Ordinary and Partial Differential Equations and Applications

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Lecture - 48
Review of Integral Transforms - III

Hello friends. Welcome to my lecture on review of integral transforms III. This is third and final lecture on the review of integral transforms.

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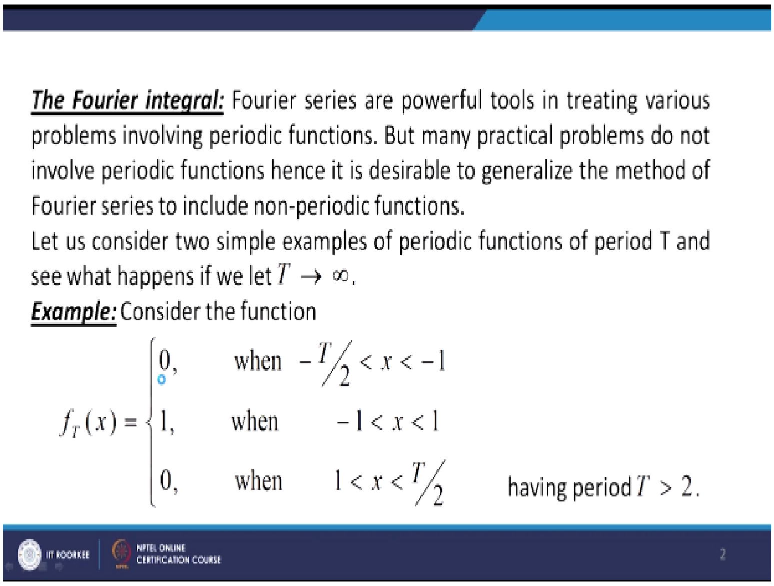

Let us look at the Fourier integral. We know that the Fourier series are powerful tools in treating various problems involving periodic functions but many practical problems do not involve periodic functions hence it is desirable to generalize the method of Fourier series to include non-periodic functions. Let us consider 2 simple examples of periodic functions of period T and see what happens if we let T tends to infinity.

Consider this function fT x=0 when –T/2<x<-1, 1 when -1<x<1, 0 when 1<x<T/2 and the period of the function fT x is T which is more than 2.

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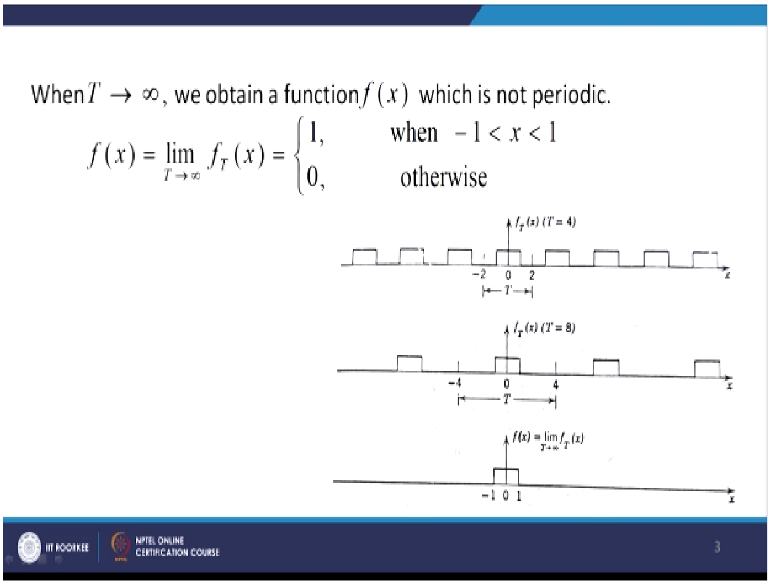

Then, we can look at the graph of fT x. This is the graph of fT x. You can see it is periodic function and here in the example we are taking T to be>2 so let us consider T to be 4. So over the interval -2 to 2 okay over the interval -2 to 2 you will see that from -2 to -1 fT x is 0 then fT x is 1 from -1 to 1 then fT x=0 from 1 to 2 and it has been extended periodically over the whole real axis using the periodicity T=4.

Here we take another value of say T=8 and then you see the graph of the function fT x from -4 to 4. This is 0 from -4 to -1, -1 to 1 it is 1 and then 1 to 4 it is 0 and then it has been extended periodically by using the period T=8. When T tends to infinity what happens, over the interval -1 to 1 the graph is fT=1 okay as T tends to infinity while over the remaining values of x from x=1 to infinity fT x tends to 0, f x becomes 0 and then from –infinity to -1 it is again 0.

And you can see that it is not periodic okay with any period so when T tends to infinity we obtain a function f x which is not periodic because f x becomes 1 when -1<x<1 and 0 otherwise.

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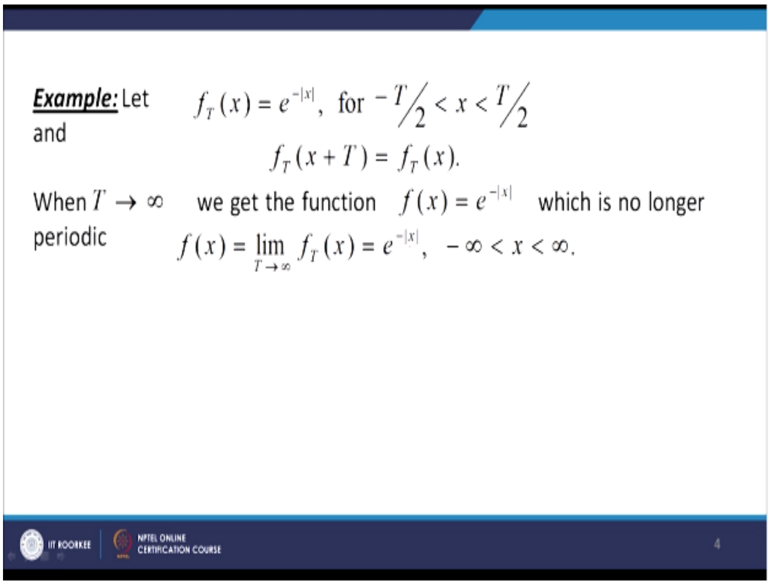

Now let us look at another example, let say fT x=e to the power –mod of x for –T/2<x<T/2 and fT x+T=fT x, so this equation tells us that fT x has the period T, when T tends to infinity we get the function f x=e to the power –mod of x, f x is the limiting function of fT x as T goes to infinity. As T goes to infinity, this interval turns to –infinity to infinity and f x becomes e to the power –mod of x.

And you can see that it is no longer periodic, f x=e to the power –mod of x. We have the graph of this function.

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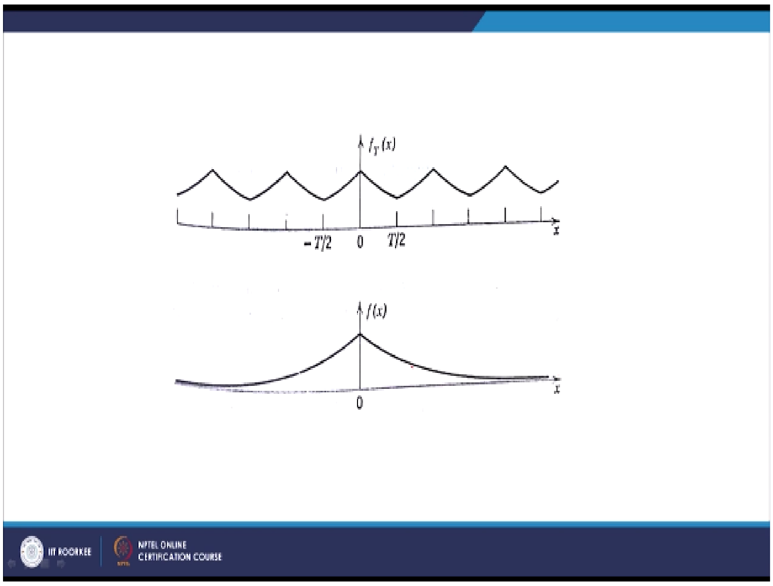

Here you see e to the power –mod of x okay, e to the power –mod of x is an even function from –T/2 to T/2. So this is the graph of the function okay from –T/2 to T/2 and then it has been extended periodically by using the period T. Now here what happens when T tends to infinity okay we have the graph of e to the power –mod of x like this, so you can see that it is not periodic anymore.

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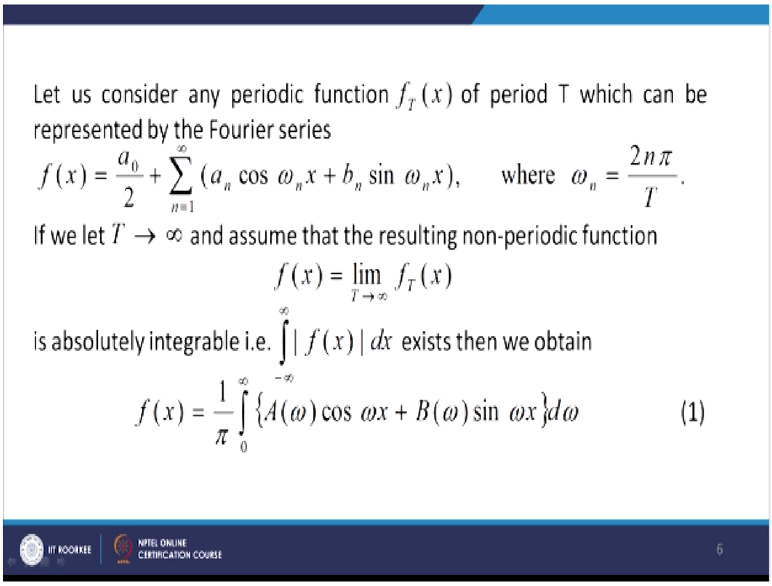

So we have to see what happens when fT x tends to say function f x as T goes to infinity then what will be the representation of f x. So that is our problem. Let us consider any periodic function fT x with period T then we can represent it by the Fourier series. Let us assume that x is a point of continuity of f, so f x=a0/2+sigma n=1 to infinity an cos omega n x+bn sine omega n x and here omega n is 2n pi/T.

Now when we let T goes to infinity let us assume that fT x tends to f x. We further assume that this limiting function f x is absolutely integrable that is the integral over –infinity to infinity mod of f x dx exists then this Fourier series while using the values of the Fourier coefficients a0, an and bn in terms of the integrals okay and taking the limit what happens is that we get the effects as this integral, 1/pi integral over 0 to infinity A omega cos omega x+B omega sine omega x d omega where A omega and B omega are given by these integrals.

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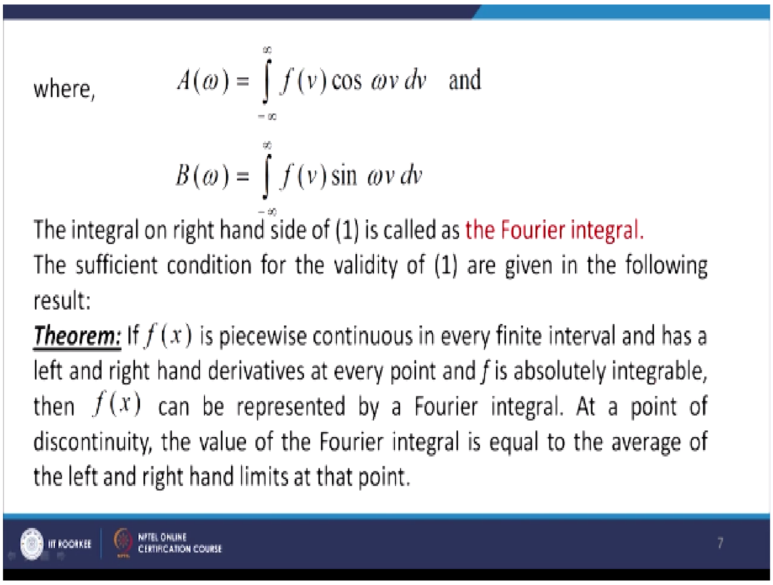

A omega is –infinity to infinity f v cos omega v dv and B omega is integral over –infinity to infinity f v sine omega v dv. The integral on the right hand side of 1 this integral okay. The integral on the right hand side of 1 is called the Fourier integral of f. The sufficient conditions for the validity of this representation of effects by the Fourier integral are given in the following theorem.

So if f x is piecewise continuous in every finite interval of the real axis n left and right hand derivatives exist at every point and f is absolutely integrable that is integral over - infinity to infinity mod of f x dx is <infinity then f x can be represented by a Fourier integral. So at a point of discontinuity the value of the Fourier integral=to the average of the left hand and right hand limits.

And that point like we have seen in the case of Fourier series representation of a function f at the point of discontinuity the sum of the Fourier series is average of the left hand and right hand limits, so here also at a point of discontinuity the value of the Fourier integral is equal to the average of left and right hand limits. At each point of continuity, the Fourier integral=f x.

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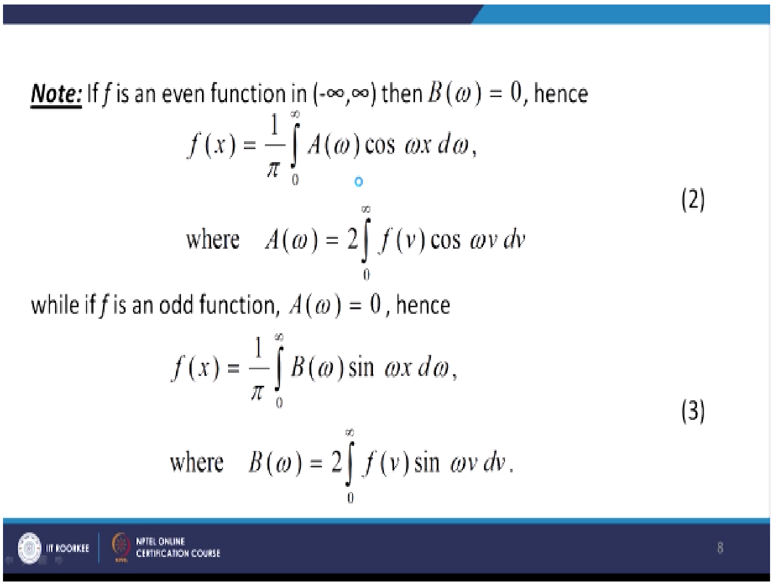

So now notice that if f is an even function in the interval –infinity to infinity then the value of b omega, b omega is given by this integral, so if f is an even function okay then f is an even function and sine omega b is an odd function so the product will be an odd function and therefore b omega will be = 0 and when b omega=0 the cosine terms will remain while sine terms will vanish.

And the A omega will become 2 times 0 to infinity f v cos omega v dv because cos omega v is even function so f v*cos omega v will be even. So we will have A omega=2 times 0 to infinity f v cos omega v so here when f is an even function the Fourier integral representation of f contains only this cosine terms, sine term is not present in the Fourier integral representation.

Similarly, if we have f to be an odd function then A omega=0 because f is odd cos omega v is even so the product is odd and B omega becomes 2 times 0 to infinity f v sine omega v dv. So here then in the Fourier integral representation of f only sine terms remain, the cosine terms vanish, so B omega becomes this and the Fourier integral representation is containing only sine term here.

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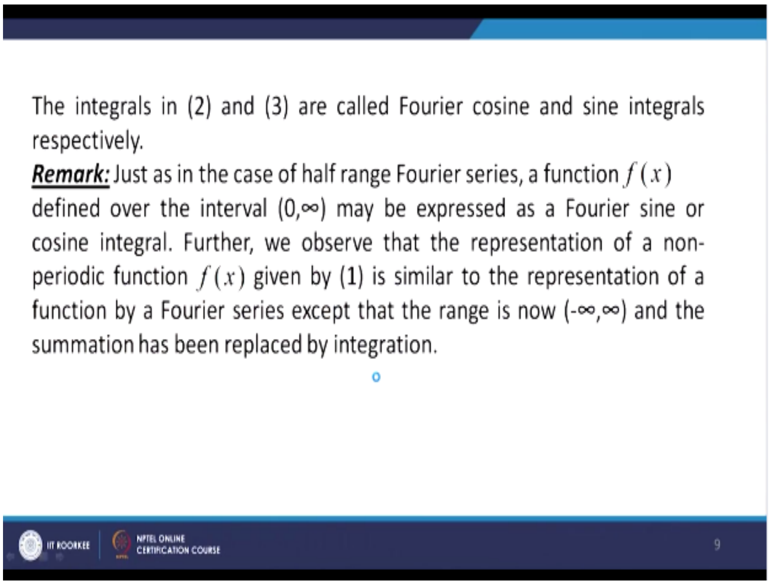

The integrals 2 and 3, these 2 integrals, integral this 2 and integral 3 are called this integral 3 and this integral 2 they are called as Fourier cosine and Fourier sine integrals. This one is called Fourier cosine integral, f x=this is called Fourier sine integral, f x=this is called Fourier sine integral of f. Now just as in the case of half range Fourier series if a function f is defined over the half range say 0 infinity then over the interval –infinity to 0 we can define it by using the even extension or the odd extension.

And then we can find the Fourier integral representation of the function f like in the case of Fourier series. So here we notice that the representation of a non-periodic function f x given by this equation 1 is similar to the Fourier series representation of the periodic function f. Here also you can see we have A omega cos omega x, B omega sin omega x like in the case of Fourier series but there we have summation here we have integration that is the difference.

So range is now –infinity to infinity and the summation has been replaced by integration.

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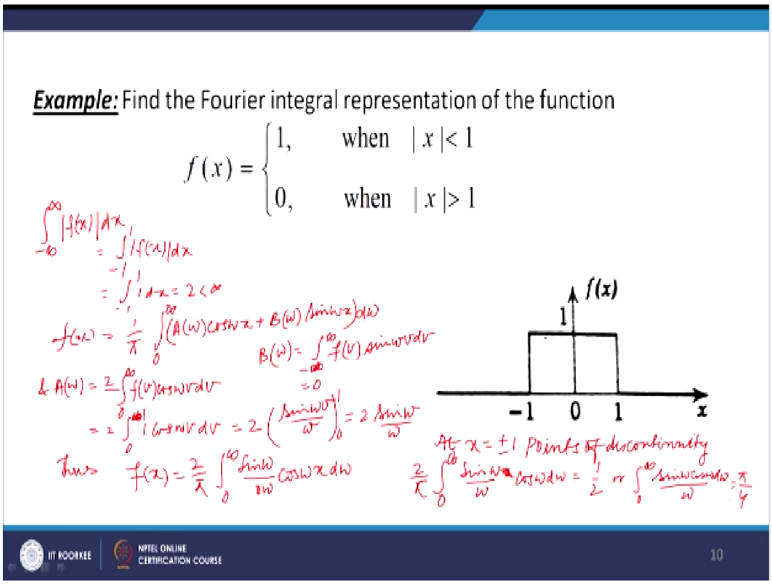

Let us find the Fourier integral representation of the function f x=1 when mod of x is<1, 0 when mod of x>1. You can see that f is piecewise continuous on every finite interval and moreover that integral over –infinity to infinity mod of f x dx=integral over you can see this function is an even function okay, f is an even function and first we reduce the interval –infinity to infinity to -1 to 1 because everywhere else it is 0.

So mod of f x dx and mod of n f x takes value 1 when mod of x is<1 so we get here -1 to 1 1 dx this=2 and which is<infinity. So f is absolutely integrable, furthermore f is having left and right hand derivatives at each point of the real axis okay. Now let us recall the definition of the Fourier integral. Fourier integral f x at each point of continuity let x be a continuity point so f x=1/pi integral over 0 to infinity A omega cos omega x+B omega sin omega x d omega.

Let x be a continuity point of f, so then the Fourier integral becomes=f x. Now here we see that B omega, f is an even function okay so B omega=0 to infinity f v sin omega v dv okay. Since f is an even function sin omega B is an odd function so the product of even and odd is odd and therefore this is –infinity to infinity so this is=0 and A omega becomes 2 times integral 0 to infinity f v cos omega v dv okay.

So we get this 2 times f v*cos omega v dv and let us use the value of f, f=1 over the interval 0, 1 and elsewhere from 1 to infinity to 0 so this is 2 times 0 to infinity 1*cos omega v dv and we get it as 2 times sin omega v/omega okay 0 to 1 because it is 1, over the interval 0, 1 it is 1, so with this 2 times sin omega/omega okay. So thus f x=2/pi 0 to infinity A omega is 2 sin omega/omega.

So this is sin omega/omega, 2 we have written outside*cos omega x d omega. This is the Fourier integral representation of f at each point of continuity of f and if you look at the interval -1 to 1 it is discontinuous at -1 it is discontinuous at 0, it is discontinuous at 1. So the points of discontinuity are x=+-1 not 0. At x=0 it is continuous so these are points of discontinuity.

So from the points of discontinuity the sum of the Fourier series is=to the average of left hand and right hand limits. Now at the point -1 left hand limit is 0, right hand limit is 1, so average is 0+1/2 that is 1/2 and at 1 the left hand limit is 1, right hand limit is 0 so again average is 1/2 so we have 2/pi 0 to infinity sin omega/omega and when x=1 cos omega x is cos omega when x is -1 again cos omega x is cos omega.

So at x=+-1 sin omega/omega cos omega d omega=f x=1/2. So we can say that integral or you can say integral 0 to infinity sin omega cos omega d omega/omega=pi/4.

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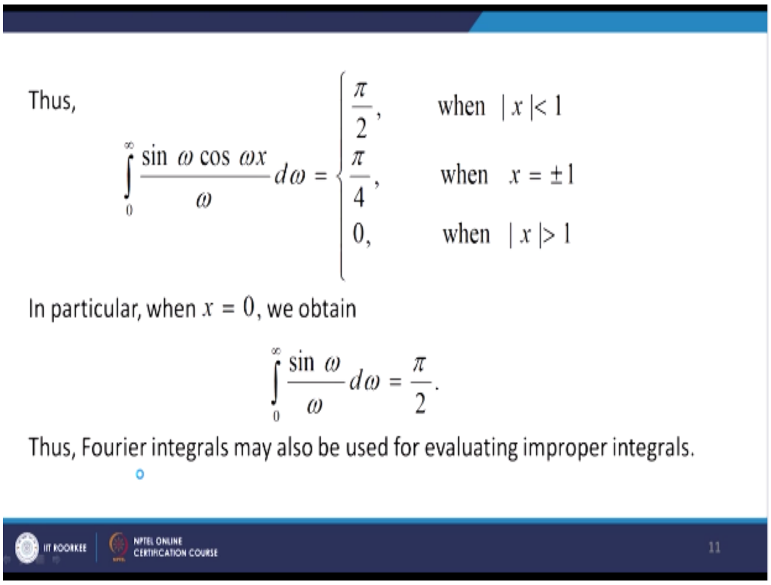

So let us see the integral 0 to infinity sin omega cos omega x/omega d omega this is=pi/2 when mod of x is<1 because when mod of x is<1 f x becomes=1 so integral 0 to infinity sin omega/omega cos omega x d omega is pi/2. At x=+-1, the value is pi/4 and when mod of x is>1 this f x=0 so integral 0 to infinity sin omega/omega cos omega x is 0. In particular, when we take x=0 you take x=0 then at x=0 f x takes value 1 and it is a point of continuity.

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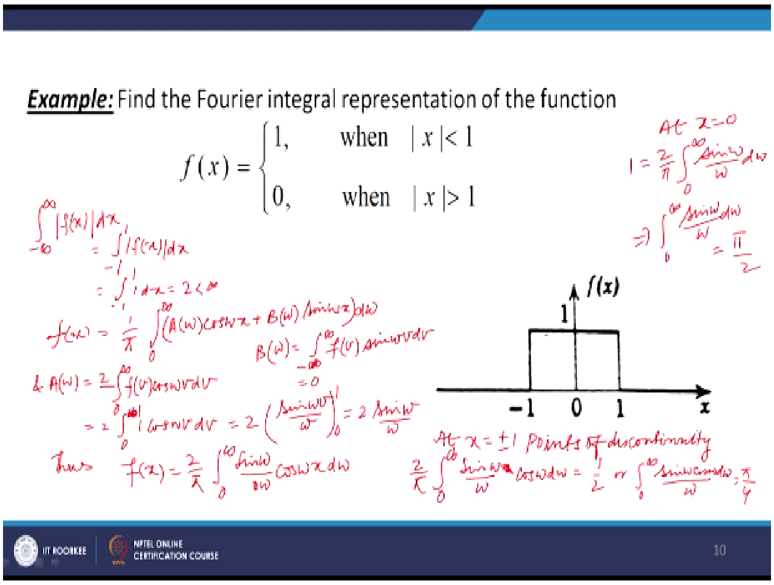

So 1=2/pi 0 to infinity sin omega/omega cos omega x becomes 1 and therefore the value becomes 1=2/pi 0 to infinity sin omega/omega d omega at x=0. It is a point of continuity and this gives you 0 to infinity sin omega/omega d omega=pi/2 okay. So you can see we can find even some improper integrals while using the Fourier integral representation of a function f.

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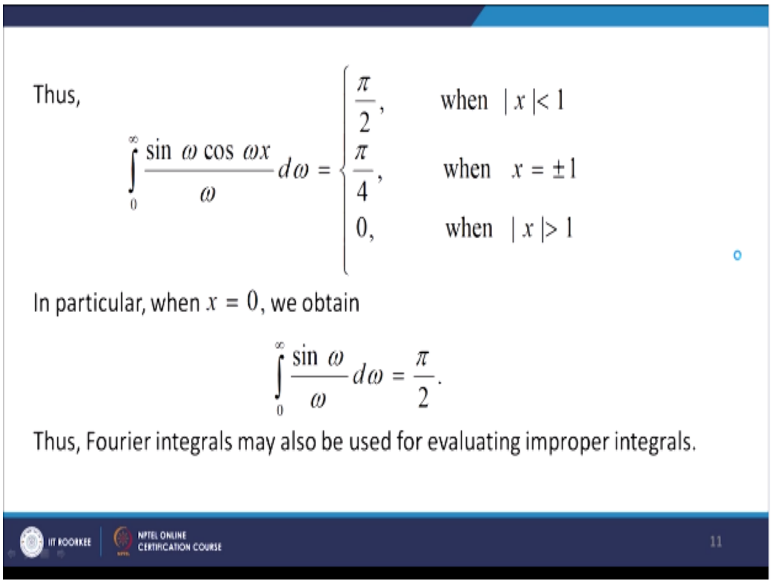

Now let us discuss the inversion formulae for Fourier transforms. We may write equation 1 as let us put the values of A omega and B omega in this Fourier integral representation. Then, we will have f x=1/pi 0 to infinity, - infinity to infinity A omega becomes f v cos omega v dv so we will have f v cos omega B*cos omega x and when you put the value of B omega you get f v sin omega B*sin omega x d omega.

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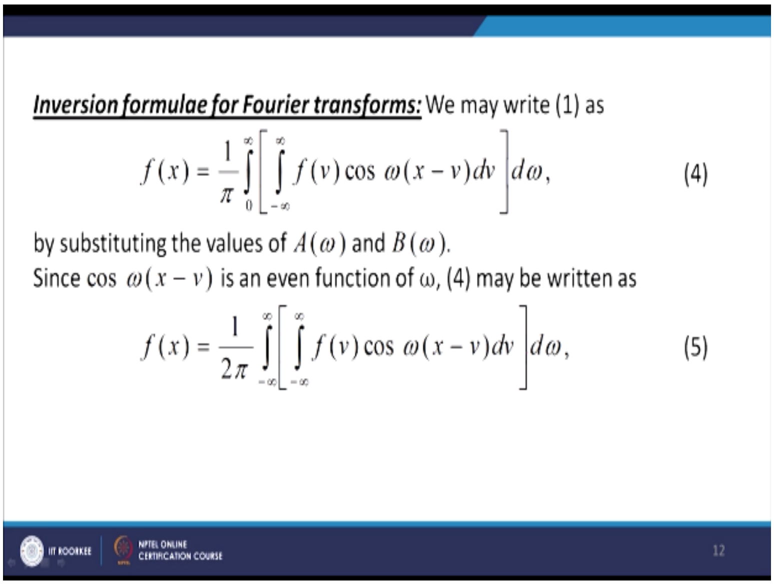

So using cos A, cos B+sin A sin B=cos A-B we have this expression 1/pi integral over 0 to infinity, - infinity to infinity f v cos omega x-v dv d omega now by substituting the values of A omega and B omega. Now here better let us look at this function cos omega x-v it is an even function of omega, the integral over omega is from 0 to infinity, so we can make it from –infinity to infinity and divide by 2 because of the fact that cos omega x-v is an even function.

So this becomes 1/2 pi-infinity to infinity, -infinity to infinity f v cos omega x-v dv d omega.

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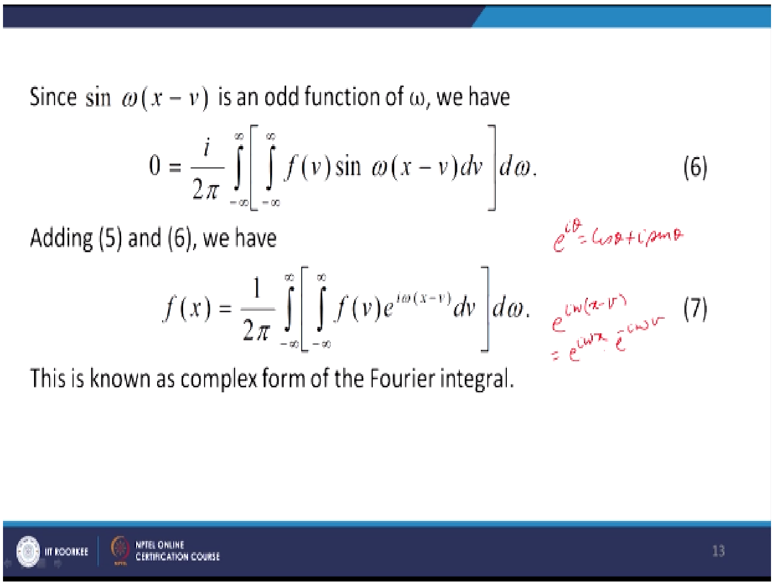

Now let us look at the fact that sin omega x-v is an odd function of omega, so if you calculate this integral, integral over -infinity to infinity, -infinity to infinity f v sin omega x-v dv d omega. Then, because this is an odd function integral over -infinity to infinity with respect to omega will be 0 so the whole thing will become 0. This is iota so 0=iota/2 pi and then this expression we have.

Now we add the equation 5 with equation 6 okay, this equation 5 and the equation 6 and use the formula e to the power i theta=cos theta+i sin theta. When we use this formal f x becomes=1/2 pi integral over -infinity to infinity, -infinity to infinity f v e to the power i omega x-v instead of theta here we have omega x-v. So this is known as complex form of the Fourier integral

Now we may write this equation in the following manner e to the power i omega x*x-v. Let us split into 2 parts, e to the power i omega x*e to the power –i omega v. The inside integral is with respect to v, so e to the power –i omega v will be kept inside, e to the power i omega x contains omega so this v we can take outside the integral with respect to v.

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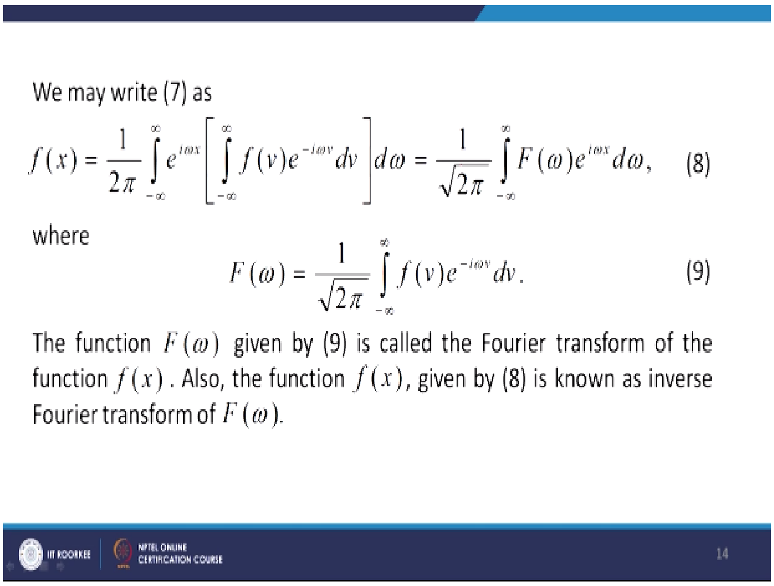

We may write (7) as

$$f(x)=\frac{1}{2\pi}\int_{-\infty}^{\infty}e^{i\omega x}\left[\int_{-\infty}^{\infty}f(v)e^{-i\omega v}dv\right]d\omega=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}F(\omega)e^{i\omega x}d\omega, \qquad (8)$$

where

$$F(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(v)e^{-i\omega v}dv. \qquad (9)$$

The function $F(\omega)$ given by (9) is called the Fourier transform of the function $f(x)$. Also, the function $f(x)$, given by (8) is known as inverse Fourier transform of $F(\omega)$.

And then what we have 1/2pi integral over –infinity to infinity for i omega x integral over –infinity to infinity f v e to the power –i omega v dv d omega. Now this can be written as 1/root 2 pi integral over –infinity to infinity F omega e to the power i omega x d omega where F omega is defined as 1/root 2 pi this i is split into 2 parts, 1/root 2 pi*1/root 2 pi, one 1/root 2 pi comes with F omega and the other 1/root 2 pi goes with the remaining integrals, so F omega=1/root 2 pi -infinity to infinity f v e to the power –i omega v dv.

Now this representation okay is called as the Fourier transform of the function F. So the F omega is called the Fourier transform of the function F and the function f x given by 1/root 2 pi -infinity to infinity F omega e to the power i omega x d omega is termed as the inverse Fourier transform of F omega. Now some authors will take here 1 okay instead of 1/root 2 pi and there we will take 1/2 pi while we use the inversion formula for Fourier transform.

So there is no ambiguity in that because where we generally use these Fourier transforms in the practical problem that is the heat conduction problems or other problems, so there first we apply the Fourier transform okay and then we apply the inversion formula for the Fourier transform. So if we multiply here by 1 and here we take 1/2 pi it is the same thing because if you take 1/root 2 pi in F omega and then while inverting you take 1/root 2 pi then also the result is the same.

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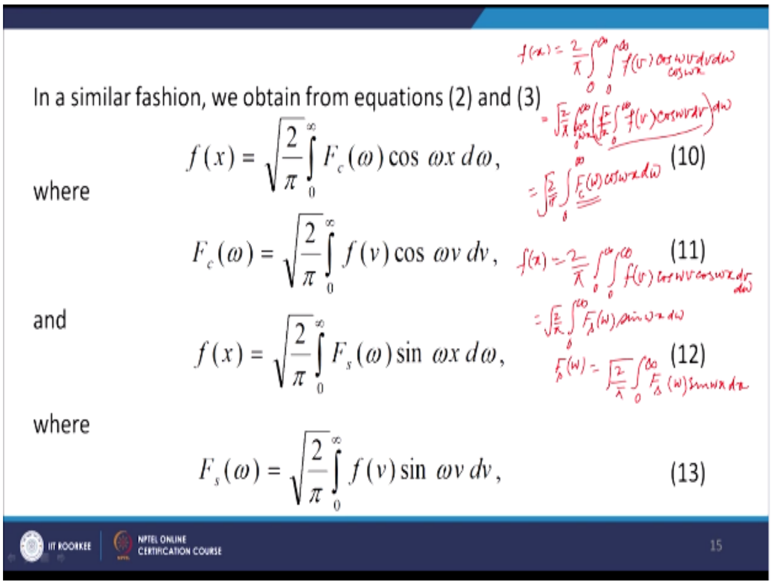

In a similar manner, we can obtain from equations 2 and 3 let us look at equation 2 and 3, yes from here okay so this is f x=1/pi 0 to infinity A omega cos omega x okay so we will get it as into 2 parts let us take it okay so if you put the values there okay in equation 2 and 3 yes so in equation 2 and 3 we have Fourier cosine, Fourier sine transforms okay we have f x=2/pi. See we get the Fourier cosine integral and Fourier sine integral by assuming F to be even and odd okay.

So from that representation okay f x becomes Fourier cosine integral 2/pi 0 to infinity, 0 to infinity f v cos omega v dv d omega. This is Fourier cosine integral okay, so you can break it into 2 parts, root 2/pi 0 to infinity and then root 2/pi 0 to infinity f v cos omega v dv*d omega okay so what we do is and then cos omega v and then cos omega x also. So cos omega x will be written here okay.

And what we will get root 2 over pi integral 0 to infinity f v cos omega v, this quantity is termed as Fc omega, so we will get under root 2/pi 0 to infinity Fc omega cos omega x d omega okay. So this Fc omega is called as the Fourier cosine transform. This c denotes cosine transform okay, Fourier cosine transform of f x okay and this f x=root 2/pi 0 to infinity Fc omega cos omega x is called as the inversion formula for the Fourier cosine transform.

Similarly, for Fourier sine integral okay Fourier sine integral we have f x=2/pi 0 to infinity, 0 to infinity f v cos omega x omega v*cos omega x dv d omega and in the similar manner I can write it as root 2/pi 0 to infinity Fs omega sin omega x d omega where Fs omega is given by root 2/pi cos 0 to infinity Fs omega, Fs omega means Fourier sine transform. So these are the Fourier sine transform and this is inversion formula for Fourier sine transform.

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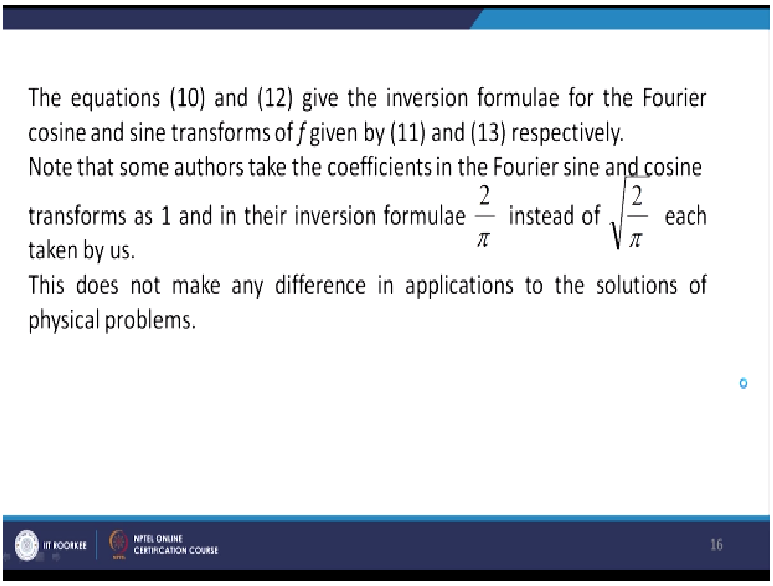

Now these equations give the inversion formulae for the Fourier cosine and Fourier sine transform of f. Some authors as I said in the case of Fourier transforms here also some authors take here is 1 and here 2/pi, here 1, here 2/pi. I have taken root 2/pi, I have split 2/pi into 2 parts, root 2/pi, root 2/pi but there is no ambiguity if you take 1 here and 2 over pi there.

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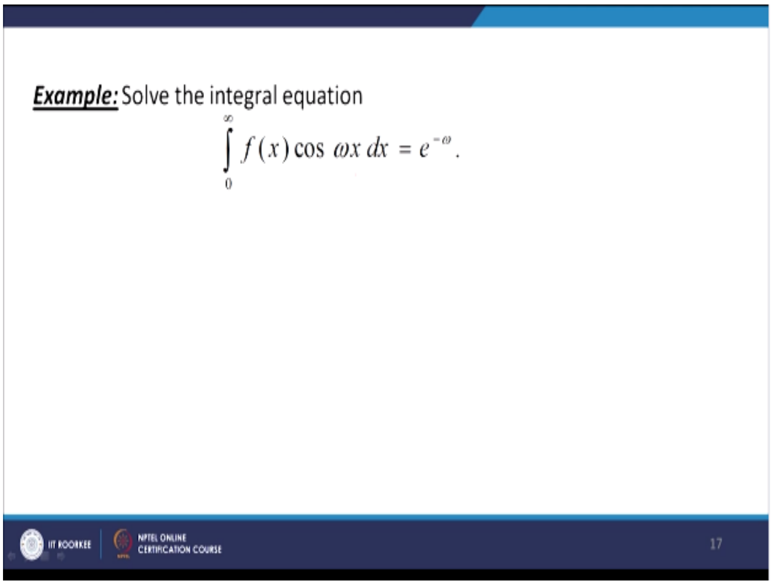

So let us solve this equation 0 to infinity f x cos omega x dx. This is an integral equation because the unknown function f x appears under the integral sine. Now let us note that this is nothing but the Fourier cosine transform of the function f. Let us look at this Fourier cosine transform, this is Fc omega root 2/pi 0 to infinity f v cos omega v dv. So let us take instead of root 2/pi let us take 1 here then Fc omega is 0 to infinity f v cos omega v dv.

Let us solve this integral equation, this is an integral equation because the unknown function f x appears under the integral sign. If you look at the integral, it is 0 to infinity f x cos omega x so it actually gives the Fourier cosine transform of the function f. If you look at this expression Fc omega root 2/pi at 0 to infinity f v cos omega v dv. So if you replace this variable of integration v/x, it is 0 to infinity f x cos omega x dx.

So what we do is let us take the coefficient here root 2/pi as 1 then take f x expression for f x we will take instead of root 2/pi 2/pi.

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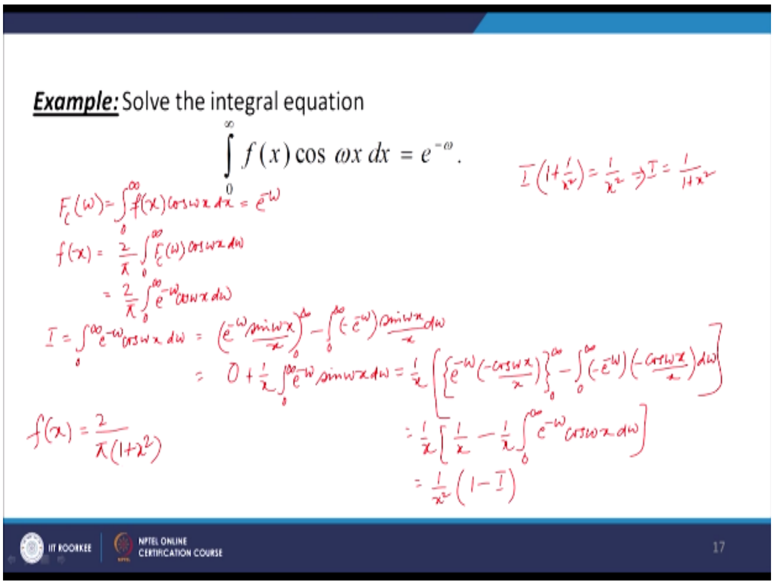

So let us say that Fc omega let us take the definition of Fc omega at 0 to infinity f x cos omega x dx okay. So if you compare with this it is e to the power –omega. Now let us find the inverse formula so 2/pi 0 to infinity Fc omega cos omega x d omega okay. So this is 2/pi 0 to infinity e to the power –omega cos omega x d omega. Let us take I=0 to infinity e to the power –omega cos omega x d omega.

Then, I can write this we can integrate by parts so e to the power –omega sin omega x/x 0 to infinity -0 to infinity -e to the power –omega and sin omega x/x d omega okay. So when omega goes to infinity e to the power –omega goes to 0 while sin omega x is a bounded function and therefore product goes to 0 and when omega is 0 sin omega x is 0 so the first expression becomes 0, this -- becomes + and when we write 1/x outside what we have 0 to infinity e to the power –omega sin omega x d omega.

So again we integrate by parts, so e to the power –omega integral of sin omega x is –cos omega x/x 0 to infinity -0 to infinity-e to the power -omega-cos omega x/x d omega. Now when omega goes to infinity e to power –omega goes to 0 cos omega axis bounded by 1 so the product goes to 0 and when we put the lower limit what we have e to the power – when we put omega is 0 this is 1 and this is -1/x so we get +1/x.

Then, this becomes ---so we get -1/x integral 0 to infinity e to the power –omega cos omega x d omega. So this is 1/x square times 1-I okay. We have denoted 0 to infinity e to the power –omega cos omega is d omega is I so 1/x square*1-I we have okay. So what we have, let us take this term to the other side then I times 1+1/x square we have=1/x square. So this gives us I=1/1+s square okay.

So what we have here f x=2/pi*1+x square, so we can solve the integral equation by using Fourier cosine transform.

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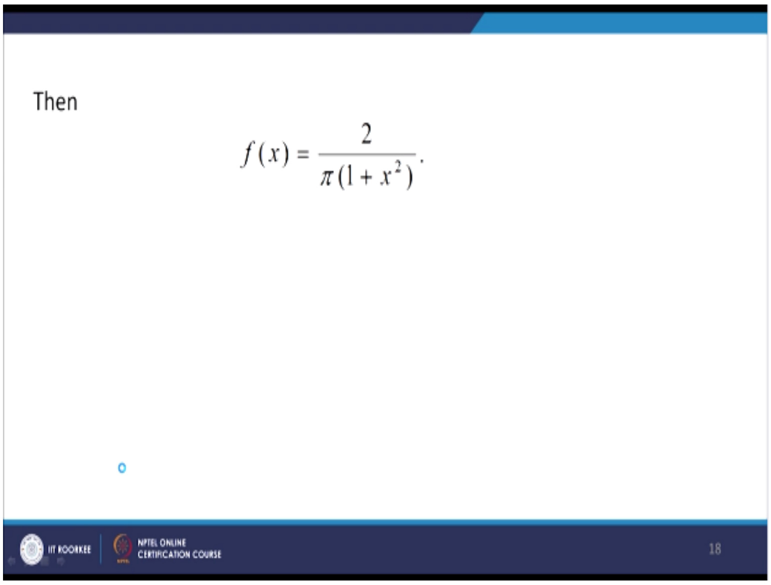

So f x is 2/pi*1+x square.

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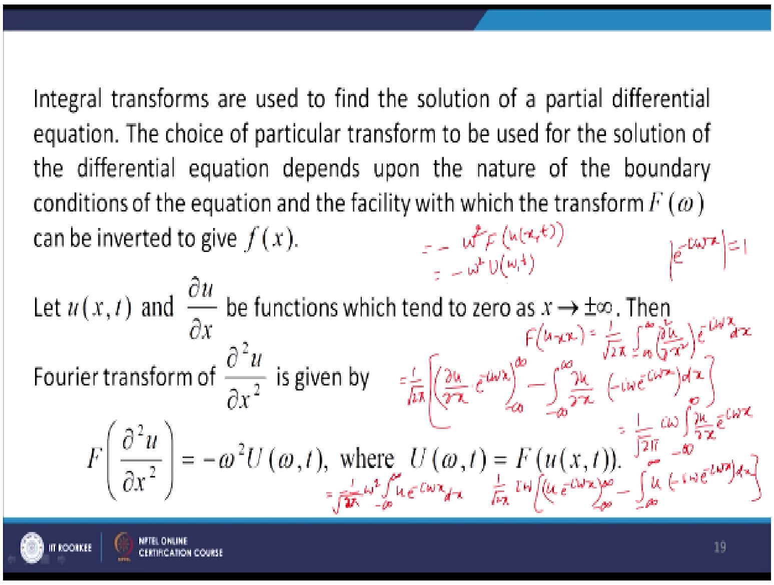

Now integral transforms are used to find the solution of a partial differential equation. The choice of a particular transform to be used for the solution of the differential equation depends upon the nature of the boundary conditions of the required problem and the facility with which the transform F w can be inverted to give f x.

Let us assume that in the problems that we consider the u x, t function, function which depends on the space variable x and time t and its derivative with respect to x that is gradient be functions which tend to 0 as x goes to +-infinity then the Fourier transform of u xx is given by this. So let us see how this Fourier transform we are getting. Let us go to the definition of the Fourier transform.

Fourier transform be defined as 1/root 2 pi/infinity to infinity f v e to the power –i omega v dv. So let us use this definition okay. So F of u x, x so let me redo here so Fourier transform of u xx=1/root 2 pi integral over -infinity to infinity e to the power -i omega x dx okay. So when we integrate by parts what we get integral of this second order derivative becomes first order derivative*e to the power –i omega x-infinity to +infinity then –infinity to +infinity*-i omega e to the power -i omega x dx.

Now let us see we have assumed that this ux we have assumed that ux goes to 0 when x goes to +-infinity and let us remember that e to the power –i omega x. Modulus of this is=1, so it is a bounded quantity so when ux goes to 0 when x goes to infinity and ux goes to 0 when x goes to –infinity because of the boundedness of e to the power i omega x, this part becomes 0 and what we get is then 1/root 2 pi i omega times integral 0 to –infinity to infinity u/x*e to the power –i omega x.

Let us integrate once more, so 1/root 2 pi i omega when we integrate once more we get u*e to the power -i omega x -infinity to infinity and then –infinity to infinity u times –i omega e to the power –i omega x dx. So again e to power –i with axis bounded, u goes to 0 when x goes to +-infinity so this part goes to 0 and this becomes then -- +, i omega comes outside it becomes i square omega square.

So we get -1/root 2 pi omega square -infinity to infinity u times e to the power –i omega x dx okay. So –omega square times U omega, t, this is nothing but –omega square times Fourier transform of u function okay so F u x, t, which we are denoting by U omega, t so –omega square U omega, t. So Fourier transform of second derivative of u with respect to x is replaced with –omega square U omega, t in the problems.

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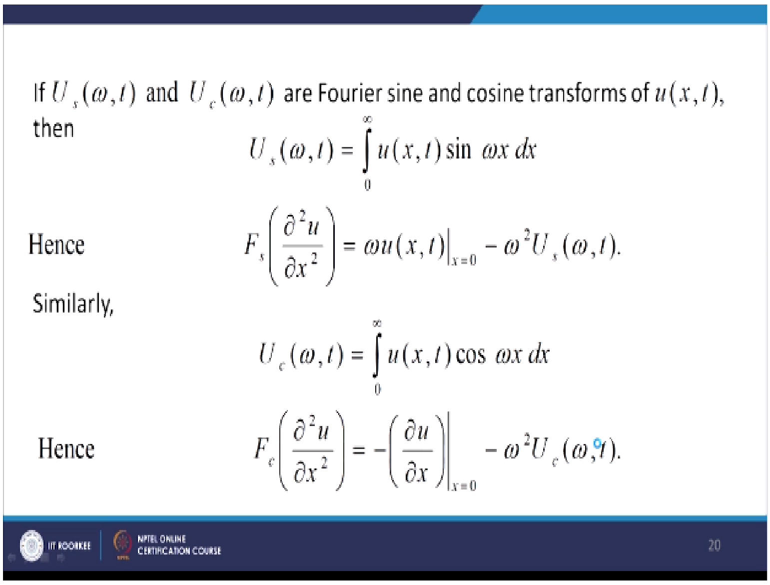

Similarly, Us omega, t represents Fourier sine transform of f x; Uc omega, t represent Fourier cosine transform of the function f x okay. So then Us omega, t is 0 to infinity u x, t sin omega x dx we are taking this coefficient instead of root 2/pi as 1. Then, Fs U x, x becomes omega u x, t at x=0-omega square Us omega, t and similarly for the Fourier cosine transform if you find the Fourier cosine transform of U x, x it becomes –Ux at x=0 –omega square Uc omega t.

Here again like we use the boundedness of e to the power –i omega x here to make this term 0, there also because of instead of e to power –omega x we will have sin omega x and cos omega x, they are bounded quantities, they are bounded by 1. So we will see that this expression again in the case of sin omega x and cos omega x goes to 0. So we can easily prove these formulas.

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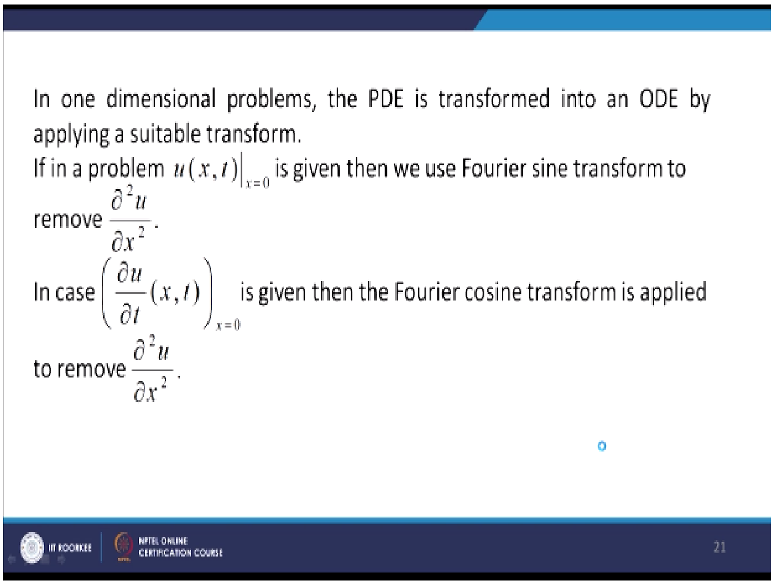

Now in one-dimensional problem the PDE is transformed into an ODE by applying a suitable transform. If in a problem u x, t at x=0 is given you see here in this formula when you take Fourier sine transform of this U x, x you are having the expression as omega*u x, t at x=0 while in the case of Fourier cosine transform you have –delta u/delta x at x=0. So if in a problem you have u x, t at x=0 is given then you use Fourier sine transform.

If in the problem the boundary condition is given as –delta u/delta x at x-0 then you use Fourier cosine transform. So which is the transform that is to be used depends on the given boundary conditions.

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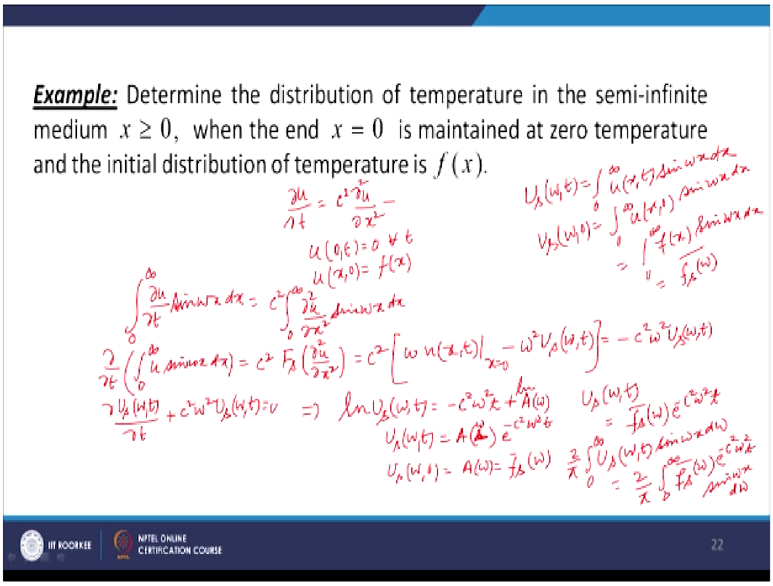

Now let us look at this problem. Determine the distribution of temperature in the semi-infinite medium x>=0, when the end x=0 is maintained at zero temperature and the initial distribution of temperature is f x. So we have this equation we have del u/del t=c square delta square u/delta x square. Now the boundary condition here is that at x=0 temperature is 0, so u 0, t=0 for all the time t and we are given initial temperature distribution as u x, 0=f x.

Now you can see we are given u x, t at x=0 when u x, t at x=0 is given we use Fourier sine transform. So let us take the Fourier sine transform of the given equation, this one okay. So taking Fourier sine transform means you multiply both sides with sin omega x and then integrate with respect to x okay. So delta u/delta t*sin omega x dx and we integrate over 0 to infinity.

Right side will be c square times integral over 0 to infinity delta square u/delta x square sin omega x dx okay. Now x and t are independent variable so here integration is with respect to x so I can write this also as this is Fourier sine transform of u/xx and we can then use the formula c square times Fourier sine transform of u/xx we have seen it is –omega u x, t at x=0 –omega square Us omega t.

So we can put the value omega times u x, t at x=0 –omega square Us omega t okay, u x, t at x=0 is given it is 0 so we get here –c square omega square Us omega t okay and left hand side is what this is Us omega t, left hand side is this, so that we have +c square omega square Us omega t. Now it can be treated as a first order differential equation du/dt+c square omega square u and then its solution will be du/u so ln Us omega, t=-c square omega square*t+some constant.

Now because it is partial derivative the constant of integration will depend on omega. So we can call it as ln A omega. So then Us omega, t will be=A omega*e to the power -c square omega square t. Now at t=0 u x, 0=f x. Let us look at this, so from here we have Us omega, t=0 to infinity we have taken u x, t*sin omega x dx. This we defined so we put t=0 in this then Us omega 0=0 to infinity u x, 0 sin omega x dx.

And this is nothing but 0 to infinity u x, 0=f x, this is Fourier transform of f x so I can write it as Fs omega bar okay. So what we will have we put t=0, so Us omega 0=A omega okay, Us omega 0 is Fs bar omega okay Fourier sine transform of f x okay. Thus, Us omega, t becomes f s bar omega*e to the power –c square omega square t. Now let us use the inversion formula for the Fourier sine transform.

Because here we are taking one coefficient okay now while inverting we will take 2/pi. So we multiply both sides by sin omega x, integrate over 0 to infinity with respect to omega and multiply by 2/pi. So 2/pi 0 to infinity Us omega, t*sin omega x d omega=2/pi 0 to infinity f s bar omega e to the power –c square omega square t into sin omega x d omega okay. Now left hand side is f x and so we get this result.

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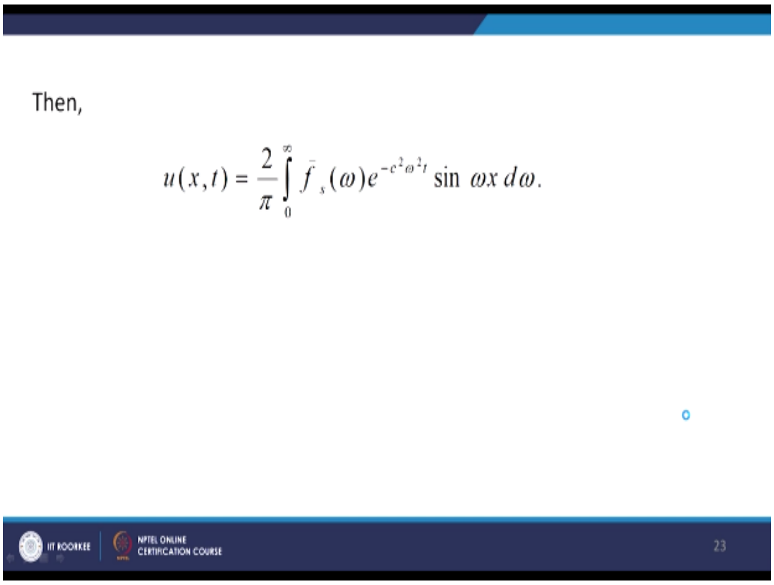

So left hand side is u x, t okay, left hand side is the inversion formula for u x, t so left hand side is u x, t. So u x, t=2/pi integral 0 to infinity f s bar omega e to the power –c square omega square t*sin omega x d omega. So this is how we solve this boundary value problem. With that I will like to end my lecture. Thank you very much for your attention.