Ordinary and Partial Differential Equations and Applications
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Lecture - 19
Regular Singular Points-IV

Hello friends welcome to this lecture. In this lecture we will continue our study of finding series solution, solutions for linear differential equation in the neighbourhood of regular singular point so if you would recall in previous lecture we have discussed the method of Frobenius to find out series solutions for linear differential equation and we have considered basically 3 case depending on the roots of indicial equation.

So in case when roots of indicial equation are not equal and not differ by integer we know how to find out the solution and that we have discussed in previous class. Now in this class we discuss the case when the roots of indicial equation say E are equal and we have seen in previous class that when roots of indicial equation here it is denoted as R1 and R2.

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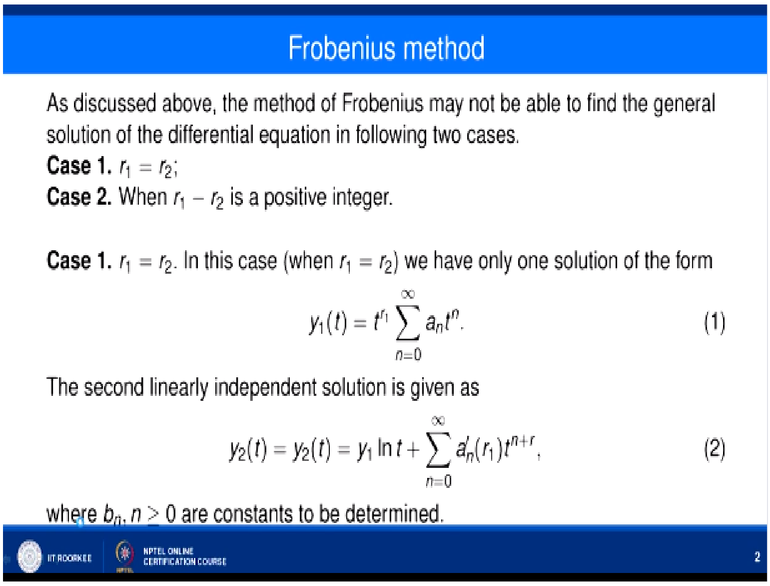

And when r1 = r2 and we have only one solution of the form that is y1t = t to the power r1 summation from 0 to infinity an t to the power n, so one solution is given like this and the second linearly independent solution is given by this y2t = y1 lnt + n = 0 to infinity an dash r1 t to the power n + r here. So here this is basically r1 here, so here the current form is already discussed in previous class. Here we take some example based on this. So here let us take one example.

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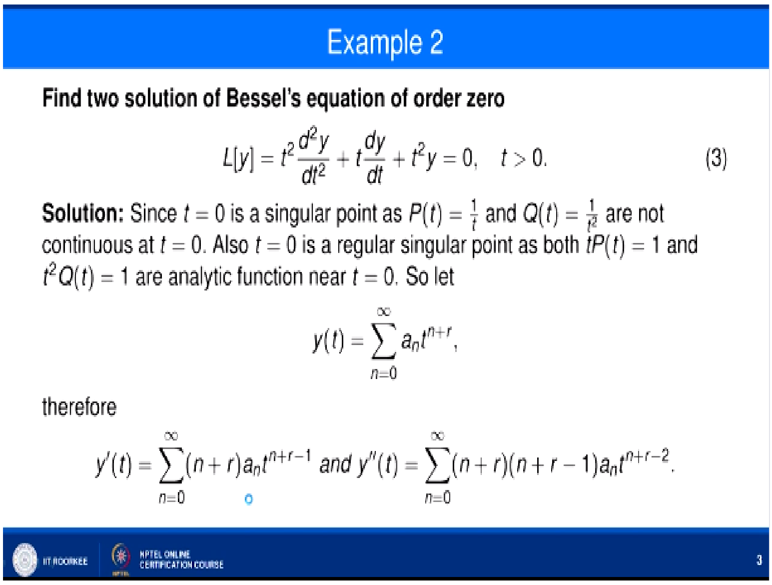

So find 2 solutions of Bessel's equation of order 0. So here Bessel's equation is given by t square d2y/dt square + t dy/dt + t square y = 0, t > 0, and if you look at the coefficient of d2y/dt square that is t square so t = 0 is a singular point and now we want to check whether it is a regular singular point or irregular singular point. Now we want to show that it is a regular singular point for that look at the tPt.

So tPt is 1 and t square Q t is 1. So both are basically constant function, so we can say that both are analytic function near t = 0, so it means that our theory can be applied here to find out solution so here let us assume yt = summation n = 0 to infinity an t to the power n + r is a proposed solution and we need to find out the values of r and the corresponding coefficients an's so that this will serve as a solution of equation number 3.

So for that you just calculate y dash t, so y dash t is from n = 0 to infinity n + r an t to the power n + r-1 and calculate y double dash t so it is n = 0 to infinity n + r * n + r -1 an t to the power n + r -2, so once we have yt, y dash t, y double dash t put it back to equation number 3.

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So we have,

$$L[y]=\sum_{n=0}^{\infty}(n+r)(n+r-1)a_n t^{n+r}+\sum_{n=0}^{\infty}(n+r)a_n t^{n+r}$$
$$+\sum_{n=0}^{\infty}a_n t^{n+r+2}=\sum_{n=0}^{\infty}(n+r)^2 a_n t^{n+r}+\sum_{n=2}^{\infty}a_{n-2}t^{n+r}.$$

Equating the sums of like power of t equal to zero, we get

t^r (*i*) $r^2 a_0 = F(r)a_0 = 0$ ✓ $\quad r(r-1)a_0 + ra_0 = 0$

t^{r+1} (*ii*) $(1+r)^2 a_1 = F(1+r)a_1 = 0$ ✓ $\quad n=1 \quad (r+1)r\,a_1 + (r+1)a_1 = 0$

t^{n+r} (*iii*) $(n+r)^2 a_n = F(n+r)a_n = -a_{n-2}, \quad n \geq 2.$

$(n+r)^2 a_n = -a_{n-2} \quad t^{n+r} \quad (n+r)(n+r-1)a_n + (n+r)a_n + a_{n-2}$

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And you will see that here it is n = 0 to infinity n + r * n + r-1 an t to the power n + r because we have already term like t square so t to the power n + r-2 and 2 t square will make it t to the power n + r + here it is t/dt. So here it is summation n = 0 to infinity n + r an t to the power n + r + n = 0 to infinity n t to the power n+ r + 2.

So here it is t square y, y is already given to you. So you can write an = 0 to infinity, an t to the power n + r. So here we have the L y = n = 0 infinity n + r, n + r-1 an t to the power n + r + n = 0 to infinity n + r an t to the power n + r + n = 0 to infinity an t to the power n + r + 2. Once we have this equation then we do the equating the sums of like power of t = 0 here. So here if you look at when you put n = 0 your powers will start from 2.

And if you look at the first 2 terms if you put n = 0 then your powers start from t to the power r. So we can collect the coefficient of t to the power r and t to the power r + 1 and t to the power r +2 then all the terms of this equation will come into picture. So first let us look at the coefficient of t to the power r = 0. So t to the power r = 0 means here look at the first term, here you put n = 0 so it is r * r -1 + here you will also get r.

So it is r * r -1 + r that is r square a0. So this we write F of r a0 = 0, so this is corresponding to t to the power r and here we are getting this r * r -1 a0 + from second term were are getting r a0 and = 0 because here if you look at the third term if you put n = 0 then the power raised t to the power r + 2 so there is no contribution from the third term. Okay.

So here we have t to the power r and we have this r square a0 and that is f of r a0 = 0 and then you look at the coefficient of t to the power r + 1 again there is only contribution from first 2 term only that is a1 corresponding to n =1 so it is r +1 r a1 + r +1 a1 = 0. So if you calculate this then here we have 1+ r whole square * a1 which I am writing as f 1+r a1 = 0 and then when you put general term t to the power n + r = 0 then what you will get here is from this term.

So write general term so it is n + r n + r -1 an + n + r an and if you look at the last term then last term is having power t to the power n + r +2. So if we replace n/n-2 then that power will be t to the power n + r. So here we write an – 2 and that is the coefficient of t to the power n + r. So if you simplify this, this and this will give you n + r whole square an so here you will get as n + r whole square an = - of an – 2 here, that we are writing here.

So here we will get 3 equations first is r square a0 = 0 and then next is f of 1 + r * a1 = 0 and last one is f of n + r an = -an -2 and here n is >= 2.

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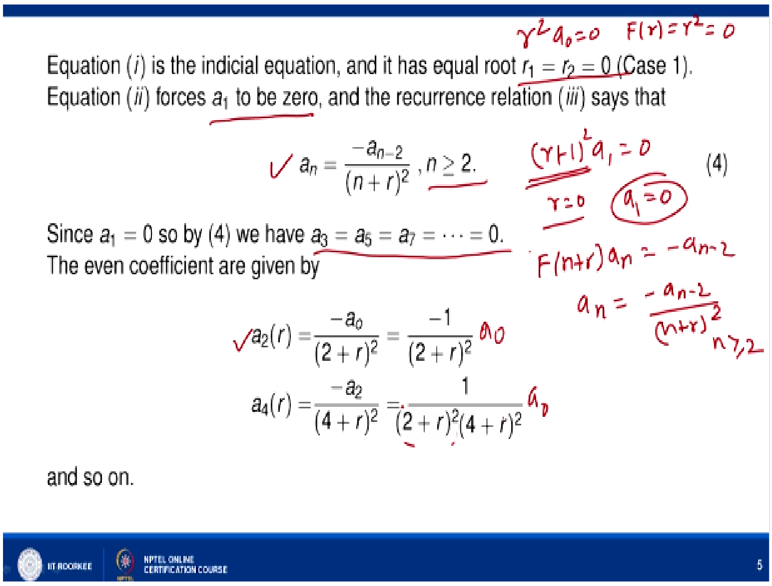

So now, so equation 1 is indicial equation and that is r square a0 = 0 and we already know that a not is never 0 so your indicial equation f of r = r square = 0 so if you look at the roots of this indicial equation are 0. So r1 = r2 = 0. So here we have equal root of indicial equation. Now if you look at the second equation that is r + 1 whole square a1 = 0 so if you take r = 0 then this term is nonzero.

So a1 has to be 0. So from second equation we say that a1 has to be 0 and if you look at the third term that is f of n + r, okay, so last one is what f of n + r an = -an-2, so here we can write an as – an -2 upon n + r whole square and that is n >= 2. So for, we have equation like this an = -an -2/n+r whole square and n >= 2. Now we already know that a1 is 0 that we have just calculated, a1 = 0.

So using this formula we can say that all the odd terms are going to be 0 because if you write a3 then a3 is given in terms of a1 – a1 upon 1 + r whole square, so here we say that since a1 is 0 so a3 is going to be 0 now a3 is 0 then using this recurrence relation we can say that all the odd terms are going to be 0. So that fix matter that all the odd coefficients are 0. Now look at the a2r.

A2r is given as using this formula 4, a2r = -a0 upon 2 + r whole square here we write -1 upon 2 + r whole square * a0. If you look at a4r, a4r = -a2 upon 4 + r whole square. Now a2 we have already calculated so it is 1 upon 2 + r square and 4 + r square a0 and so on. So in this way we can calculate our even order coefficients.

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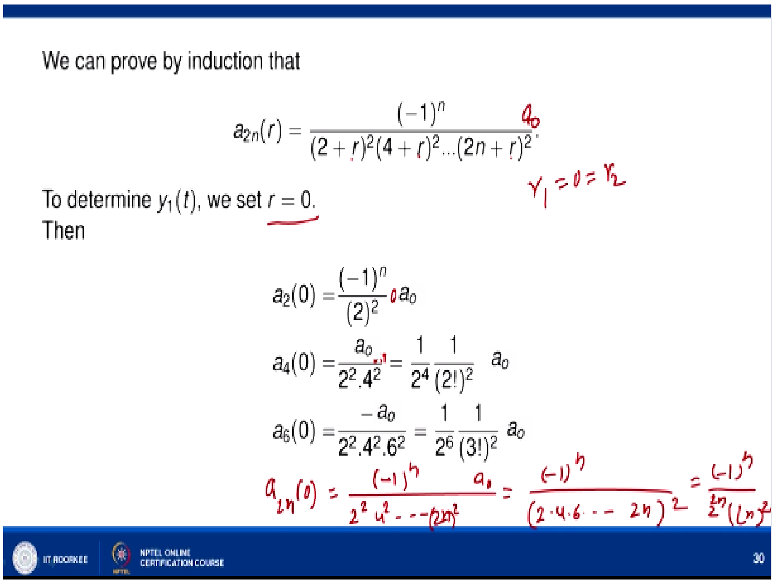

And we can write this formula as a2nr = -1 to the power n 2 + r whole square 4 + r whole square and so on 2n + r whole square * a0 and to determine y1t, we simply put r = 0 because here we have 2 roots r1 = 0 = r2, so to find out one solution we can put r = 0 and we can find out a20 = -1 to the power n and when put r = 0 all these term will be sample vanishing so it is 2 square 4 square and so on.

So a2n 0 is going to be 2 square 4 square up to 2n whole square so or we can write it a2 as -1 to the power n 2 square a0 and a 4 0 as 1 upon 2 square 4 square a0 and so on. So in this way you can calculate here a2n 0 = here – 1 to the power n and if you look at it is 2 square 4 square and so on n square and if you simplify and a0 and it is – 1 to the power n and here if we look at it is what 2, 4, 6 and so on 2n.

Here it is 2n whole square and whole square and this we can write as -1 to the power n, 2n * factorial n whole square. Here that we can simplify here, if we take 2 out then it is 1 2 3 and up to n. So here we can write -1 to the power n and 2 to the power 2 n will come out and factorial n whole squares. So that an * a0 we are writing here.

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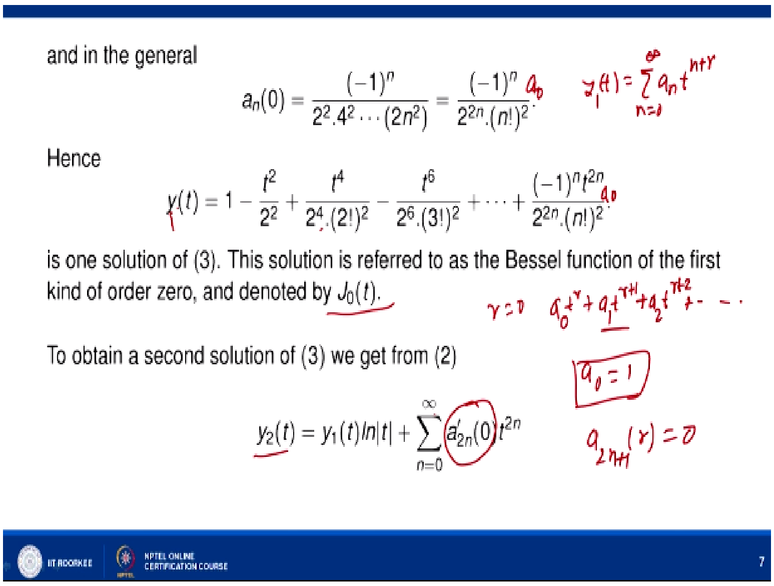

So here we got a2 n0 as -1 to the power n a0/2 to the power 2n factorial n whole square. So now we already know that all odd terms are 0 so we can write y1t as 1- you simply write it y1t = summation an t to the power n + r and is from 0 to infinity. So here this we can write as let me write it here. So here we can write a0 t to the power r + a1 t to the power r + 1 + a2 t to the power r + 2 and so on.

Now we already know that these terms are gone, odd terms are gone and r = 0 here so we can write a0 let us say that if we take a0 as 1 then we can write out solution as y1t = 1–t square upon 2 whole square. Here we are using the value of A2 that is value of A2 we have already calculated. Value of a2 is –1 upon 2 square.

So here we are using 1-t square upon 2 square + a4 a4 as 1 upon 2 to the power 4 factorial 2 whole square and so on, so in this way we can calculate y1t. So y1t is one solution of 3, that we have already calculated and this solution is referred as Bessel's function of the first kind of order 0 and we denote as J not t. So now to obtain a second solution we have already seen that our second solution y2t can be written as y1t * ln modulus t + n = 0 to infinity a dash 2 n 0 t to the power 2n.

Here I am writing a 2n dash 0 because we already know that a2n +1 r is 0, irrespective of whatever value in the n. So here what is left out is only the even terms. So here we need to calculate the derivative of aa2n r.

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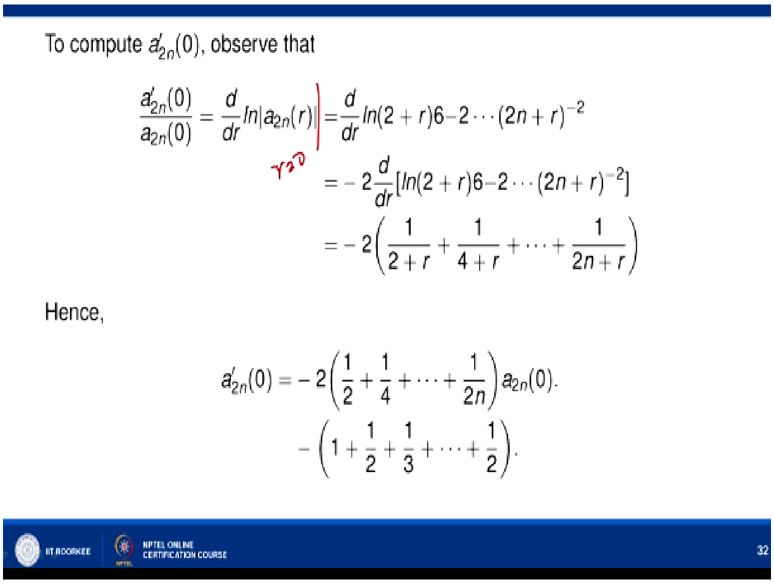

So to calculate a derivate of a2n 0 so here we use this thing. We already calculated a2nr like this, so here let us assume without loss of generality that a not = 1 right. Then a2nr is given by this and if you want to calculate a-2n at 0 for that we simply look at that a dash 2n0/a2n0 that can be written as d/dr ln of a2n r modulus of a2nr log and that you calculate at r = 0 and using the, let me do it here.

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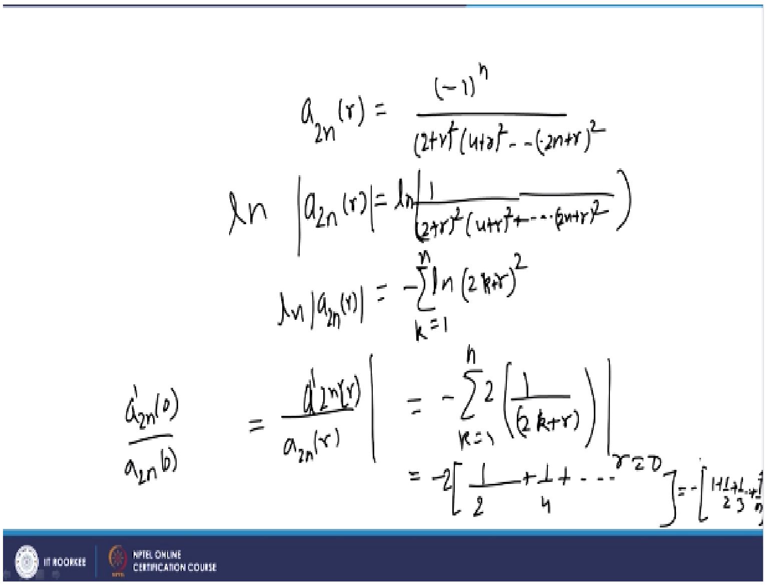

So here a2nr is given as -1 to the power n, it is I think it is written here. It is 2+r whole square. Let me write it here, 2 + r whole square, 4 + r whole square and so on, 2n + r whole square. So here let us take the modulus of a2n r that will get rid of this minus sign so it is 1 upon 2 + r whole square, 4 + r whole square and so on 2n + r whole square and then take the ln, so here if you take the ln here then it is what, ln of -1 that is gone.

So –ln of the summation will come, sorry the summation is outside this, so here you will get summation here we can write 2 + r 2n+r whole square and is from, if you write k, so let me write k here, 2k + r k is from 1 to n. so we can write ln of modulus of a 2nr = -of k = 02n ln of 2k+r whole square, is it okay. Then we can take the differentiation. So if we take ln of modulus of a 2n r.

Now take the derivative. So derivate will be 1 upon a2n r and a dash 2n and here it is r. Now we take r = 0 here = here it is what – and then we can summation k = 02n and 2 you can take it out, 2 and this is 1 upon 2 k + r and k is from 0 to infinity and that you calculate at r = 0. So when you simplify, it is k = 1 not 0, so it is from k = 1. So here when you put it is what minus and here when you put k = 1 then it is what, 1 upon 2 is already out -2 and here it is what, 2 + 1 upon 4 +1, so you will get.

And when you take 2 out then it is what it is given as -1+1/2 +1/3 and last is 1/n, so here we can get this as a dash 2n 0/a2n 0 like this, and this we denote as hn so that we are going to write it like this.so here we have calculated a dash 2n0/a2n0 as -2 1 upon 2 + r +1 upon 4 + r and so on and when you put r = 0 then all these terms will simple cancel out and you take 2 inside then a dash 2n0 = -21/2+1/4 and so on a2n 0.

Now this I am writing as -1 + 1/2 + 1/3 and so on, a2n0. So here if we set the bracket term as Hn.

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Setting

$$H_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \quad (5)$$

we see that

$$a'_{2n}(0) = \frac{-H_n(-1)^n}{2^{2n}.(n!)^2} \qquad a_{2n}(0) = \frac{(-1)^n}{2^{2n}(\lfloor n)^2}$$

and thus

$$y_2(t) = y_1(t) ln|t| + \sum_{n=0}^{\infty} \frac{(-1)^{n+1} H_n t^{2n}}{2^{2n}.(n!)^2}$$

is a second solution of (3) with H_n given by (5).

So we can write Hn as 1 + 1/2 + 1/3 + 1/n then we can write a dash 2 n 0 = -Hn * -1 to the power n/2 to the power 2n factorial n whole square. This we are writing as the value of a2n0 that is -1 to the power n/2 to the power 2n factorial n whole square. So using the value of a2n0 we can write a dash 2n0 as this. So y2t = y1t ln0 t + n = 0 to infinity – 1 to the power n + 1 Hn t to the power 2n/2 to the power 2n factorial n whole square and that is going to be your second solution of Bessel's equation of order 0.

But if you look at this Bessel's solution, second solution contains the term ln modulus t and because of this it is having unboundedness near the point t = 0 so generally we ignore this second solution but for sake of completeness we are putting the second solution like this, okay. So now look at the last case, the second case where the method of Frobenius may not give the second solution of the Frobenius form.

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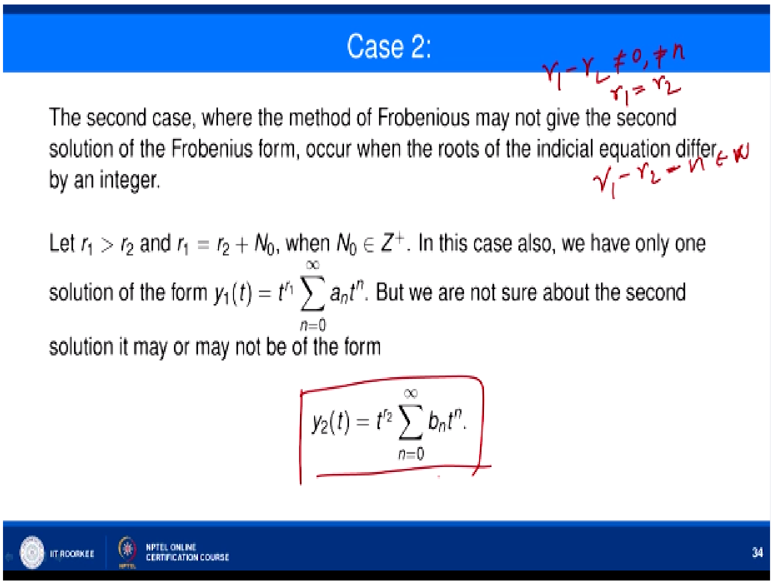

This occurs when the roots of the initial equation differ by a integer so here we have already seen that when r1-r2 is nonzero, not integer then we already know that we have discussed in previous lecture and the case when r1 = r2 that is difference is 0 we have already discussed one example right now and then last case when r1-r2 = some n and is some integer that we want to discuss it now.

So here we already assume that r1 is > r2, so let us take r1 = r2 + some n0, n0 is some integer, positive integer and in this also we have only 1 solution of the form that is y1t = t to the power r1 summation n = 0 to infinity an t to the power n and we are not sure about the second solution why, because that we are going to discuss but we are not sure about the second solution.

We will take this form or not, it may take this form, it may not take this particular form that depend on the difference of r1-r2.

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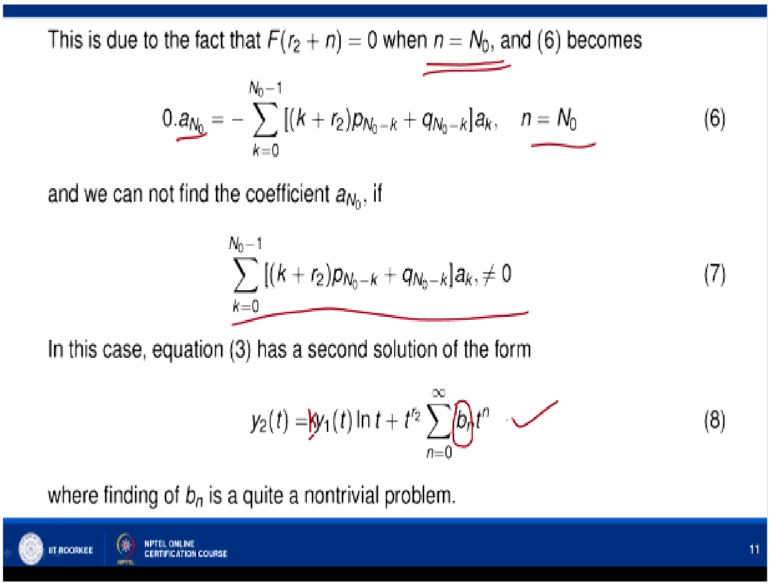

So here we will not get the second solution because if you remember the recurrence solution, recurrence solution is what f of r2 + n * an + r2 = -summation k = 0 to n0 -1, this term is there, so from recurrence relation we know that if f of r2 + n = 0 when n = n0. So when n = n0 then the coefficient of a n0 is basically 0. So here this is 0 and here if you look at the righthand side it is –k from 0 to n0 -1, k + r 2 Pn0 – k+ qn0 –k * ak.

That is, I am writing the recurrence relation for n = N0 and we cannot find the coefficient an0 if the right hand side is nonzero. So if righthand side is nonzero then this equation cannot be satisfied. So it means that in this case when this righthand side is nonzero we do not have any solution of the form given by Frobenius. So in this case equation 3 has a second solution of this form y2t = y1t lnt * sum constant + t to the power r 2 summation n = 0 to infinity bn tn.

So it is up to us how to calculate this bn and this constant. So many times it may happen that your constant k maybe 0. So the second solution may happen that we do not have term involving lnt, but that will depend how to find out this kn bn and one method to find out this is the method of variation of parameters so once we know one solution we can use method of variation of parameter to find out the other solution.

So it means that when this righthand side is nonzero then we do not have no method to find out this coefficient a0 and we simply say that second solution will not be the form of forming a series solution and it will take the form of 8 where it may or may not involve the term y1t lnt and it is up to us how to calculate this bn.

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But if (7) is not true that is

$$\sum_{k=0}^{N_0-1}[(k+r_2)p_{N_0-k}+q_{N_0-k}]a_k=0 \quad (9)$$

then we can choose any value for the coefficient a_{N_0}. In particular we choose $a_{N_0}=0$(what happen if we choose $a_{N_0}\neq 0$) and once we have a_{N_0} we can use the recurrence relation to find other coefficients $a_n(r_1)$ for all $n\geq 1, n\neq N_0$. In this case when (9) is true we have both the solution given in Frobenius series form.

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But if this is 0 means if the righthand side of this equation is 0 then of course we can say that equation is satisfied means 0 * an0 = this term which is already = 0 in this case we can choose any value of the coefficient a0, so in particular we will take the value 0. So here we take a0 as 0 value. If we take a nonzero value, then we will get a part of y1t it means that when we take an0 as nonzero then when you calculate all the coefficient.

And when you write the general solution then you will say that y2 will contain a part of y1t. So to avoid this thing we generally assume that an0 = 0 so here we say that once we have an0 then we can use our recurrence relation to find out further coefficients an0 + 1 and so on. So in this case when this right hand side is = 0 we can find out all the coefficient and we say that our solution is given in terms of series solution of the form of given by Frobenius.

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Example 3

Find two solution of Bessel's equation of order $\frac{1}{2}$

$$L[y]=t^2\frac{d^2y}{dt^2}+t\frac{dy}{dt}+(t^2-1/4)y=0, \quad t>0. \quad (10)$$

Solution: Since $t=0$ is a singular point as $P(t)=\frac{1}{t}$ and $Q(t)=\frac{1}{t^2}(t^2-1/4)$ are not continuous at $t=0$. Also $t=0$ is a regular singular point as both $tP(t)=1$ and $t^2Q(t)=t^2-1/4$ are analytic function near $t=0$. So let

$$y(t)=\sum_{n=0}^{\infty}a_nt^{n+r},$$

therefore

$$y'(t)=\sum_{n=0}^{\infty}(n+r)a_nt^{n+r-1} \text{ and } y''(t)=\sum_{n=0}^{\infty}(n+r)(n+r-1)a_nt^{n+r-2}.$$

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So let us take one example based on this thing that root differ by integer so here let us take Bessel's equation of order 1/2 so here it is t square d2y/dt square + tdy/dt + t square -1/4y = 0 here, I am assuming that is positive. So here we can easily see that t = 0 is a regular singular point as Pt = 1/t and Qt = 1 upon t square * t square -1/4 and not continuous at t = 0. So first thing is to check whether it is a singular point or not.

So once it is singular point then look at the next thing that is it is regular singular point or not. For that you look at tPt and t square Qt. So look at tPt, tPt is = 1 and t square Qt is t square – 1/4 and we can say that these 2 are analytic function, so we can say that t = 0 is a regular singular point. Once it is regular singular point then we can propose our solution is of the form yt = summation n = 0 to infinity an t to the power n + r.

And we can find out the solution y dash t = n from 0 to infinity, n + r an t to the power n+r -1 and ytt, y double dash t = n = 0 to infinity n+r n+r -1 an t to the power n+r -2, put it back to equation #10 and we can have Ly as summation n = 0 to infinity, n+r n+r -1 an t to the power n+r because t square is already there and second term is n = 0 to infinity n + r an t to the power n + r + 0 to infinity an t to the power n + r +2-1/4 n = 0 to infinity an t to the power n+r. So last 2 term is corresponding to this term the t square -1/4 * y.

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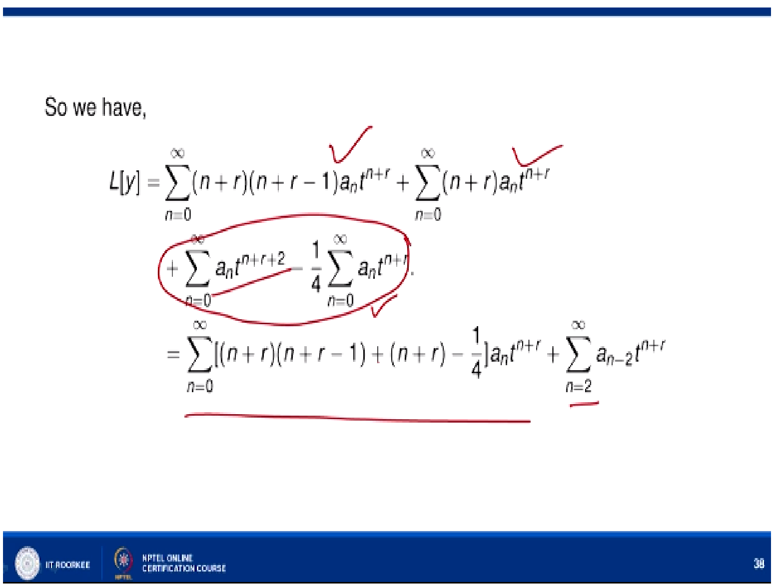

So when you simplify this we simply say that this term, this term and the last term will give you the coefficient of t to the power n +r and this term means third term is going to give a term having power t to the power n + r + 2. So we can write this as from n = 0 to infinity n+r

* n+r -1 + n+r-1/4 an t to the power n+r and from n = 2 to infinity an-2 t to the power n+r. So again this we can simplify and we can say that it is n+r whole square.

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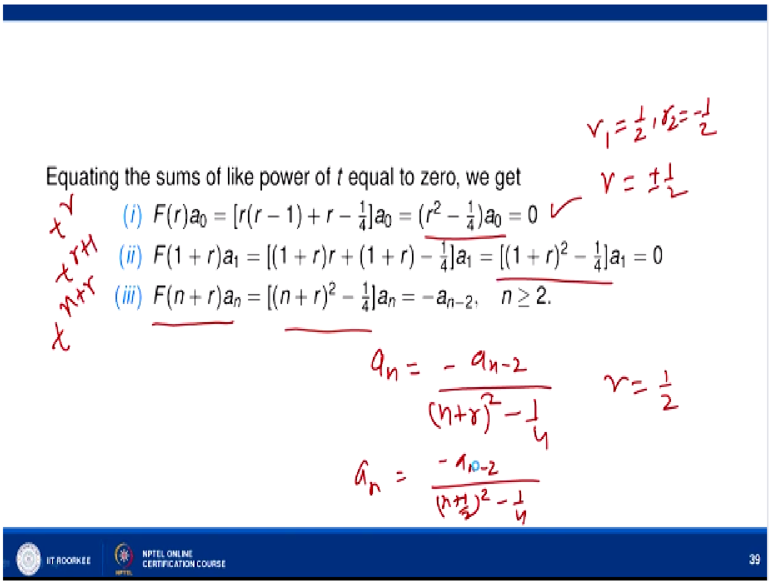

So we can write that equating the sums of like powers of t = 0, so here first let us take t to the power r. If you look at the coefficient of t to the power r that we can obtain from by putting n = 0 when put n = 0 then it is r * r-1 +r-1/4. So it is r square – 1/4 r square -1/4 * a not = 0 and when you look at t to the power r +1 and then again there is no contribution from this term and we will look at here by putting n = 1.

When you put n = 1 then 1+r 1+r -1 +1 +r -1/4 a1 = 0, so we have F of 1+r a1 = 0 and F of 1+r is 1+r whole square -1/4 a1 = 0, the last is t to the power n+r and here we have F of n+r an which is given as n+r whole square -1/4 an and there is a contribution of last term also here it is +an -2. So here when you take the other side it is –an -2. So here we will get these 3 equations.

First is the indicial equation which will give you the values of r for which we can find out the solution and that when you look at it is nothing but r = +/- 1/2 so r1 is the bigger value that is 1/2 and r2 = -1/2, so here we have 2 roots of indicial equation which are differed by n positive integer that is 1.

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Equation (i) is the indicial equation, and it implies that $r_1 = \frac{1}{2}, r_2 = -\frac{1}{2}$ (Case 1).
$r_1 = \frac{1}{2}$ and set $a_0 = 1$ Equation (ii) forces a_1 to be zero, and the recurrence relation (iii) says that

$$a_n = \frac{-a_{n-2}}{(n+r)^2 - \frac{1}{4}}, n \geq 2. \qquad a_1 = 0 \qquad (11)$$

Since $a_1 = 0$ so by (13) we have $a_3 = a_5 = a_7 = \cdots = 0$.

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So let us look at the analysis of this, so here I said that r1 = 1/2, r2 = -1/2 so let us find out the solution corresponding to r1 = 1/2 and let us set that a not = 1 so that this solution should not come back. So now look at the second solution and if you put r = 1/2 then what you will get, here when you put r = 1/2 then you can say that this term is not going to be 0. So even has to be 0 for this. So here we say that the second equation forces a1 to be 0.

So we can say that since a1 = 0 then by recurrence relation that is obtained by the last equation that is this thing so here we can simply say that an = -an-2 and if you simplify this is what n + r whole square -1/4 and when r =1/2 when you can simply say that an = -an-2 and it is what, n+1/2 whole square -1/4 you can further simplify this. Okay, so we can write an as –an-2/n+r whole square -1/4. So from this we can simply say that since a1 = 0 then all term will be 0.

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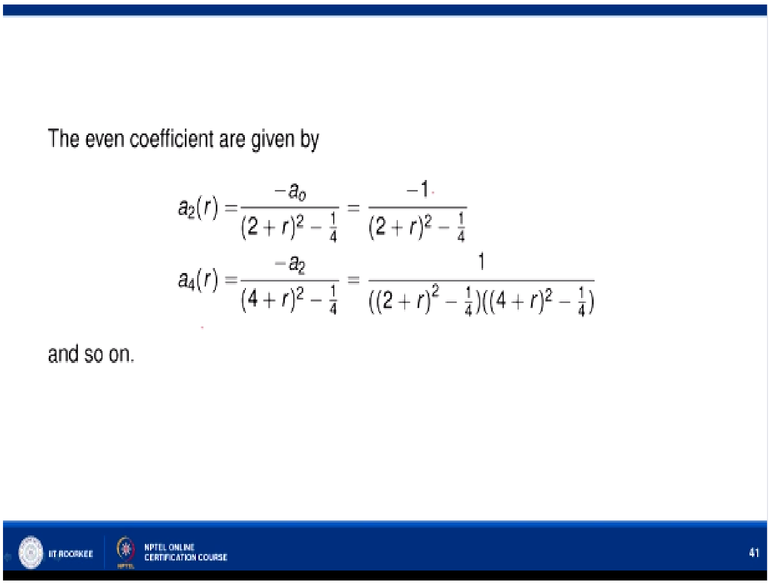

Then even coefficient we need to calculate so let us calculate a2 r, a2r = -a0 upon 2+r whole square-1/4 and we can simply say that it is -1 upon 2 + r whole square -1/4 and similarly we can calculate a4r and all that.

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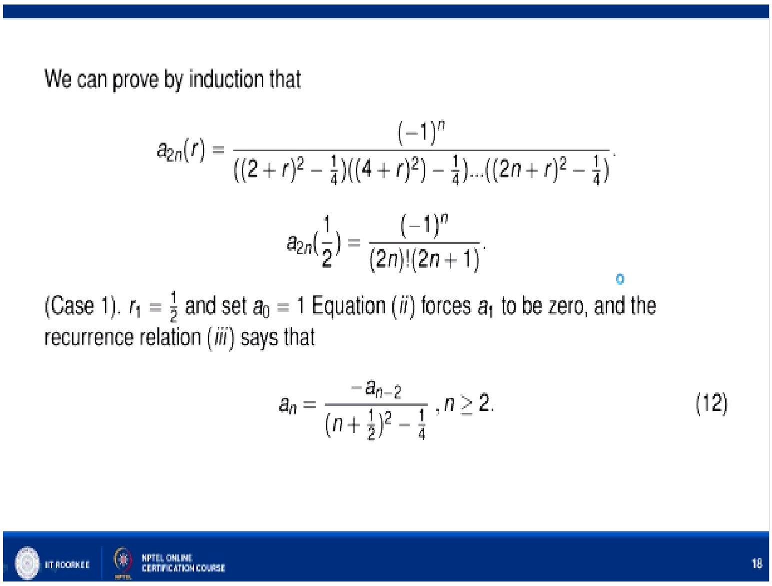

And we can prove by induction that a2nr is given by -1 to the power n, 2+r whole square -1/4, 4+r whole square -1/4 and so on. So this is the formula of a2nr when r is some values. We already know that our first solution is corresponding to r1 = 1/2 just put the value of r1 = 1/2 and put set = a not = 1, we can say that this equation is given like this.

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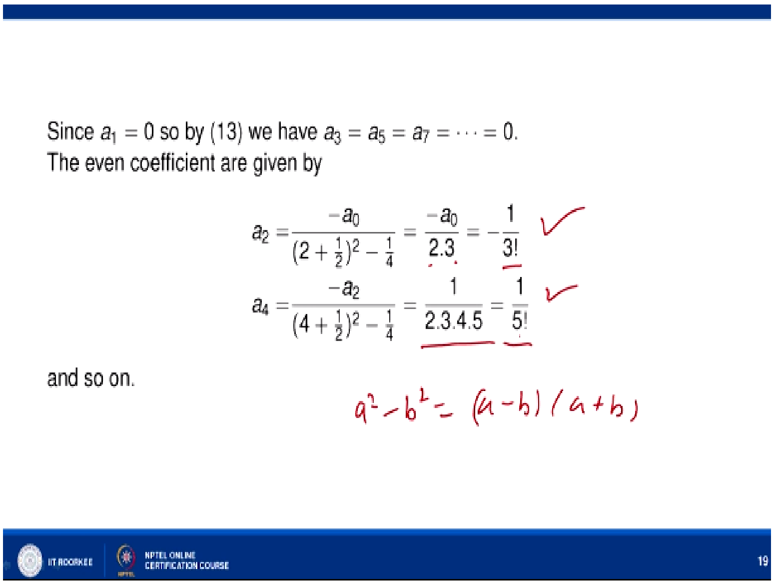

So here we have a2 as -1 upon factorial 3, a4 as 1 upon factorial 5 you can simply calculate a4 as –a2 upon 4+1/2 whole square-1/4 and we can write it like this. In fact, you will use the formula of a2-b2 as a-b a+b, so we can say that here it is, in a2 it is –a0 upon 2 * 3 a0 is 1, so you can write -1 upon factorial 3 and a4 it is what –a2 upon 4+1/2 whole square -1/4 which we simplify and we can write it 1/factorial 5 and so on.

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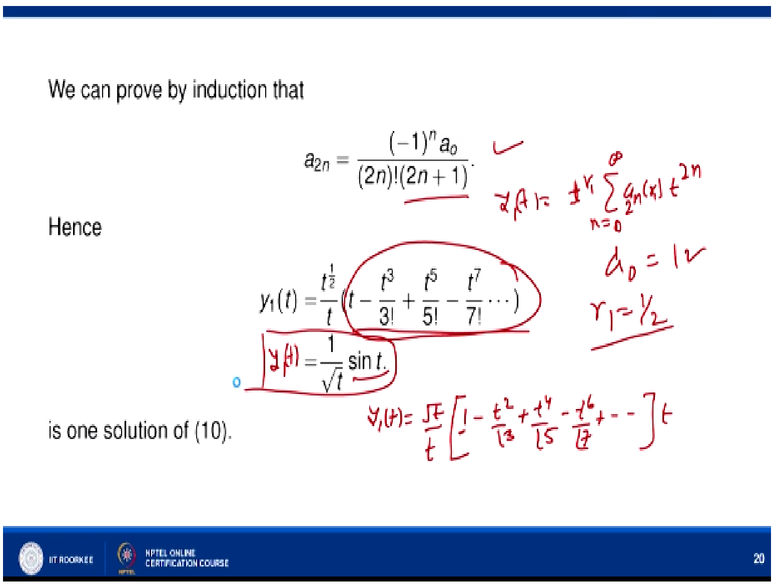

So here we can prove by induction that a2n is going to be -1 to the power n/factorial 2n * 2n+1 that you can prove it and once we have a2n we can easily find out the first solution y1t as t to the power r then it is t to the power 1/2 and upon t t – t cube upon factorial 3 and so on, So, y1t = t to the power r1 summation a2n r1 t to the power 2n and is from 1 to infinity, basically it is from 0 to infinity.

And a0 we are assuming as 1, so that we are writing it here. So we are writing y1t = t to the power, so r1 is 1/2 her. So here we have already calculated a2n as -1 to the power n/factorial 2n 2n + 1. So we can write down our solution as y1t = t to the power r1 summation n = 0 to infinity a2n r1 to the power 2n. So here a0 is already we assumed as it is 1 and r1 = 1/2 then we can put as root t * a0 that is 1 and a1 is 0 so that we are cancelling out.

A2 we have calculated as -1 upon factorial 3 so t square + a4 is 1 upon factorial 5 * t to the power 4 - so on. So now if you look at this then we can multiply and divide by t so we can multiply by t here so we can write this as t to the power 1/2/t and in the bracket it is t–t cube/1 factorial 3 + t 5 upon factorial 5 and so on, and we already know that this is an expansion of sin series and we can write this as t1/2/t 1/root t and this is sin t.

So y1t is given as 1 upon root 2 sin t, so that is our one solution and we can find out the other solution look at the other solution here, so it means that in this case our solution is given by 1/root t sin of t and if you look at in this case r1-r2 is basically 1, so we need to find out our second solution. The second solution may not be of this form maybe some other form let us look at this analysis and that analysis we are going to do in a next lecture. So here we conclude and will discuss it in next lecture. Thank you for listening us thank you.