Ordinary and Partial Differential Equations and Applications

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Lecture – 53
One Dimensional Wave Equations and its Solutions- I

Hello friends, welcome to this lecture, in this lecture, we will discuss very important second order linear partial differential equation homogeneous equation and that is known as wave equation and if you recall at the time of classification, we have discussed the problem like l of u = f of xyzpq. So let me write it here, so we consider the following second order semi linear PDE; l of u + g of x yu ux uy.

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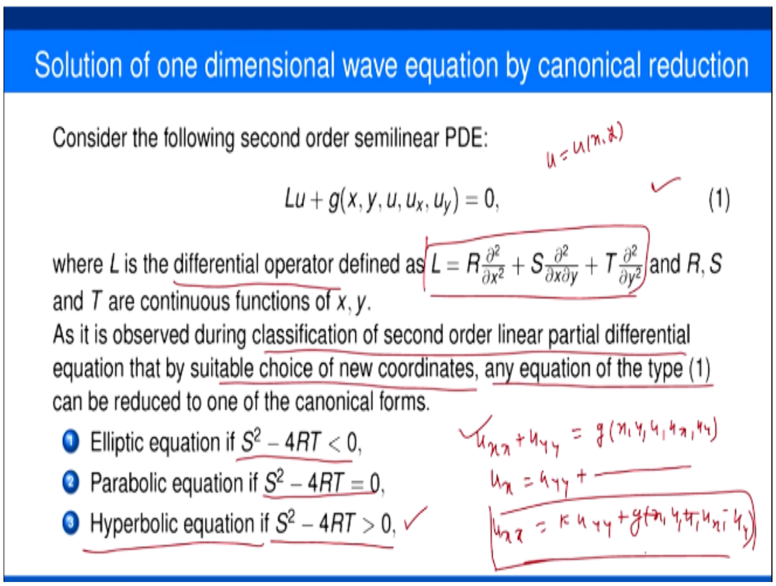

Here, this u is a function of x and y and l is the differential operator in fact, linear differential operator defined as la s R dou of 2u dou 2 dou of x square + S dou of 2 dou of x dou of y + T dou of 2 dou of y square and these coefficient R, S, T are continuous function of xy, in fact we call this kind of equation as linear equation, if it is linear in highest order; highest order derivatives.

So, here highest order derivatives second order and they are linear in terms of say, xy we simply write it that xy that this is a linear differential equation. So now, we have already observed that during classification of second order linear partial differential equation that by suitable choice of new coordinates, which we call as characteristics, any equation of the type 1 this kind of equation can be reduced to one of the standard canonical form that is elliptical equation.

Here, your form is uxx + uyy = some function of x y u ux uy and parabolic equation means say, ux = uyy + g, similarly hyperbolic equation means uxx = some constant times uyy and so on, so here elliptic equation, we call this is as elliptic equation, if S square – 4RT is < 0, where S is the coefficient of mix partial derivative that is dou of 2 upon dou of x dou of y and R is say, coefficient of dou of 2 upon dou of x square.

And T is the coefficient of the other double derivative that is dou of 2 upon dou of y square, so if S square – 4RT is < 0, then we call this equation as elliptic equation, parabolic equation when S square – 4RT = 0 and hyperbolic equation is when we call as S square – 4RT > 0 and we say that when we have these equations, then with the help of characteristics, we can convert equation of these type into a prototypical problem.

And in this lecture, we will discuss one prototype that is corresponding to hyperbolic equation and we say that if our problem is hyperbolic equation, then we can always reduce hyperbolic problem into this one that uxx = uyy, some constant times uyy + some function gx yu ux and uy. Now, in this lecture we simply consider the homogenous part that is uxx = kuyy, where is k is some constant.

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Consider the one- dimensional wave equation given as follows:

$$u_{tt} - c^2 u_{xx} = 0. \qquad (2)$$

Equation (2) represent a prototype equation of hyperbolic type second order linear partial differential equation in two independent variables. Here, $S = 0$, $R = -c^2$, $T = 1$, therefore $S^2 - 4RT > 0$. Choosing the characteristic lines

$$\xi = x - ct, \eta = x + ct. \qquad (3)$$

The chain rule of PDE gives

$$u_x = u_\xi \xi_x + u_\eta \eta_x = u_\xi + u_\eta$$

$$u_t = u_\xi \xi_t + u_\eta \eta_t = c(u_\eta - u_\xi)$$

So here, we are considering the one dimensional wave equation given as follows that is utt – c square uxx, here t we consider as the time and x is consider as the space variable, so here we call it one dimension because it is one dimensional in space, so we call one dimension wave equation means, utt – u square uxx = 0 because we have only one dimension space variable is involved, so now this equation 2 represent a prototype equation of hyperbolic type second order linear partial differential equation in 2 independent variable.

And you can check easily that here the S is = 0, R, you can consider as –c square and T = 1 and you can calculate the value of S square -4RT which is coming out to be positive, so it means that this wave equation is a hyperbolic equation and you can see that any equation of hyperbolic type can be convert into this kind of simpler form. Now, here we have already discussed in fact, it is done in this lecture that in this case, we can choose our characteristic lines.

It means that new coordinate x as like xi = x – ct and eta = x + ct here, so when we use these characteristics then we can write down our derivative uxx utt in terms of new xi and eta and you can write uxx u xi Xi x + u eta x, X then you can write it; u xi + u eta and ut = u xi xi t + u eta eta t, here we are simply using the chain rule that u is a function of xy and xy is the function of x eta. So, here we are finding all the other way round that u is a function of xi eta and xi is the function of x and y.

So, I want to find out derivative of u with respect to x and u xi xi x u xi xi y, so that is what we have written here, so ux is to u xi xi x + u eta eta because u is a function of xi and eta; xi and eta are the function of x and y, so we are writing here like this, here y is your t, so we can write ux as u xi + u eta and ut as c times u eta – u xi.

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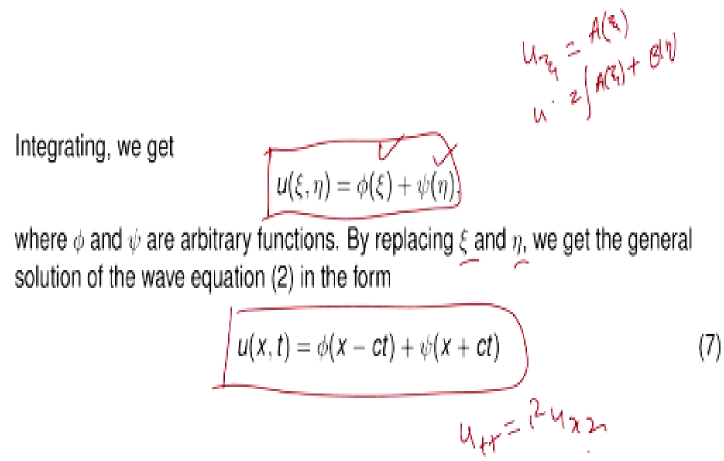

So, if you write down in operator form, then we can write dou/ dou x as dou/ dou xi + dou/ dou eta and dou/ dou t, you can write c times dou / dou eta – dou / dou xi, so you can write dou of 2/ dou x square is kind of square of this operator that is dou/ dou eta + dou/ dou xi whole square operating on u, when you simplify you will get this term that u xi xi + 2 u xi eta + u eta eta and similarly, you can calculate dou 2u/ dou t square and that is coming out to be c square * u xi xi – 2 u xi eta + u eta eta.

When you put it into this equation that uxx = c square utt, I think it is the other way round, it is 1/; it is c square; utt =; let me write it here, it is utt = c square uxx, so when you use this, you can see that your equation is now reduced to 4u xi eta = 0 because here utt is what? Here, utt is c square u xi xi – 2u xi eta + u eta eta = c square times u xi xi + 2u xi eta + u eta eta, now these will be cancel out, what you will have is this that 4 times c square u xi eta is = 0.

Now, c square is just a constant and which we have taken as non-zero, so c can be taken out, in fact, 4 also you can take out you have u xi eta = 0, so this is the; this is why we always try to find out a new quadrant xx, where our equation can be modify in a simpler format. Now, here this wave equation utt = c square uxx is now reduced to u xi eta = 0 which is quite easy to sort and if you can solve this, if you integrate with respect to xi, and then eta, you will get the following formula that u xi eta = phi c xi + xi eta.

In fact, when you integrate with respect to eta, then u xi is = is a constant basically because it is 0, so if you look at the constant, then the constant will be; constant with respect to xi here, so you will get A xi here and then when you integrate this, you will get what? It is a indication of A xi + some constant with respect to eta, so B eta, you will get some kind of formula then, so we can write this as some function of xi and some function of eta.

Here phi and xi are some arbitrary function, now by replacing xi and eta because these are intermediate variables, so we can write xi and eta in terms of x and t, in fact, we already know xi and x– ct and eta as x + ct, so we can write our solution uxt as phi of x – ct + psi of x + ct of this equation that this utt = c square uxx, so this wave equation has the following form here, phi and psi are some arbitrary function and we can use initial and boundary condition along with this equation to find out these arbitrary variable that is phi and psi.

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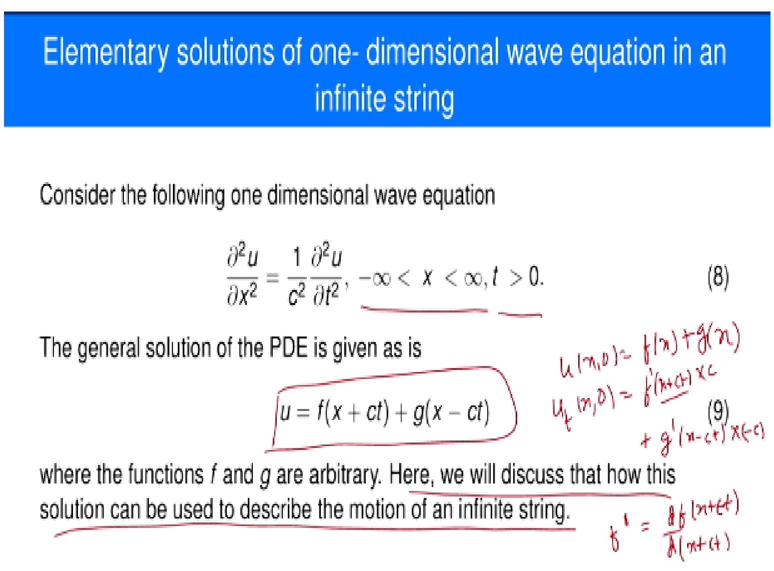

Now, so this is; this we have obtain using only the characteristics equation, so here now how we can utilise this to find out a solution of wave equation defined in a infinitely. So, consider the following one-dimension wave equation; dou of 2u/ dou x square = 1 upon c square dou 2u/ dou t square, here x is lying between – infinity to infinity and t is a time which is > 0 and at time t = 0, we have certain conditions.

So, we already seen that the general solution of the PDE is given as u = f of x + ct + g of x – ct, where f and g are arbitrary constant, here I have just taken f and g, in previous slide, we have taken phi and psi, so no problem because these are arbitrary constant which we have to say, remote to in order to that it satisfies certain given condition. So, here we will discuss that how this solution can be used to describe the motion of and an infinite string.

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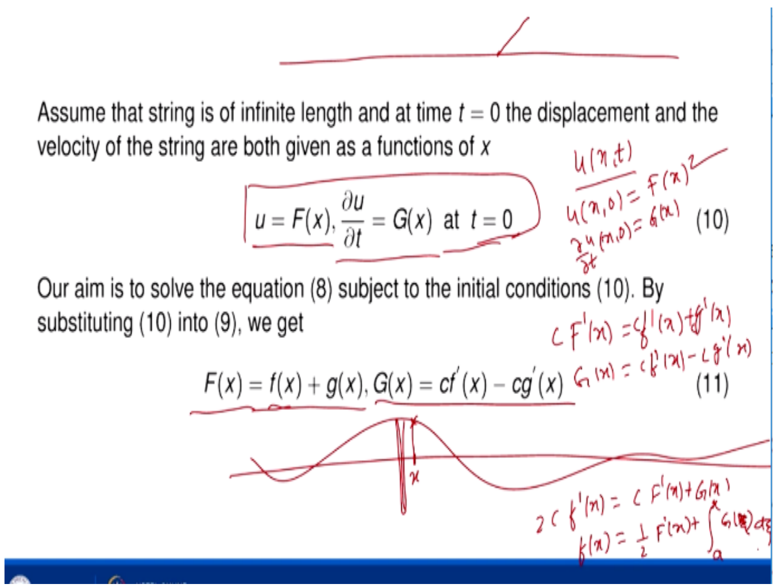

So, let us move here and suppose that the string is of infinite length, so here we have a string and at time t = 0, the displacement and the velocity of the string are both given as the function, so it means that at t = 0, it is not in an equilibrium position, in fact it is an disturb position, you have an infinite string, your take some point and just disturb this, so it means that it may not looking like this, it may be of something like this, may be this or any form.

So, here we say that this is your x axis and you simply say that at this point, your displacement is this and it may not be in a static position, it may have some velocity and to we say that at point x your displacement is given by f of x and at this point, it must; it may have some velocity of displacement, it may move like this, so it means that we have some velocity also, so we call it dou of u/ dou of t = g of x at t = 0.

At initial point t = 0, it may have displacement from the equilibrium position that is this x axis and it may have some velocity, we call this as g of x at t = 0, so this is the initial condition which we are talking about. Now, we want to find out a solution satisfying these initial conditions, so it means that you find out your small f and small g in terms of F and G. So, we already know that u = fx means, u is a solution; u is a function of x and t.

So, it means that what is given here that is u of x0 is F of X and dou u/ dou t means, dou u / dou t at x, 0 is your g of x, so using this, you put t = 0 in previous here, you should put t = 0, then ux0 will be what? U x0 is simply f of x g of x and you can write it here that f of x + gx is = ux0 that is f of x. Similarly, if you differentiate with respect to t, what you will get? Utx 0 is =; when you differentiate you will get fc f with respect to x + ct.

If you differentiate this, what you will get? In fact, let me to it, so here when you differentiate this, you differentiate fx + ct with respect to x + ct and then you differentiate x + ct with respect to t that is your c will come in to picture, similarly your g, so you will get g dash x - ct and then then you would differentiate x – ct with respect to -c that will give you –c, so derivative; this derivative f dash means d/d of x + ct of f; x + ct.

Because xf is a function of only x + ct, so f dash means derivative of f of x + ct with respect to x + ct and then you use chain rule and you differentiate x + ct with respect to t, so you will get utx0 as this, so and put t = 0. So, when you put t = 0, you will get c f dash x- c g dash x, so we have 2 relations and 2 unknown to remove, so with the help of this you need to find out small f and small g and since in second equation, we have derivative.

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By integrating the second of these relations, we get

$$f(x) - g(x) = \frac{1}{c}\int_{b}^{x} G(\xi)d\xi$$

where b is arbitrary.

From this equation and the first of the equations (11), we obtain the expressions for

$$f(x) = \frac{1}{2}F(x) + \frac{1}{2c}\int_{b}^{x} G(\xi)d\xi$$

$$g(x) = \frac{1}{2}F(x) - \frac{1}{2c}\int_{b}^{x} G(\xi)d\xi$$

So, we differentiate the first relation and you can say that f dah x = small f dash x + g dash x and then you write down c here and then you add, so in this way you can get f of x – g of x = 1 upon c, when we differentiate, okay let me do it, so here we have this and your Gx is = c f dash x – c g dash x and when you sum this up, we will cancel out, so we will get 2c f dash x = c f dash x + g of x but we want to find out fx, so we simply integrate it.

And when you integrate, you will get 1 upon 2 f dash f of x + integration G, now integration you are doing with respect to x, so let us take that it is starting from some arbitrary value, let us say a to some x here and G xi d xi, so that is what we have writing here that I can write fx - gx =n1 upon c b to x G xi d xi, here b is is just arbitrary constant. Now, similarly, you can simplify and you can get f as as 1/ 2 f of x +1 / 2c b to x G xi d xi.

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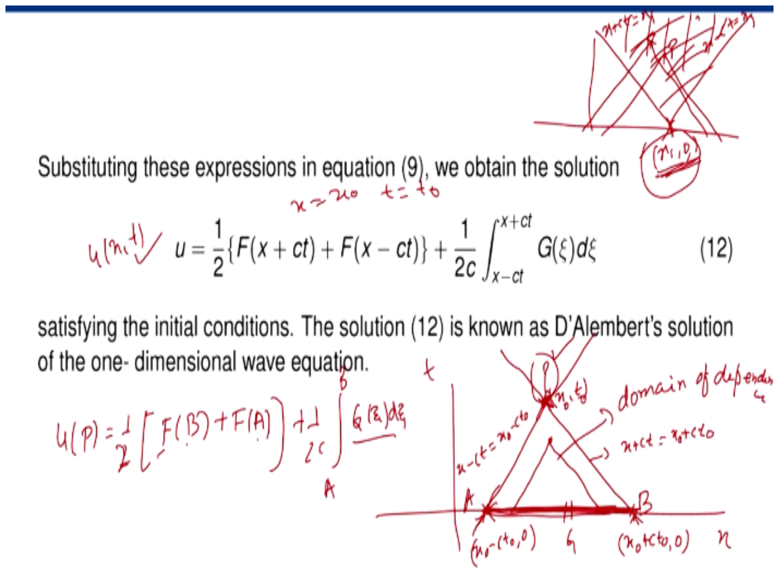

And gx as 1 upon 2 f of x -1 upon 2c b to x G xi d xi, so once we have arbitrary value, the value of arbitrary constant by assuming that your equation will satisfying initial condition that is u = f of; ux0 = f of x at t = 0 and utxo is = g of x and once we have f of x and g of x, then we can write down our relation, solution uxt as follows that uxt is = 1 upon 2 f of x + ct + f of x – ct + 1 upon 2c x – ct to x + ct G xi d xi here.

So, it means that at a point, you can now find out, so here u represent u of xt, so it means that solution u of xt is written as 1 upon 2 f of x + ct + f of - ct +1 upon 2 c x – ct to x + ct G xi d xi and here f and g is already known because it is given in terms of initial condition and this solution 12 is known as D'Alembert's solution of the 1 dimensional wave equation in fact, wave equation in infinite string,

Now, if you look at here, I can draw this here, here we have this x, here we have t axis and here we have the point x of x and t at which we want to find out the solution and at this point, if you draw your characteristics, so characteristics are what? X - ct and x + ct here, so x – ct, you can write it a line passing through this, so if it is, so it means here we have x – ct, let me write it point x0 t0, so x - ct = x0 – ct0.

So, here this is the point x0 – ct0, 0 and here this line is x + ct = x0 + ct0 and this point is given as x0 + ct0, 0,so it means that we look at the point say, x0 t0 and we want to find out our solution at this particular point, then we can draw 2 characteristics passing through this points, x0 t0 and one characteristic is this that is x – ct = x0 – ct0 and another characteristic is x + ct = x0 + ct0 and we say that it forms a kind of triangle, whose name is pab, where p is a point at which we want to find out the solution.

And a and b are points on x axis, so if you look at your equation number 12, I can look at the solution as u at p = 1 upon 2 and here F x + ct is a point here, so F of B + F of x - ct is this point F of A, we are looking at the solution at x0 t0, so everywhere it is x is replaced by x0 and t is replaced by t0, so I can write it 1 upon 2 FB + F of A +1 upon 2c, now, x - ct is basically you're A to B is G xi d xi, so it means that if you want to find out a solution at p, I must know the value of F at point A and B.

So, it means that F value I should know at these value and the value of G between A to B means, in this entire reason I need to know the value of G, so we say that any point from give you a reason, right and we say that it is the line AB on which your solution at point P depends and we call this domain as domain of dependence, so it means that if you take any point P, then the solution will depend on the data of F and G given on this line AB and the endpoint of AB.

So and one more important point that if you take any point in this triangle, say here then here if you find out a solution again your solution will depend something like this, so it means that if you take any point inside your triangle, your domain of dependence is a subset of this AB, so this reason; this domain we call as domain of dependence, so it means that solution of P is depending on the line AB and which you can obtain by drawing 2 characteristic line passing through this P.

And it will form a triangular region and the baseline is called domain of dependence and similarly, I can find out say, region of influence, if you look at the same feature again, so here we have this point P and we say that this will determine some kind of domain of dependence, now if you look at any point this, then I can have a say, characteristic line like this and characteristic line like this, right.

Here, this characteristic line, if you take this point as say, x1, 0, then this characteristic line is what? X + ct = x1 and this is what x – ct = x1, so we say that if you this make a triangular region, now what this reason is; we call this reason, this reason as a reason of influence of x1, 0, why? Because if you take any point in this region, right and draw your say, characteristic, then this point will come into picture.

So, it means that the solution in this reason, if you take any point in this region, the solution at that particular point is influenced by the point x1, 0, so we call this region; upper region, this region as the region of influence of this point x1, 0 because if you take any point in this region say, here and if you draw your domain of dependence that must contain the point x1, 0, so we say that the point on x axis will define a reason in which every point, if you take any point in this region, the solution must depend on x1, 0.

So, we call that this region is your region of influence of x1, 0 and similarly if you take any point here P, then it will give a triangular region and you say that the base will give you a domain of dependence that solution at P will depend on this base line, so we call this as domain of dependence and this region as region of influence. So, it means that that is the basic thing which we want to connect with these D'Alembert's solution of one-dimension wave equation.

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If the string is released from rest, $G \equiv 0$, then equation (12) reduces to

$$u = \frac{1}{2}\{F(x+ct) + F(x-ct)\} \qquad (13)$$

showing that the subsequent displacement of the string is produced by two pulses of shape $u = \frac{1}{2}F(x)$, each moving with velocity c, one to the right and other to the left.

Now, we will move forward and we say that if the string is released from rest, it means that the initial at time t = 0, if that string will not have any kind of velocity, it is just a static kind of thing that your; it is static and it may be displaced, right, then in this case, your solution is given by u = 1/2 F of x + ct + F of x - ct and if you look carefully, then it is what? It is basically it is this displacement at a given point x and t is now produced by 2 pulses of shape.

That is u = F of x + ct and f of x – ct, so basically these are 2 disturbances and when you join together, look at the resultant, then that will give you the shape at the point x, t, let us say that 1F of x + ct is moving in one side and other F of x - ct is moving the other side and we say that these are 2 disturbance moving in opposite direction and the resultant will give you the linear combination of 1/2 of F of x + ct + F of x – ct.

So, we say that basically the subsequent displacement of the string is produced by 2 pulses of shape; u = 1/2 F of x + ct and 1/2 F of x – ct, each moving with velocity c, one of the right to the right and other to the left, right and to understand this phenomena let us consider that what actually it is happening in the equation number 13.

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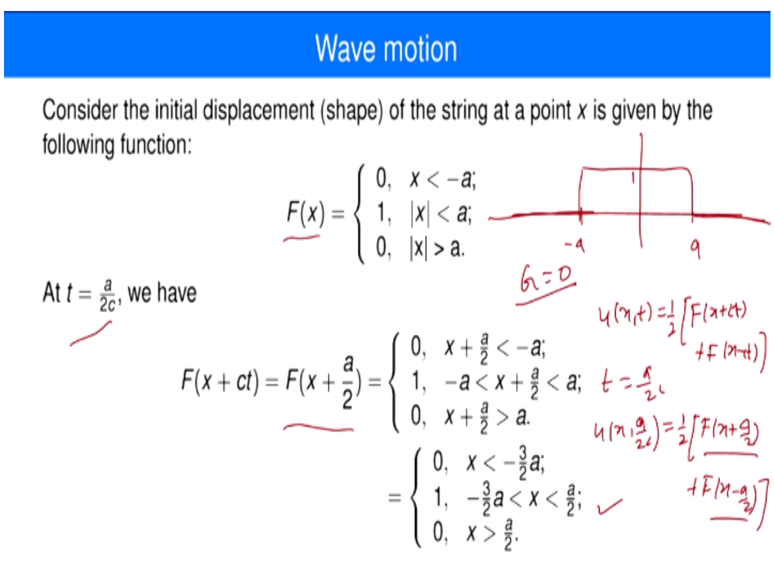

So, let us say that your initial shape is given by this, so you initial shape let us assume that here we have the point -a and here we have a and we say that your initial displacement is 0 and rest it is 0, so it means that your string is looking like this here we have it is infinite string, all other places between –infinity to -a and a to infinity it is lying on the rest but between –a to a, it is basically the one position.

And so, that is what we say that F of x is given a 0, x is < -a, 1 when mod of x is < a and 0 when mod of x is bigger than a, in fact x is bigger than a, so here these initial displacement given and we assuming that G is =0, it means that it will not have any velocity. Now, to look at here, let us look at some times like t = a/ 2c, so at time t = a/2c because this now it is fully to say, oscillate free to move.

So, when we are make it free to move, then we try to find out what should be the shape at different, different times. So, at t = 0 it has the same, now let us say that t = a/2c what should be the shape? So, here we know that at any point your shape is uxt = 1/ 2 F of x + ct + F of x – ct. Now, at t = a/2c, I want to write, so it means that ux a/2c will be =1/2 F of x + ct is a/2, so x + a/2 + F of x – a/2, I just want to find out the value of F of x + a2 and F of x – a2.

So, let us find X out x +; f of F of x + a2, so using this definition, I can write 0 when x + a/2 is < -a and 1 when x + a/2 lying between - a and a and 0 when x + a/ 2 is > a and when you can simplify, you will have this relation that F of x + a/2 is 0, when x is lying between -3/2 a and 1 between -3/2a to a/2 and when 0 is; when x is bigger than a/2.

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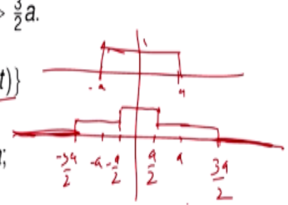
$$F(x-ct)=F(x-\frac{a}{2})=\begin{cases}0, & x-\frac{a}{2}<-a;\\ 1, & -a<x-\frac{a}{2}<a;\\ 0, & x-\frac{a}{2}>a.\end{cases}$$

$$=\begin{cases}0, & x<-\frac{1}{2}a;\\ 1, & -\frac{1}{2}a<x<\frac{3}{2}a;\\ 0, & x>\frac{3}{2}a.\end{cases}$$

so at $t=\frac{a}{2c}$, the shape is $u=\frac{1}{2}\{F(x+ct)+F(x-ct)\}$

$$=\frac{1}{2}\{F(x+\frac{a}{2})+F(x-\frac{a}{2})\}=\begin{cases}0, & x<-\frac{3}{2}a;\\ \frac{1}{2}, & -\frac{3}{2}a<x<-\frac{1}{2}a;\\ 1, & -\frac{1}{2}a<x<\frac{1}{2}a;\\ \frac{1}{2}, & \frac{1}{2}a<x<\frac{3}{2}a;\\ 0, & x>\frac{3}{2}a.\end{cases}$$



Similarly, you can find out the value of F of x – a/2 and you can simplify and you can have this that 0 when x is lying between minus -1/2a and 1 between -1/2 a to 3/2 a and 0, when x is bigger than 3/2a and what we want to write; at t = a/2 c, the shape u is given by some of these 2 and if you want to draw what you will get that your initial; let me write it here, this will give you the initial step, so that is –a to a.

So, here –a to a and here it is -a and here we have -3 a/2, here –a, here it is a/2; -a/2 bite, a, 3a/2, so let us look at the shape; the shape is right now is this, it is one here. Now, here 0 between x, so till here it is 0 and here also it is 0 and between -3/2 a2 -1/2a, it is ½, so let us say that this disturbance is 1/2 and then between –a/2 to a/2; -a/2 to a/ 2 it is 1, so let us say that it is 1 and then again it is this thing.

So, now your disturbance is now move like this that between -a/2 to a/ 2, it is 1, and between -3/a to –a/2 is just 1/2 and similarly here, so it is kind of a symmetric shape we are moving, now it is the time at t = a/ 2c.

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At $t = \frac{a}{c}$, we have

$u(x,t) = \frac{1}{2}\left[F(x+a) + F(x-a)\right]$

$$F(x+ct) = F(x+a) = \begin{cases} 0, & x+a < -a; \\ 1, & -a < x+a < a; \\ 0, & x+a > a. \end{cases}$$

$$= \begin{cases} 0, & x < -2a; \\ 1, & -2a < x < 0; \\ 0, & x > 0. \end{cases}$$

$$F(x-ct) = F(x-a) = \begin{cases} 0, & x-a < -a; \\ 1, & -a < x-a < a; \\ 0, & x-a > a. \end{cases}$$

$$= \begin{cases} 0, & x < 0; \\ 1, & 0 < x < 2a; \\ 0, & x > 2a. \end{cases}$$

Now, look at here; now look t = a/c, then t = a/c, your uxt will be what; uxt is = 1/2 of F of x + ct; x + ct is a, so F of x + a + F of x –a, so I need to find out the value of x + a; F of x + a and F of x – a, so using the same technique we can find out F of x +a, is this, 0, when x is < -2/ a and 1 between –2a to 0 and 0 when x is bigger than a, similarly F of x - a also.

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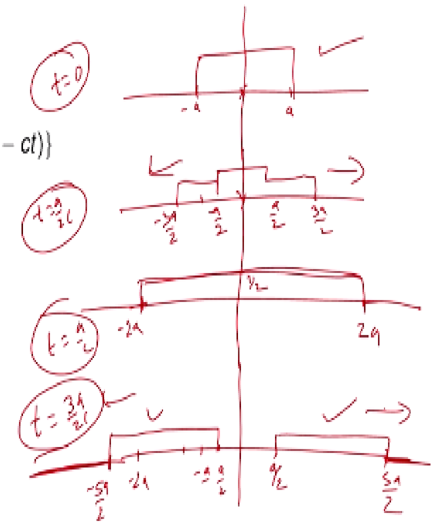
so, at $t = \frac{a}{c}$, $u = \frac{1}{2}\{F(x + ct) + F(x - ct)\}$

$= \frac{1}{2}\{F(x + a) + F(x - a)\}$

$$= \begin{cases} 0, & x < -2a; \\ \frac{1}{2}, & -2a < x < 2a; \\ 0, & x > 2a. \end{cases}$$



And when we can write uxt, I can write it here that it is 0, when x is < -2 a and 1/2 when -2a to 2a and 0 when; so let me again repeat the same thing, so here it is t = 0, t = a/2c and t = a/c, so it is -a to a, rest it is 0 and here it is say, here it is this and -3a/ 2 to 3a/2 and –a/2 to a/2, so this we have already seen. Now, here at t = a/ c between -2a to + 2a, it is 0 here, here also and between -2a to a, it is 1/2 only, so it is now reduced to 1/2.

So, it means that initially shape is this, now when your string is vibrating then your disturbance is trying to move away from the initial position that is from this point and it is moving towards this and this, so we say that it is your magnitude is now divided into parts and one is moving here and one is moving here. So, at t = a/c, now it is almost moved here, now let us consider one more point that is t = 3a/ 2c

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At $t = \frac{3a}{2c}$, we have

$$F(x+ct) = F(x+\frac{3}{2}a) = \begin{cases} 0, & x+\frac{3}{2}a < -a; \\ 1, & -a < x+\frac{3}{2}a < a; \\ 0, & x+\frac{3}{2}a > a. \end{cases}$$

$$= \begin{cases} 0, & x < -\frac{5}{2}a; \\ 1, & -\frac{5}{2}a < x < -\frac{a}{2}; \\ 0, & x > -\frac{a}{2}. \end{cases}$$

$$F(x-ct) = F(x-\frac{3}{2}a) = \begin{cases} 0, & x-\frac{3}{2}a < -a; \\ 1, & -a < x-\frac{3}{2}a < a; \\ 0, & x-\frac{3}{2}a > a. \end{cases}$$

$$= \begin{cases} 0, & x < \frac{1}{2}a; \\ 1, & \frac{1}{2}a < x < \frac{5}{2}a; \\ 0, & x > \frac{5}{2}a. \end{cases}$$

And we want to see what should be the behaviour here, so to find out let us say t = 3 a/ 2c and here we express it is F of x + 3/2a, and we have calculated it like this F of x - ct that is F of x – 3/ 2a and we have calculated like this.

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so, at $t = \frac{3a}{2c}$, the shape of the string become

$$u = \frac{1}{2}\{F(x+ct) + F(x-ct)\}$$

$$= \frac{1}{2}\{F(x+\frac{3}{2}a) + F(x-\frac{3}{2}a)\}$$

$$= \begin{cases} 0, & x < -\frac{5}{2}a; \\ \frac{1}{2}, & -\frac{5}{2}a < x < -\frac{1}{2}a; \\ 0, & -\frac{1}{2}a < x < \frac{1}{2}a; \\ \frac{1}{2}, & \frac{1}{2}a < x < \frac{5}{2}a; \\ 0, & x > \frac{5}{2}a. \end{cases}$$

Then we can write down at t = 3a/ 2c, the shape of this string becomes u = 1/2 of F of x + ct + F of x – ct, which is nothing but 1/2 F of x + 3/2a + F of x -3/2 a and 0 between say 0 when x is < -5/2 a and 0 when x is > 5/2 a and between -5/2 t0 –a/ 2, it is 1/2 let me draw it here, so -5 pi/2 somewhere here, -5pi/2, -2a and somewhere here it is –a/2, somewhere here –a/2 and here it is a/2 and somewhere here, it is 5 pi/2, so it is -5/2 is 0, no problem.

And then between -5 pi/2 to -1/2a, it is 1/2 so between here to – a/2, it is 1/2 similarly, here a/2 to 5 pi/2, it is 1/2 and sorry, here then 0 between -1/2 to 1/2 and again 1/2a to 5 pi/2, it is 1/2 and so on, so I can draw like this whatever we have drawn is at t = 3/2c and t = q/2, now if you look at carefully, what is happening, so basically it is initial disturbance and disturbance is created say, symmetric about this.

Then at as time passes from 0, then this disturbance is now trying to distribute into 2 equal parts and trying to move away from each other and if you look at; if you look at the areas is a still 2a, right that you can calculate and when you look at t = a/c, your, it is spread in a longer period but is still area is your 2a only and if you look at in at t = 3a/ 2c, now this disturbance is say, move apart and now it is distributed into 2 equal disturbance, one is moving this side and one is moving this side, is that okay.

So, here you can say that here the initial disturbance is now distributed into equal disturbance and area will remain the same, it means that disturbance is now distributed exactly into 2 parts and one is moving this side and one is moving other side, so we can say that here it is kind of a wave propagating in 2 different directions and this you can see very easily, when you have some kind of pond or some kind of gathering a water.

And if you put some kind of stone, then you will see that there is some kind of vibration coming to be; some kind of vibration is coming and these vibrations are moving from the point where your stone fall, so from the point of disturbance, it will try to move in all the directions and moving away from that and it is looking like a wave that wave is moving from that particular point of disturbance.

And since we are considered only as a one dimensional, so here we have only left hand side and right hand side, so here it is the disturbance is moving in opposite direction and having the equal kind of behaviour, so we call this kind of behaviour as wave, so we can say that here F of x = ct is 1/2 wave and in this other 1/2 wave, so we say that here because of this we can say that it is the sum is; sum of 2 equal disturbance or say, wave.

One wave is moving right and one wave is moving left, so here we have seen that we had taken a particular example to show that solution is behaving like a wave, how it disturbance is; say, distributed in 2 equal part and moving away to each other, so here I will stop and so in this lecture, what we have discussed; we have discussed the use of characteristic lines to find out the solution of an infinite string and that solution we call as D' Lambert's solution that is given by the following thing.

That uxt = 1/2 F of x + ct + F of x - ct +1 upon 2c x - ct to x + ct is a design and we have discussed domain of dependence and reason of influence and in continuation, we will discuss some more kind of a domain for example, here we have considered an infinite string, what happen if this string is not infinite, in fact it is a semi-infinite or finite or some kind of 2-dimension region that also we want to discuss and that will discuss in coming lectures. So, here I will stop, thank you for listening us, thank you.