Higher Engineering Mathematics
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Lecture – 02
Symbolic Representation of Statements – II

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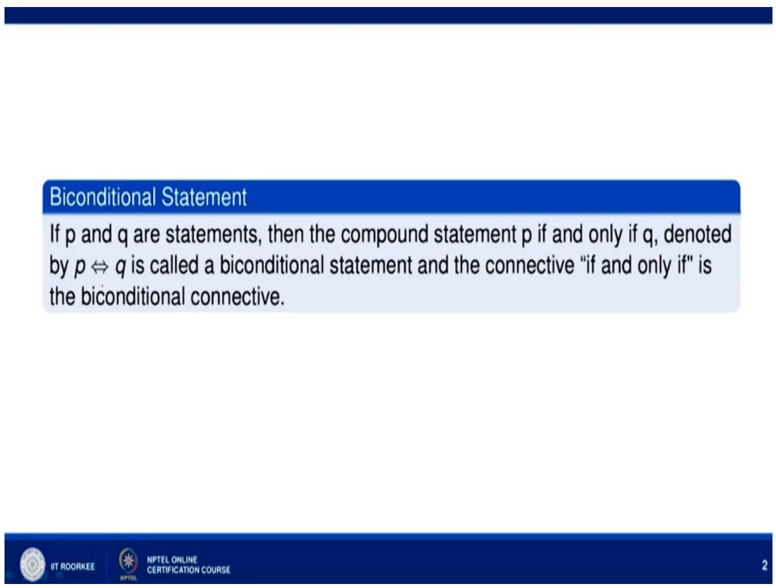

Hello friends welcome to my second lecture on Symbolic Representation of Statements. Let us define a bi-conditional statements if p and q are any two statements then the compound statements p if and only if q denoted by $p \Leftrightarrow q$ is called a bi-conditional statement and the connective "if and only if" is the bi-conditional connective. So, when we are given two statements p and q, $p \Leftrightarrow q$ is called a bi-conditional statements.

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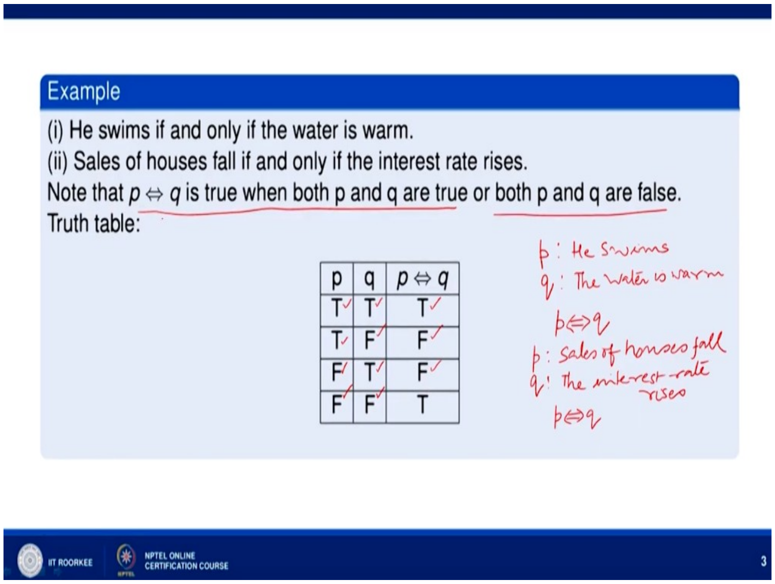

Now, let us consider an example, “He swims if and only if the water is warm,” so let us consider the statement he swims to be p, let us say p is he swims and q is the water is warm. Then he swims if and only if the water is warm can be symbolically represented as p⇔q similarly, “Sales of houses fall if and only if the interest rate rises. So, let us say let p : sales of houses fall and let q : the interest rate rises, then the sales examples sales of houses fall if and only if the interest rate rises can be symbolically represented as p⇔q.

Now, p⇔q is true to when both p and q are true or both p and q are false. So truth table for p ⇔q is if p is T, q is T then p⇔q is T, when p is true, q is false, the p is implies and implied by q, truth value is false, when p is false, q is true then p⇔q, hence truth value false and then when p is false q is false, the truth value of p⇔ q is true, so that is T. p ⇔q is true when both p and q or both of them are false.

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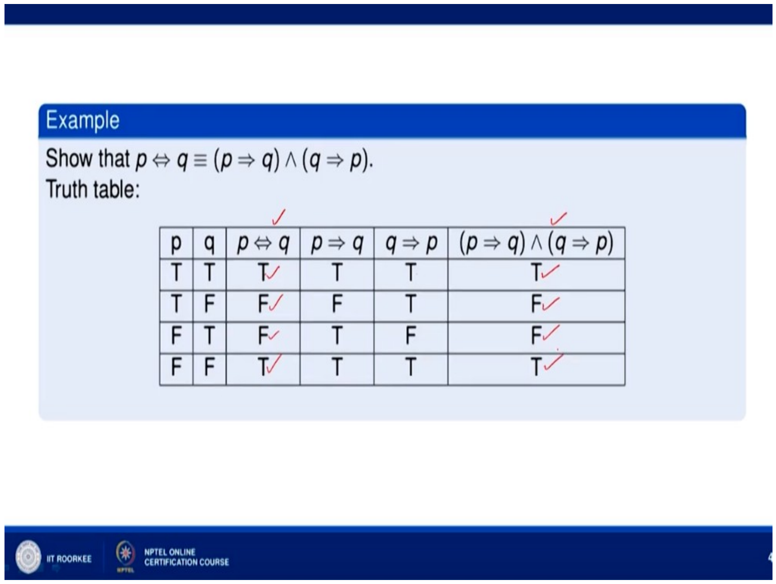

p	q	$p \Leftrightarrow q$	$p \Rightarrow q$	$q \Rightarrow p$	$(p \Rightarrow q) \wedge (q \Rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

Now, let us show that p⇔q≡ (p⇒q) ∧ (q⇒ p), so let us form the truth table p q has truth values TT TF FT FF then p⇔q the bi-conditional statement p⇔q has truth values for TTT for TFF FTF FFT just, now we have discussed that if p and q both are truth value is T then p ⇔q will have truth value T and its truth value will also be T in the case when p and q both have truth values F otherwise it is always F.

Now truth value of p⇒q we know it is only F when p is true q is false otherwise it is always true, so TT gives T, TF now you see p is T, q is F so TF gives F and then FT gives T, FF gives T similarly, we can write the truth value is of q implies p from here so they are TT FT. Now we from this two columns, p⇒q, q⇒p there truth values from there truth values we can find the truth is of p⇒q and q⇒p, so for TT we get T FT we get F TF we get F and TT gives T.

Now, you can see this column and this column we can compare, so here we have T, here also we have T, here we have F here we have F this is F this one is also F and here we have T and here also we have T, so the truth value is of the bi-conditional statement p ⇔q are same as the truth is of the statement (p⇒q) ∧ (q⇒ p).

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Example

Show that $p \Leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$, using the truth table.

Truth table:

p	q	$p \Leftrightarrow q$	$p \vee q$	$p \wedge q$	$(p \vee q) \Rightarrow (p \wedge q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	~~F~~ T	F	F
F	F	T	F	F	T

We can also show the above equivalence by using algebra of propositions.

Now, we can consider another example $p \Leftrightarrow q \equiv (p \vee q) \Rightarrow (p \wedge q)$ so this is equivalent to this one, $p \vee q \Rightarrow p \wedge q$, let us see the truth table p and q have truth values TT TF FT FF, then $p \Leftrightarrow q$, when p and q both are true it is T it is F for TF or FT and for FF it is T. Now $p \vee q$, so this is TT we get T here TF gives T here FT should give T not F, it should be T, p and q TT gives T, TF gives F, FT gives F, FF gives F, then $p \vee q \Rightarrow p \wedge q$.

So, implication means whenever $p \vee q$ is true and $p \wedge q$ is false we will get F otherwise it is always true, so $p \vee q$ is true and $p \wedge q$ is true, so implication gives T and when $p \vee q$ is true $p \wedge q$ is false we get F here, now this one is T and this one is false, so we get F here, so this should be F, this should be T, so $p \vee q$ is true and $p \wedge q$ is F so $p \vee q \Rightarrow p \wedge q$, so if it is true and this is false then the truth values is false and here we have $p \vee q$, F and q false so F F here.

So we get T here so this is not F this is T we can also show now we can see the truth values is of $p \Leftrightarrow q$ are same as the truth value is of this implication TT FF FF TT so they are logically equivalent. Now this equivalence can also be shown the equivalence of $p \Leftrightarrow q$ and $(p \vee q) \Rightarrow (p \wedge q)$ can also be proved using algebra of propositions.

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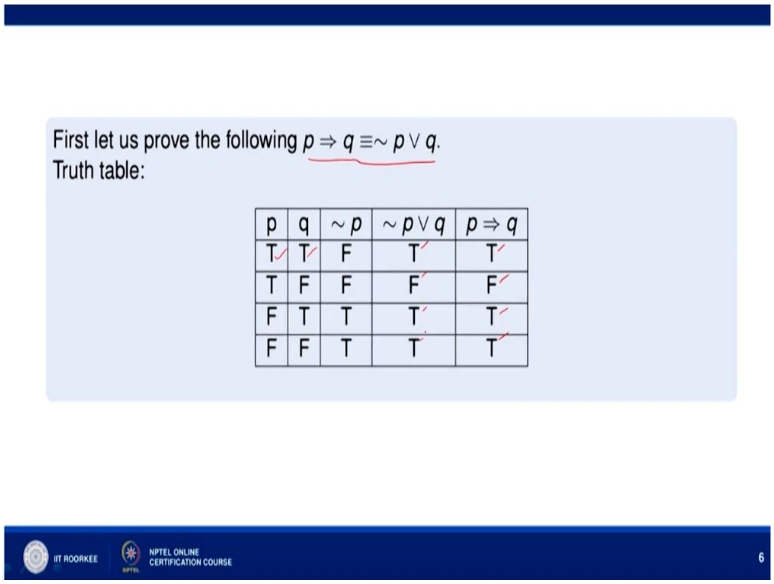

So for that let us first show that p⇒q≡ ~p∨q so these are the truth values of p and q TT TF FT FF ~p will be then have truth values F FT T, ~p∨q will have truth values as, FT, here so we will have T here, FF means F here, TT is T, TF is T, now p⇒q so TT will give T, TF will give F, FT will give T, FF will give T, so now you can see the p⇒q this are the truth values, T F TT and not p or q, TF TT they are same so they are logically equivalent, so p⇒q≡~p∨q we will use this.

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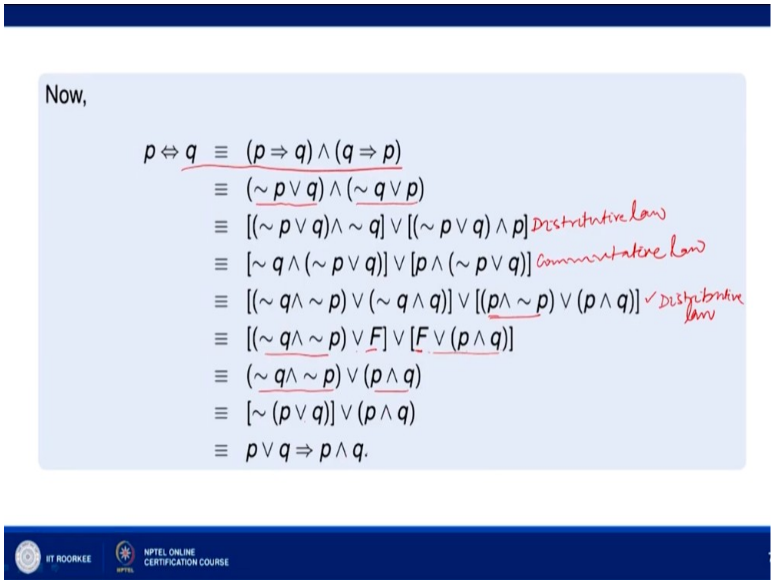

In order to prove the logically equivalence of $p \Leftrightarrow q$ and $p \vee q \Rightarrow p \wedge q$ so let us show that by using algebra propositions. This we have earlier shown, $p \Leftrightarrow q \equiv p \Rightarrow q \wedge q \Rightarrow p$ this we have early approved. Now $p \Rightarrow q$ we have just now shown, $p \Rightarrow q \equiv \sim p \vee q$, so we have $\sim p \vee q$ here and we have also $\sim q \vee p$, now let us take use distributive law.

So this is $\sim p \vee q$ be distribute over $\sim q \vee p$, so we have $\sim p \vee q$ and then we are distributing over this one, so $[(\sim p \vee q) \wedge \sim q] \vee [(\sim p \vee q) \wedge p]$ so this is obtained by using distributive law and then we have here, so we are using commutative law here, we are using commutative law here so we using commutative law so $\sim p \wedge (\sim p \vee q)$ we write here and here also we are commutative law, so $p \wedge (\sim p \vee q)$ so commutative law we are using here. Now after that what we have here, $\sim p \vee q$, so we are again using distributive law, so $(\sim q \wedge \sim p) \vee (\sim q \wedge q)$ and then here we are again using distributive law, so $(p \wedge \sim p) \vee (p \wedge q)$, so this is again obtained by using distributive law.

Now, $(\sim q \wedge \sim p)$ we have here $(\sim q \wedge q)$ is false this we have earlier in algebra proposition this we earlier have done in the first lecture, so $(\sim q \wedge q)$ is F and p and $\sim p$ both cannot be true so we have the truth value F and then here $\vee (p \wedge q)$ now $(\sim q \wedge \sim p)$, now we this is $(\sim q \wedge \sim p) \vee F$ the truth value is the truth value is $(\sim q \wedge \sim p)$ and here we have truth value of this $F \vee (p \wedge q)$ this is $(p \wedge q)$ this is again by algebra propositions $p \wedge q$ and then we have now $(\sim q \wedge \sim p)$ this compliment law it is $\sim (p \vee q) \vee (p \wedge q)$.

So we have here now we use this result we use this result this one $(\sim p \vee q) \equiv p \Rightarrow q$ so we have now you can see here this is $\sim(p \vee q) \vee (p \wedge q)$ so we are using and that this is $(p \vee q) \Rightarrow (p \wedge q)$ so this is how we prove the equivalence of $p \Leftrightarrow q$ and $(p \vee q) \Rightarrow (p \wedge q)$ by using algebra of propositions.

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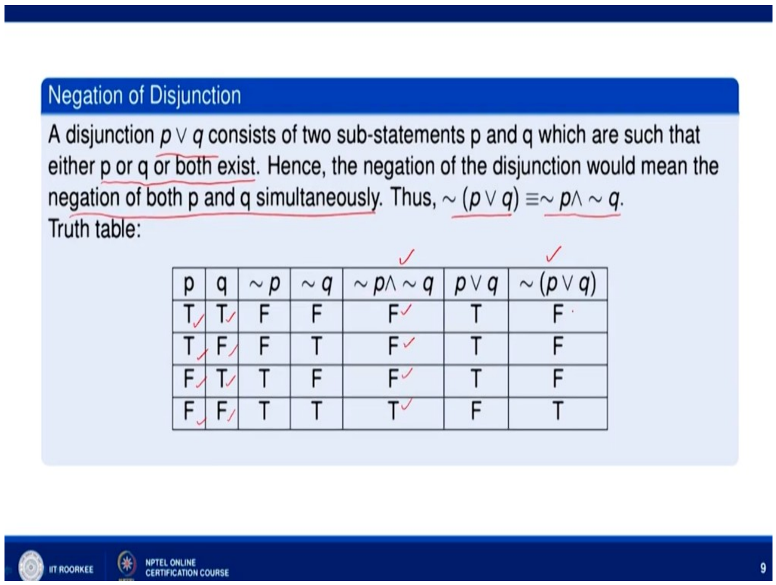

Now, negation of compound statements negation, let us consider negation of conjunction, conjunction p and q consist of two sub-statement p and q both of which exist simultaneously hence the negation of the conjunction would mean negation of at least one of the two sub-statements thus $\sim(p \wedge q)$ is $\sim p \vee \sim q$ this can proved by the truth table. So we can show the logical equivalence of $\sim(p \wedge q)$ and $\sim p \vee \sim q$ so truth values of p and q are given in is two columns, $\sim p$ has truth values of FF, TT and here they are FT FT then $\sim p$ and $\sim q$ we can write the truth values.

So, $\sim p \vee \sim q$, for FF we have F, FT is T, TF is T TT is T and then $p \wedge q$, so TT gives T, TF gives F FT gives F FF gives F. Now $\sim(p \wedge q)$ gives for T we get F for F we get T so this are F T T, now you can see this column and this column, their truth values are same for all possible cases FF TT TT TT, so $\sim(p \wedge q) \equiv \sim p \vee \sim q$, so at least one of the two statements must have negation.

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Negation of compound statements

Negation of conjunction: A conjunction $p \wedge q$ consists of the two sub-statements p and q, both of which exist simultaneously. Hence, the negation of the conjunction would mean the negation of at least one of the two sub-statements. Thus, $\sim(p \wedge q) \equiv \sim p \vee \sim q$. This can be proved by the truth table:

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$	$p \wedge q$	$\sim(p \wedge q)$
T	T	F	F	F	T	F
T	F	F	T	T	F	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T

Now, negation of disjunction, disjunction p or q consist of two sub-statements p and q which are such that either p or q or both exist, hence the negation of the disjunction would mean the negation of both p and q simultaneously, so $\sim(p \vee q) \equiv \sim p \wedge \sim q$, so p, q have these truth values, these truth values, now ~p then will have truth values F FT T, ~q will have truth value F T F T and $\sim p \wedge \sim q$ will have truth value for FFF FTF TFT TTT and then $p \vee q$ will give truth values TT give T TF gives T FT gives TFF gives F.

Now, we can see $\sim(p \vee q)$ is for T we have F here, so this is F this is also F, this is F and this is T, now you can see the truth values of this column and this column, so $\sim(p \vee q)$ have truth value is FF FT and $\sim p \wedge \sim q$ also have the same truth value, so they are logically equivalent, so $\sim(p \vee q) \equiv \sim p \wedge \sim q$.

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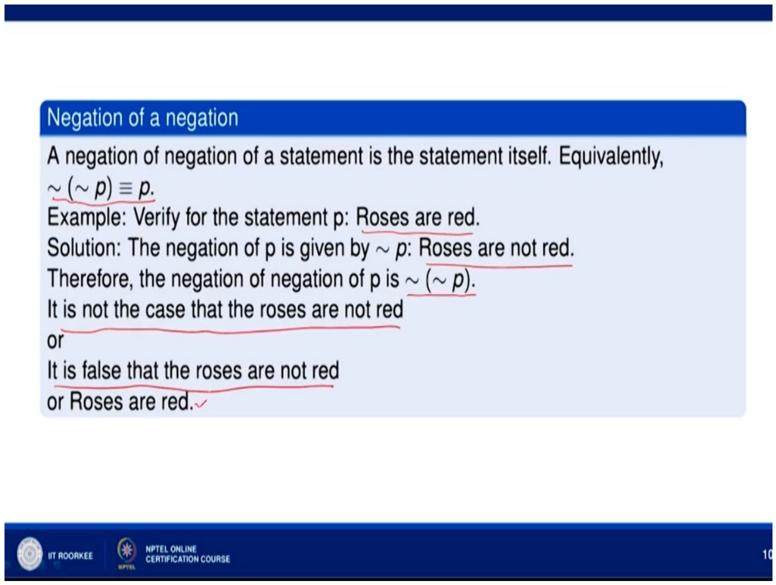

Now negation of a negation, a negation of negation of a statement is the statement itself equivalently ~(~p)≡ p, let us consider this statement p to be 'Roses are red' then ~p is given by 'roses are not red' negation of negation of p is ~(~p), so negation of 'roses are not red' describes it is not the case that the 'roses are red' it is not the case that the 'roses are not red' or it is false that the 'roses are not red' which is same as 'roses are red' ~(~p)≡ p.

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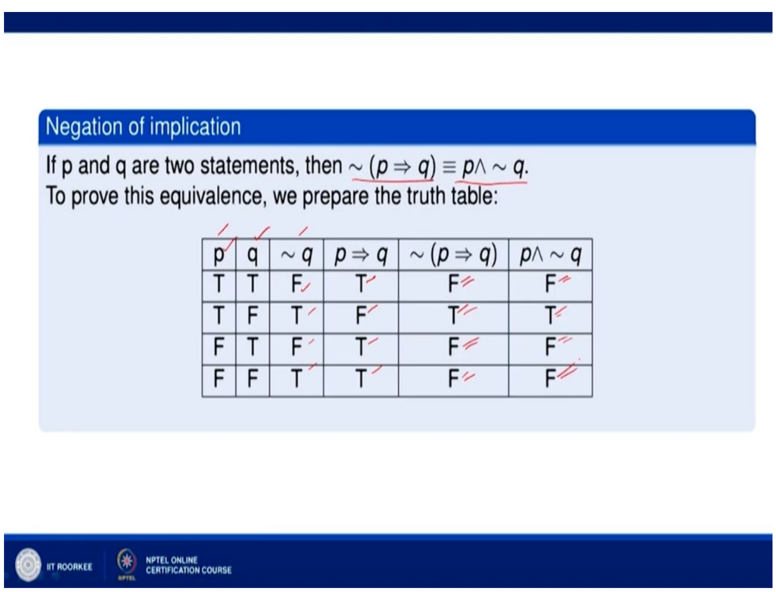

Now, negation of implication if p and q are two statements then negation of $\sim(p \Rightarrow q) \equiv p \wedge \sim q$, so to prove this equivalence we prepare the truth table, so these are the truth values of p and q, $\sim q$ have truth values FT FT, $p \Rightarrow q$ have truth values T F TT and $\sim(p \Rightarrow q)$ have truth values F T F F, then $p \wedge \sim q$ let us consider this p and this is $\sim q$, so $p \wedge \sim q$, so T and F is F, T and T is T, F and F is F, F and T is F. Now you see the T truth values of $p \wedge \sim q$ and $\sim(p \Rightarrow q)$, they are same FF TT FF FF,so $\sim(p \Rightarrow q) \equiv p \wedge \sim q$.

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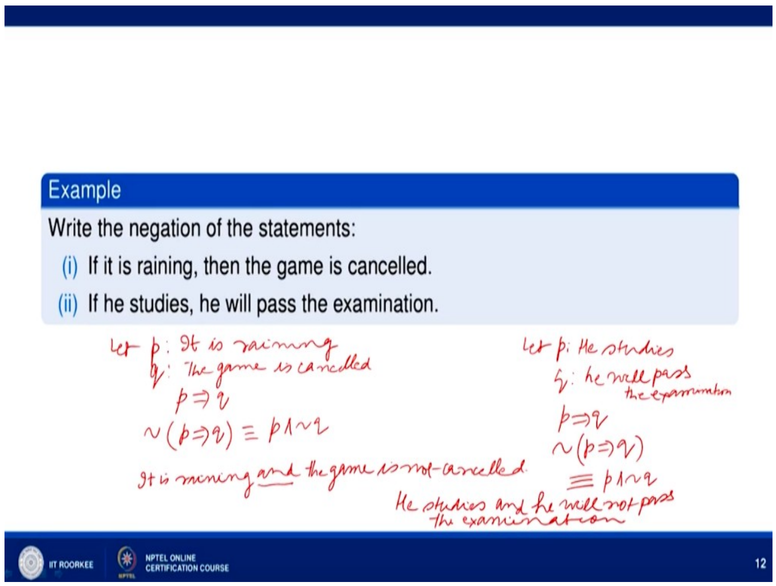

Now, let us write the negation of the statements 'if it is raining then the game is cancelled', so let us say let p: it is raining' and q: the game is cancelled then the given statement can be described symbolically as $p \Rightarrow q$, if p then q, now let us see $\sim(p \Rightarrow q)$ we have just now seen $\sim(p \Rightarrow q) \equiv p \wedge \sim q$, so $p \wedge \sim q$ means 'it is raining and the game is not cancelled', so p is 'it is raining' this symbol represents and correct logical connective this is $\wedge \sim q$, q is the 'game is cancelled' $\sim q$ is 'the game is not cancelled'.

So it is raining and the game is not cancelled, similarly, the negation of the second statement we can find, we can write p: he studies and q: he will passed the examination then the given statement can be written as $p \Rightarrow q$, so $\sim(p \Rightarrow q) \equiv p \wedge \sim q$, $p \wedge \sim q$ means 'he studies and he will not pass the examination' 'he will not pass the examination' so this is negation of the second statement.

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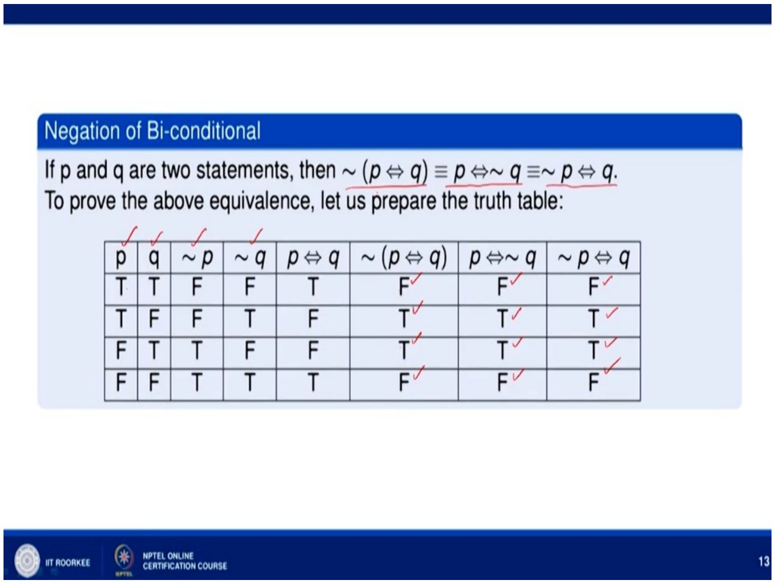

Now, negation of bi-conditionally p and q are two statements ~(p⇔q)≡p⇔~q≡~p⇔q, so let us prove the above equivalence, so these are the truth values of p and q TT TF FT FF ~p has truth values FF TT, ~q have truth FT FT we know p⇔q has truth value T, if both p and q are true or both of them are false, so TT gives T, TF gives F FT gives F, FF gives T and ~(p⇔q).

So ~(p⇔q) will then have truth values F T T F, p⇔~q, so this is p, ~q is this one so TF, TF give F and here TT, TT gives T then FF gives T and FT gives F, similarly in ~(p⇔q) this is ~p and we have q here so F here T here so we get F here then F here so we get T here, then T here T here so we get T here, T here.

So we get F here, now you can see the truth values is of negation of implies so they are FT TF and here FTT F and here also FTT F, so ~(p⇔q)≡p⇔~q≡~p⇔q.

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We note that

$$\begin{aligned} \sim(p \Leftrightarrow q) &\equiv \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)] \\ &\equiv \sim[(\sim p \vee q) \wedge (\sim q \vee p)] \\ &\equiv \sim[(\sim p \vee q)] \vee \sim[(\sim q \vee p)] \quad \text{Complement law} \\ &\equiv (p \wedge \sim q) \vee (q \wedge \sim p) \end{aligned}$$

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Now ~p become also not this ~(p⇔q) is ~[(p⇒q)∧ (q⇒p)] we have order that one ~(p⇔q) is ~[(p⇒q)∧ (q⇒p)], so we can put that equivalence here, so [(p⇒q)∧ (q⇒p)], now negation of p⇒q we have seen this is logically equivalent to ~p∨q and q⇒p is similarly equivalent to ~q∨p so we have this much ~(~p∨q).

Now, let us use compliment law, so negation of this and this will be negation of this or negation of this so ~(~p∨q) or ~(~q∨p) so this here we are using compliment law, so then ~(~p∨q) then again by complement law becomes p∧~q and the ~(~q∨p) again by using compliment law gives q∧~p, so this also comes by compliment law, so ~(p⇔q)≡(p∧~q)∨(q∧~p).

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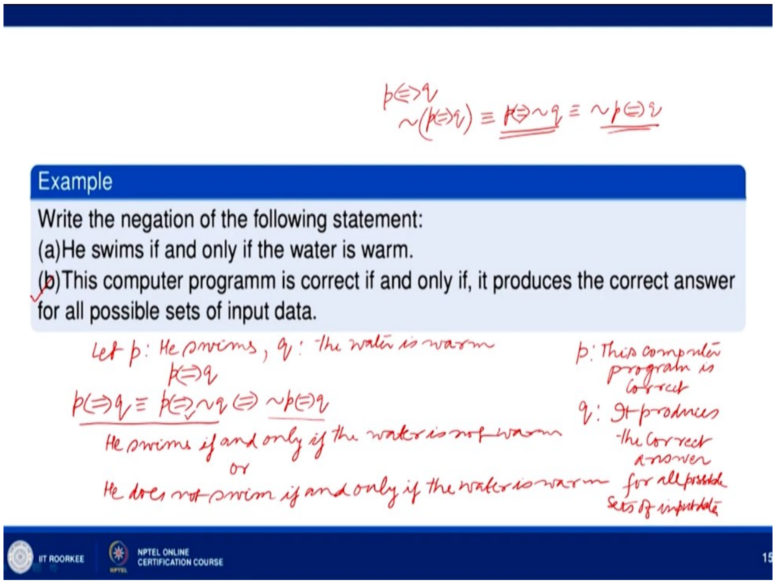

Now, let us write the negation of the following statement 'he swims if and only if the water is warm', so let say let p: he swims and q: water is warm then the given statement can be express as $p \Leftrightarrow q$, 'he swims if and only if the water is warm', now $\sim p \Leftrightarrow q$ we get can write we go to this one $\sim p \Leftrightarrow q$ is $p \Leftrightarrow \sim q$.

So, let us write, $p \Leftrightarrow q \equiv p \Leftrightarrow \sim q \equiv \sim p \Leftrightarrow q$, so we can say, first we can use this equivalence this one, so $p \Leftrightarrow \sim q$, so he swims if and only if, if I am using this equivalence, if and only if the water is not warm and if I use this equivalence $p \Leftrightarrow q \equiv \sim p \Leftrightarrow q$, then we will get the following 'he does not swim if and only if water is warm'.

So negation of the given statement can be one of these two, anyone of these two he swims if and only if the water is warm or he does not swim if and only if the water is warm, similarly let us say for the second statement, let p: this computer program is correct and q: it produces the correct answer for all possible cases sets of input data, then the given statement in the case b is can express that $p \Leftrightarrow q$, so $\sim(p \Leftrightarrow q) \equiv p \Leftrightarrow \sim q$ and $\sim p \Leftrightarrow q$.

So again if we want to write $\sim(p \Leftrightarrow q)$ it could be either this two $p \Leftrightarrow \sim q$ or $\sim p \Leftrightarrow q$, so if I use this one this equivalence $p \Leftrightarrow \sim q$, then $p \Leftrightarrow q$ will be giving us the statement 'this computer program is correct' and if and only if 'it does not produce the correct answer for all possible sets of data', and if I use this one then I will say ~p, ‘so this program is not correct if and only if it produces the correct answer for all possible sets of input data’, so we can easily say that.

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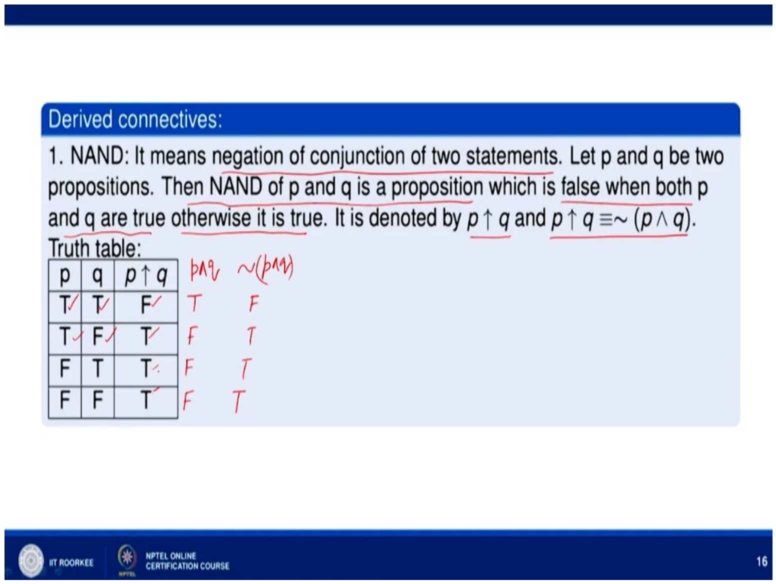

So, let us now consider derived connectives Nand, it means negation of conjunction of two statement, now we have already define conjunction of two statements, now we have already define conjunction of two statements, if p and q are two propositions then Nand of p and q is a proposition which is false when both p and q are true, when both p and q are true then Nand of p and q is false otherwise it is always true and we denoted by $p\uparrow q$, so $p\uparrow q \equiv \sim(p \wedge q)$.

So, this is truth values of p and q, these are truth values of p and q, so $p\uparrow q$ when p and q both are true then its value is false otherwise it is always true, so in all other cases it is true, now let us prove the logical equivalence of $p\uparrow q$ and $\sim(p\wedge q)$, so let us write the truth values for $p\wedge q$, for TT we get T, TF is F, FT is F and FF is F so $\sim(p\wedge q)$, so negation of T is F, for F we write T here, so you can see the truth values of, $p\uparrow q$ they are F TTT, and here also they are F TTT so this is they are logically equivalent.

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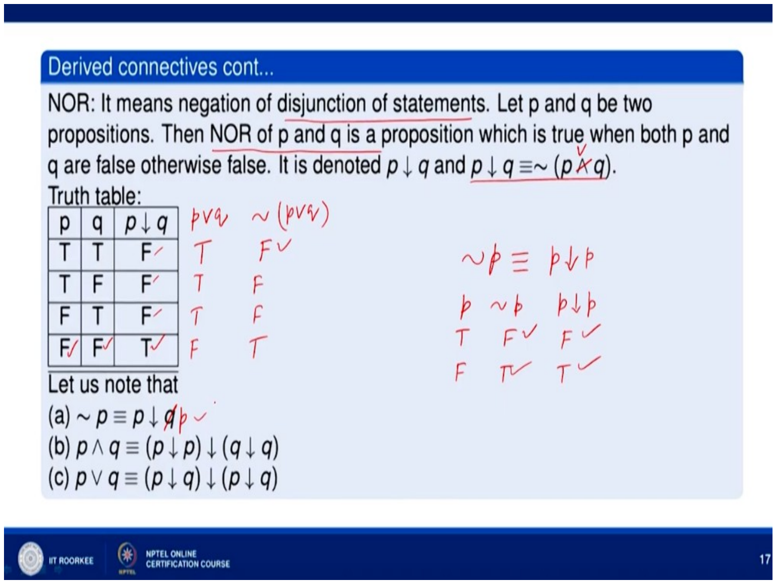

So, let us now discuss the derived connective NOR it means negation of disjunction of statements disjunction of statements we have already discuss let p and q be two proposition then NOR of p and q is a proposition which is true then both p and q are false, otherwise false so when p and q both are false, then NOR of p and q is true otherwise it is always false so in other three cases its value, truth value is false. Now we have to show that $p{\downarrow}q \equiv \sim(p \vee q)$.

So, let write $p \vee q$ here so for TT we have T, TF means T, FT means T, FF means F and $\sim(p \vee q)$ then we will see you F F F and T, so these truth values are same as the truth values of $p{\downarrow}q$ so they are logically equivalent. Now let us note that $\sim p \equiv p{\downarrow}p$ we can easily show this, so let us write so that $\sim p \equiv p{\downarrow}p$, so p has truth values T and F, $\sim p$ has truth values F T and $p{\downarrow}p$, so when they are both T then $p{\downarrow}p$ has truth value F, so this case will be F, when it is FF then we will have T, so they are same, so they are logically equivalent.

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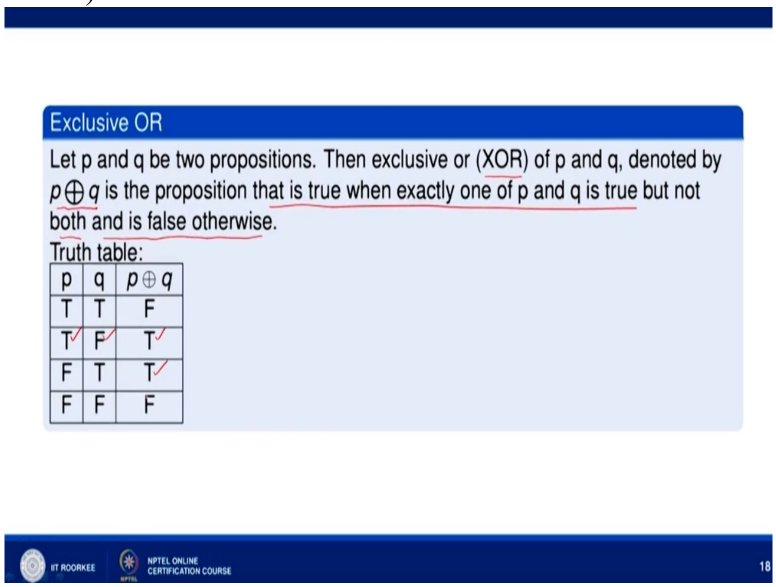

Similarly, let us now consider the third case exclusive OR, so let p and q be two proposition then exclusive OR which we also write a XOR of p and q denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true but not both and is false otherwise, so if the truth table or exclusive OR is $p \oplus q$ is the proposition that is true when exactly one of p and q is true, so p is true q is false then $p \oplus q$ is true here also it is F it is true and then it is false otherwise, so when exactly one of the two, p or q is true then its value is true so for TT it is false for FF also it is false, so this is the case exclusive or which we also write as XOR.

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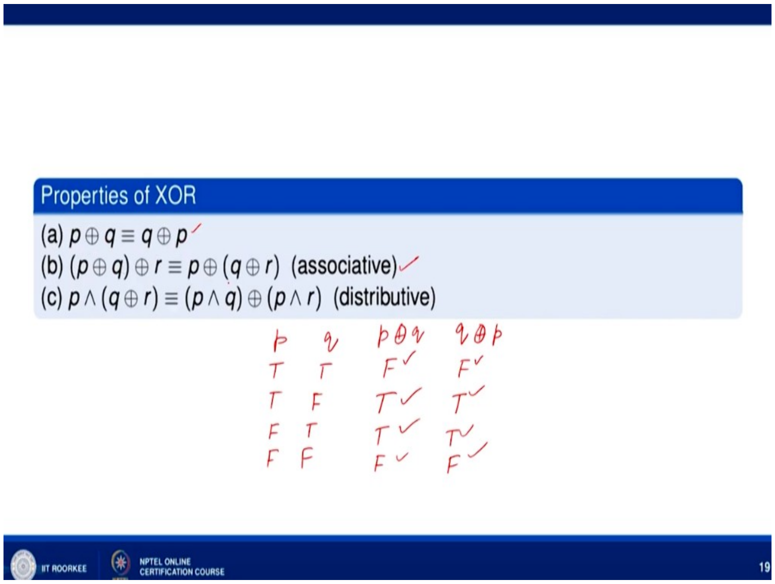

And the properties of XOR are, p⊕q≡ q⊕p and then it also follows the associative law it also follows the distributive law we can see the first case, so p, q let us say p, q they have truth value is TT TF then FT and then we have FF, so then we can write the truth values for p⊕q so we have seen that TT gives F, TF FT gives TT and FF gives F, so either one p or q should be true, so then q⊕p .

So again for q is true and p true we have F here for q false and p true, it is true here and then for q true and p false it is true here and then for here qF and pF it is again F here, so the truth value is for p⊕q≡ q⊕p, so this is the proof of the first property. Similarly, we can show the prove of the associative property and the distributive property this is the third case exclusive or in the derived connectives so with that we come to the end of this lecture thank you very much for your attention.