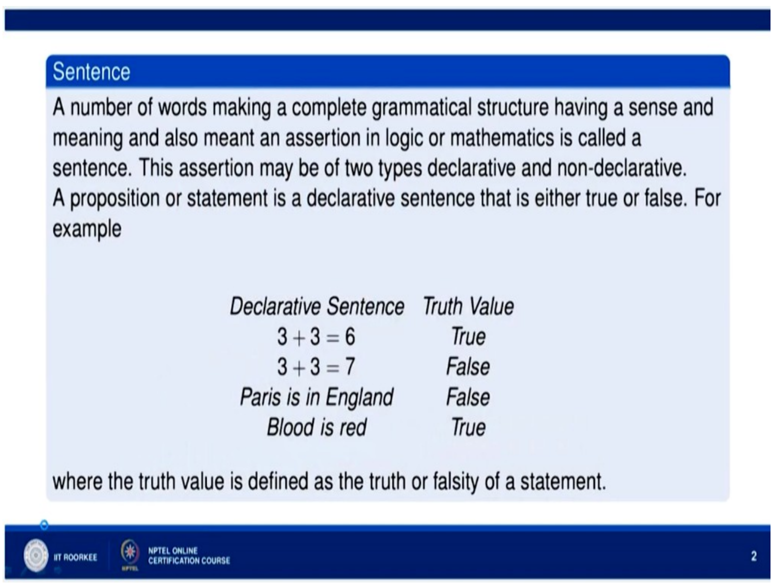
Higher Engineering Mathematics
Prof. P.N. Agrawal
Indian Institute of Technology Roorkee
Department of Mathematics
Lecture 01
Symbolic Representation of Statements – I

Hello friends, welcome to my lecture on Symbolic Representation of Statements. There will be two lectures on this topic and this one of those two lectures. First we define what we mean by a sentence.

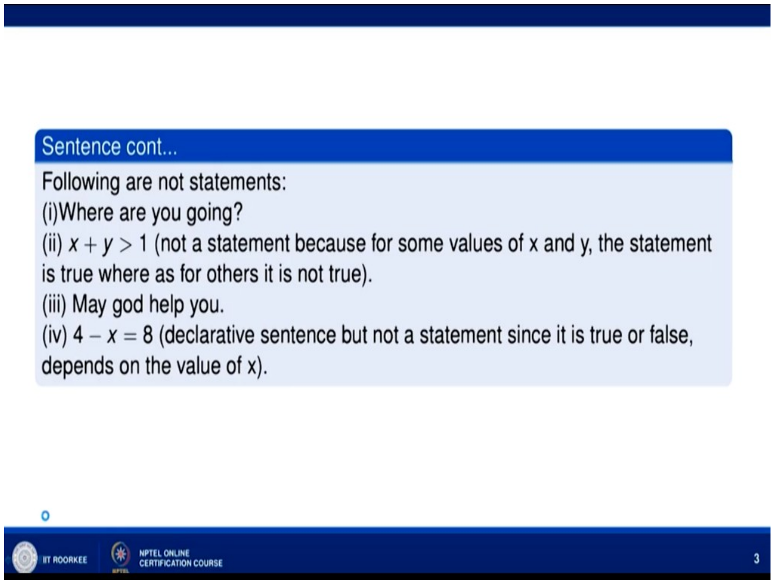
(Refer Slide Time: 00:42)



A number of words making a complete grammatical structure having a sense and meaning and also meant an assertion in logic or mathematics is called a sentence. This assertion may be of two types declarative and non-declarative. A proposition or statement is a declarative sentence that is either true or false. For example, let us consider the equation 3+3=6, it is a declarative sentence and it is truth value is true, it is a true result.

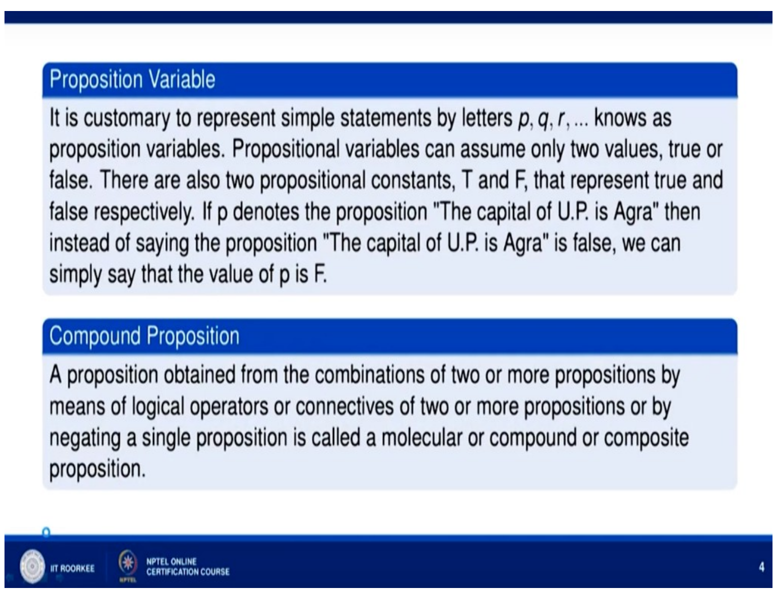
Then the next equation, let us consider 3+3=7, it is a declarative sentence again but 3+3≠7 so it's truth value is false. Paris is in England the truth value is false. Blood is red the truth is true. So, where the truth value is defined as the truth or falsity of a statement, we know that blood is red is a true statement so the truth value is true, Paris is in England is a false statement, so its truth value is false.

(Refer Slide Time: 01:58)



Let us now consider some more examples, "Where are you going?" it is an interrogative sentence, so it is not a statement. x+y>1 it is not a statement because for some values of x and y the statement is true while for others it is not true. May God help you, it is a wish, so it is not a statement. 4-x=8, it is a declarative sentence but it is not a statement, because it is true or false depends on the value of x.

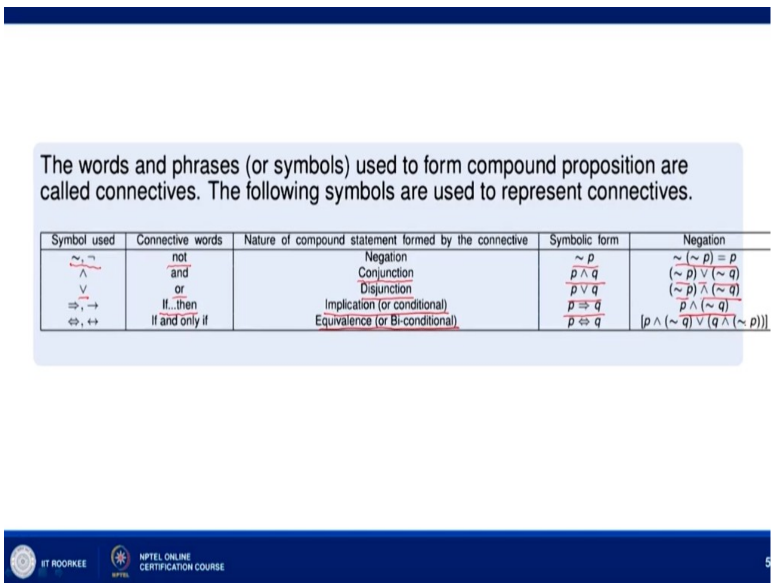
(Refer Slide Time: 02:45)



Now, let us define proposition variable, it is customary to represent simple statements by letters p,q,r… known as propositional variables. Propositional variable can assume only two values, true or false, there are also two propositional constants and they are T and F, they represent true and false respectively. If p denotes the proposition “The capital of U.P. is Agra” then instead of saying the proposition “The capital of U.P. is Agra” is false, we say that simply say that the value of p is F.

p denotes this statement “The capital of U.P. is Agra” it is a false statements, so we say that the value of p is F. Compound proposition - A proposition obtained from the combinations of two or more propositions by means of logical connectives or logical operators of two or more propositions are by negating a single propositions is called a molecular or compound or composite proposition.

(Refer Slide Time: 03:51)



The words and phrases (or symbols) used to form compound proposition are called connectives. The following symbols are used to represent connectives.

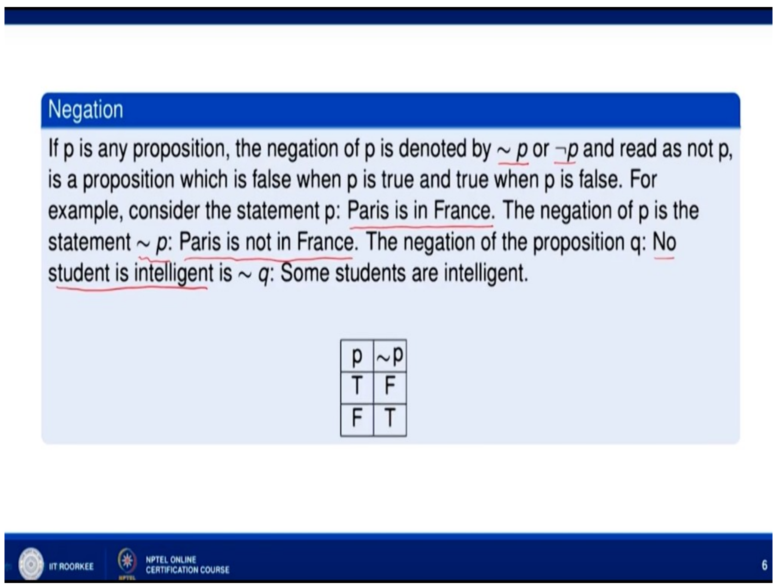
Symbol used	Connective words	Nature of compound statement formed by the connective	Symbolic form	Negation
$\sim, \neg$	not	Negation	$\sim p$	$\sim(\sim p) = p$
$\wedge$	and	Conjunction	$p \wedge q$	$(\sim p) \vee (\sim q)$
$\vee$	or	Disjunction	$p \vee q$	$(\sim p) \wedge (\sim q)$
$\Rightarrow, \rightarrow$	If...then	Implication (or conditional)	$p \Rightarrow q$	$p \wedge (\sim q)$
$\Leftrightarrow, \leftrightarrow$	If and only if	Equivalence (or Bi-conditional)	$p \Leftrightarrow q$	$[p \wedge (\sim q) \vee (q \wedge (\sim p))]$

Let us look at the symbols and phrases which are used to form compound propositions, they are called connectives. The following symbols are used to represent connectives. Symbol used, if when we use this symbol, okay, it is not, we can also use this symbol, is it not, when we use these symbols it means not, nature of compound statement found by the connective negation, symbolic form, ~p, ~(~p)=p.

And then this symbol is used for the connective word ‘and’ okay, and it is a conjunction, nature of compound statement found by the connective is conjunction and we write symbolically p∧q. Now ~(p∧q) is ~p∨~q and this symbol is used for or connective word ‘or’ and the nature of compound statement found by the connective is disjunction, okay, symbolic form is p∨q, ~(p∨q) is ~p∧~q. Now this is implies, implies can also be written like this. Now it means connective words are 'if then' and the nature of compound statement found by the connective is implication or conditional.

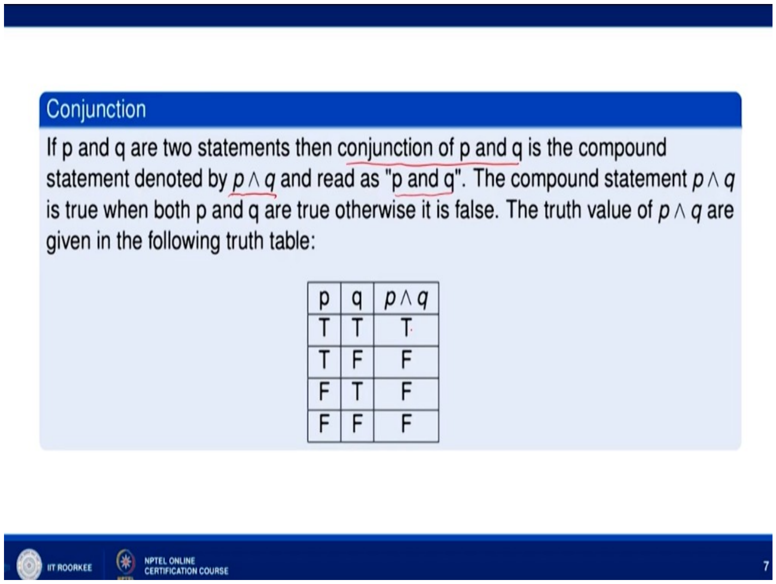
So, symbolic form is $p \Rightarrow q$, okay, $\sim(p \Rightarrow q)$ is p∧~q. Now, this is implies and implied by and connective words are if and only if nature of compound statement is equivalence or bi-conditional, symbolic form is $p \Leftrightarrow q$ and $\sim(p \Leftrightarrow q)$ is p∧~q or q∧~p as we shall see later in this lecture.

(Refer Slide Time: 05:53)



Now, if p is any proposition, the negation of p is denoted by ~p, so we can use this symbol or we can also use this symbol for not p and read as not p is a proposition which is false when p is true so not p means if p is true then not p is false and if p is false then not p is true. For example, consider the statement p: Paris is in France, the negation of p is the statement ~p: Paris is not in France, the negation of the proposition q, let us say the proposition q is, No student is intelligent, then ~q: Some student are intelligent. Now, let us see the truth table of p and q. This statement p if it is true then not p, not p will be F and then if p is false then not p will be true, so this is the truth table for not p.

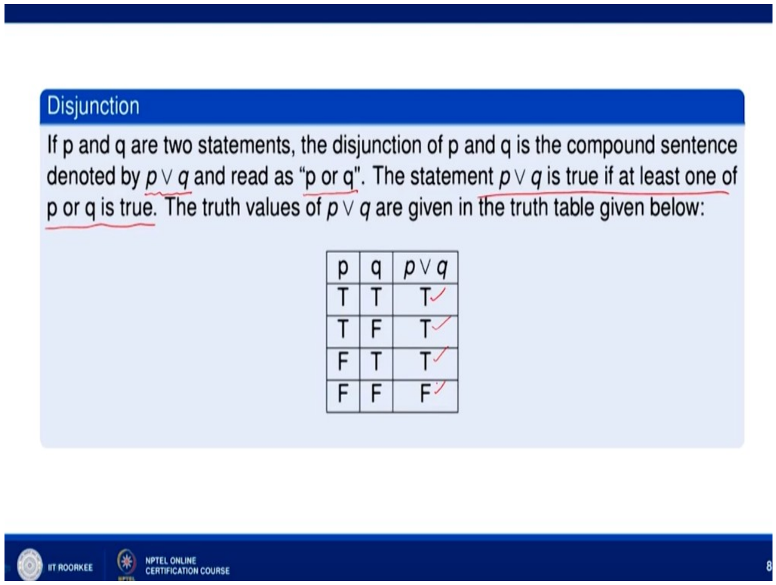
(Refer Slide Time: 07:08)



Conjunction: If p and q are two statements then conjunction of p and q, conjunction of p and q is the compound statement denoted by p^q and read as p and q, the compound statement p^q is true when both p and q are true, when both p and q are true otherwise it is false. The truth value of p^q is, are given in the following truth table, so you can see the statement p if it is true q may also be true.

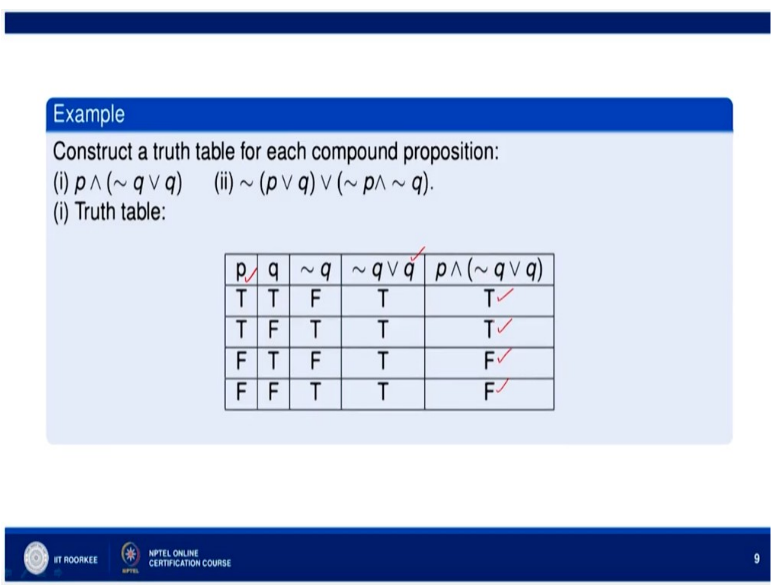
So we have TT and TT we have then p and q is true, because we have said here the compound statement p^q is true when both p and q are true otherwise it is false. Now if the statement p is true and q is false then p^q is false, if the statement p is false the statement q is true then the statement p^q, compound statement p^q is false, if p is false q is false then again the conjunction of p^q is false.

(Refer Slide Time: 08:14)



Disjunction: If p and q are two statements, the disjunction of p and q is the compound statement denoted by p∨q, and read as "p or q". The statement p∨q is true if at least one of p or q is true. The statement p∨q is true if at least one of p or q is true. The truth values of p∨q are given in the truth table given below, so if p is true and q is true then p∨q is true if p is true q is false then p∨q is true, if p is false q is true then p∨q is true, if p is false q is false then p∨q is false okay.

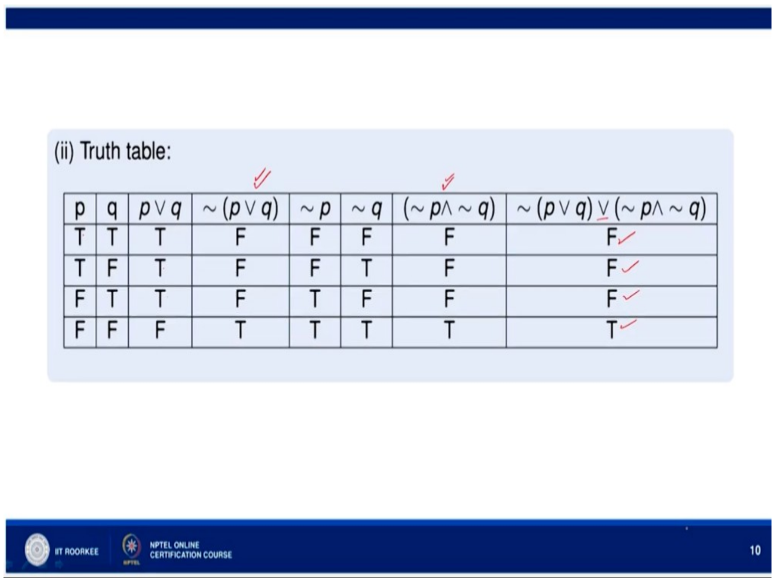
(Refer Slide Time: 09:05)



So let us construct a truth table for the compound propositions. One given by p∧(~q∨q) and the second one is ~(p∨q)∨(~p∧~q). Let us first write the truth table for the first compound proposition. So p and q, the truth values of p and q are of the type TT, TF, FT, FF. Now, let us write the truth values for negation of q, so ~q, the truth values are for T it is F, for F it is T, for T it is F, for F it is T, then ~q∨q.

So negation of q or q gives for F and TT, for T and FT, for F and TT, for T and FT, so all of them are T. Now then we write the truth values for the compound proposition p or p∧(~q∨q). So, let us consider this one p and this one, ~q∨q. So T and TT, so T and T again T, now F and T F, and F and T F, so the truth values for the compound proposition p∧(~q∨q) are TT FF. Let us go to the next compound proposition.

(Refer Slide Time: 10:43)

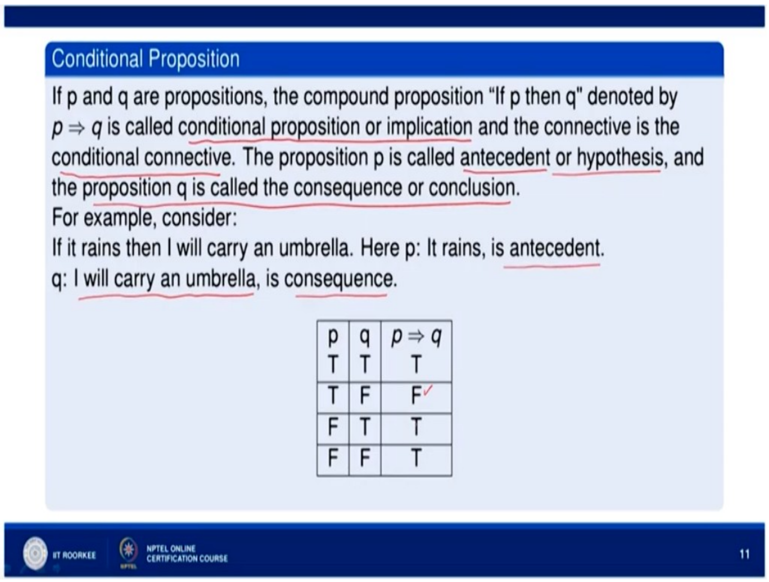


p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$(\sim p \wedge \sim q)$	$\sim(p \vee q) \vee (\sim p \wedge \sim q)$
T	T	T	F	F	F	F	F
T	F	T	F	F	T	F	F
F	T	T	F	T	F	F	F
F	F	F	T	T	T	T	T

So, now we will write the truth values for ~(p∨q)∨(~p∧~q). Let us first write this, so the truth values for p q are TT TF FT FF, then p∨q will have truth values T T T and F for these four cases and then ~(p∨q) will give for T it is F, for T it is F, for T it is F, for F it is T. Now ~p, when p is T, ~p gives F, when it is T again it is F, when it is F it is T, when F it is T, ~q. Now for T it is F, for F it is T, for T it is F, for F it is T, then we can consider the compound proposition ~p∧~q, so for FF it is F, for FT it is F, for TF it is F, for TT it is T.

And then we consider ~(p∨q), ~(p∨q) we have found here and then we consider or ~p∧~q, ~p∧~q are value, truth values are here so we can now consider ~(p∨q) or ~p∧~q. So we just consider this column and this column, now FF so when you have all here FF gives F, and then FF gives F, and then FF give F, and TT give T, so these are the truth values for the compound proposition ~(p∨q)∨(~p∧~q).

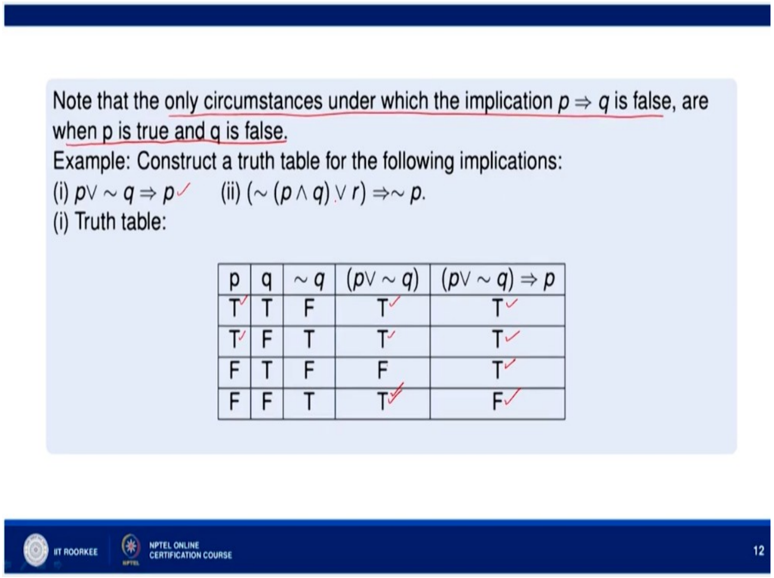
(Refer Slide Time: 12:51)



Now Conditional Proposition: If p and q are two propositions, the compound proposition if p then q denoted by $p \Rightarrow q$ is called conditional proposition or implication, conditional proposition and or implication, and the connective is the conditional connective. So the proposition p is called the antecedent, the proposition p is called the antecedent or hypothesis, and the proposition q is called the consequence or conclusion. For example, consider this statement, "If it rains then I will carry an umbrella," "If it is rains then I will carry an umbrella."

So here if p: it rain p will be an antecedent, I will carry an umbrella if q: I will carry an umbrella then q is consequence or conclusion. Let us write the truth table for this case, so let say pq have truth values TT TF FT FF then $p \Rightarrow q$, the conditional connective $p \Rightarrow q$, here for TT we will have T, TF is F, FT it is T, FF it is T that means only in the case where p is true q is false $p \Rightarrow q$, is F otherwise it is always true. So when this statement, p is true and q is false the truth value of $p \Rightarrow q$ is F otherwise it is always true.

(Refer Slide Time: 14:34)



Note, that the only circumstance we have written this statement here, note that the only circumstances under which the implication p⇒q is false are when p is true and q is false. Example, construct a truth table for the following implications, let us construct a truth table for this implication. p∨~q⇒p so the truth value of p and q are TT TF FT FF, then ~q will have truth values F, this will be T, this will be F, this will be T, and then we have compound proposition p∨~q, p has value, truth value T, ~q has truth value F.

So, T or F give T, then T or T gives T, then F or F gives F, and F or T gives T. And here now we write the implication and we have seen that if p is true q is false, then p⇒ q, the truth value of p ⇒q is false otherwise it is always true. So, here p or q, p∨~q, the truth value is T and the truth value of p is also T, so this means p∨~q⇒p we will have truth value T.

Now here the truth value is T, here the truth value is T again truth value here will be T, now here the truth value is F, here the truth value is F so the truth value of this implication will be T, here truth value is T, now you can see, let us look at this, “The only circumstances under which the implication p⇒q is false, are when p is true and q is false.” So here p∨~q it is true while p is false and therefore, the implication p∨~q⇒p is false, so the truth values of the implication p∨~q ⇒p are TTT and F. Now let us go to the second implication, ~(p∧q)∨r⇒~p, so let us form the truth table for that.

(Refer Slide Time: 16:58)

(ii) Truth table:

p	q	r	$p \wedge q$	$\sim(p \wedge q)$	$\sim(p \wedge q) \vee r$	$\sim p$	$(\sim(p \wedge q) \vee r) \Rightarrow \sim p$
T	T	T	T	F	T	F	F
T	F	T	F	T	T	F	F
F	T	F	F	T	T	T	T
F	F	F	F	T	T	T	T
T	T	F	T	F	F	F	T
T	F	F	F	T	T	F	F
F	T	T	F	T	T	T	T
F	F	T	F	T	T	T	T

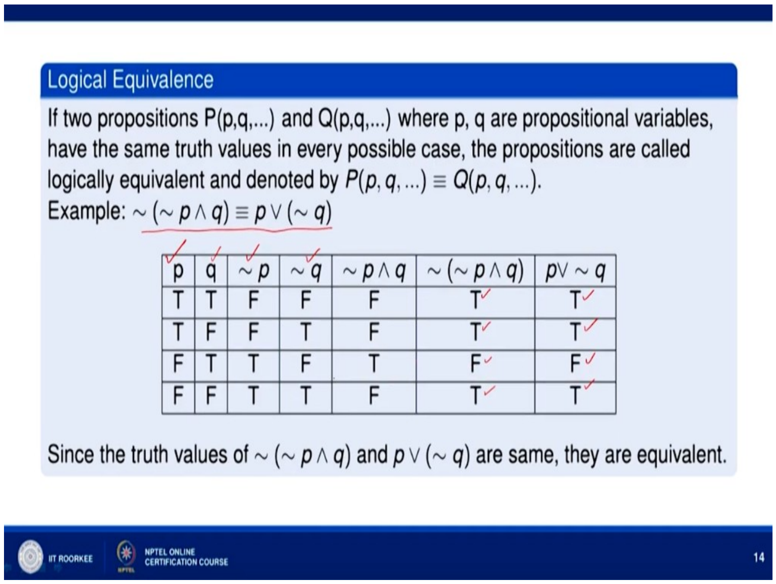
IIT ROORKEE NPTEL ONLINE CERTIFICATION COURSE 13

So, now here we have three statement p q and r, so p q and r can have true or false values, false statements, there may be false, true or false statements, so their truth values are TTT, there will be eight cases for the three statement pqr, so there will be TTT TFT FTF FFF TTF TFF FTT FFT, there eight cases when pqr three statements are involved. So truth values is of the case p and q, truth values of p and qr, now p^q, TT will give you T, TF will give you F, FT will give you F, FF will give you F, TT will give you T, TF will give you F, FT will give you F, FF will give you F.

Now, ~(p^q), negation is easy you can just write for T you can write F, for F you can write T, T gives F, F gives T, then here T, here T, here F, here T, here T and here T. Now ~(p^q)∨r, so we have to combine this one and this, so when it is F it is T ~(p^q)∨r gives T, then here it is T, here it is T, so we get T here, here it is T here it is F, so we get T again and then here T here F we get T again, FF gives F, TF gives T, and then TT gives T, and then we have here T and T again we get T. Now ~p, for T we get F, this T gives F, this F gives T, this F gives T, this T gives F, this T gives F, for F we have T here, for F we have T here.

Now, ~(p^q)∨r, we have the truth values this and we also have the truth values of ~p. So we can now write the truth values for the implication ~(p^q)∨r⇒~p, so when it is true here it is false, so we will have F here, it is true it is false so we will have F here, here it is true here it is true so we have true and here it is true here it is true so we have true again, FF gives T, TF give F. So, whenever ~(p^q)∨r⇒~p we have if this is true, and this is false the truth value will be false. So, now ~(p^q)∨r if it is true and ~p is true the implication will give T and here again T we have here again T we have so get T here, so these are the truth values of this implication.

(Refer Slide Time: 20:19)



p	q	$\sim p$	$\sim q$	$\sim p \wedge q$	$\sim(\sim p \wedge q)$	$p \vee \sim q$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	F	T	T	F	T	T

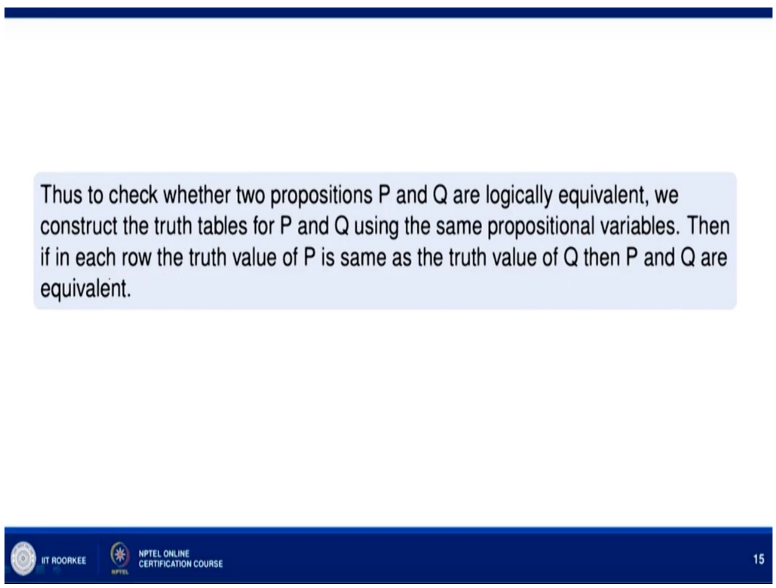
Now, logical equivalence if we have two propositions P which are denoted by the propositional variable p q and so on and another proposition, let us say Q is denoted by the propositional symbols, propositional variable p q and so on where p q are propositional variables they have the same truth values, if two propositions have the same truth values in every possible case the propositions are called logical equivalent and denoted by P(p,q,..)≡Q(p,q,…).

So, logical equivalence means two propositions p and q will be logically equivalent if their truth values are same in every possible case. For example, let us look at this this example, so ~(~p^q)≡p∨(~q). Let us prove this, so let us form the truth table for this case, so p q may have these following four cases TT TF FT FF ~p gives for TF, here again F, here T, here T, ~q gives for TF, this one is T, this is F, this is T, then negation of p and q.

So, let us consider this column and this column, so ~p^q, so F T means F, here FF means F, TT means T, and TF means F. Now, ~p^q, so negation of this truth values, will for F we write T and for F we write T, for T we write F, for F we write T. Then let us find the truth values for p or not q, so we consider this column p and ~q, p∨(~q), ok, so T F means p∨(~q) gives T, then T and T or T gives T, then F or F gives F, then F or T give T, so these are the truth values. TT FT and you can now see the truth values this ~(~p^q) they are TT FT.

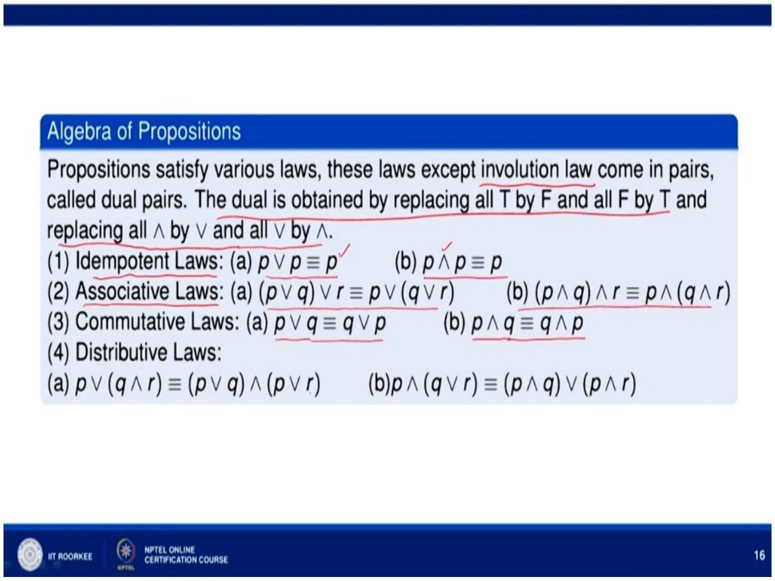
So in every case, let us look at the first case where p and q have truth values TT, we have the truth values of ~(~p^q) where T, p or negation of q it also has truth values of T and then in the second case when p is T q is F again we can see truth values of this proposition and this proposition are same, truth values in the third case are again same, forth they are again same, so truth values of ~(~p^q) and p∨~q are same in every possible case, and therefore they are logically equivalent.

(Refer Slide Time: 23:38)



Now, let us go to again explaining this logical equivalence. So this check whether two propositions P and Q are logically equivalent, we construct the truth tables for P and Q using the same propositional variables, then in each row, then if in each row the truth value of P is same as the truth value of Q then we say P and Q are logically equivalent.

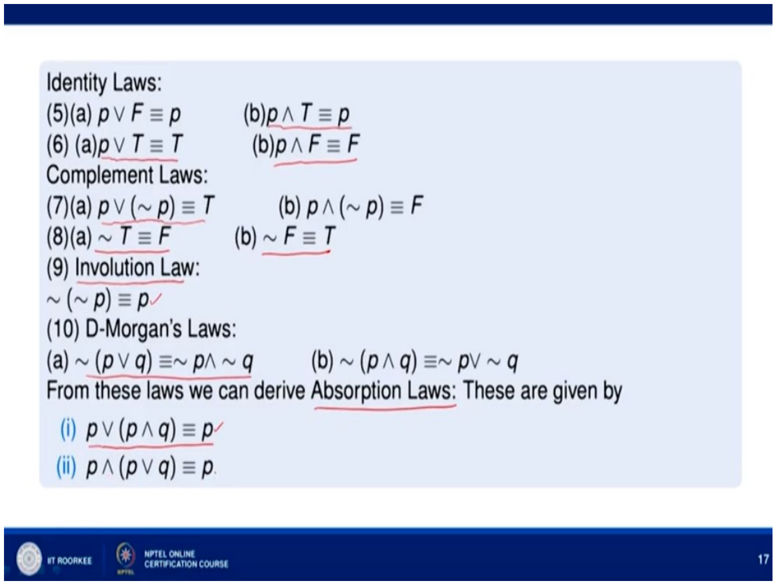
(Refer Slide Time: 24:08)



Algebra of propositions. Now propositions satisfies various laws, this laws except involution law, except involution law, these laws come in pairs we called them as dual pairs. The dual is obtained by replacing all T by F, the dual is obtained by replacing all T by F and all F by T and replacing all $\wedge$ by $\vee$ and $\vee$ by $\wedge$. First we look at Idempotent laws, so $p \vee p \equiv p$, now this $\vee$ is replaced by $\wedge$ so $p \wedge p \equiv p$, so this is logical, this proposition this is dual of this one.

So, they are come in dual pairs. And then associative laws $(p \vee q) \vee r \equiv p \vee (q \vee r)$ this is associative law. Now we replace $\vee$ by $\wedge$, so $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$, so these also comes in dual pairs. Commutative laws $p \vee q \equiv q \vee p$ and its dual is $p \wedge q \equiv q \wedge p$, then distributive laws $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$, its dual pair $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$.

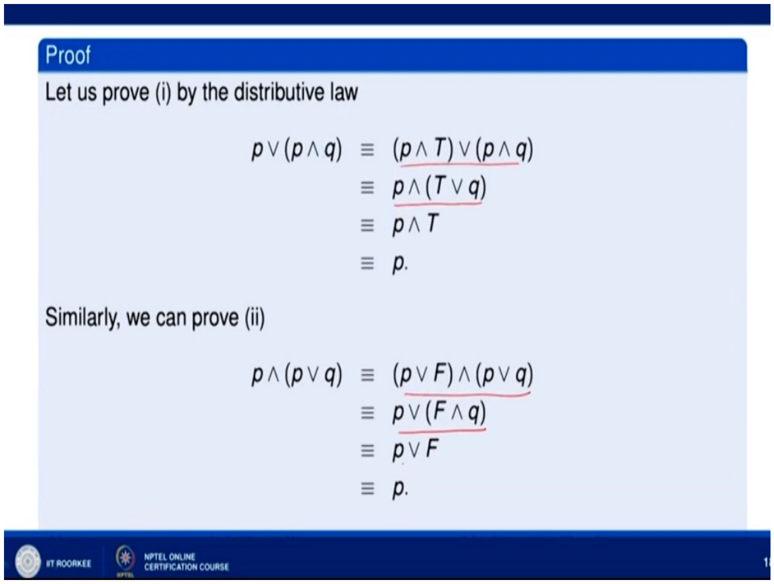
(Refer Slide Time: 25:56)



Now, identity laws $p \vee F$, $p \vee F \equiv p$, then this or is replaced by end, F is replaced by T, so its dual is $p \wedge T \equiv p$ and here $p \vee T \equiv T$, its dual pair is $p \wedge F \equiv F$. Now compliment laws $p \vee (\sim p)$, $p \vee (\sim p) \equiv T$, its dual is $p \wedge (\sim p) \equiv F$, now, we know that negation of T, $\sim T \equiv F$, then its dual is $\sim F \equiv T$. Now this is involution law, so $\sim(\sim p) \equiv p$, there is no dual of this, excepting involution law all other laws come in dual pairs.

So $\sim(p \vee q) \equiv \sim p \wedge \sim q$, its dual is $\sim(p \wedge q) \equiv \sim p \vee \sim q$. Now form these laws we can derive absorption laws, absorption laws are p or p and q, $p \vee (p \wedge q) \equiv p$ and then $p \wedge (p \vee q) \equiv p$. Now let us prove this absorption law 1, now let us prove absorption law, so we begin with the proof of the absorption law 1 so we have to show that $p \vee (p \wedge q) \equiv p$.

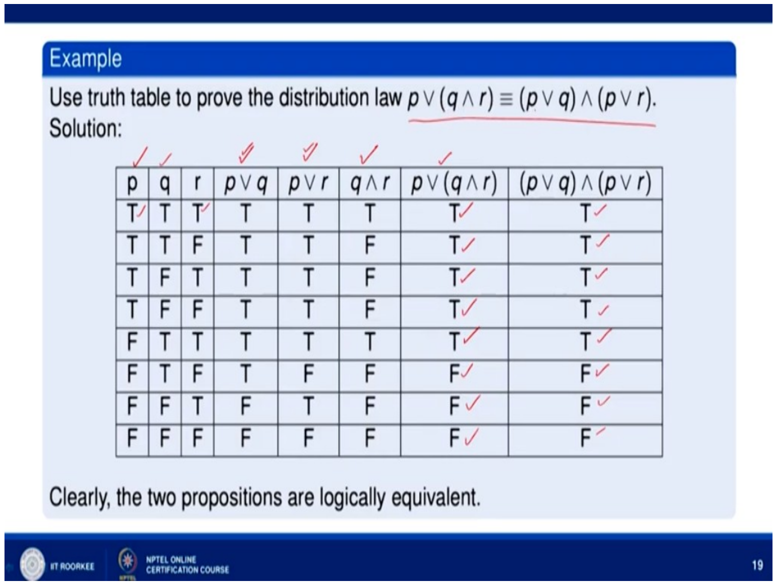
(Refer Slide Time: 27:51)



So, let us first write p or p and q, $p\vee(p\wedge q)$, p we can write as $p\wedge T$ by the identity law, so we have p and T here, $(p\wedge T)\vee(p\wedge q)$ and this is what $p\wedge(T\vee q)$ by the associative law. So and then we have T or q, $T\vee q$ equal to T, now $p\wedge T$ equal to p, so we have $p\vee(p\wedge q)\equiv T$.

Similarly, we can prove the second one, second one is $p\wedge(p\vee q)\equiv p$, let us prove this so $p\wedge(p\vee q)\equiv p$, now p can be written as $p\vee F$, p or F we can use this, $p\vee F$ equal to p, so $(p\vee F)\wedge(p\vee q)$, now we use distributive law, so this is equivalent to $p\vee(F\wedge q)$, this is equivalent to this by distributive law. So, now $F\wedge q$ is equal to F so $F\wedge q$ equal to F, so we have $p\vee F$, $p\vee F$ equal to p, so have here p, so we can use absorption, we can prove absorption laws by using the identity laws.

(Refer Slide Time: 30:05)



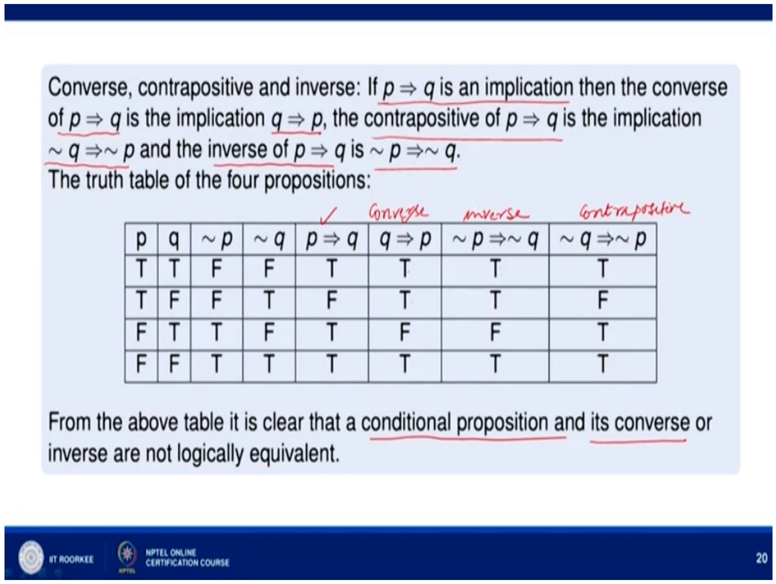
p	q	r	$p \vee q$	$p \vee r$	$q \wedge r$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	T	F	F	F
F	F	F	F	F	F	F	F

Now, let us prove this distribution law, let us prove this distribution law where p∨(q^r) we have show is logically equivalent to (p∨q)^(p∨r), so the truth values for p q r, they are eight possible combinations TTT TTF TFT TFF FTT FTF FFT FFF, and prq, so we take first column, second column, so prq will have truth values TTT TTT FF and for p∨r, p∨r so we take first column and third column, so when we have TT here we have T here, when we have TF we have T here, we have TT we have T, TF gives T, and then FT gives T, FF gives F, FT gives T, FF gives F.

So, these are the truth values for p∨ r and then we can similarly write truth values for q^r. q^r gives you, q^r so for T and T we get T, TF gives F, and then FT gives F, FF gives F, TT gives T, TF gives F, FT gives F, FF gives F. So these are the truth values for q and r and then we can write the truth values for p∨(q^r) similarly, so we consider first column and we consider this column, so this one is 6th column, so considering first column and 6th column the truth value is are TTT TTFFF, and then we can consider the truth values for p or q and q p or r.

So for that we need to consider this column, forth column which is p∨q and p∨r, this fifth column, so from forth and fifth column the truth values for (p∨q)^(p∨r) are TTT TT FFF, now you can see the truth values of p∨(q^r), they are these once and they are the truth values for p or q and p or r, they are also TTTT TF FFF, so in every possible case truth of we find the truth values of the two side p∨(q^r), and (p∨q)^(p∨r) to be same so there are they are logically equivalent.

(Refer Slide Time: 32:53)



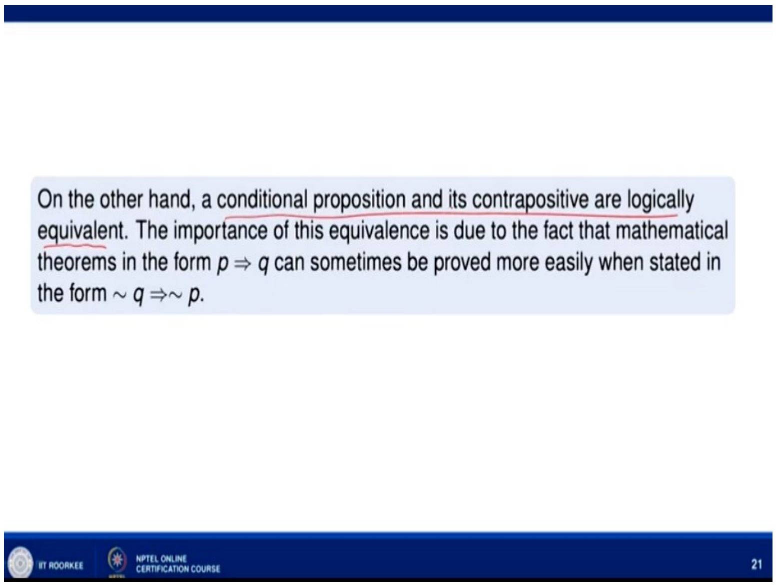
Now, let us consider converse of a proposition, contraposition, contrapositive and inverse, so if p $\Rightarrow$q is an implication then the converse of p$\Rightarrow$q is the implication q$\Rightarrow$p. Let us consider the converse of the implication p$\Rightarrow$q, so p implies q gives the converse as q$\Rightarrow$p, the contrapositive of p$\Rightarrow$q, the contrapositive of p$\Rightarrow$q is the implication, ~q$\Rightarrow$~p that is the contrapositive and the inverse of p$\Rightarrow$q is ~ p $\Rightarrow$~q.

So, let us now consider the truth table for this four propositions, p$\Rightarrow$q its converse q$\Rightarrow$p, contrapositive of p$\Rightarrow$q that is ~q$\Rightarrow$~p, and the inverse of p$\Rightarrow$q that is ~p$\Rightarrow$~q so there are four propositions. So their truth values let us write, so p q have truth values TT TF FT FF, then ~p has truth value is FF TT, ~q have truth values FT FT and then p$\Rightarrow$q, we know that p$\Rightarrow$q is having a truth value false only in the case when p is true q is false, otherwise it is always true, so TF TT and then we can write truth values for the implication q$\Rightarrow$p.

So, they are TT FT by the same reasoning and then we write the truth values for the implication ~p$\Rightarrow$~q they are TT FT, and then we write truth values for ~q$\Rightarrow$~p, so they are TF TT. Now you can see a conditional proposition, this is conditional proposition, p$\Rightarrow$q, and its converse, its converse is the case q$\Rightarrow$p, this is converse, this is converse and ~q$\Rightarrow$~p that is contrapositive and this inverse, inverse of p$\Rightarrow$q is ~p$\Rightarrow$~q, this is inverse, so the truth values of the conditional proposition they are TF TT, the truth value is of q$\Rightarrow$p are TT FT.

So $p \Rightarrow q$ and $q \Rightarrow p$ they are not logically equivalent. Now, let us look at this $p \Rightarrow q$, $\sim p \Rightarrow \sim q$ inverse of $p \Rightarrow q$, the truth values are TT FT so they are not same in every possible case, but when you look at the truth values of $\sim q \Rightarrow \sim p$ you see that they are same as the truth values of $p \Rightarrow q$, so p implies q and it is contrapositive $\sim q \Rightarrow \sim p$ they are logically equivalent.

(Refer Slide Time: 36:13)



So, we have written here that a conditional proposition and it is contrapositive are logically equivalent and we use this fact quite often when we prove mathematical theorems using the in order to prove that $p \Rightarrow q$, we often consider it is contrapositive that is we show that $\sim q \Rightarrow \sim p$, so that is that make some time the proof of the theorem easy. So with that I would like to end my lecture thank you very much for your attention.