INDIAN INSTITUTE OF TECHNOLOGY
ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)
Discrete Mathematics

Module-02
Logic

Lecture-05
First order logic (2)

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In this lecture we continue our discussions on first-order logic that we started in the previous lecture, in the previous lecture we ended by introducing two quantifiers namely Universal quantifier and existential quantifiers we also listed down some propositions involving the these quantifiers namely Universal and existential bounty quantifiers and predicates, so this is continuation of first-order logic.

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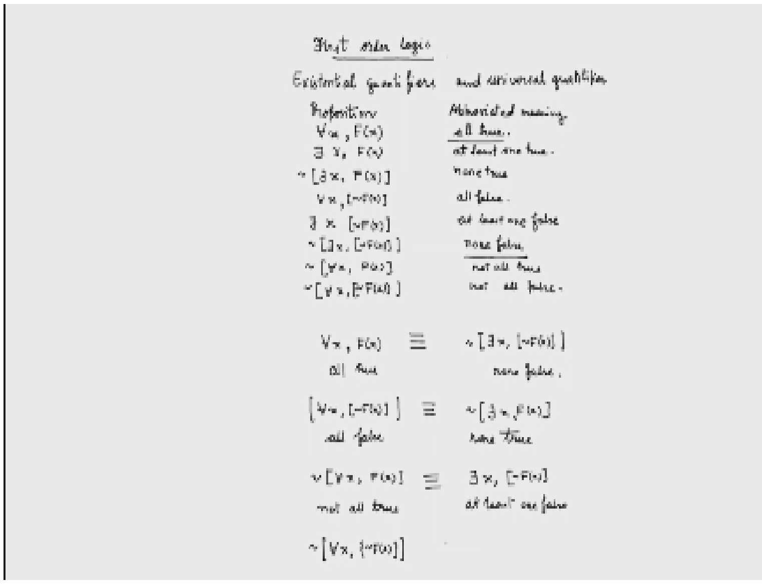

We are discussing two quantifiers namely Existential quantifiers and existential quantifier and Universal quantifier, now I list down some statements involving these quantifiers the first one is for all x F (x) now if this proposition is true that means for all x in the universe the predicate Fx is true, so we write the abbreviated meaning as all true next there exists x Fx if this segment is true that means there is at least one x such that Fx is true, so we write the abbreviated meaning as at least one true third I write quickly not of there exist x Fx this means none true forth for all x_2 of Fx.

That means all false 5^{th} there exists x_0 Fx this means at least one false then not of there exists x such that knot of Fx that means null false then knot of their for all x Fx this means not all true and lastly not all false, now we have seen these propositions in the last lecture now what we can do is that to group these propositions into equivalent propositions, for example all true and non false all true is for all x Fx and non false is negation of there exists x such that naught of x is true, so all true and non false should be same.

So they are equivalent now our question at this point is that can we by using the rules of logic that we have developed derive the equivalence of these two propositions the answer is yes.

But we will do that after we have grouped these 8 propositions into 4 groups, so here we have all true and non false another two propositions are all false and none true all false is given by for all x negation of a x and none true is given by negation of there exists x Fx our common sense says that they should be equal but the question is that, how do we prove it analytically the fourth one is not all true, so that is negation of for all x Fx, so this is not all true and on the other side we have at least one false that means there exists x_0 of Fx again we expect them to be equivalent and finally we have not off for all x_0 of Fx which is not all false.

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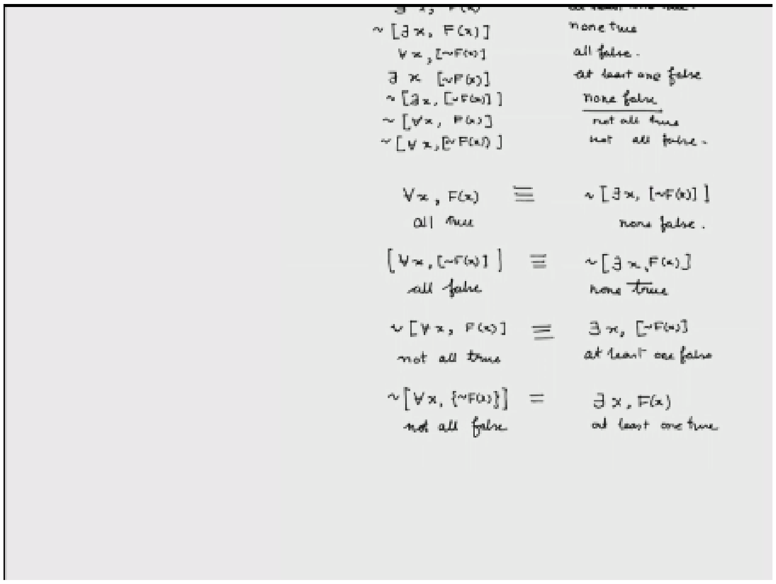

And on the other side we have there exists x Fx that means at least one is true we expect them to be equal, now the question is how do we prove these equivalence is to do to do that we go back to De Morgan's laws now De Morgan's laws says that.

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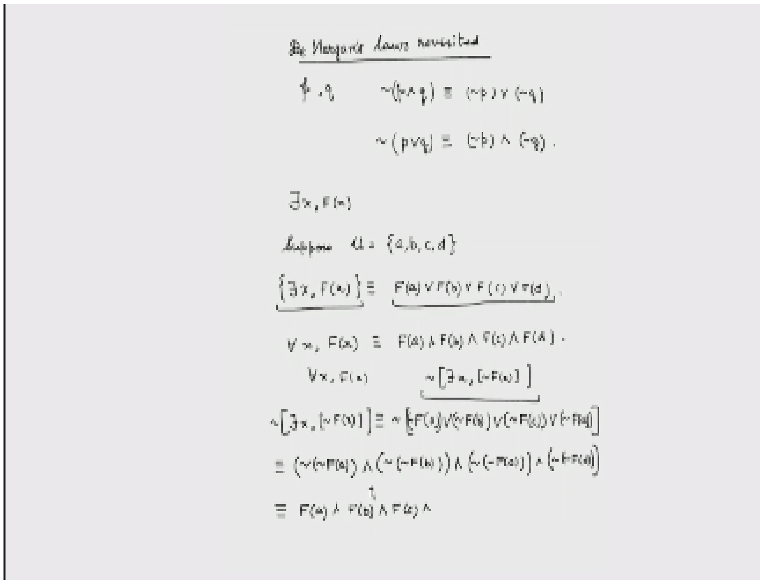

If I have two propositions p and q then p and q_0 is equivalent to P_0 or Q_0 and not of p or q is equivalent to not of p and not of q, if we look at the proposition there exists x Fx this proposition is true if when x varies over the whole universe we find one instance where Fx is true, now let us try to understand this by restricting the universe to something very small let us suppose the universe new consists of only four elements suppose U = a b c and d therefore we see that when I say four there exists x Fx this statement is equivalent to stating that if a or Fb or Fc or Fd the question is why.

The reason is what I have already told that the statement in the right hand side is going to be true if there is one instance for which Fx is true, now there are only four possible instances and for each of them I can put the value of x in Fx, so then I will get Fa Fb Fc and Fd Fa Fb Fc and Fd are propositions, so if at least one of them is true then the proposition in the right hand side is true and well a proposition in the left hand side is also true and it is false if and only if all F a Fb Fc and Fd are false and that will also mean the left-hand-side falls because there will exist no x.

For which Fx is true therefore these two propositions are same now on the other hand if I have something like this for all x Fx then this is equivalent to Fa ∩ b ∩ Fc sorry we call it and Fd now let us look at the first the first expression that we started with that is for all x Fx and not of there exists x such that not of x, now suppose I start with this proposition not of there exists x not of Fx this is equivalent to not of Fa or Fb or Fc or Fd restricting the universe to just a set a b c d now we can use De Morgan's law and write that this is equal to not of f.

And I am sorry I have to make a small change over here this will be not of Fa or lot of Fb or v of Fc or not of Fd, now if I use De Morgan's law I will get v of v of Fa yes and v of not of Fb and v of Fc and v of Fd this and since I know that v of Fa is Fa itself, so this is same as Fa and Fb and Fc and Fd which is same as for all x Fx thus we have established that for all x Fx is equivalent to v of there exists x v Fx, now we will prove the other equivalences that we have stated in the beginning of this lecture let us consider the equivalence for all x_0 of Fx.

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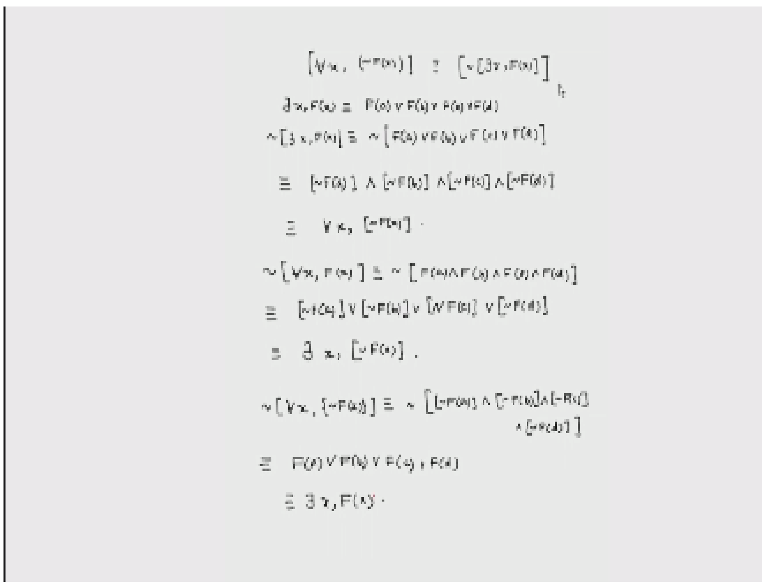

v of there exist x Fx again we are restricting our universe to just four elements ABCD there exists x Fx is equivalent to Fa or Fb or Fc or Fd not of there exists x Fx is equivalent to v of Fa or Fb or Fc or Fd which is equivalent to by De Morgan's law v of Fa and v of Fb and not of Fc and not of Fd and which is of course equivalent to for all x not of Fx, if we take up the next equivalence.

We start from not off for all x Fx which is equivalent to not of Fa and Fb and Fc and Fd which is equivalent to not of Fa or v of Fb or v of Fc or v of Fd which means there exists x such that not of effects and the last one v of for all x not of Fx is equivalent to not of not of Fa and v of Fb and v of Fc and not of Fd which is equivalent to Fa or Fb or Fc or Fd which means for all x Fx thus we see that we can prove many equivalences involving the quantifiers by using De Morgan's law next we move on to describing some more proof techniques by using the quantifiers.

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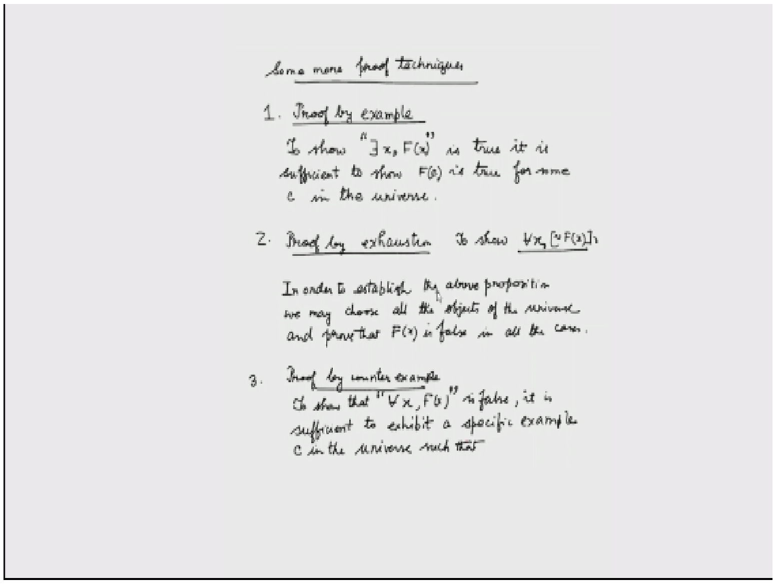

One technique is group by example to show there exists x Fx is true it is sufficient to show Fc is true for some see in the universe second technique is proof by exhaustion to show for all x_0 of Fx is true we choose to show that in this case we have we have to show for all x such that not of Fx is true then in order to show that, so here we have to prove that for all x_0 of Fx is true in order to do that we may choose to exhaust all the elements of the universe and prove that Fx is false everywhere and that will prove the proposition for all x_0 of Fx and the last technique that we discuss is called proof by counter.

Example to show that for all x Fx is true to show that for all of Fx is false it is sufficient to exhibit a specific example see in the universe such that if see is false, so suppose I have a proposition for all x Fx, now what we can do is that we may search for one instance in the universe such let us call it c such that Fc is false then of course for all x Fx this proposition is false this is called proof by contradiction, now we will move on to an example of a proof by exhaustion and approve by contradiction.

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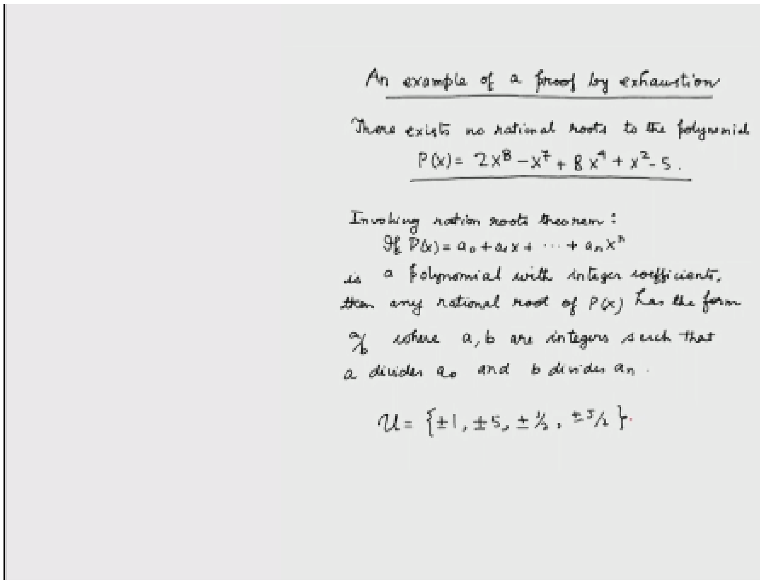

Now suppose we would like to prove the statement there exists no rational roots to the polynomial P x = 2x to the power 8 – x^7 +8 x^4 + x^2 – 5 , now of course if we have to exhaust the whole set of rational numbers we will not be successful because the set of rational numbers is infinite, but we can invoke a theorem called rational roots theorem which says as follows if px equal to a 0 + a1 x + and so on, a n x^n is a polynomial with integer coefficients then any rational root of px has the form a by b where a,b are integers such that a divides a0 and b divides in if you use this theorem to the polynomial.

Under consideration then we will see that our universe reduces to only + -1 5 + - ½ half and + - 5 / 2 and we can evaluate the polynomial P(x) at all these points and see that Pc is not equal to 0 for all c belonging to you this is where we are exhausting all the choices by reducing the universe and in this way we are proving that there is no rational route to the polynomial px the second example that we discuss is a is an example of proof by counter example.

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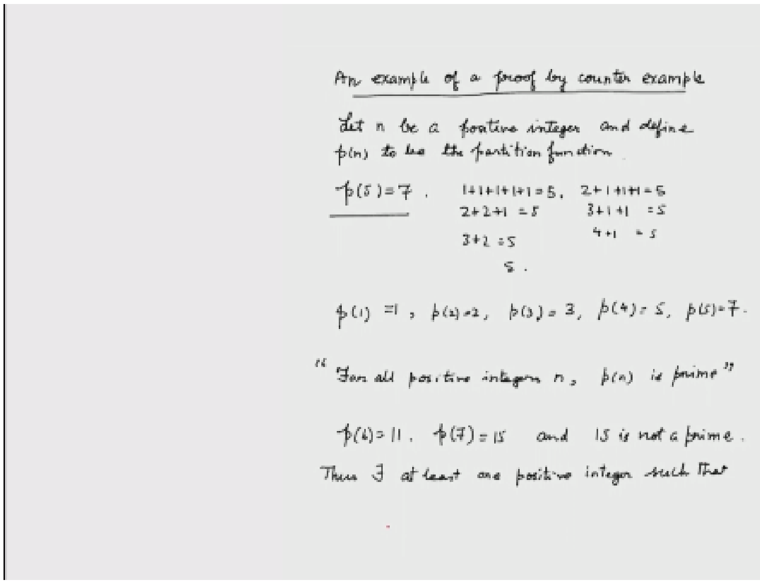

Now let n be a positive integer and define P n to be the partition function now a partition function on a positive integer is a function which gives the count of the number of ways that integer can be written as sum of positive integers without taking order into account for example if we take P 5 P5 is 7 this is because 5 can be written as 1 + 1 +1 +1 + 1 2 + 1 + 1 +1 2 + 2 + 1 3 + 1 + 1 3 +2 4 +1 and 5 itself, so there are 7 ways of writing 5 as sums of positive integers and therefore we write P5 is 7.

Now if we do calculate P, P values of from 1 onward then we will find that P_1 is 1 P_2 is 2 P_3 is 3 P_4 is 5 P_5 is 7 now suppose we form a proposition from this that is for all positive integers n P n is prime suppose, we are asked to prove or disprove this proposition then what we do is that we start from 6 onwards, so if you see it is P_6 you will be able to see that P_6 is 11, so we cannot say anything, but then if we calculate P_7 we will see that P_7 is 15 and 15 is not a ‘ therefore there exists positive integers.

Such that the partition function on it does not return a prime number and therefore the statement is false thus the proposition under consideration is false this is an example of proof by counter example the counter example is the number seven whose p-value is 15 and which is not true which is not prime and therefore the proposition is not true we stop here today thank you.

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Acknowledgement
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