INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)

Discrete Mathematics

Module-02
Logic
Lecture-04
First order logic (1)

With
Dr. Sugata Gangopadhyay
Department of Mathematics
IIT Roorkee

In today's lecture we will start discussions on first order logic.

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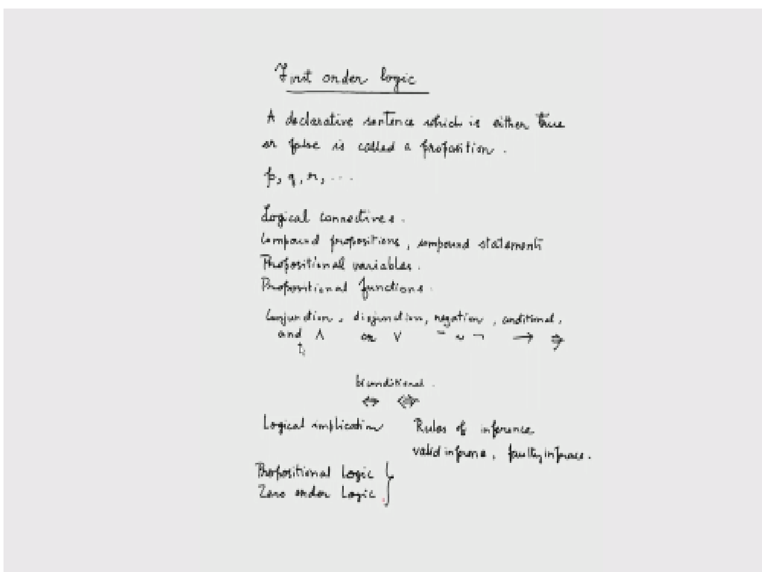

So far whatever we have discussed involved propositions and some logical connectives. Let us recall a declarative sentence which is either true or false is called a proposition. Now we denoted propositions by the lowered case letters P, Q, R and so on. We also introduced logical connectives which are used to combine several propositions to obtain compound propositions.

And sometimes we call those compound propositions as propositional functions assuming that the original propositions are in fact variables.

So we introduced compound propositions which are sometimes referred to as compound statements. We also introduced propositional variables which we denote by P, Q, R and so on and propositional functions. Now the logical connectives that we introduced are conjunction, disjunction, negation, conditional, and at the end bi-conditional. Conjunction is also called AND denoted by a wedge.

Disjunction is called OR denoted by a V, negation is either denote by over line or a ~ or a symbol like this, conditional by a right arrow like this, and bi-conditional as a two-sided arrow or a symbol like this. We also introduced another idea of logical implication, that is an implication which is always true and rules of inference. Now a logical inference involves two types, valid and invalid. So valid inference and fault inference.

Now whatever logic assistant that we have discussed based on these and the proofs methods of proof all together is called propositional logic or zero^{th} order logic. Now what we see at this point is that, by using propositions, simple propositions and this framework of propositional logic we cannot express everything that we would like to express. That is why we introduce something which is marginal than a propositional which is called an open proposition or a predicate.

We will soon discuss what we mean by a predicate, but what happens is that with this marginal form of propositions we can use the logical connectives and inferences and methods of proof, and build up a more powerful logical framework which is called the first order logic or predicate logic.

Now first let us look at what we mean by a predicate now we often have some propositions like this3/4 is a rational number we can have another proposition like ½ is a rational number yet another as √ 2 is a rational number now what we note over here that in all these cases we are considering rational numbers and we are putting some numbers in the beginning and making a statement that number is a rational number let us see this again ¾ is a rational number.

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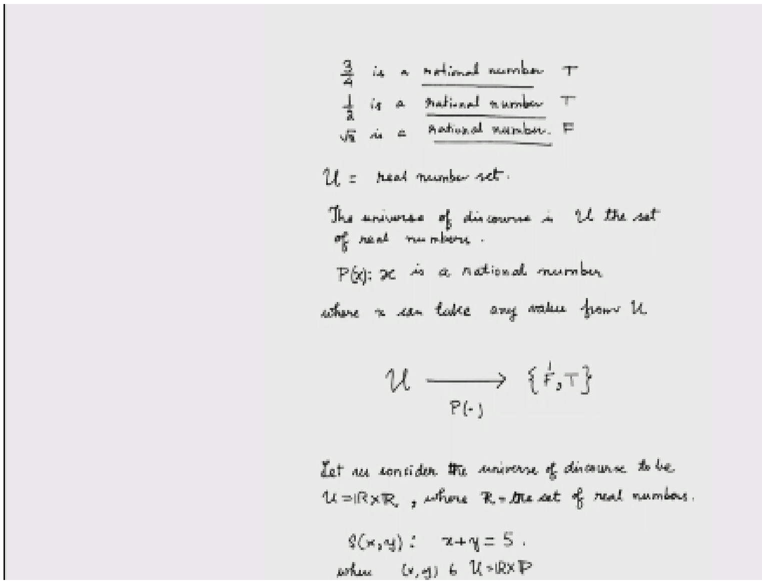

½ is a rational number √2 is a rational number now what we see over here is that we are considering numbers and we can probably fix our discourse in the set u of real numbers we are picking up numbers from u and asking and stating that number is a rational number sometimes this proposition is 2 and sometimes the proposition fault depending on the number that we gave chosen for example in this case the 1sr proposition ¾ is a relational number is true ½ is a rational number is true but √2 is a rational number is false.

We can write this in a more compact way as stating that our universe of this course is U the set of real numbers and we are tilting a is statement x is a rational number where x can take any value from u we can also write this as P(x) now we see that the P(x) that I have written here is a not a proposition because it is not meaningful to specify a truth value to this statement if we do not fix x.

So x is a rational number that x may take any value from u sometimes it will be true and sometimes it will be false however if I keep on putting values of x from u then I will get propositions which has got a specific truth values so this P(x) can be thought of as a function from the set u that is the universe of this course that we have to fix before we start any discussion and to the set got any 2 symbols t and f.

And we can suppose that T designates true and F designates false so here we are looking at functions from u the universe of this course to the set F , T and this function is P which can take the values x and the so the function is x is a rational number when I put that the truth value of that will be F or T and this is how I am thinking p as a function now which is quite possible that I have got more than 1 variables varying over a universe of discourse.

For example let us consider the universe of discourse to be R + R where R is set of n numbers now we consider the preposition and open preposition are predicate of the type S x, y x +y =5 so here the sentence is x + y = 5 where x, y where is so far the universe of this course are simply will universe U, so if we want to formulize S will be associated a function from U x U to a set F T this is also a predicate, now we are in a position to define predicate in a general framework.

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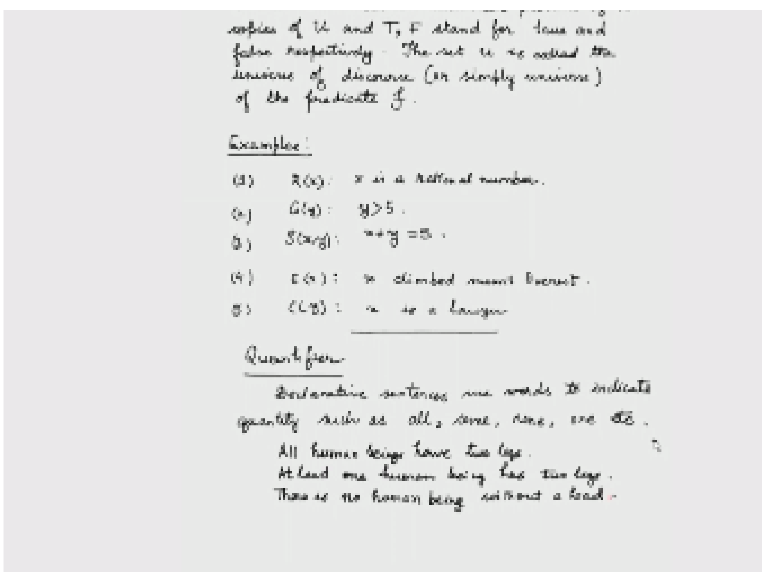

So it predicate are an open preposition in n variables from a set new is a function F from U^n which is essentially the Cartesian product of U we can n times 2 a 2 symbol said T F as where T stands for true and F stands for false the set true is called the universe of this course or simply universe of where predicate now let us look at some examples now we have already seen that R(x) x is a rational number, is predicate G(y) y > 5 is also predicate 3 S(x, y) x + y = 5 is also >.

Now here we have to remember that whenever we talk about predicate we have to be careful in what we are assuming as the universe in case of R(x) that universe can be rational numbers can be real numbers can be complex numbers and so on in case of G(y) is there also we see that it is tacitly assume that it is the number y is a number and in case of x the second one which is also assume or understood that xy are going to be numbers it maybe real's complex or any other numbers on the other hand we can have predicates like this.

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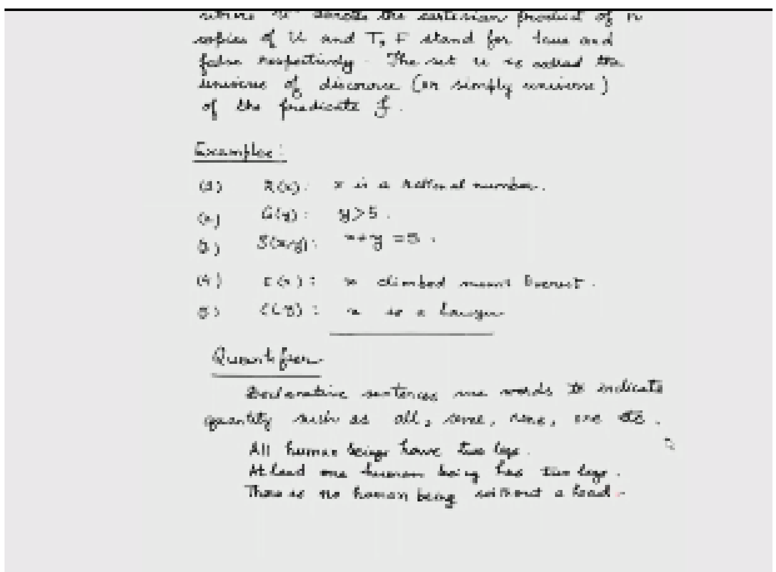

Ex, x claimed Mount Everest here we have see that it is of course possible to have the universe as numbers but is not going to be very meaningful, because if I say that x varies over a real numbers then no real number has ever claim mount Everest, so it makes no meaning in this case the universe might be the set of all human beings and among those set of human beings there are some proof Claim mount Everest.

And of course many others who have never claimed mount Everest like this we can have other predicates let us look at cy x is a layer and so on we have many other predicates like this, once we have understood the definition of predicate we will move on to a definition of another very useful notion which is called a quantify. We observe that declarative sentences use words to indicate quantity such as all, some, none, one, etc.

For example we have statement like all human beings have two legs or we can say at least one human being has two legs or there is no human being without a head here we are seeing that our universe of disperse is the set of all human beings and we are sometimes saying that all of them has two legs or at least one of them have two legs or none of them is without a head. So these are somehow indicating some quantities of human beings having something. Now this idea gets formalized to two specific quantifies which are called universal quantifier and existential quantifier.

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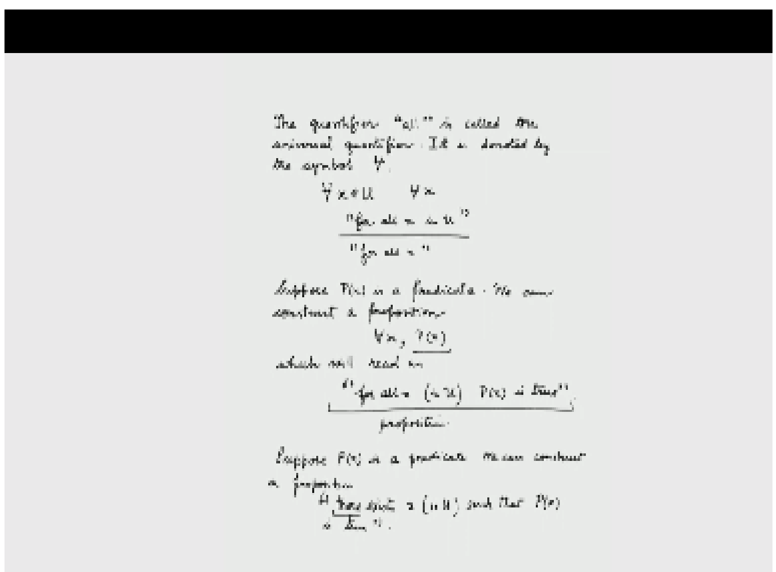

Quantifier all is called the universal quantifier it is denoted by the symbol inverted a if u is our universe of discourse then the phrase for all x belong in to u or simply for all x we will mean well we write this as for all x in u, so this inverted a x belong in to u or inverted a x will designate the phrase for all x and u or if u is already specified and understood in a simply say for all x.

Now suppose bx is a predicate we can construct a preposition denoted by this which we need as for all x I am writing px is free now what happens for all x for u dx is true that we denote that this although dx is the predicate for all x px is a preposition this is true for each and every f in the universe of the source vx is true for all preposition is false by universal quantifier we come to extensional quantifier suppose ex is a predicate we can construct a preposition.

They are exist x in u such that ex is proved this phase they are exist is written in a compact notation as there exist x in u or simply there exist x and a complete preposition is written as there exist x this symbol which is signifies they exist is called the existential quantifier now once we have these quantifiers and predicates and off course now let me miss down some sentences before I end this lecture and in the next lecture we look at the sentences more carefully and more details so we have sentences like this for all x fx essentially all true here we have things like exist.

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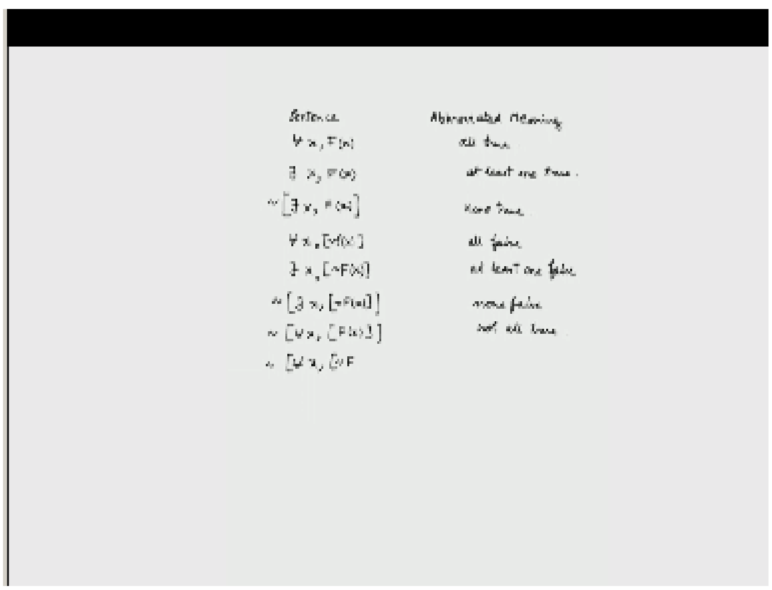

X fx which mean at least one now I can have negation of these they exist x fx which means that there exist x if that fx is true negation of that so these means if this is true and that means that non true for all x not of x negation of x which means all false there exist x such that negation of fx which means at least one false then negation of there exist x negation of fx this means none false negation of all x fx which means not all true and lastly negation of for all x negation of fx which means not all false in the next lecture we will consider sentences of these types and this is the end of the present lecture thank you.

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For Further Details **Contact**

Coordinate, Educational Technology Cell
Indian Institute of Technology Roorkee
Hoorkee-24/667
Email:etcell@iitr.ernet.in,etcell.iitrke@gmail.
Website: www.nptel.iim.ac.in

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Prof pradipta Banerji
Director,IIT Roorke

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Dr.Sugata Gangopadhyay
Dept of Mathematics
IIT Roorkee

Production Team
Neetesh Kumar
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