Ordinary and Partial Differential Equations and Applications

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Lecture - 50
Laplace Equation - II

Hello friends. Welcome to my lecture on Laplace equation. This is second lecture on Laplace equation. In the previous lecture, we had considered 2-dimensional Laplace equation in Cartesian coordinates and polar coordinates and we did one example on polar coordinates in that lecture. Now in this lecture, we begin with another example on Laplace equation in polar coordinates.

And then we shall be doing Laplace equation in 3 dimensions and there we shall be considering Laplace equation in cylindrical polar coordinates. So let us begin with the Laplace equation, example on Laplace equation in polar coordinates.

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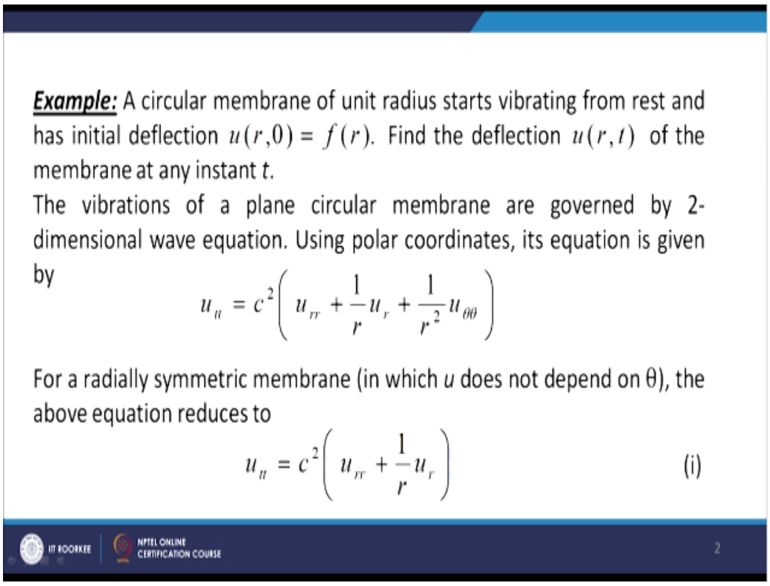

Let us take up the case of a circular membrane of unit radius starts vibrating from rest and has initial deflection u r, 0=f r. We have to find the deflection u r, t of the membrane at any instant t. Now the vibrations of a plane circular membrane are governed by 2-dimensional wave equation. Using polar coordinates as you know, its equation is given by u tt=c square u rr+1/r ur+1/r square u theta theta.

Now for a radially symmetric because circular membrane that we are considering is the radially symmetric membrane, so here u does not depend on theta and therefore the above equation reduces to this term containing u theta theta will become 0 and we shall have u tt=c square u rr+1/r ur.

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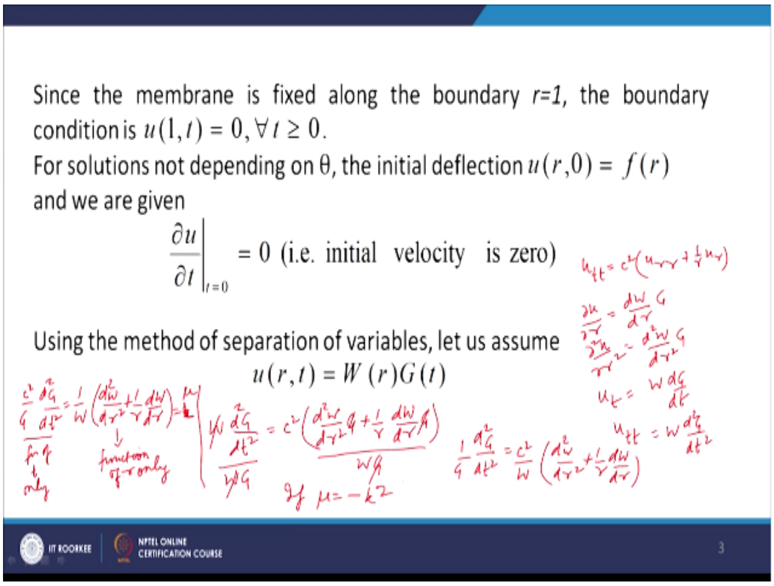

Now since the membrane is fixed okay since the membrane is fixed along the boundary r=1, the radius of the membrane is given=1, so the membrane is fixed along the boundary r=1 and therefore the boundary condition is u 1, t=0 for all t>=0. Now for solutions not depending on theta because we have to find a solution which is not depending on theta the initial deflection given is u r, 0=f r.

We are given u r, 0=f r and we are given that it starts initially at rest, it starts vibrating from rest, so we have initial velocity=0 that is dou u/dou t at t=0, now let us apply the method of separation of variables so using the method of separation of variables let us assume that u r, t can be written as a function in r that is W r* a function in t that is G t and then we put it into the differential equation u tt=c square u rr+1/r ur.

So we put it in here u tt=c square times u rr+1/r ur. Now when you find partial derivative of u with respect to r what you get is dW/dr*G and when you find second order derivative you get d square W/dr square*G and when you find partial derivative with respect to t that is ut what you get is W*dG/dt and u tt then will become W*d square G/dt square. So let us put these values here in the equation, this equation okay and then divide the equation by WG okay.

What we will get is a function we will have the following, see let me put it here so u tt=W*d square G/dt square and the right hand side will be c square*u rr so c square*u rr means d square W/dr square*G+1/r dW ur, ur=dw/dr*G okay. Now both sides we divide by WG okay. When we divide by WG what do you notice, this W will cancel you will get d square G/dt square=c square times this G will cancel with this G.

And this G we will have 1/W d square W/dr square+1/r dW/dr okay or we can write it as 1/c square here 1/G is also there, so we have c square/G*d square G/dt square=1/W d square W/dr square+1/r dW/dr. Now you can see left hand side is a function of t alone, this is the function of t only while the right hand side is the function of r only, r and t are independent variables, so they can be equal provided they are equal to a constant okay.

So let us take the constant to be k then we have 2 situations, we are taking k= or you can say let me take here instead of k you can take another constant say mu okay.

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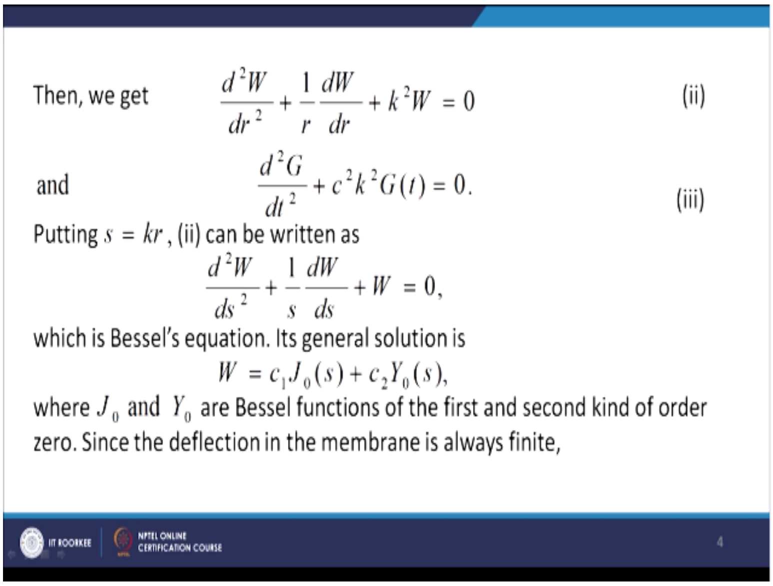

So if you take mu=-k square gives you these equations d square W/dr square+1/r dW/dr+k square W=0. The other equation is d square G/dt square+c square k square G t=0. Now why we have taken –k square, mu=-k square because if you take there can be 3 possibilities mu is 0, mu is>0 and then mu is<0.

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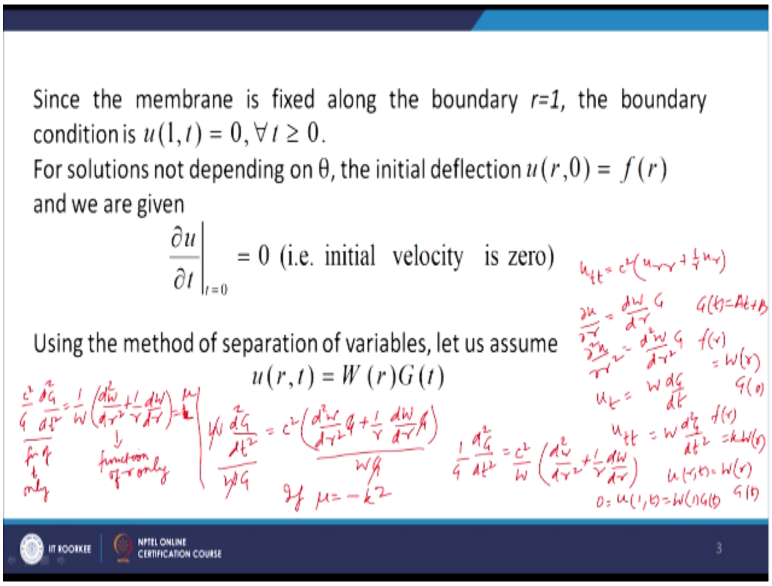

If you take mu=0 then d square G/dt square will give you G t=At+B okay, at t=0 we have f r okay, u=f r we are given, u r, 0=f r so f r will be=now you will have W u r, t=W r*G0 okay. Now Wr=solution in r so this means G0 must be 0, Wr cannot be 0 otherwise f r will be 0 so dr G0=0 and G0=0 so this means that f r=Wr*G0 so G0 must be some constant okay. So this means at f r=some constant times Wr okay and when we have u 1, t=0 so u 1, t=0 means we have u r, t=Wr*Gt okay.

So when you put r=1 what you get u 1, t and u 1, t=0, this is = W1*GT okay. So there what do we get Gt cannot be 0 otherwise u will be 0 so W1=0, so what we get is these situations mu=0 and mu>0 that is mu=some constant k square, they lead us to solutions which are not possible where the boundary conditions that we are taking do not provide us solution, they provide us trivial solution.

So what we do is we take mu=-k square and when we take mu=-k square we get these 2 equation, so putting s=kr in this equation what we get is this equation 2 can be written as d square W/ds square+1/s dW/ds+W=0 and this equation we know is a Bessel's equation.

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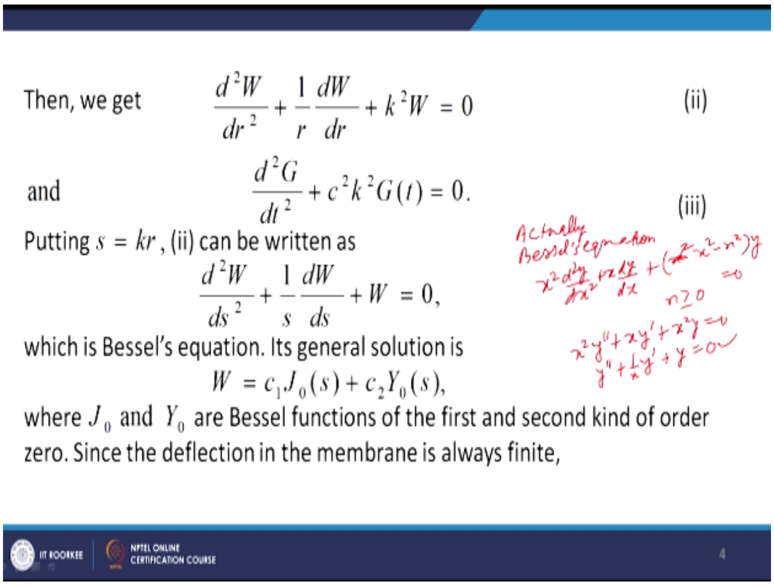

Actually, the Bessel's equation is of this form. Bessel's equation is x square d square y/dx square+x times dy/dx+x square-n square*y=0 and n is>=0, n>=0 is a parameter so here what is happening is you take n=0 then you get x square y double dash+xy dash+x square y=0 or you get y double dash+1/x y dash+y=0. So this equation is of this form where we are taking n=0 and that is why its general solution is W=c1 J0 s+c2 Y0 s where J0 and Y0 are Bessel functions of first and second kind of order 0.

Now we are given that the deflection in the membrane is always finite okay, so Y0 become infinite as s approaches 0 and therefore what we will have c2 will be taken=0.

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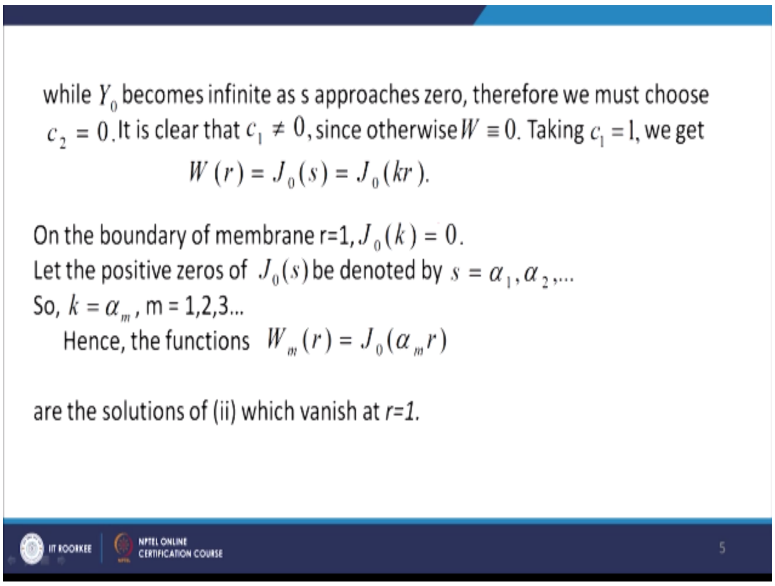

So c2 becomes 0 and then what we have W=c1 J0 s, now this W=c1 J0 s the constant c1 will be absorbed while we write the general solution u r, t=Wr*G t. So this even will be absorbed in the constants which will come in dt part so we can take without any loss of generality c1=1, so we have Wr=J0 s, so J0 s=J0*J0 kr because we are taking s=kr, now on the boundary of the membrane r=1 and so what we have J0 k on the boundary of the membrane r=1 so J0 k=0.

Because on the boundary, we know that u 1, t=0 okay u 1, t=0 means u 1, t is W1*G t okay.

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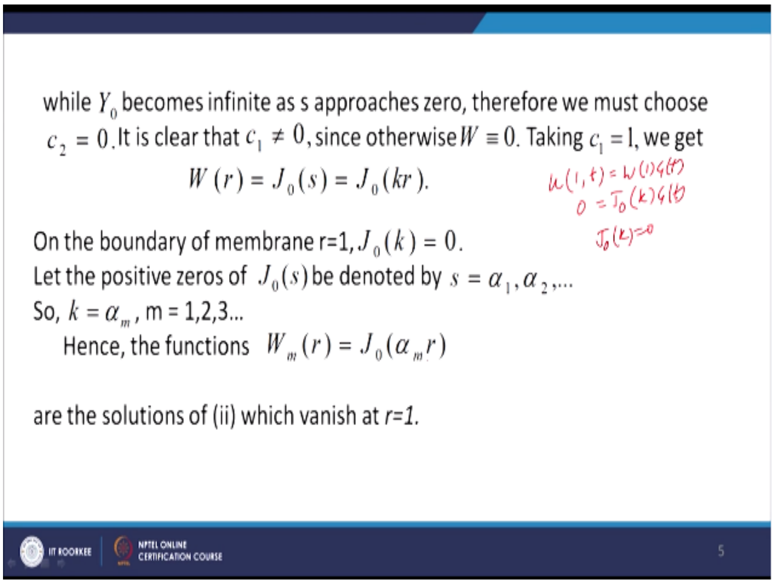

Now u 1, t=W1*G t and W1=J0 k*G t, so either J0 k is 0 or G t=0 if G t=0 then u r, t will be 0 so trivial solution we will have and therefore we must take J0 k=0 and J0 k=0 means the 0 is of the valid Bessel function of order 0. So let the positive 0s of J0 s be denoted by s=alpha 1, alpha 2 and so on so then k=alpha m where m=1, 2, 3, and so on. Hence, the functions Wm r=J0 alpha m r, for each value of the root of J0 s=0 we will have a Wr function.

So we write Wm r=J0 alpha m r. They are the solutions of the equation 2 this equation, which vanish at r=1. Now hence the general solution of the equation 1 satisfying the boundary conditions is now this part d square G/dt square+c square G t=0 is here we have the auxiliary equation m square+c square k square=0 okay.

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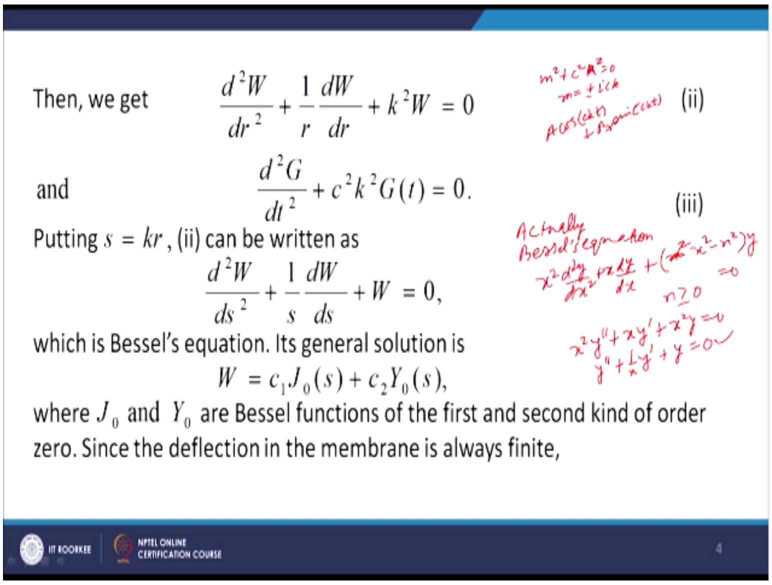

So m=+-i*ck okay and so its general solution is of the type cos some constant*cos ckt+some constant A times cos ckt+B times sin ckt.

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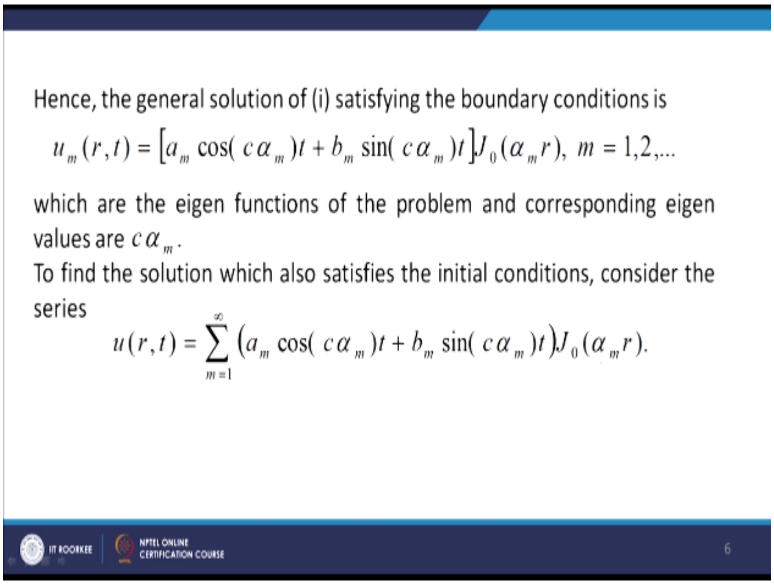

And we write that as in this form okay, am cos c alpha m t+bm sin c alpha m t*J0 alpha m r because here k becomes alpha m so we write it as cos c alpha m t and the constants A and B which we take in the solution corresponding to this equation, equation number 3 they will change with each value of m so we write them as am and bm okay. So now this um r, t are then the eigen function of the problem and the corresponding eigen values are c alpha m.

Now to find the solution which also satisfy the initial conditions let us consider the series u r, t=sigma m=1 to infinity am cos c alpha m t+bm sin c alpha m t J0 alpha mr, m takes values from 1 to infinity.

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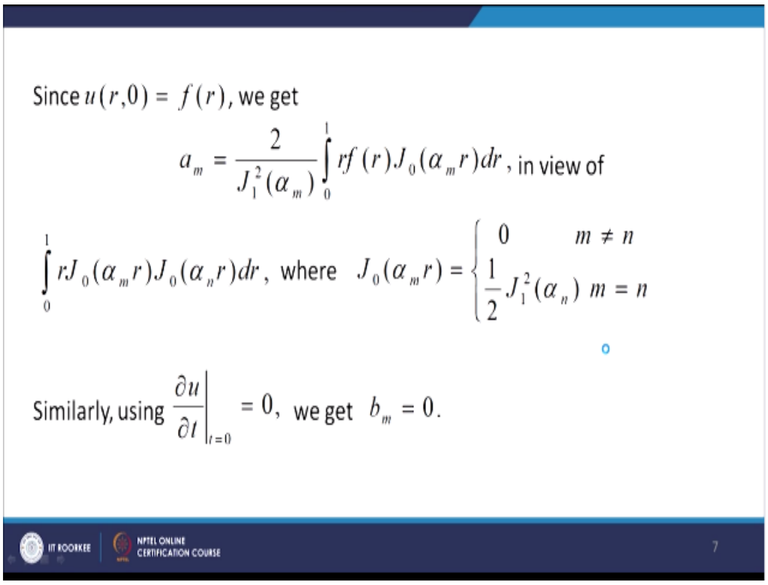

Now here let us use the initial condition u r, 0=f r, so when you put t=0 what do we get here, this part becomes am, this part becomes 0 so we get sigma m=1 to infinity am J0 alpha mr.

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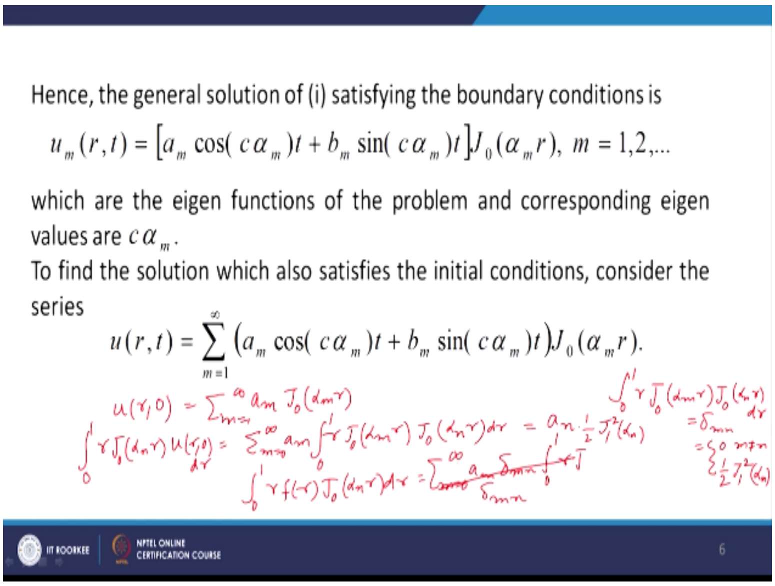

And what we do is that this is putting t=0 ur=0=sigma m=1 to infinity am times J0 alpha mr then we multiply both sides with r times J0 alpha n r and then integrate with respect to r over the interval 0 to 1 okay. So u r, 0 is given to be=f r, u r, 0 is given to be f r so let us put its value there and we will get it as 0 to 1 r f r J0 alpha nr dr=summation m=0 to infinity, now am here we have integral 0 to 1 r J0 alpha mr J0 alpha nr.

We know that Bessel functions are orthogonal with respect to the weight function r so J0 alpha mr J0 alpha nr they are orthogonal to each other with respect to the weight function r so we have the value of this integral is 0 when m!=n and when m=n we write delta mn okay integral 0 to 1 r J0 let us put it like this we have delta mn okay let me write it as this is okay 0 to 1 r J0 alpha m r J0 alpha n r dr=delta mn which is=0 when m!=n okay.

And which is equal to when m=n we will have 0 to 1 r J0 alpha n square or J0 square alpha n r so that value will be 1/2 J1 square alpha n okay. So this will be equal to I can write it as okay so 0 to 1 this is = delta mn so what we will get this will be = m=n so an times 1/2 J1 square alpha n okay so we can get the value of an.

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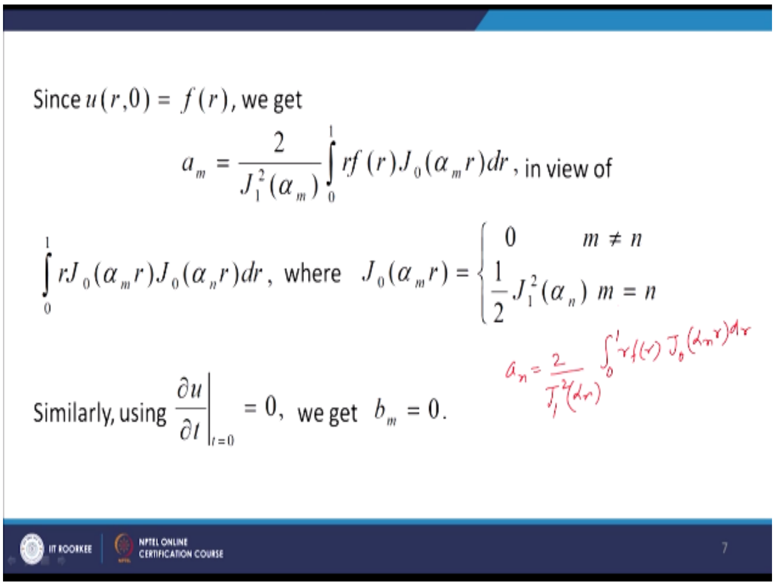

And thus an=you see an=2/J1 square alpha n integral 0 to 1 r f r*J0 alpha n r dr okay. Now n is any integer from 1 to infinity, so we can replace n by m and therefore am=2/J1 square alpha m*integral 0 to 1 r f r J0 alpha m r dr in view of the orthogonal property of this integral 0 to 1 r J0 alpha m r J0 alpha n r which is=0 when is m is !=n and 1/2 J1 square alpha n when m=n.

Since similarly using the partial derivative of u with respect to t at t=0 what we get b m=0, you see we have now this, you differentiate with respect to t so we will have here am-sin c alpha m*t then c alpha m and then bm cos c alpha m*t*c alpha m and J0 alpha m r.

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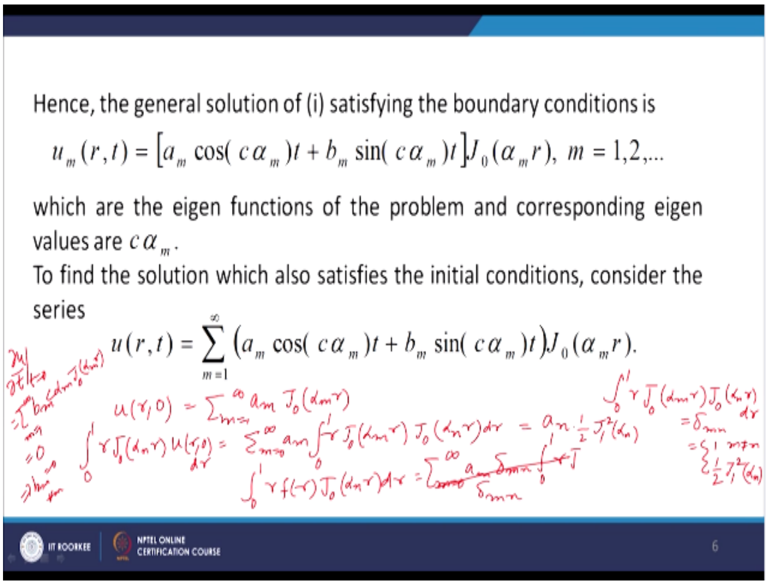

So then when you differentiate this with respect to t and put t=0 what we get cos term will become sin and so when we will put t=0 this time this term will vanish and here we will get sigma m=1 to infinity bm cos c alpha m*t*c alpha m. So cos c alpha m*t will become 1 so bm c alpha m J0 alpha m r and delta/delta t this is = 0 because we start with the vibrating with rest so this is = 0 so this imply that bm=0 for all m.

And so we get u r, t=sigma m=1 to infinity am cos c alpha m t +bm sin c alpha m t, bm becomes 0 so am cos c alpha m t*J0 alpha m r where am is given by this expression.

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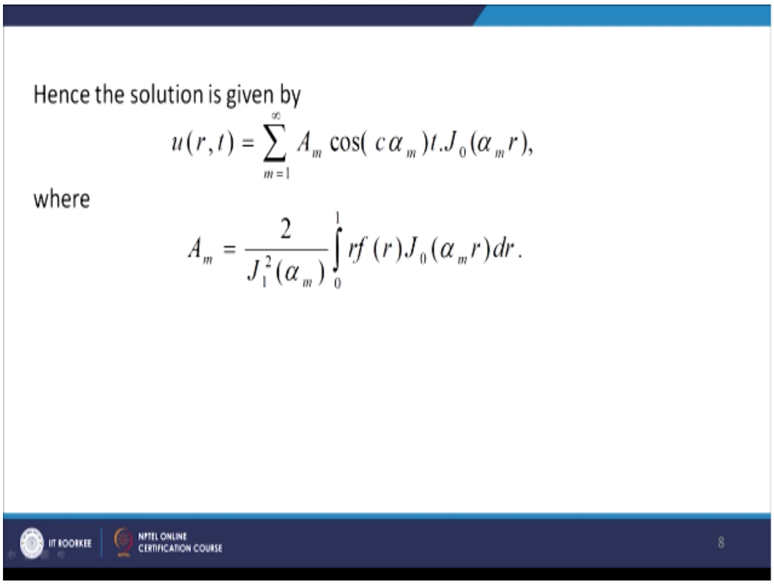

So what we get is u r, t=sigma m=1 to infinity Am cos c alpha m t J0 alpha m r where m is given by 2/J1 square alpha m 0 to 1 rf r J0 alpha m r dr. So we can omit this one, this is how we solve this problem.

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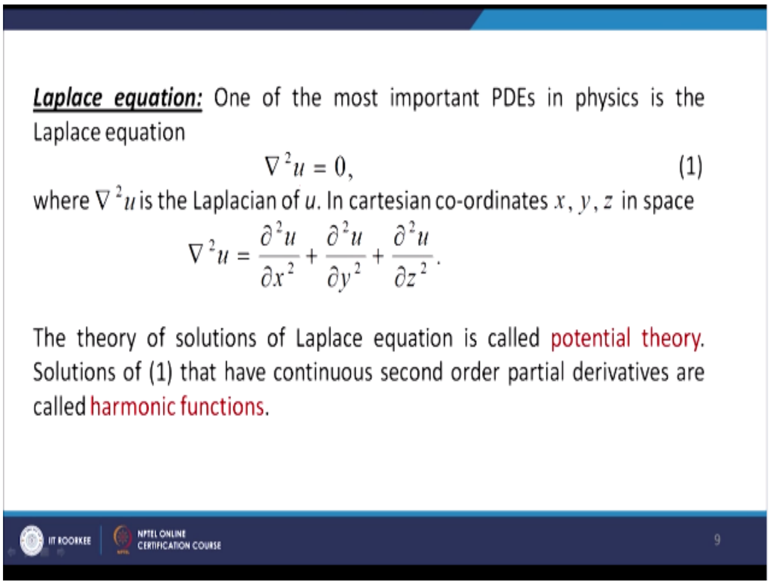

Let us go to Laplace equation in 3 dimensions. So one of the foremost important partial differential equation in physics is the Laplace equation del square u=0 where del square u is the Laplacian of u. In Cartesian co-ordinates x, y, z in space del square u is dou square u/dou x square+dou square u/dou y square+dou square u/dou z square. The theory of solutions of Laplace equation is called potential theory.

And the solutions of this equation 1 that have continuous second order partial derivatives, they are called as harmonic functions.

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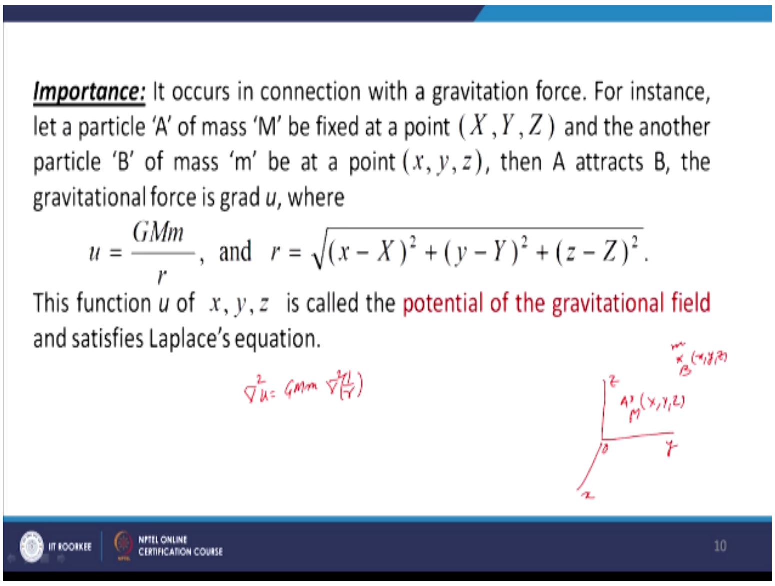

Now this Laplace equation occurs in connection with gravitational force. For instance, let us take a particle A of mass M which is fixed at a point X, Y, Z so let us say we have a point here x, y, z and particle is fixed at this point whose mass is m okay and another particle B is here at the point x, y, z of mass m okay, here we have mass to be M, so then this point in the particle at A at rest the particle at B okay and the gravitational force is grad u where u is GMm r/r, r is the distance between the point A and the point B okay.

So x-X whole square+y=Y whole square+z-Z whole square, this function u of x, y, z okay. This function u of x, y, z is called the potential of the gravitational force and satisfies Laplace equation. We can show that del square u=GMm is a constant, so we have del square 1/r and del square 1/r=0.

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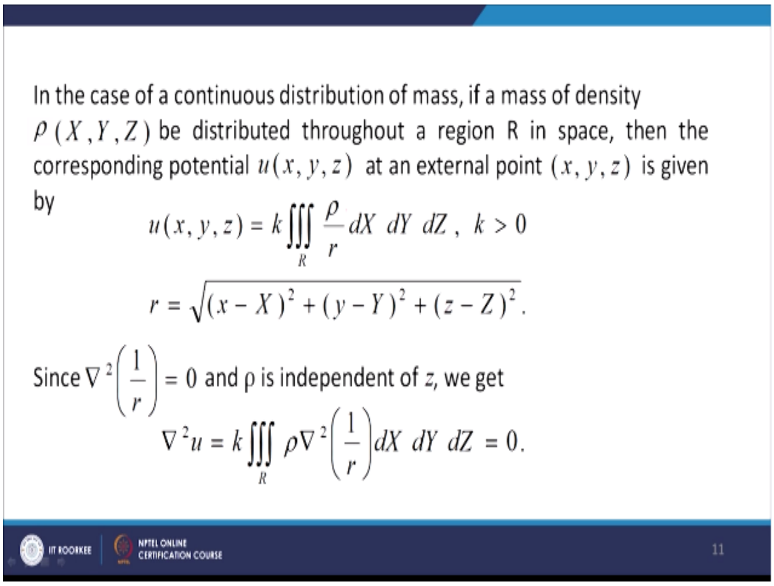

So in the case of a continuous distribution of mass if a mass of density rho X, Y, Z be distribution throughout the region R in space then the corresponding potential u x, y, z at an external point x, y, z is given by u x, y, z=k the volume integral over the region R and rho/r dX dY dZ. Now here we have r=under root x-X whole square+y-Y whole square+z-Z whole square.

Since del square 1/r=0 and rho is independent of x, y, z we get del square u=k times integral over the region r*rho del square 1/r dX dY dZ=0.

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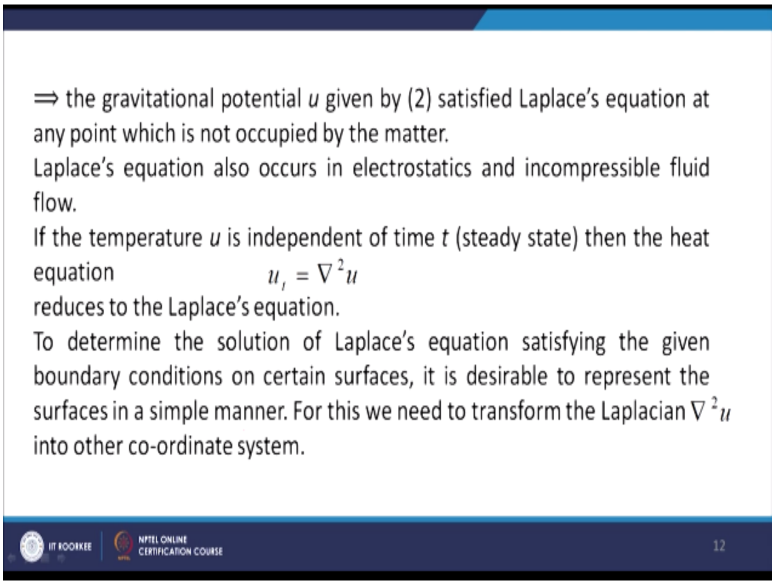

Now this implies that the gravitational potential u given by 2 satisfies Laplace equation at any point which is not occupied by the matter. Laplace equation also occurs in electrostatics and incompressible fluid flow. If the temperature u is independent of time that is we are in the steady state, then the heat equation ut=del square u reduces to the Laplace equation. Now to determine the solution of Laplace equation satisfying the given boundary conditions on certain surfaces, it is desirable to represent the surfaces in a simple manner.

For this we need to transform the Laplacian del square u into other coordinate systems.

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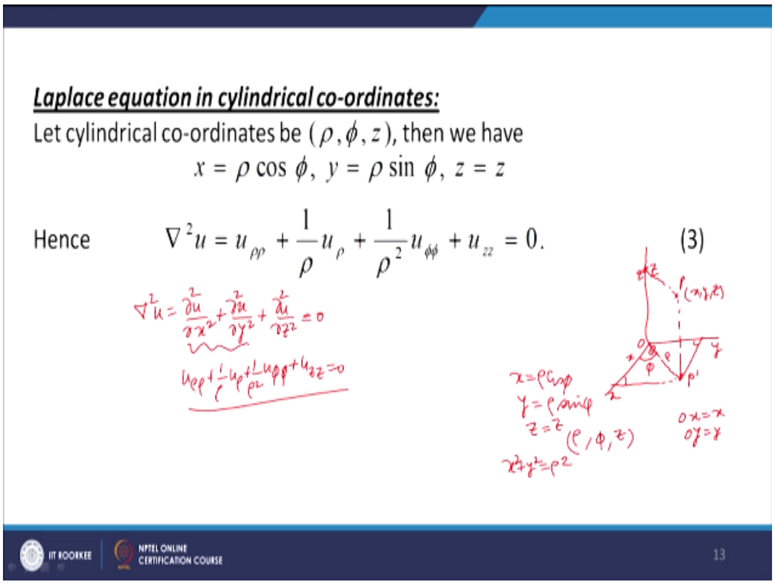

So let us look at the Laplace equation in cylindrical coordinates. The cylindrical coordinates we are taking as rho, phi, z. If you take any point x, y, z in space draw the perpendicular from the point is let us say this is p on to the x-y plane so this is p dash join o to p dash, draw through p dash lines parallel to x and y axis okay. Then, this angle is pi/2, this angle is also pi/2 and what we have this is your rho okay.

Op dash is rho and the angle that op dash makes with x axis this is phi okay. Now this is x, y, z so what we have ox, ox=x and oy=y okay and draw through p line parallel to this op dash, so this is your oz okay. Then, what happens this is your oz now what do we have, this x this is rho so x=rho cos phi and y=rho sin phi and z=z, so the co-ordinates x, y, z are related to the cylindrical, these coordinates are called cylindrical coordinates, rho phi z.

They are called as cylindrical coordinates and they are related to the Cartesian coordinates x, y, z by the relations x=rho, cos, phi; y=rho, sin, phi; z=z and so from here we can see x square+y square=rho square okay. Now we have the del square u=this is del square u in Cartesian coordinate del square u=0 and we have seen that when we change from Cartesian coordinates x, y to the polar coordinates r, theta this u xx+u yy becomes u rr+1/r ur+1/r square u theta theta.

So instead of r theta now here we have rho and phi, so we get for this part u rho rho+1/rho u rho+1/rho square u phi phi okay and z does not change, z remains the same so we get u zz=0. So this is Laplace equation in cylindrical polar coordinates.

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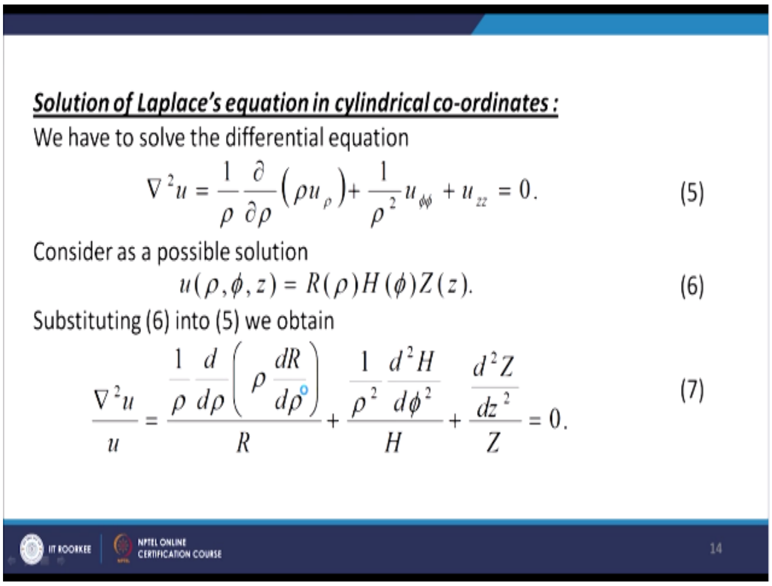

Now let us see how we solve this equation in cylindrical polar coordinates. Again by the method of separation of variables, we will solve it.

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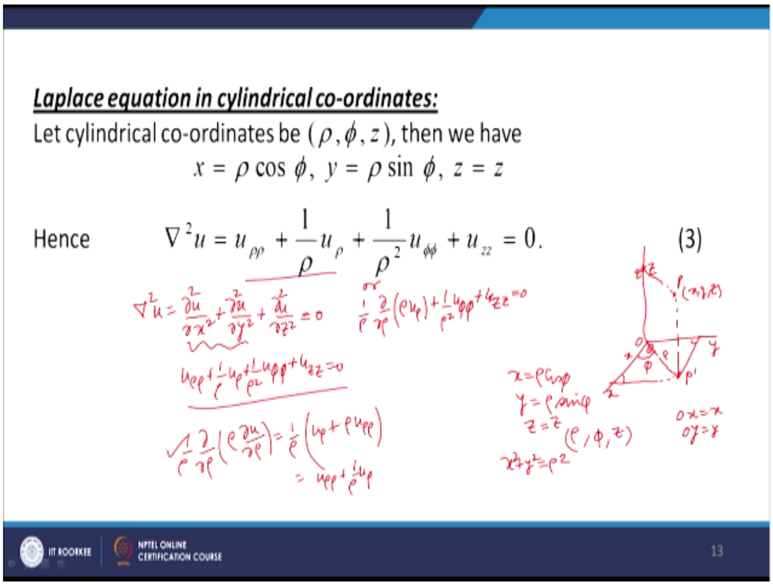

So we have to solve the equation, we can write this equation you can see we can write this equation also as 1/rho if I multiply the partial derivative of u rho with respect to rho and multiply 1/rho then what do we get, this is = 1/rho times derivative of rho with respect to rho is 1 so we get u rho and then we get rho u rho rho. So what we get here u rho rho+1/rho u rho.

So u rho rho+1/rho u rho this part of the Laplace equation in cylindrical polar coordinates can also be expressed as 1/rho del/del rho rho u rho okay. So I can also write it as 1/rho so this+1/rho square u phi phi+u zz=0 this is another way of writing this equation okay. So what we do is we have 1/rho del/del rho rho u rho+1/rho square u phi phi+u zz=0. Now let us consider a possible solution r rho*H phi*zz okay.

R is a function of rho only, H is a function of phi only, Z is a function of z only. When we substitute this solution u rho, phi, z into this equation and divide by RHZ then what do we get del square u/u, u is RHZ, so we will get 1/rho d/d rho rho dR/d rho/R+1/rho square d square H/d phi square/H+d square Z/d z square/Z=0. Now what we can do, this term and this term let us say this is term 1 okay this is term 2 okay.

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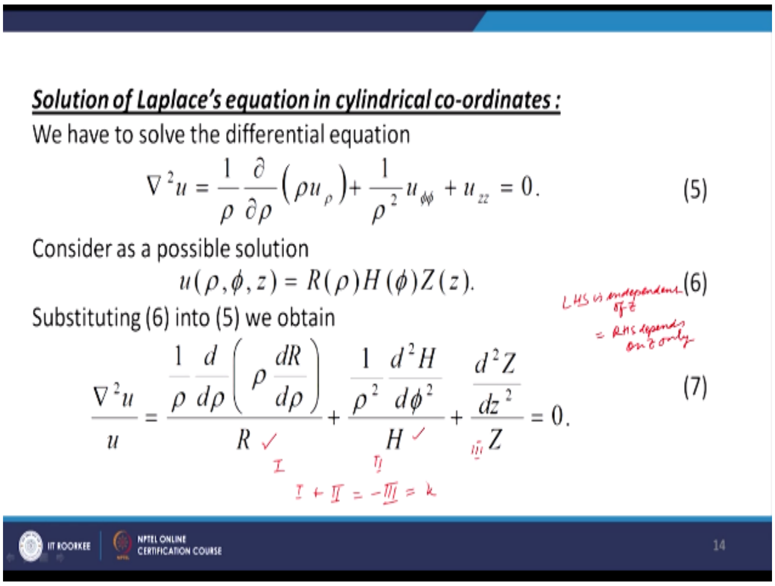

So if you write I+II and this is III okay=-III written term I and II on the left, take term III to the write okay then you have the left side depends only on rho and phi, left hand side is independent of Z okay. Right hand side depends on Z only okay, left hand side is a function of r and phi which are independent of Z and r and phi themselves are independent with each other, so each side will be equal to a constant so I+II=-III=some constant we have.

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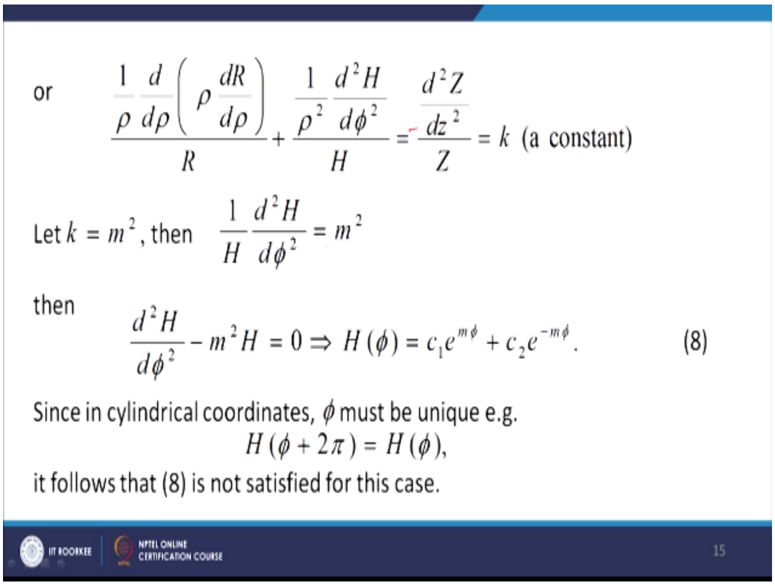

Now let us stake so this I have written this is the first term, this is second term=d square Z/dz square/Z=let us put equal to this is – here –d square z/dz square will be dividing by R, H and Z so we get this equation. This is equal to some constant. Now let us take the constant to be = m square when we take the constant to be m square then take this constant is m square and here we take 1/H d square H/d phi square=m square another constant.

Then, d square H/d phi square-m square H gives you H phi=c1 m phi e to power m phi+c2 e to power –m phi. Now in cylindrical coordinates, the phi must be unique, so H phi+2 phi should be=H phi but here if you replace phi/phi+2 phi you do not get back H phi and therefore this solution is not appropriate for our problem. So we consider 1/H d square H/d phi square to be equal to –m square.

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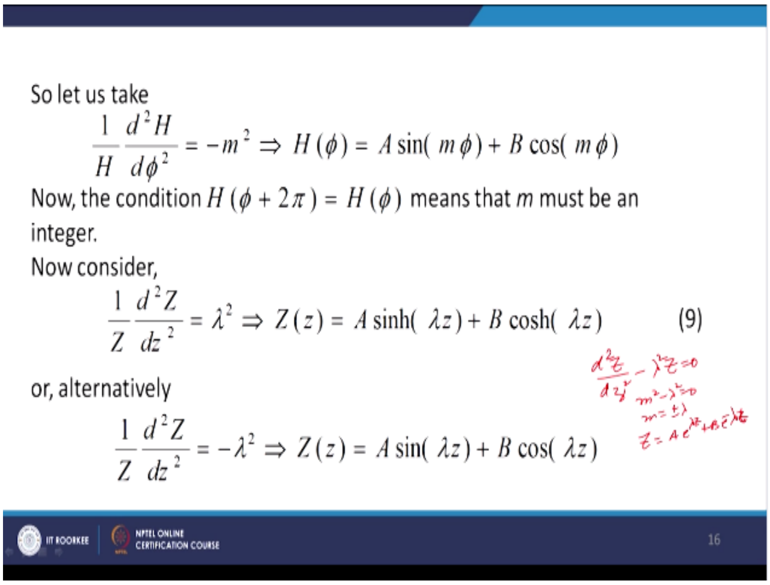

And when we consider –m square we get H phi=A sin m phi+B cos m phi. Now H phi+2 phi=H phi, we can see from here because H phi consist of some trigonometric functions. Now for this H phi+2 phi to be=H phi it is necessary that m be an integer. So for the condition H phi+2 phi=H phi we have to assume that m is an integer. Now consider 1/Z d square Z/dz square=lambda square.

So here what do we notice d square Z/dz square is –k Z okay, let us take k to be equal to –lambda square, so when we take –lambda square Z z=A sin hyperbolic because this is d square Z/dz square-lambda square*Z=0. So its auxiliary equation is m square-lambda square=0, m=+1 lambda so it has 2 distinct real roots and therefore we have Z=some constant A times e to the power lambda Z+B times e to the power –lambda Z.

And this can also be written in terms of sin hyperbolic cos hyperbolic functions, so we have A sin hyperbolic lambda z+B cos hyperbolic lambda z. If you choose this k to be lambda square, then d square Z/dz square will be equal to –lambda square z. So if we choose –lambda square we get the solution as A sin lambda z+B cos lambda z. Now corresponding to these 2 different solutions we will have 2 different solutions of the given equation.

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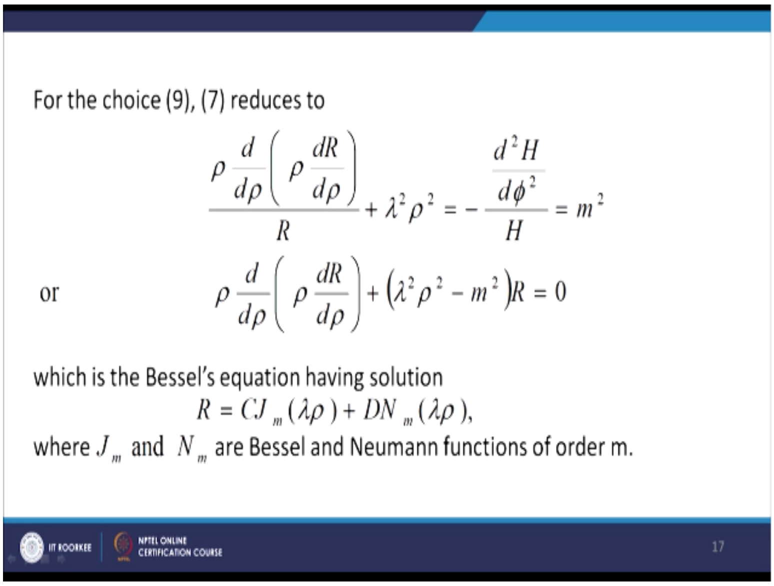

Now here rho d/d rho rho dR/d rho and now when we choose here this quantity if we choose it to be lambda square okay, we have chosen this to be lambda square okay. So this we choose as lambda square and d square Z/dz square/Z if you choose lambda square then we will get this one, rho square and then lambda square rho square because we had chosen it as m square so lambda square rho square-m square*R=0.

This is the case this one when we choose here this one 1/Z d square Z/dz square when we chose a lambda square. So we get this equation and this is Bessel's equation and where the solutions are R=CJm lambda rho+DNm lambda rho, C and D are constants, Jm and Nm are Bessel functions and Neumann functions of order m.

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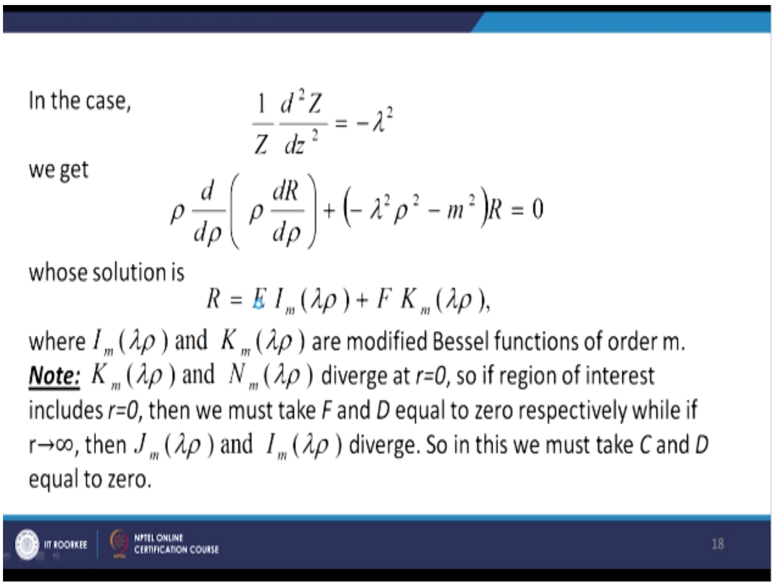

And if you take 1/Z d square Z/dz square to be –lambda square, you get here –lambda square rho square-m square*R whose solution is R=E times Im lambda rho+F times Km lambda rho, E and F are constants, Im lambda rho and Km lambda rho are modified Bessel functions of order m. Now let us note the following, Km lambda rho and Nm lambda rho this Nm lambda rho, they diverge at R=0.

So if region of interest includes R=0 then we must take this F or this D=0 respectively while if R tends to infinity then Jm lambda rho and Im lambda rho diverge, so in this case we must take C and D=0, this C0 and this E0 this E0. So we have to see the problem and then decide whether if the region includes R=0, if the region includes R=0 and if it is appropriate to choose –lambda square you will have this solution.

In this solution, you will take this F=0, for your problem this is the solution then you will have this solution, in this solution you will take D=0 and if the region includes the case of R tending to infinity and your equation is this, you choose E=0 if your equation is this you choose C=0, so this is how we do it.

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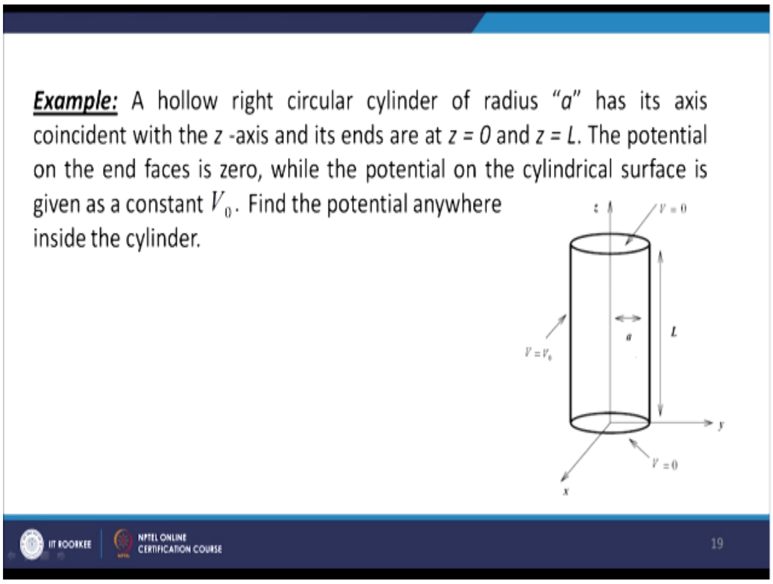

Now let us consider a hollow right circular cylinder of radius a which has its axis as coincident with z-axis and its ends are at z=0 and z=L this is z=L, this is z=0, the potential on the end surface is 0, potential on this face V=0 potential on this face V=0 and potential on this cylindrical surface is given as a constant V0, on this curved surface potential is given as V0. Now we have to find the potential anywhere inside the cylinder.

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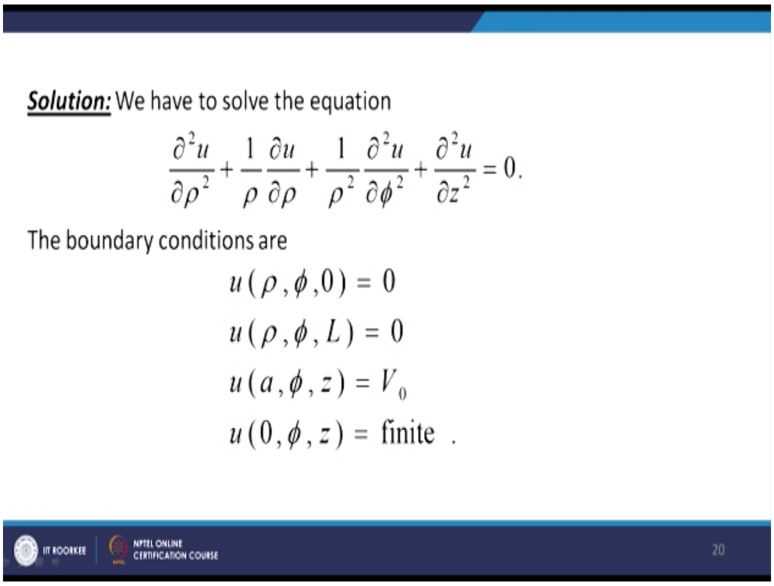

Now here in this solution we have taken potential as u okay. So I have denoted it by u, so we have to solve the equation del square u/del rho square+1/rho del u/del rho+1/rho square del square u/del phi square+del square u/del z square=0. The boundary conditions are now when z=0 the end faces are at 0 potential okay and faces this end this are at 0 potential, so rho, phi, 0 is 0, u rho, phi, L is 0 and when rho is equal to a on the curved surface potential is V0.

And when rho=0 here this one okay here at this point is R=0 so here the potential is finite.

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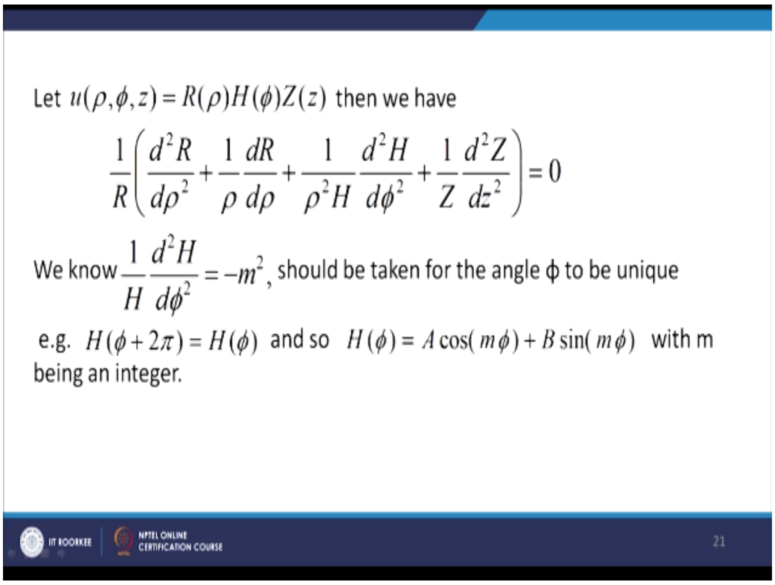

Now let us consider u rho, phi, z=R rho H phi Z z then we have this equation. We know that 1/H d square H/d phi square=-m square because if you take +m square you get e to the power m phi and e to the power –m phi where we notice that when H phi+2 pi!=H phi so we have to consider –m square and there when this consist of cos m phi+B sin m phi, H will be equal to A cos m phi+B sin m phi.

So H phi+2 phi will be=H phi provided m is an integer, so here for H phi+2 pi to be=H phi, m has to be taken an integer.

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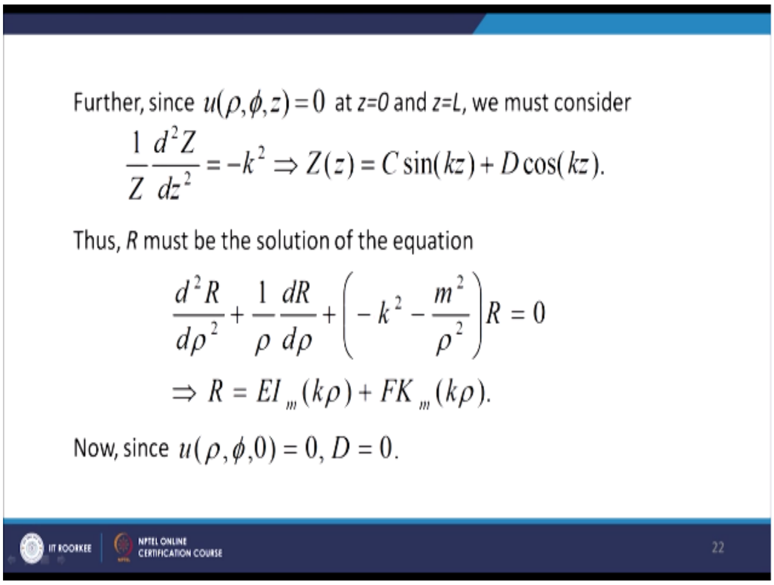

And u rho, phi, z=0 at z=0 and z=L therefore 1/Z d square Z/dz square will have to be taken –k square and +k square will not do so that will lead us to a trivial solution. So Z z=C sin kz+D cos kz. Now R therefore must be a solution of this equation because we have taken it as –k square so we have –k square here and this is taken as –m square so we get –m square/rho square R, so this is the Bessel’s equation of order m and its solution is R=EIm k rho+FKm k rho.

Now u rho, phi, 0=0, at z=0 u is 0, so u is 0 means D must be 0 okay.

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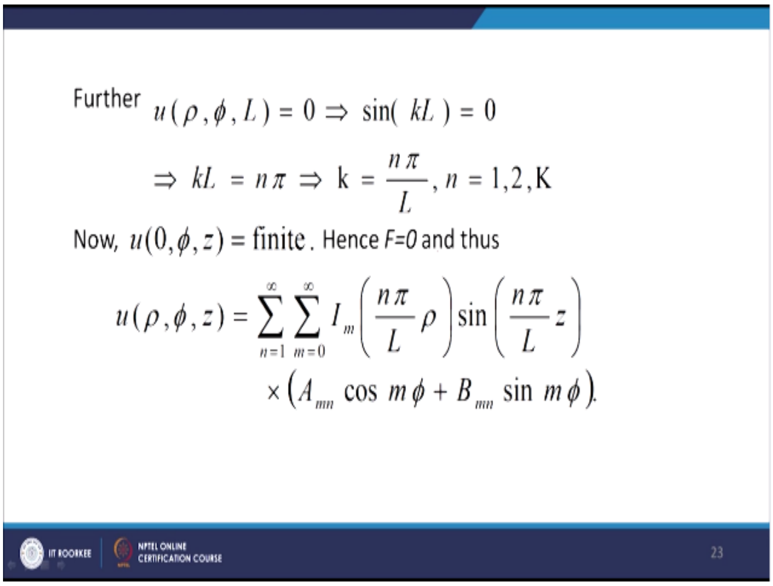

And u rho, phi, L is 0 at z=L u rho, phi, z is 0, so we shall have sin kL=0 sin kL=0 will give you k= n pi/L, n=0 will lead us to a trivial solution when n is negative integer because of sin function sin-theta is –sin theta, so –sin will be absorbed in the constants and therefore we take positive integral values of n. Now u 0, phi, z=finite so u 0, phi, z=finite means the region includes R=0 and when region includes R=0 we have to consider F=0 this constant F=0.

So F is 0 and therefore u rho, phi, z becomes sigma n=1 to infinity and there is from 0 to infinity Im and pi/L rho sin n pi/L z A mn cos m phi+B mn sin m phi. This m comes from H phi and n comes from this one this equation okay from here we get this n. So there will be double summation.

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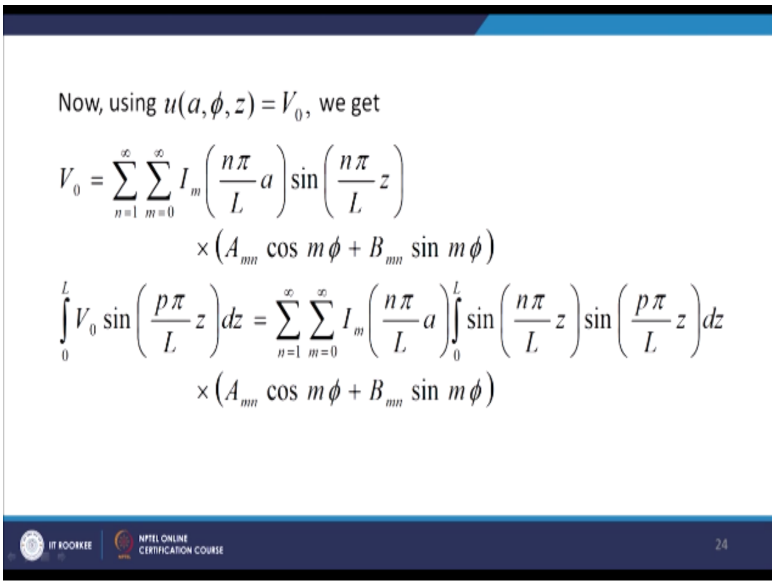

Now we put here u a, phi, z=V0, let us use this equation, put rho=a, when rho=a, u a, phi, z=B0 so B0 will be equal to this Im and pi L here we will have A and this is what we will get. Now multiply both sides by sin p pi/L z and then integrate over 0 to L. So we will have left side like this, right side will integral 0 to L sin n pi/L z sin p pi/L z dz and now this integral is L/2 okay when n = p and when is !=p this integral is 0.

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$$\Rightarrow -V_0\left[\cos\left(\frac{p\pi}{L}z\right)\frac{L}{p\pi}\right] = \sum_{m=0}^{\infty} I_m\left(\frac{p\pi}{L}a\right)\frac{L}{2} \times \left(A_{mp}\cos m\phi + B_{mp}\sin m\phi\right)$$

$$or,\ \frac{V_0}{p\pi}\left(1-(-1)^p\right) = \frac{1}{2}\sum_{m=0}^{\infty} I_m\left(\frac{p\pi}{L}a\right) \times \left(A_{mp}\cos m\phi + B_{mp}\sin m\phi\right)$$

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So we will have the following, -V0 cos p pi/L z=L/p pi this is L/2 this and then –V0/p pi cos p pi is -1 to the power p we have this equation and this we have half L we have cancelled both sides, this L with this and we get this equation Im p pi/L this one okay.

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$$\text{or} \quad \sum_{m=0}^{\infty} I_m\left(\frac{p\pi}{L}a\right)\left(A_{mp}\cos m\phi + B_{mp}\sin m\phi\right) = \begin{cases} \dfrac{4V_0}{p\pi}, & \text{if p is odd} \\ 0, & \text{if p is even} \end{cases}$$

Hence, if p is odd

$$\sum_{m=0}^{\infty} I_m\left(\frac{p\pi}{L}a\right)\left(A_{mp}\cos m\phi + B_{mp}\sin m\phi\right) = \frac{4V_0}{p\pi}.$$

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Or we can say sigma m=0 to infinity Im p pi/L a Amp cos m phi+Bmp sin m phi=4V0/p pi where p is odd 0 when p is even. Hence, if p is an odd integer okay we have sigma m=0 to infinity Im p pi/L a Amp cos m phi+Bmp sin m phi=4V0/p pi. Now left hand side contains cos m phi sin m phi terms but right side does not contain cos m phi sin m phi terms, therefore m has to be taken 0.

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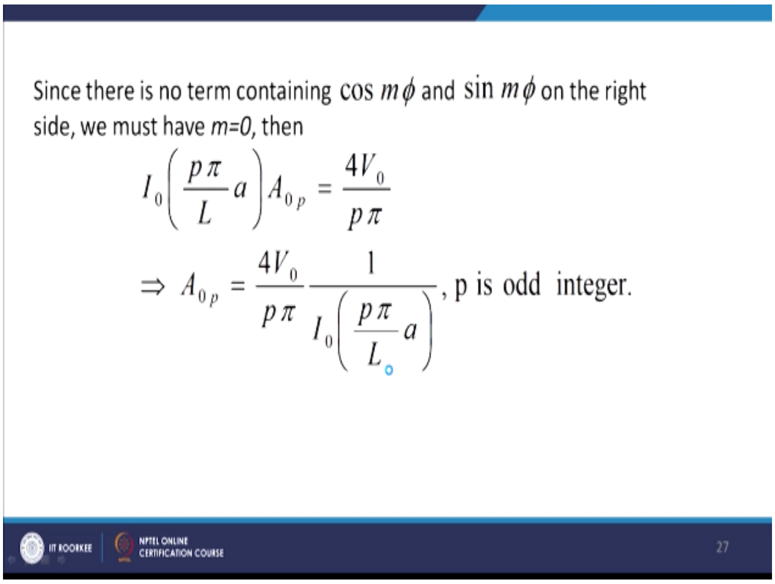

So we have I0 p pi/L a A0 p=4V0/p pi and this gives us the value of A0 p, A0 p==4V0/p pi 1/I0 p pi/L a, p is an odd integer.

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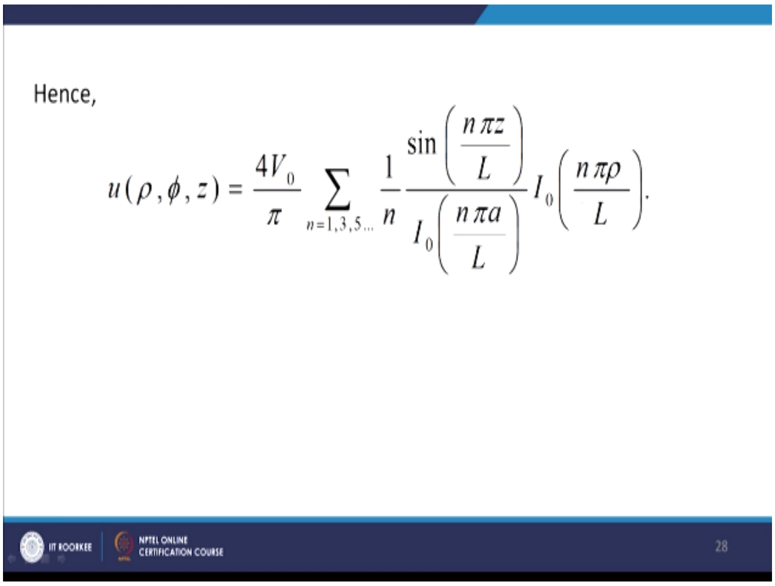

And this leads to the solution u rho, phi, z= 4V0/pi where n runs over odd integral values*1/n sin n pi z/I0 n pi a/L*I0 n pi rho/L. So this is an example on the cylindrical polar coordinates, when we convert the Laplace equation into cylindrical polar coordinates. In the next lecture, we shall consider spherical polar coordinates and will solve Laplace equation in spherical polar coordinates. Thank you very much for your attention.