Ordinary and Partial Differential Equations and Applications

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Lecture - 25
Critical Points and Paths of Non-Linear systems

Hello friends, welcome to the lecture on critical points and paths of non-linear systems. So, in this lecture we shall consider real autonomous system which is non-linear.

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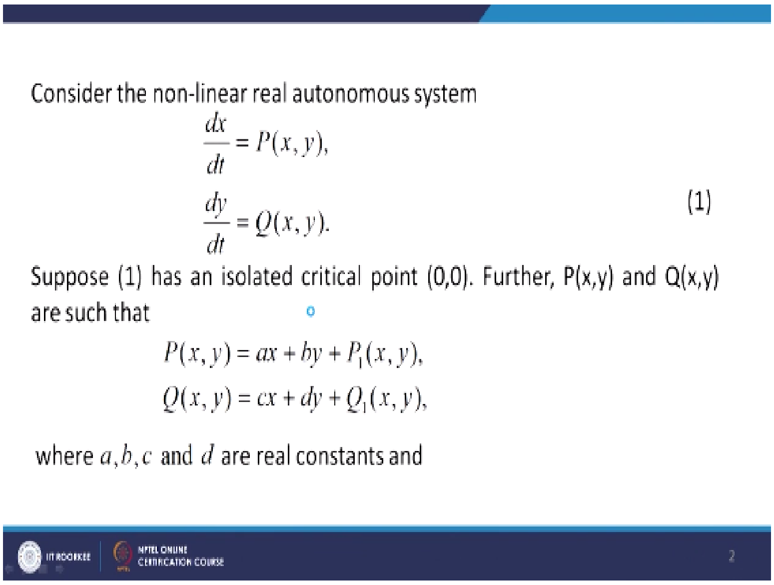

Say for example, dx/dt = P(x,y), dy/dt – Q(x,y). Now, suppose the system has an isolated critical point (0,0) further P(x,y) and Q(x,y) are such that P(x,y) = ax + by + P1(x,y), Q(x,y) = cx + dy + Q1(x,y) where a, b, c and d are real constants and they satisfy the condition that ad – bc is not = 0.

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Condition I: $ad - bc \neq 0$

Condition II: Further, $P_1(x, y)$ *and* $Q_1(x, y)$ have continuous first partial derivatives for all (x,y) and are such that

$$\lim_{(x,y)\to(0,0)}\left(\frac{P_1(x,y)}{\sqrt{x^2+y^2}}\right) = \lim_{(x,y)\to(0,0)}\left(\frac{Q_1(x,y)}{\sqrt{x^2+y^2}}\right) = 0.$$

Thus, we may write (1) as

$$\frac{dx}{dt} = ax + by + P_1(x, y),$$
$$\frac{dy}{dt} = cx + dy + Q_1(x, y), \qquad (2)$$

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For the functions P1(x,y) and Q1(x,y) we have the following condition. P1(x,y) and Q1(x,y) have continuous first order partial derivatives for all x,y and are such that P1(x,y) over under root x square + y square as x,y (0,0) goes to (0,0) and Q1(x,y)over under root x square + y square as x,y goes to (0,0) also goes to 0. So, P1(x,y) and Q1(x,y) satisfy these conditions then the system (1) can we written as dx/dt = ax + by + P1(x,y), dy/dt = cx +dy + Q1(x,y).

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where $a, b, c, d, P_1(x, y)$ *and* $Q_1(x, y)$ satisfy the above conditions.
If $P(x, y)$ *and* $Q(x, y)$ in (1) can be expanded in power series about (0,0), the system (1) becomes

$$\frac{dx}{dt} = P(0,0) + \left(\frac{\partial P}{\partial x}\right)_{(0,0)} x + \left(\frac{\partial P}{\partial y}\right)_{(0,0)} y + a_{12}x^2 + a_{22}xy + a_{21}y^2 + ...,$$

$$\frac{dy}{dt} = Q(0,0) + \left(\frac{\partial Q}{\partial x}\right)_{(0,0)} x + \left(\frac{\partial Q}{\partial y}\right)_{(0,0)} y + b_{12}x^2 + b_{22}xy + b_{21}y^2 + ...$$

Since P(0,0)=Q(0,0)=0, this system is of the form (2), where $P_1(x, y)$ and $Q_1(x, y)$ are the terms of higher degree.

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Where a, b, c, d, P1(x,y), Q1(x,y) satisfy the conditions 1 and 2. That is ad – bc is not = 0, P1(x,y) and Q1(x,y) have first orders continuous partial derivatives and P1(x,y) over under root x square + y square. Q1(x,y) over under root x square + y square to (0,0) as x,y goes to (0,0).

Now, if P(x,y) and Q(x,y) in the system (1) can be expanded in the power series about the point (0,0) then we can write the system 1x dx/dt =P(0,0) + partial derivative of P with respect to x at (0,0) into x, partial derivative of P with respect to y at (0,0) * y and then second order terms so a 12 * x square a 22 * xy a 21 * y square and then higher order terms.

And similarly dy/dt can be written as Q(0,0) + partial derivative of Q with respect to x at (0,0) * x partial derivative of Q with respect to y at (0,0) * y and then second order terms and so on. So, b12 * x square b22 * xy + b21 y square and so on. Now, since (0,0) is a critical point of the nonlinear system we are so P(x,y) and Q(x,y) at (0,0) must vanish and so therefore p(0,0) and Q(0,0) is = 0 and then this system, okay this system is of the form (2).

The system is of the form (2), where a is partial derivative of P with respect to at (0,0). b is partial derivative of P with respect to y(0,0), c is partial derivative of Q with respect to x(0,0) and d is partial derivative of Q with respect to y(0,0). P1(x,y) is a series of terms a12 x square + a22 xy + a 21 y square and higher degree terms and Q1(x,y) is b12 x square + b22 xy+ b21 y square and higher degree terms in in (x,y).

Now, we can see that P1(x,y) and Q1(x,y), if the dy P1(x,y) by under root x square + y square and Q1(x,y) under root x square + y square then as (x,y) goes to (0,0), they got to 0.

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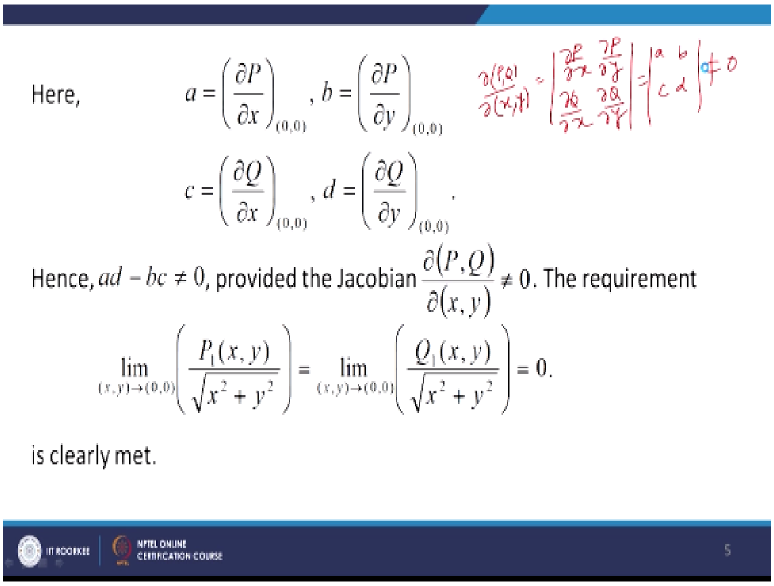

So, here a, b, c, d are this ad – bc therefore is not 0 provided this Jacobian is not 0. We know that the Jacobian of (P,Q) with respect to (x,y) is of this form. We define like this. Okay, so a is partial derivative of x with respect to partial derivative of P with respect to x at (0,0) similarly bc and d and ad – bc is not = 0 it means that the Jacobian is not 0. So, the requirement this is clearly met because P1(x,y) and Q1(x,y) contains second and higher order terms.

So when we square + y square and take the limit as x,y tends to (0,0) the limit becomes 0. So, this requirement is met and this requirement is also met.

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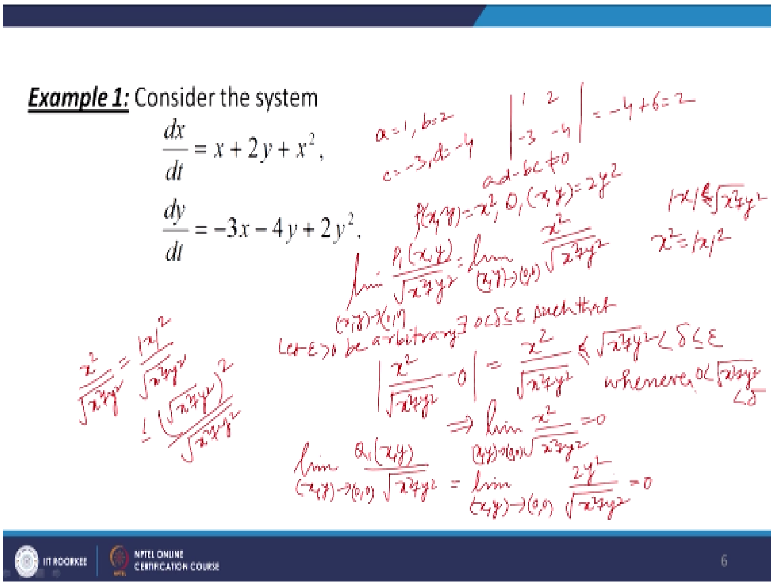

Now, let us consider the system dx/dt = x + 2y + x square dy/dt = - 3x – 4y + 2y square. You can see that here a = 1, b = 2, c = -3 and d = -4 and so 1, 2, -3, -4, a, b, c, d, okay. This is -4 and then + 6 = 2. So, ad-bc is not = 0, okay. So, our non-linear system is such that the coefficients of xy that is ab in the first equation, cd in the second equation satisfy the condition ad-bc not = 0. P1(x,y) = x square, Q1(x,y) = 2y square.

Since, P1(x,y) and Q1(x,y) are polynomials and x and y of degree 2, okay they have continuous first order partial derivatives and further P1(x,y) over under root x square + y square. Let us find the limit of this as (x,y) tends to (0,0). We shall show that this limit is 0. So, this is limit (x,y) tends to (0,0) x square over under root x square + y square. Now, we know that in order to find this limit, okay we go by epsilon delta definition.

So, in order to show that this limit is 0, let us begin within epsilon which is positive number. So, let us epsilon > 0 be arbitrary, then we have to find a delta > 0 such that mode of x square over under root x square + y square – 0 can be made < epsilon whenever under root x square + y square is > 0 but <delta. So, this is = x square upon under root x square + y square.

Now, this I can write us under root x square + y square because x square + y square is under root this is < or =, actually you see mode of x is = under root x square + y square, okay. And x square, I mean x is real is = mode of x is square. So, x square is = so this is < or =. Mode of x is < or = under root x square + y square and x square is mode of x square.

So, this is what, this is nothing but x square on under root x square + y square = mode of x square upon under root x square + y square and mode of x is < or = to under root x square + y square, so under root x square + y square x square/under root x square + y square. So, what you get is under root x square + y square, so this is this. And this is < delta, okay this is < delta and delta is < or = epsilon.

So, what we do is let epsilon be > 0 then mode of under x square upon under root x square + y square – 0 can be made < epsilon whenever 0 is < under root x square + y square < delta and delta is to be chosen, so < or = epsilon. So, they are a just 0 < delta < or = epsilon such that mode of this is < 0. So, this implies that limit of x square upon under root x square + y square as (x,y) tends to (0,0) = 0.

So, in a similar manner we can show that limit (x,y) tends to (0,0) Q1(x,y) over under root x square + y square which is = limit (x,y) tends to (0,0). Q1 (x,y) is = 2y square upon under root x square + y square. We can show that this also 0. So, P1(x,y) and Q1(x,y) satisfy the condition (2). That is they have continuous first order partial derivatives and P1(x,y) over under root x square + y square goes to 0 as (x,y) goes to (0,0) and Q1 (x,y) over under root x square + y square also goes to 0 as (x,y) goes to (0,0).

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Further, $P_1(x,y)$ and $Q_1(x,y)$ have continuous first order partial derivatives for all (x,y).

From the requirement (ii), the non-linear terms $P_1(x,y)$ and $Q_1(x,y)$ in (2) tend to zero more rapidly than the linear terms $ax+by$ and $cx+dy$. Hence, it is suspected that the behaviour of the paths of the system (2) near (0,0) could be similar to that of the paths of the related linear system

$$\frac{dx}{dt} = ax + by, \qquad \frac{dy}{dt} = cx + dy, \tag{3}$$

obtained from (2) by neglecting the non-linear terms. In general, this is actually true.

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Now, from the requirement 2, on P1(x,y) and Q1(x,y) that they tend to 0. P1(x,y) over Q1(x,y)/and x square + y square tend to 0 as (x,y) tend to (0,0) it means that they tend to (0,0) more rapidly than the linear terms ax + by and cx + dy. Hence we suspect that the behavior of the paths of the system (2) the system, non-linear system 2 could be similar to that of the paths of the related linear system.

dx/dt = ax + by, dy/dt = cx + dy which is obtained from the system 2 by neglecting the non-linear terms and in general it turns out that this is actually true.

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Theorem: Consider the non-linear system (2), where a, b, c, d, P_1 and Q_1 satisfy the requirements (i) and (ii) above. Consider also the corresponding linear system (3). Both systems have an isolated critical point at (0,0). Let λ_1 and λ_2 be the roots of the characteristic equation

$$\lambda^2 - (a+d)\lambda + (ad - bc) = 0$$

of the linear system (3). Then it turns out that

1. The critical point (0,0) of the non-linear system (2) is of the same type as of the linear system (3) in the following cases:

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Let us discuss this theorem, consider the non-linear system (2), where a, b, c, d, P1 and Q1 satisfy the requirements (1) and (2) above. That is ad – bc is not = 0 P1(x,y) and Q1(x,y) have continuous first order partial derivatives P1(x,y) over under root x square + y square and Q1(x,y) over under root x square + y square tends to 0 as (x,y) tends to (0,0).

So, consider also the corresponding linear systems (3) then both the systems have an isolated critical point at (0,0). Let us take the cross consider characteristic equation lambda square – (a+d) * lambda + (ad-bc) = 0 and let us say that lambda (1) and lambda (2) are its roots then it turns out that the critical point (0,0) of the nonlinear system (2) is of the same type as that of the linear system (3) in the following cases.

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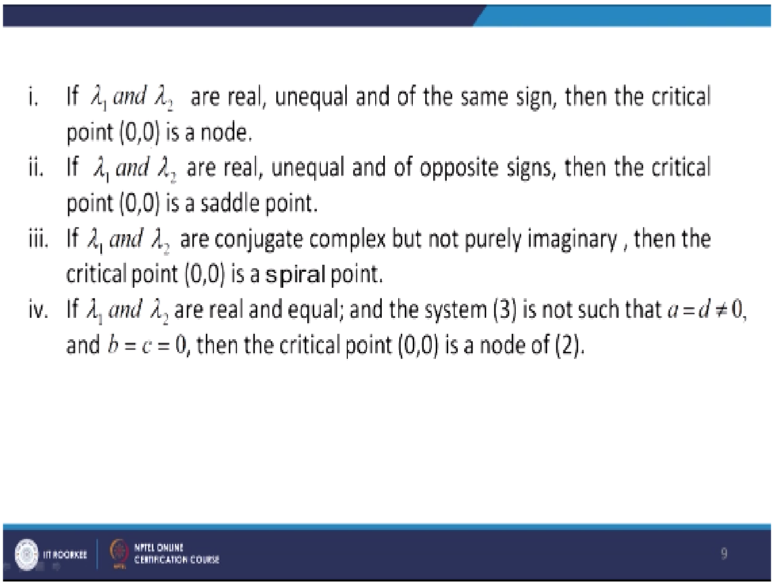

If lambda 1 and lambda 2 are real, unequal and are of the same sign then the critical point (0,0) is a node, we know that. For the linear system (3) if lambda 1 and lambda 2 are real, unequal and are of the same sign, then the critical point (0,0) is a node and same is true for the non-linear system. So, if lambda 1 and lambda 2 are real, unequal and of the same sign then the critical point (0,0) is a node.

If lambda 1, lambda 2 are real, unequal and are opposite signs then the critical point (0,0) is a saddle point. So, in the case of non-linear system also if lambda 1 and lambda 2 are real, unequal and or of opposite signs then the critical point (0,0) will be a saddle point. Same is the nature as for the linear system here also in the case of non-linear system. If lambda 1 and lambda 2 are conjugate complex but not purely imaginary, then the critical point (0,0) is a spiral point.

If lambda 1 and lambda 2 are real and equal and the system (3) is not such that, okay. System (3) means again let us see the system (3) is the corresponding linear system, this one. So, if it is not of this type. a = d not = 0 and b = c = 0. We have discussed the situation in the case of a linear system. There were 2 possibilities. One possibility is that in the case of real and equal roots was that a = d not = 0 and b = c = 0.

And the other case that we are discussed was in the case of all other possibilities which lead to equal roots which are real, okay we had discussed the type of the critical point. So, here if lambda 1 and lambda 2 are real and equal and the system associated linear system is not such that a = d not = 0 and b = c = 0, then the critical point (0,0) is a node of the non-linear system.

Now, the critical point (0,0) of the system (2). That is the non-linear system is not necessarily of that. So, here there is a difference between the type of the critical point in the case of non-linear system and the associated linear system. The critical point (0,0) of the non-linear system is not necessarily of the same type as that of the linear system (3) in the following cases.

If lambda 1 and lambda 2 are real and = and the system is such that a = d not = 0 and b = c = 0 then although (0,0) is a node of (3) it may be either a node or a spiral point of the non-linear system. If lambda 1 and lambda 2 are purely imaginary, then although (0,0) is center of (3) it may be either a center or a spiral point of the non-linear system.

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Remark: Although the type of the critical point (0,0) is the same for (2) as it is for (3) in the cases (i)-(iv) in the above theorem, the actual appearance of the paths may be somewhat different. For example, the fig.1 shows a typical saddle point for a linear system whereas, fig.2 suggests how a non-linear saddle point might look. A certain amount of distortion is clearly present in the latter, but nevertheless the qualitative features of the two configurations are the same.

Now, let us look at this remark. Although the type of the critical point (0,0) is the same for (2) as it is for (3) in the cases (1) to (4) in the above theorem. 1, 2, 3, 4 in this theorem. The actual appearance of the paths may be somewhat different. For example, the figure 1 shows a typical saddle point for a linear system you can see.

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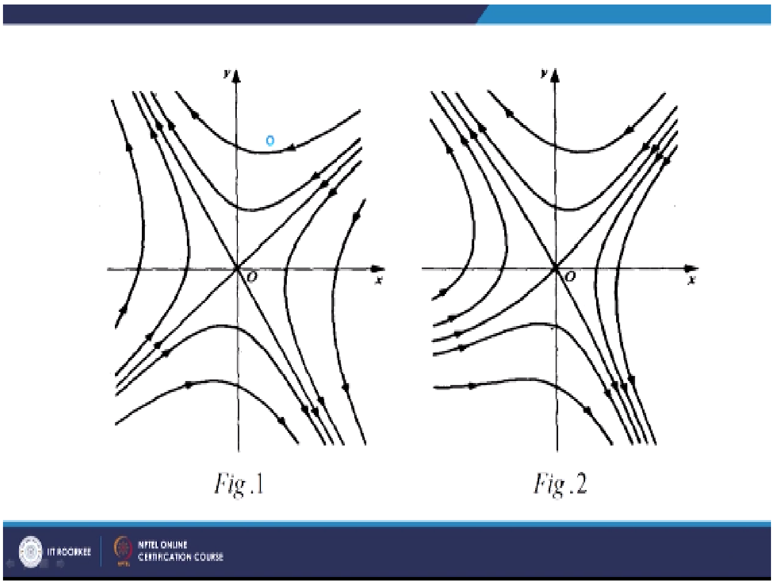

This (0,0) here is a saddle point for the linear system here also (0,0) is a saddle point but the appearance of the saddle point are the paths, okay is different. They are not linear they are curves, non-rectilinear paths. So, the figure 1 shows a typical saddle point for a linear system whereas figure 2 suggests how a non-linear saddle point might look. A certain amount of distortion is clearly present in the latter figure, in the figure 2.

But nevertheless the qualitative features of the 2 configurations are the same. So, we can see here, here we have 2 half line paths, one half line path here. They all enter and approach (0,0) as t goes to + infinity and then we have other 2 half line paths they also enter and approach (0,0).

But this is the half line path for t tends to infinity, these 2 are the half lines paths for t tends to – infinity and these are the non-rectilinear path which are one half line, one of the 4 half lines paths. And here we do not have half line paths, we have rectilinear paths which enter and approach (0,0) you can see here.

There is one non-rectilinear path and there is another non-rectilinear path which enter and approach (0,0) as t goes to + infinity and to goes to – infinity and these non-rectilinear paths are these half line path. Half this non-rectilinear path which approach enter (0,0). So, actual appearance of the saddle point here in the case of non-linear system may be somewhat different.

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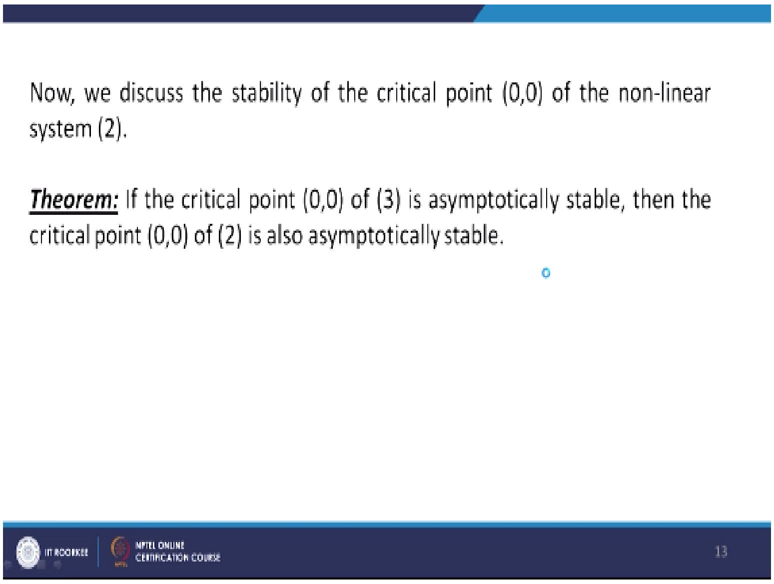

Now we discuss the stability of the critical point (0,0) of the non-linear system (2). If the critical point (0,0) of the non-linear, if the linear system (3) is asymptotically stable. See, we have the non-linear system dx/dt = ax + by + P1 (x,y), dy/dt = cx + dy + Q1(x,y). We are linear system, we are considering is dx/dt = x + by and dy/dt = cx + dy. So, if the critical point of the linear system is asymptotically stable, then the critical point (0,0) of the non-linear system is also asymptotically stable.

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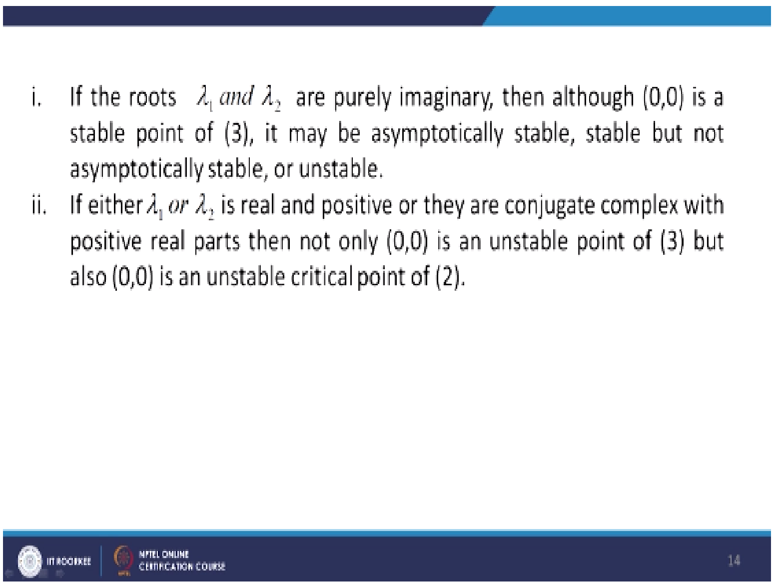

Now, here are the exceptions. If the roots lambda 1 and lambda 2 are purely imaginary. lambda 1 and lambda 2 of the equation are purely imaginary then although (0,0) is a stable point of the linear system. It may be asymptotically stable, is stable but not asymptotically stable or in the stable point, critical point for the corresponding non-linear system.

If either lambda 1 or lambda 2 is real and positive or they are conjugate complex with positive real parts, then not only (0,0) is an unstable point of the linear system but also (0,0) is an unstable critical point of the non-linear system (2).

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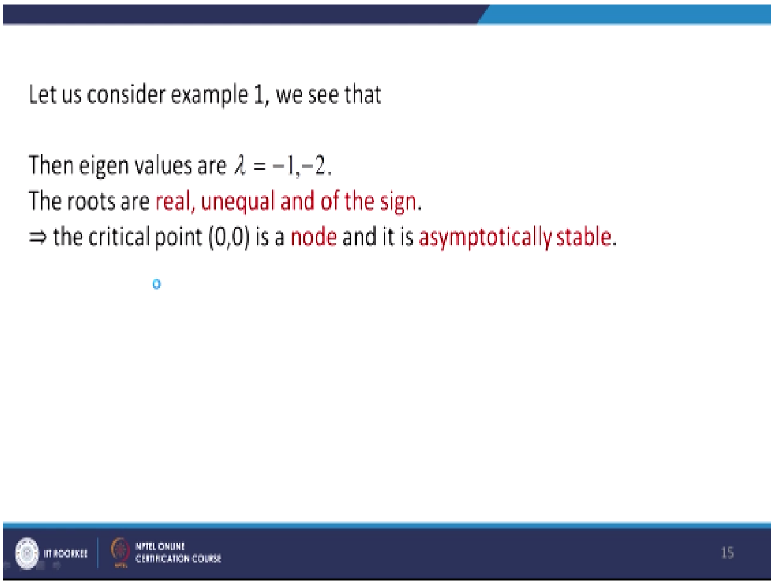

So, Let us consider example 1, we see that let us go to the example 1.

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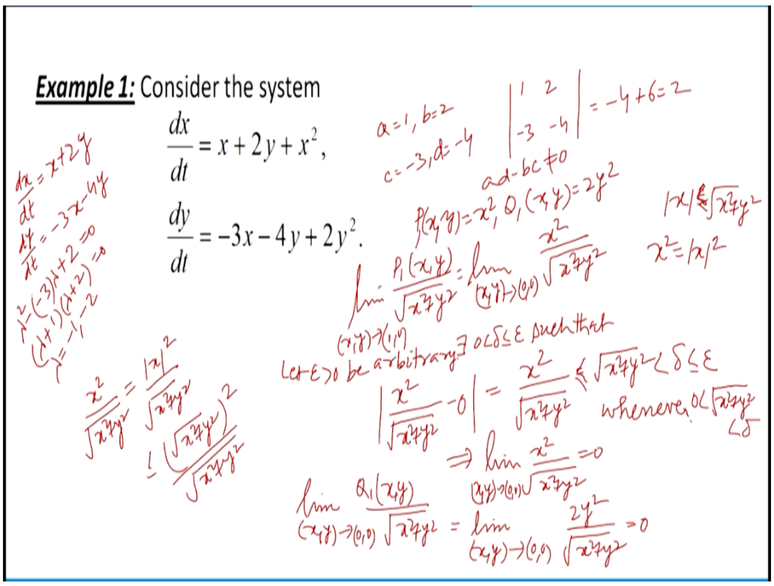

Let us go to the example 1. So this example 1 dx/dt = x + (2*.y) + x square 2, dy/dt = -(3*x) – (4*y) + 2*(y square 2). So let us consider the type of the critical point (0, 0) here. So let us write the corresponding linear system dx/dt = x + (2* y) dy/dt = - (3 * x) – (4 * y). So the Eigen values here are lambda square 2 – a + d, so a is 1, d is -4, so – (3 * lambda) + (a*d – b*c) which is 2 = 0. So, (lambda square 2) + (3 * lambda); so (lambda +1) * (lambda + 2) = 0.

So Eigen values are -1 and -2. Ok, so now let us see what is the type of the critical point at (0, 0). Let us go to this theorem.

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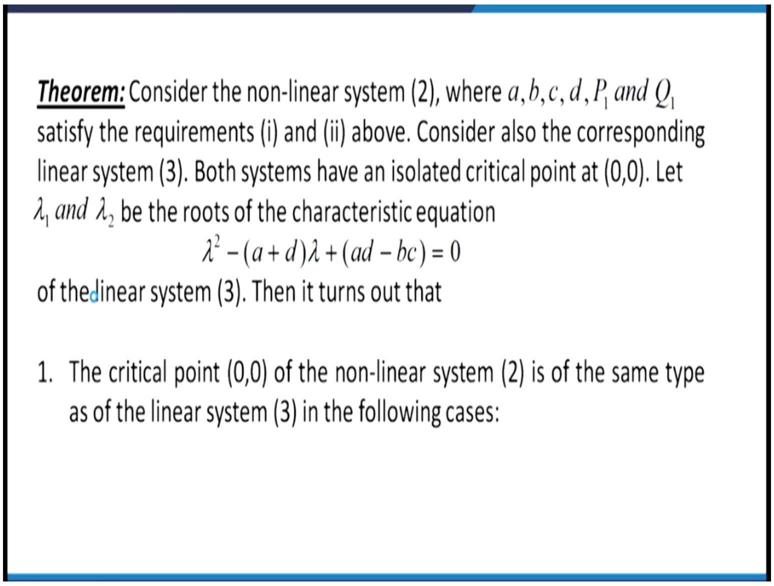

It says that, the critical point (0, 0) of the non-linear system (2) is of the same type as that of the linear system (3) in the following cases.

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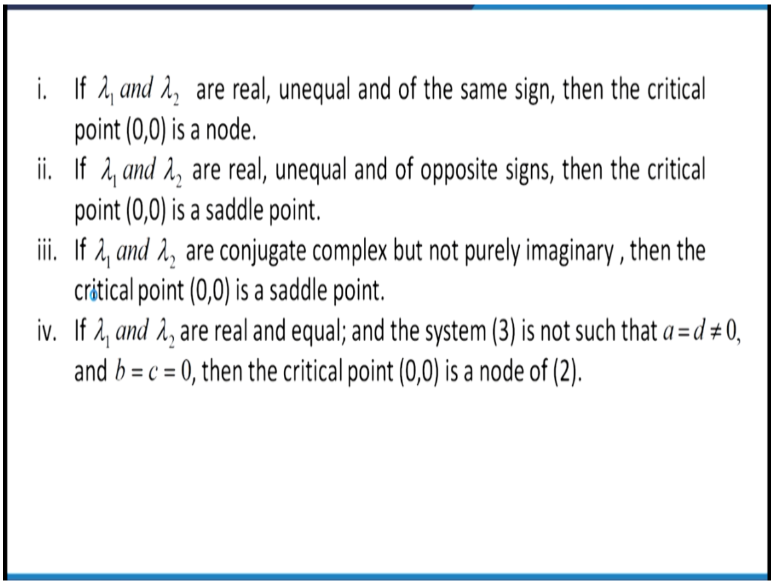

If lambda 1, lambda 2 are real, unequal and of opposite signs, then the critical point (0, 0) is a saddle point. Oh sorry we have to go to (i). If lambda 1, lambda 2 are real, unequal and of the same sign, then the critical point (0, 0) is a node. So here the roots are -1 and -2 which are real and equal and have the same sign, so the critical point (0, 0) is a node.

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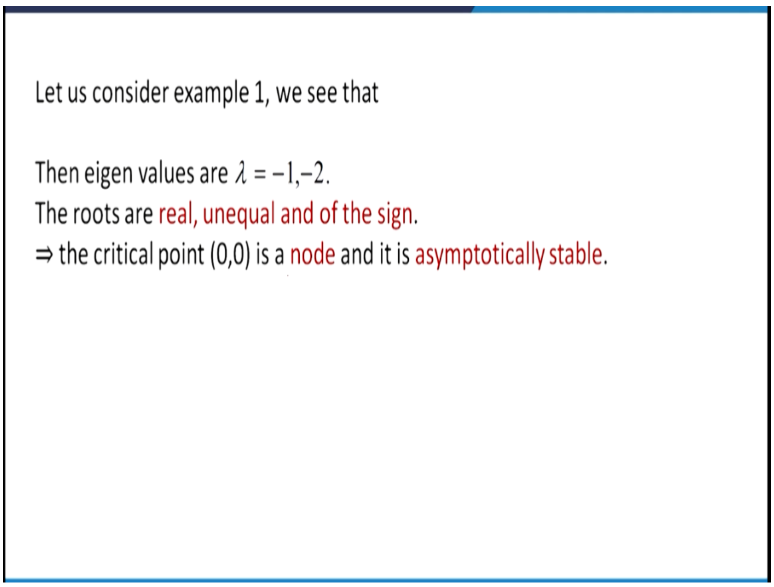

So the roots are real and equal and of the same sign, so the critical point (0, 0) is a node. So now we have to go to the stability of this.

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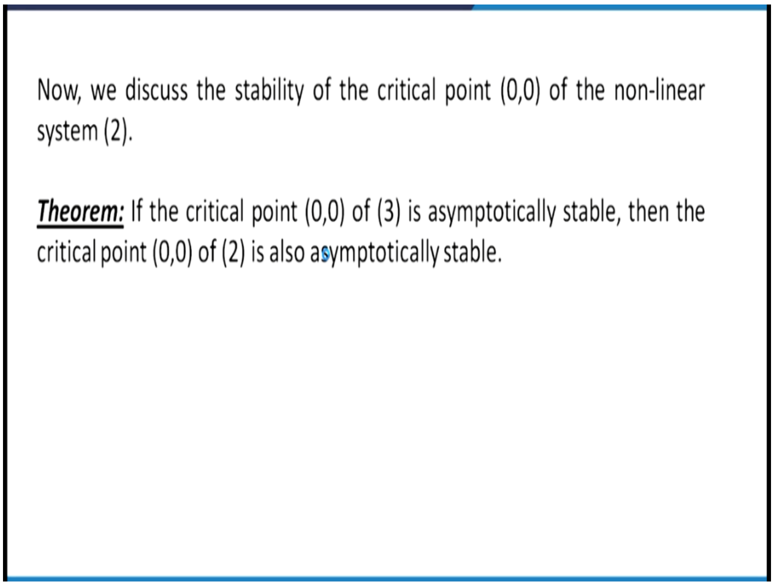

If the critical point (0, 0) of (3) is asymptotically stable then critical point (0, 0) of 2 is also asymptotically stable. So let us see, in the case of this, regarding the stability we know that the Eigen values are lambda = -1 and lambda = -2. So it is a asymptotically stable critical point of the linear system. Dx/dt = x + dy, dy/dt = cx + dy and so (0, 0) is also asymptotically stable point in the case of the non-linear system of example (1).

So the critical point (0, 0) is a node and it is asymptotically stable.

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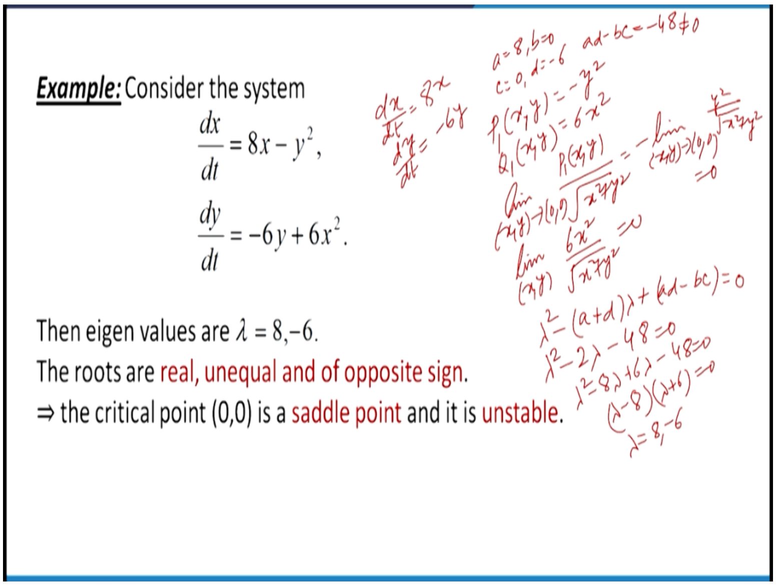

Now let us consider the system dx/dt = (8*x) – (y square 2),dy/dt = -(6 *y) + (6 * x square 2). So here we will have, the corresponding linear system will be dx/dt = (8*x), dy/dt = -(6 *y). So a = 8, b = 0, c = 0, d = -6. So (a*d)-(b*c) = -48 which is not 0. And therefore first condition (a*d)-(b*c) != 0 is met.

P1(x, y) = -(y square 2), Q1(x, y) = (6 * x square 2). So again P1(x, y) and Q1(x, y) are polynomials in x and y of degree 2 each, so they have continuous first order partial derivatives. Further as we have shown in example (1), P1(x, y) over under root (x square 2 + y square 2), limit (x, y) tends to (0,0) P1(x, y) over under root (x square 2 + y square 2) = – limit (x, y) tends to (0,0) y square 2 over under root (x square 2 + y square 2).It will be 0.

And similarly limit (x, y) tends to (0,0) (6 * x square 2) which is Q1(x, y) upon under root (x square 2 + y square 2) will be 0. So the second condition is also met. Now let us see the (()) (24:32) equation is, (lambda square 2) – (a + d) * lambda + (a*d –b*c) = 0. So (lambda square 2) – (a + d), a = 8 and d = -6, so we have (lambda square 2) – (8 -6) so we have (2*lambda); (lambda square 2) - (2*lambda), and (a*d) = -48.

So we have (lambda square 2) - (8*lambda) + (6 * lambda) – 48 = 0 and this gives us lambda – 8) * (lambda + 6) = 0. So the Eigen values are lambda = 8 and -6. Now the Eigen values are real and equal and are of opposite sign. So the point (0, 0), let us go to the theorem, if lambda 1, lambda 2 are real, unequal and of opposite signs, then the critical point (0, 0) is a saddle point. So it is a saddle point, (0, 0) is a saddle point.

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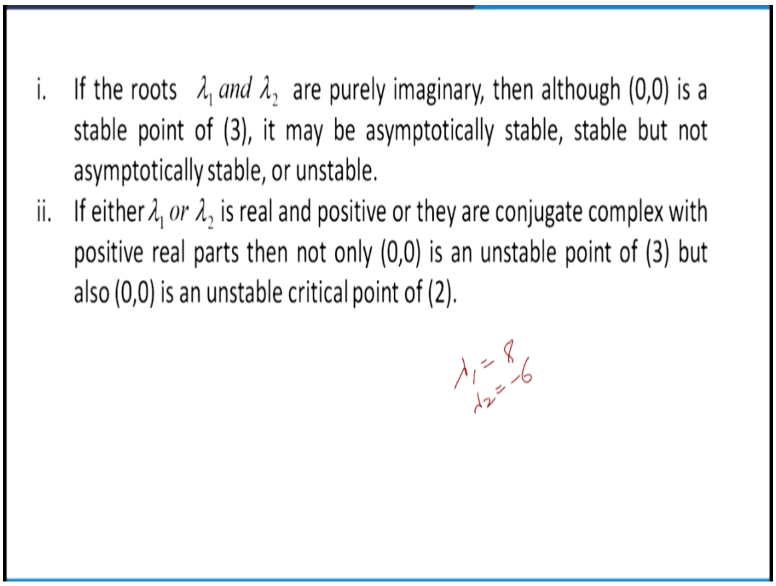

So now we go to regarding stability, we have if the critical point (0, 0) of (3) is asymptotically stable, then the critical point (0, 0) of 2 is also asymptotically stable. Ok, here if either lambda 1 or lambda 2 is real and positive or they have conjugate complex with positive real parts then not only (0, 0) is an unstable point of (3) but also (0,0) is an unstable critical point of (2).

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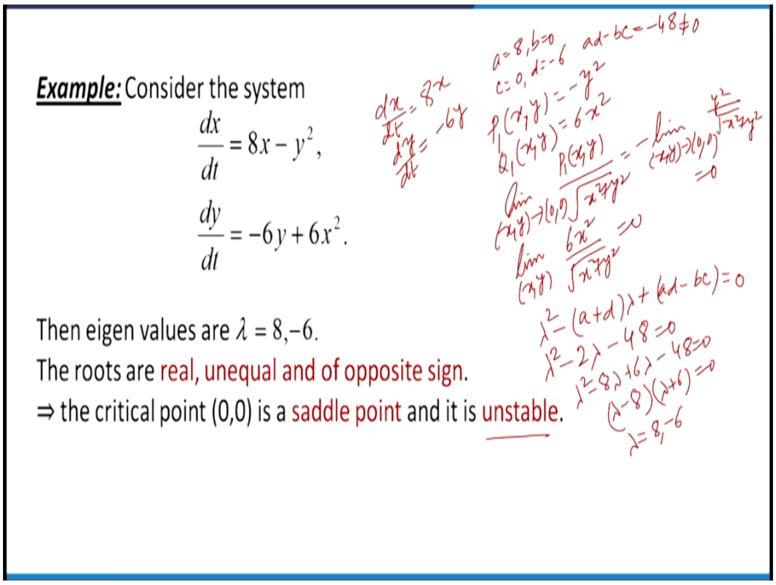

So here we have seen lambda 1 = 8, lambda 2 = -6. So one of the 2 Eigen values is positive, real and positive, and therefore (0, 0) is an unstable point of (2), unstable point of this system. So this is unstable.

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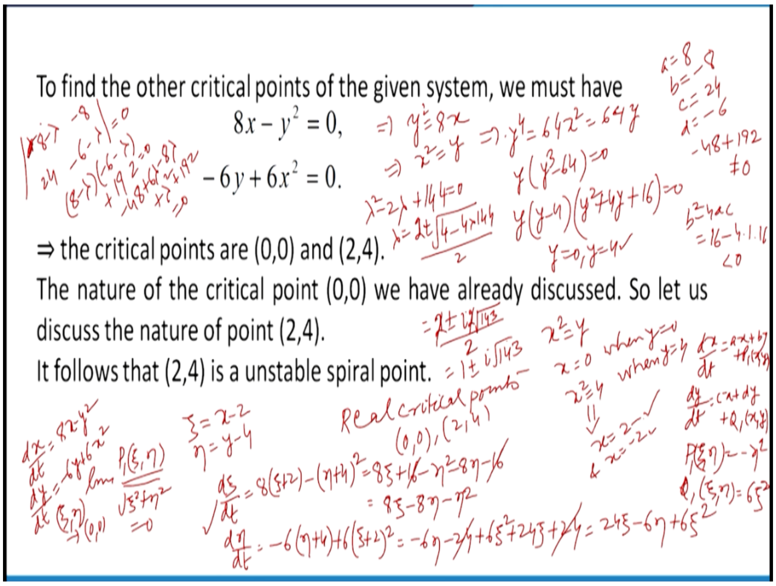

Now let us find other critical points, real critical points of this system. We have seen dx/dt = (8*x) – (y square 2), dy/dt = - (6 *y) + (6 * x square 2). So we have only considered the real critical points (0,0) but it has other critical points also. Let us find those other real critical points and see what is the nature of the critical point there. And so for that we know the critical points of a system is given by dx/dt = P(x, y)/ (dy/dt) = Q(x, y).

P(x, y) = 0, Q(x, y) = 0.So we shall have (8*x) – (y square 2) = 0 and – (6 *y) + (6 * x square 2) = 0. So this will give you (y square 2) = (8 * x) and here (x square 2) = y. So we have here, this will give you (y to the power 4) = (64 * x square 2). (x square 2) = y, so (64 * y). So (y times (y to the power 3)) – 64 = 0. So this is y * (y -4) * (y square 2), this is (y to the power 3) – (4 to the power 3), so (y square 2) + (4*y) + (4 square 2), so 16 = 0.

So now the real solutions of this equation, you can see are y = 0 and y = 4. This equation (y square 2) + (4 * y) + 16 does not give us the real solution because (b square 2) – (4* a* c) = 16 – 4*1*16 which is negative. So equation (y square 2) + (4 * y) + 16 does not give us the real value of y. The real values of y are 0 and 4. And x square 2 = y, so x = 0 and y = 0. And x square 2 = 4 when y =4. Now this gives you 2 values of x, x = 2 and x = -2.

Let us see x= 2. If you take x = 2 then y square 2 = 16 is fine, so x = 2 is admissible but x = -2 is not admissible because y square 2 becomes -16. So corresponding to 4, x will be taken as 2. So we have 2 real critical points are (0, 0) and (2, 4). Now the nature of critical point (0, 0) we have already found. So let us see the nature of the other real critical point (2, 4). So we have x = 2, y = 4, let us translate this point (2, 4) to (0, 0) by putting Xi = x – 2 and Eta = y -4.

Then this x = 2, y = 4 will to Xi, Eta plane at the point (0, 0). Now let us write the given equations in the form of Xi and Eta.

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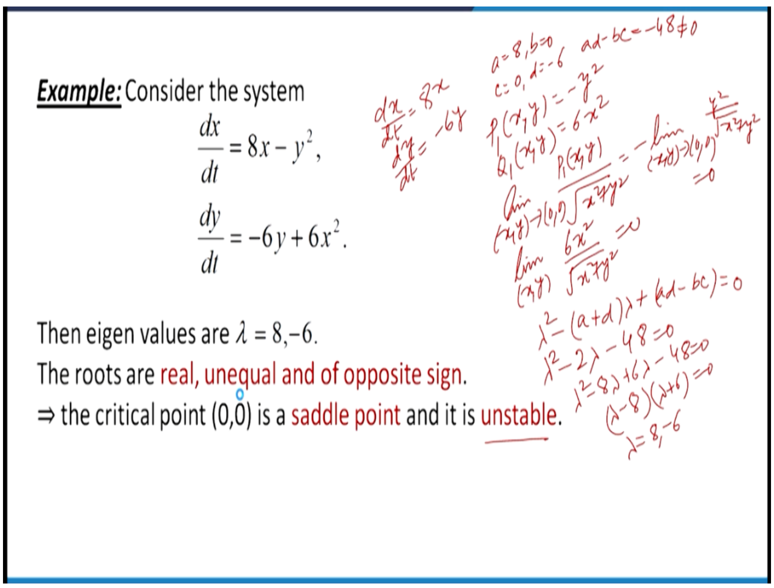

So we have dx/dt = (8*x) – (y square 2) and – dy/dt = -(6*y) + (6* x square 2). Now you can see here dXi/dt = dx/dt, so we have dXi/dt = (8*x), so 8 times x = Xi +2 – (y square 2), y square is (Eta + 4) square 2. So we have 8*Xi + 16 – Eta square 2 – 8* Eta -16. So we will get this, dXi/dt = 8 * Xi – 8 * Eta – Eta square 2. And dEta/dt = dy/dt. So dy/dt we can write as dEta/dt and then -6 times y= Eta +4 plus 6 times x square 2, ((Xi + 2) square 2).

So we get -6 * Eta – 24 + 6* Xi square 2 + 24 * Xi + 24. So this cancels with this and we have 24 * Xi – 6 * Eta + 6 (Xi square 2). So now you can see, dXi/dt = 8*Xi – 8* Eta – Eta square 2, dEta/dt = 24*Xi – 6*Eta + 6 * (Xi square 2). This system is of the same type as we have taken dx/dt = (a*x) + (b*y) + P1(x, y) and dy/dt = (c*x) + (d*y) + Q1(x, y). This system is same as this system.

You can see a*d – b*c is != 0. Here we have a = 8, b= -8, c= 24 and d = -6. So a*d, a*d = -48, a*d – b*c will be -48 + 192, so this is not 0 and you can see P1(x, y) is actually P1 (Xi, Eta) here, P1 (Xi, Eta) = -Eta square 2 and Q1 (Xi, Eta) = 6* Xi square 2. So P1 (Xi, Eta) over under root Xi square 2 + Eta square 2, as (Xi, Eta) goes to (0, 0). Similarly, Q1 (Xi, Eta) over under root Xi square 2 + Eta square 2 goes to (0, 0) as (Xi, Eta) goes to (0, 0). So both condition 1 and 2 are met.

And therefore the solution of this system, the critical point of this system dXi/dt, dEta/dt that is (0, 0) in (Xi, Eta) plane will be same as the corresponding linear system. The Eigen values here for this system are, we have 8Xi-8 Eta, so 8, – lambda, – 8, so we have here 24 and -6, -lambda =0. So we have to find Eigen values here. We have 8 – 8- lambda then -8, 24-6-lambda, we will get here (8-lambda) *(-6-lambda) and then we have here + 192 = 0.

So what equation we are getting? -48 + (6* lambda) – (8*lambda) and then + lambda square 2 + 192 =0. So lambda square 2 – (2* lambda) + 144 = 0. Now (b square 2) – (4* a* c) is negative here, so it will give you complex, lambda = [2 + or – under root (4 – 4*144))]/ 2. So it will give you (2 + or – i)/2. Here what we get, we can get 4 common and so we will get 2 here and then under root 143. So this 2 will cancel, and we will get 1 + or – i square root 143.

So 2 Eigen values are conjugate complex alpha + -i * beta, alpha is positive. It is 1. So we can see, so the nature point, so in this case lets go to theorem 1. If lambda 1 and lambda 2 are conjugate complex but not purely imaginary, since alpha = 1, alpha 1 alpha 2 are not purely imaginary, then the critical point (0, 0) is a saddle point. So (0, 0) in the (Xi, Eta) plane is a saddle point.

So (0, 0) in the (Xi, Eta) plane, (2, 4) has gone to (0,0) in the (Xi, Eta) plane, so it is a spiral point. If lambda 1 and lambda 2 are conjugate complex but not purely imaginary, then the critical point (0, 0) is a spiral point, so (2, 4) is a spiral point.

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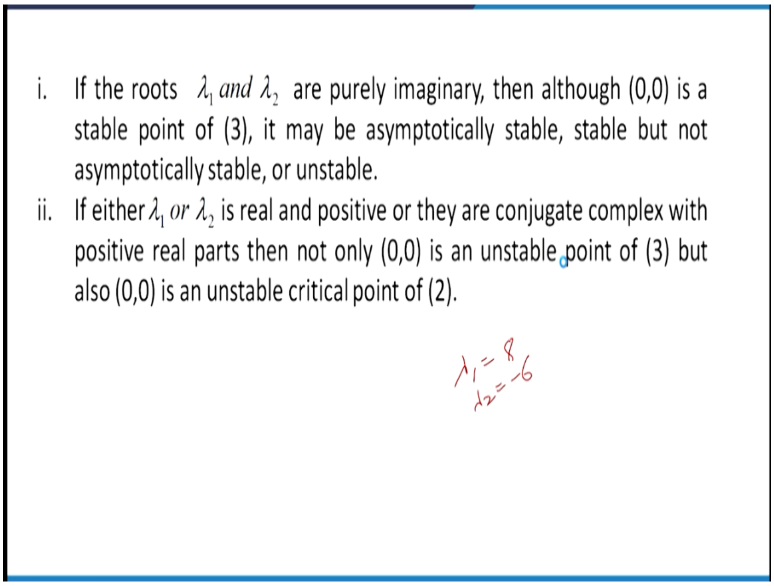

And regarding stability, if either lambda 1 or lambda 2 is real and positive or they are conjugate complex with positive real parts then not only (0,0) is an unstable point of (3) but also (0,0) is an unstable critical point of (2). So (2,4) or you can say (0,0) in the (Xi, Eta) plane is an unstable point. So it is unstable spiral point. Now let us come back to the (x, y) plane. (0, 0) is the point in the (Xi, Eta) plane but in the (x, y) plane its (2, 4).

So the nature of (2, 4) is that it is a unstable spiral point. Thank you very much for your attention.