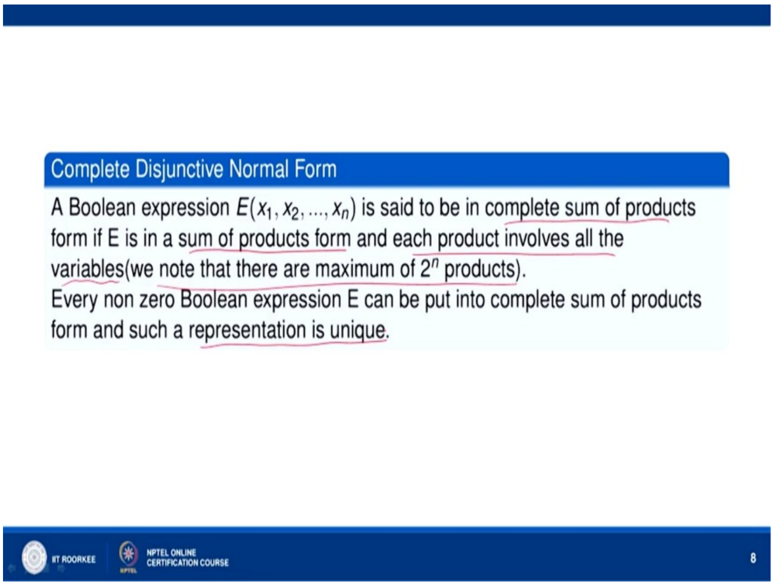
So this is $(x_1\,x_2+x_1\,x_2')$ okay $(x_1\,x_2+x_1\,x_2')$ and we have (x_3+x_3') and here we have

$(x_2 x_1 + x_2 x_1^{'})(x_3 + x_3^{'})$ okay, so how much is that now again we use this formula so we have $x_1 x_2 x_3 x_1 x_2 x_3^{'}$ and we get x_1 and $x_2 x_3^{'}$ and we get $x_2 x_1^{'} x_3^{'}$ here what we get $x_2 x_1 x_3$ and we get $x_2 x_1 x_3^{'}$ we get $x_2 x_1 x_3^{'}$ and we have $x_1 x_2^{'} x_3^{'}$. Now let us say x_1, x_2, x_3 can be combined and we get from a plus a equal to a we get $x_1 x_2 x_3$.

So these 2 can be combined to get x_1, x_2, x_3, now which others are same, let us see we have… yes so this one you see $x_1 x_2 x_3^{'}$ and here also we get $x_1 x_2 x_3^{'}$ so these two can be combined and we get $x_1 x_2 x_3^{'}$, so what we will have $x_1 x_2 x_3$ this and this together give you $x_1 x_2 x_3$ then $x_1 x_2^{'} x_3$ then so I have written this and then this one and this one can be combined and we get x_1 and $x_2 x_3^{'}$ and then we get $x_1 x_2^{'} x_3^{'}$ and then we get $x_2 x_1 x_3^{'}$ and we have $x_2 x_1^{'} x_3^{'}$.

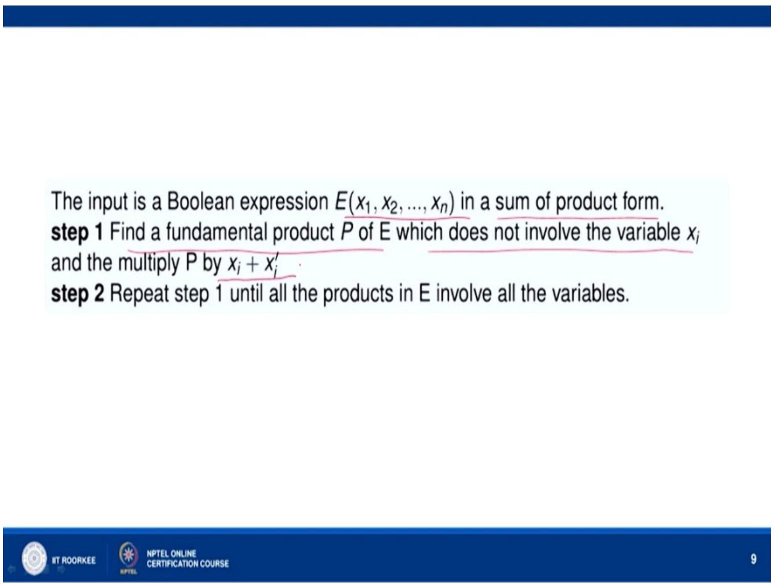
Okay so this is in the sum of products canonical form in three variables but there are two variables only we wanted in three variables so what we do, we multiply x_1 we take $x_1 x_2$ is $x_2^{'} x_3 + x_3^{'}$ and $x_2 .1 + x_1^{'} . x_1 + x_3^{'}$ because we know a plus $a + a^{'} = 1$ so this is the SOP form in three variables.

(Refer Slide Time: 26:50)



Now, a complete disjunctive normal form or we call it complete sum of products form. A Boolean expression E is called in a complete sum of products form if E is in a sum of products form and each product involves all the variables, we note that there are maximum of 2 the power n products. Every non zero Boolean expression E can be put into complete sum of products form and such a representation is always unique okay.

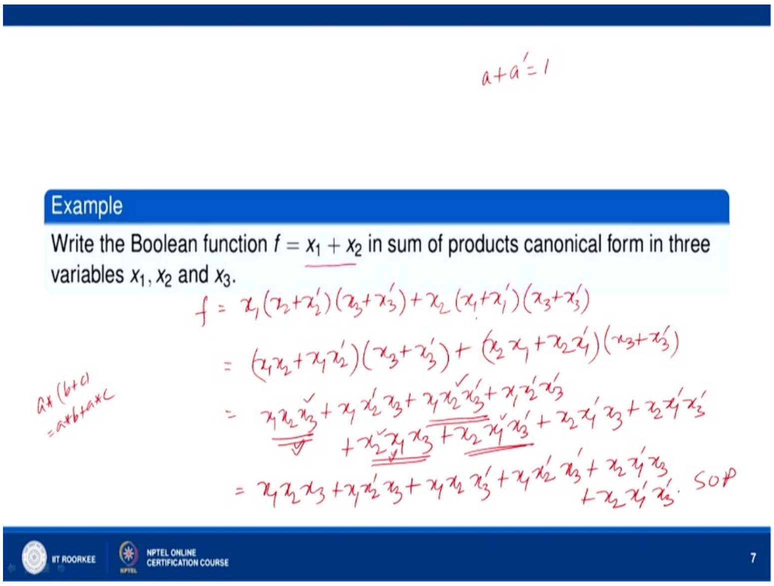
(Refer Slide Time: 27:25)



Now, let us look at this problem so what is the algorithm to bring a Boolean expression to complete sum of products form. The Boolean expression suppose it is $E = x_1, x_2, \ldots, x_n$ first we bring it to some of products form and then find a fundamental product P of E which does not involve the variables x_i Suppose there is a, you consider the term fundamental product P

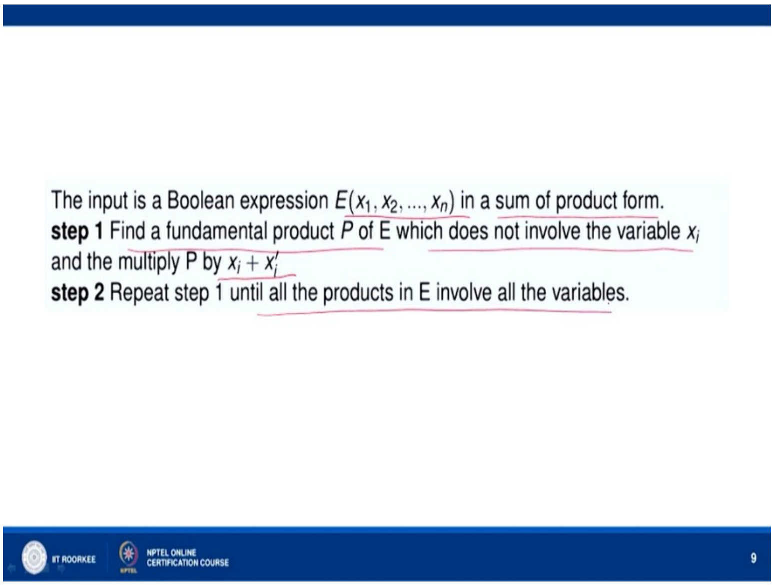
of E which does not involve the variable x_i and multiply P by $x_i + x_i^{'}$ like we did just now in the case of this problem okay.

(Refer Slide Time: 28:01)



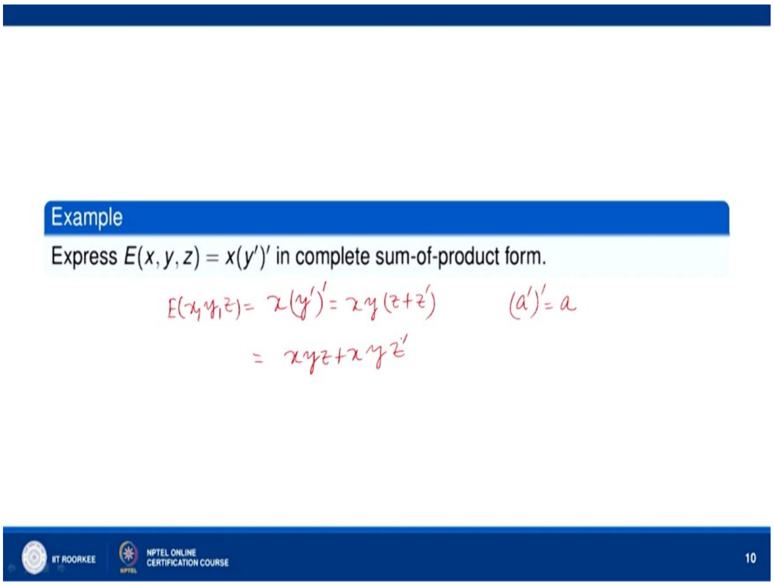
We multiplied x_1 by $x_2 + x_2^{'}$ and $x_3 + x_3^{'}$ because $x_2$and x_3 are not involved in the term x_1 and then considered $x_2 * (x_1 + x_1^{'}) * (x_3 + x_3^{'})$ because x_1 and x_3 were not involved in x_2 okay.

(Refer Slide Time: 28:26)



So, it take a fundamental product P of E which does not involve the variable x_i multiply P by $x_1 + x_1^{'}$. Repeat step 1 until all the products in E involve all the variables.

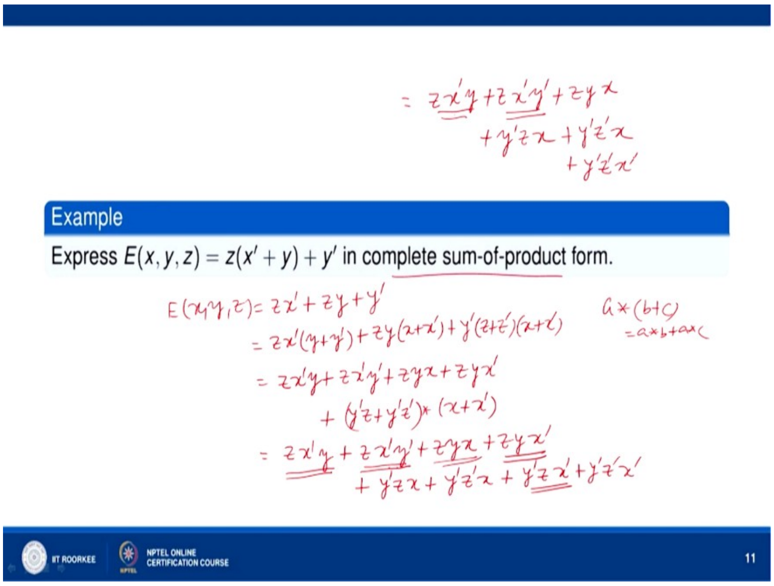
(Refer Slide Time: 28:41)



Now, let us see a problem on this suppose we have this one $E(x, y, z) = x(y')'$ okay so $E(x, y, z) = x(y')'$. Now $(a')' = a$ okay we know it is equal to a okay so by Involution law this is $x.y$. Now $(y')x.y$ x and y two variables are there z is not there so we multiply

$y(z+z')$ okay and we get this as $(xyz+xyz')$ okay, now every term involves all the variables xy and z, so it is complete sum of products form okay.

(Refer Slide Time: 29:31)



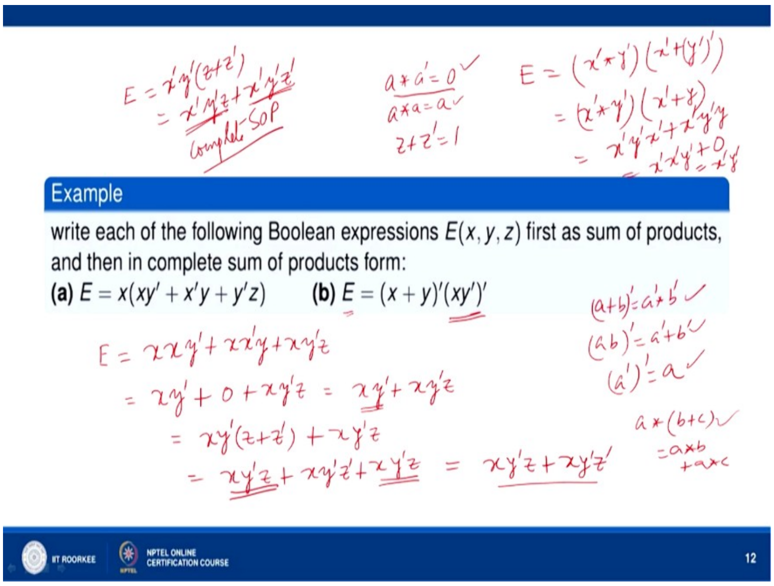
Now, we go to this problem say $E(x, y, z) = z(x'+y) + y'$ okay, so we have to bring it to complete sum of product form so we have $E(x, y, z) = z, a*(b+c) = a*b+a*c$ so we get

$zx'+zy+y'$ okay we have to bring it to complete sum of product form and see z and $x$are there y is not there so we write zx', $y+y'$ here we have zy so we will multiply it by $x+x'$ and then $y'(z+z')(x+x')$ because x and z are not there okay.

So, this is equal to $zx'y+zx'y'$ then we have zyx we are using distributive law and then we have zyx' okay and here what we have $y'*(z+z')$ so $(y'z+y'z')(x+x')$ so what we get $zx'y+zx'y'+zyx+zyx'$. Now again $a*(b+c)$ so we get ab that is $y'zx+y'z'x+y'zx'+y'z'x'$ okay. Let us see if there are similar terms so here we see $zy'x'$ and here we have $zy'x'$ okay $a+a=a$ okay.

So we can combine these two and yes so these two terms can be combined and we get this answer to be$zx'y$ okay and this plus this, $a+a=a$, so $zy'x'+zyx+zyx'$, okay this one zyx' and zyx' can be combined and we get. So we have already written, okay so we do not have to write again, so we can erase that and then we have yes so this one and this one can be combined and we get $zx'y$, okay this one and this one can be combined to get this okay then we have zyx which is here, then we have $y'zx$okay and then we have $y'z'x$ and then we have $y'z'x'$. So each term involves all the three variables xy and z okay this is incomplete sum of product form.

(Refer Slide Time: 33:38)



Now, let us write each of the following Boolean expression first as sum of products and then incomplete sum of products form, okay so $E=x.xy'$ so we have xxy' and then we have xxy' okay and then we have $xy'z$ okay. Now, we know that $a*a'=0$ okay so and $a*a=a$, so we get $xx=x$, so xy' we have plus $xx', x*x'=0$, so we get 0 here, plus $xy'z$okay. Now so this is equal to $a+0=a$, so we get $xy'+xy'z$.

Now, in this term second 2nd term x, y, z all three variables are there but in the first term only x and y are there xy' so we consider it to be equal to xy' and z is missing so $z+z'$ okay plus $xy'z$ and we get $xy'z$ by distributive law plus $xy'z'+xy'z$ okay. Now, this and this can be combined to get $xy'z$ is $a+a=a$ so $xy'z+xy'$ this is. So the E has been written in complete sum of products form. Now, let us take E equal to b part let us take so $(x+y)'$.

Okay we recall DeMorgan's law $(a+b)'=a'*b'$ okay, so we will get $(x'*y')$ okay and so $(a*b)'=a'+b'$ so we get here $x'+(y')'$ okay for this okay, so this now then be use the formula $(a')'$ and we have here $x'*y'=x'y'$okay. So now we can use the formula $a*(b+c)=a*b+a*c$, so this is x' into y', $x'*y'$is $x'y'$so $x'y'y'$ we get and then $x'y'$ we get, okay now $yy'=0$.

So, we get this as 0 and this $x'x'$ okay $a*a=a$ so this is $x'x'y'$ okay plus $x'y'=0$ so we get here $x'x'$ is x' so $x'y'$ okay. Now, but we have here E is a function of the variables xyz and after reducing we get $x'y'$ two variables z is missing here, so we write $E=x'y'(z+z')$ and we get $x'y'z+x'y'z'$ okay that is the complete SOP.

So, complete sum of products form okay we first apply DeMorgan's law then Involution law then Distribution law and then we apply these laws, $a'*a=0, a*a=a$ and since we are getting $x'y'$, okay z is missing their so we take $x'y'z+z'y'$ and $z+z'=1$ okay, so we get $x'y'z'+x'y'z'$. Now here you can see both the terms contain all the three variables x, y, z and therefore, it is in the complete sum of products form. So that is all in this lecture thank you very much for your attention.