INDIAN INSTITUTE OF TECHNOLOGY
ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)

Discrete Mathematics

Module-02
Logic
Lecture-02
Logical Inferences

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In today's lecture we will study logical inferences.

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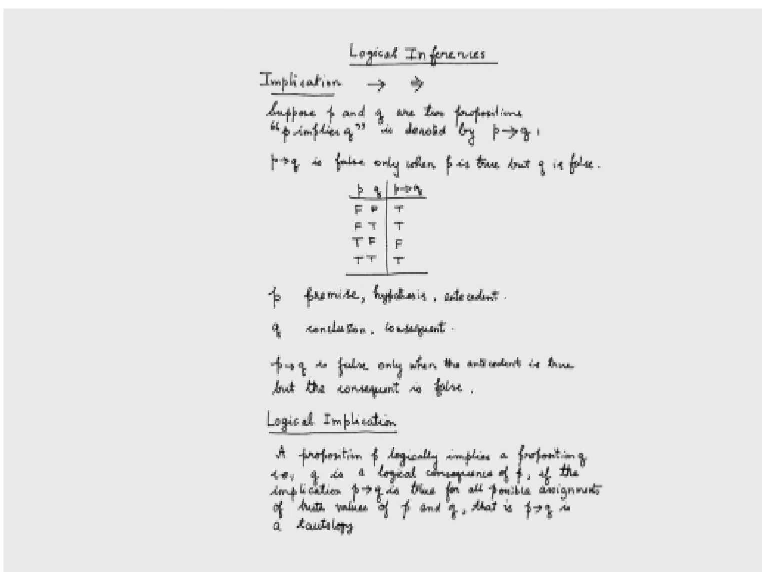

Now before going to the rules of inference we will take a look at the connective that we have discussed in the previous lecture namely implication. The connective implication is denoted by an arrow like this or an arrow like this. We will be using the former symbol to denote implications. Suppose P and Q are two propositions, then the statement P→Q is denoted by P→Q.

What we have seen is that this proposition namely P→Q is false only when P is true, but Q is false. The truth table of P→Q is written as --. Now we will introduce some terminologies here, the left hand side proposition in the compound statement P→Q namely P is called premise or hypothesis or antecedent. The right hand side expression or proposition Q is called conclusion or consequent.

So we can reward what we have said just now that is the statement P→Q is false only when the antecedent is true, but the consequent is false. We now move on to another type of implication which is called a logical implication. A proposition P logically implies a proposition Q that is Q is a logical consequence of P, if the implication P→Q is true for all possible assignments of truth values of P and Q.

In other words, P→Q is a tautology. Here let us recall that a tautology is a proposition which is always true irrespective of the values of the propositional variables involved in it now we define the word inference.

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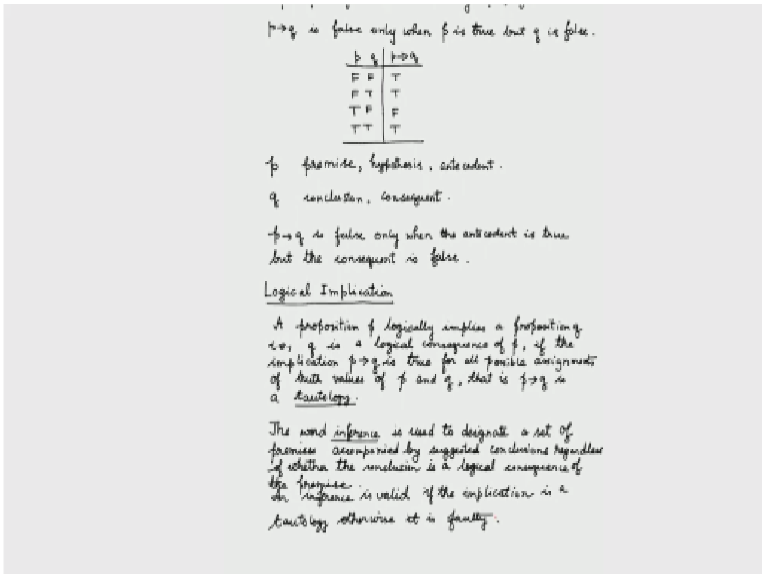

The word inference is used to designator a set of premises accompanied by suggested conclusions regardless of whether the conclusion is a logical consequence of the premise and inference is valid if the implication is a tautology otherwise it is faulty let us have a relook of at what I have stated suppose we are having some propositions.

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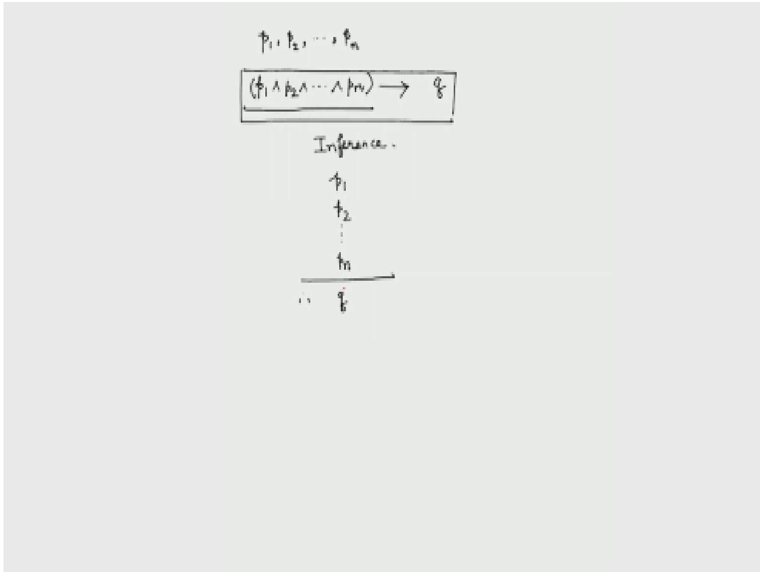

Which we call premises and denote by P1, P2, Pa and what we can do is that we can combine then all with a conjunction to get a single premise P1 and P2 and so on and Pn so this is the combined premise that we have and we write another proposition what we call is a conclusion or a consequent and we join them by a implication symbol now this total thing will be called an inference.

We can write this also in a in another from which is writing PI's one blow the other in a least and then write a horizontal line and then write therefore q both these notions will mean the same thing now we will call an inference valid if the statement given here that is a implication is a tautology otherwise we say that it is not valid as a rule of inference we will have a list of valid inference.

And we will also have some very common faulty inferences or fallacies we will move on to this writer now so let us go to the next page here we have rules of inferences related to the language of proposition.

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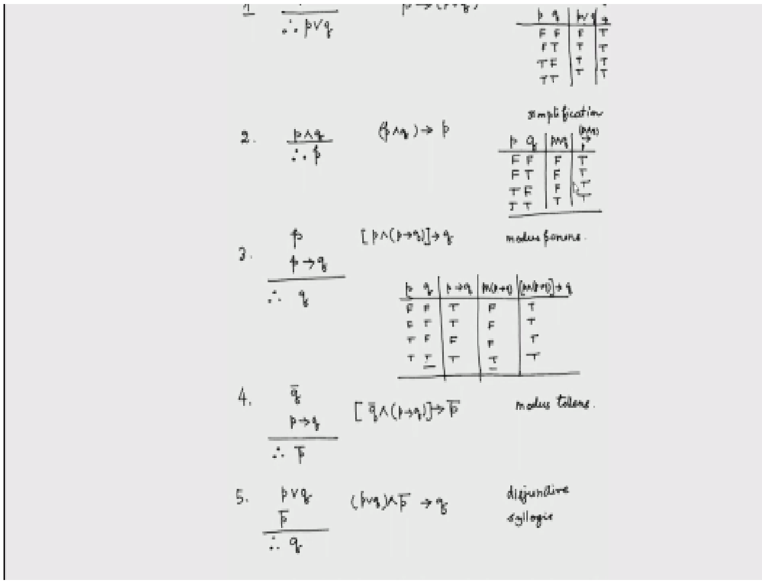

The first one is called addition here we see that if is essentially p → p U q now p, q are propositions p or q which is true if p is true therefore if p is true then p or q ids always true and thus the implication p → p or q is always true and therefore this implication is a tautology and he4nce a valid inference we can quickly draw a truth table and check this fact the last column is corresponding to the preposition p → p or q.

And bc is that always true second p and q therefore p in to logical form this is p and q → p this called simplification, now we can again try truth table to check that it is in detail tautology for that we take all the possible values of p and q which is F F, F T, T F and T T and T and p and q and finally p and q → p, p and q is of course F F, F and T and here we see that.

If the anti event is false then implication is always true and at the end anti event is true and consequent is also true therefore the implication is true and therefore we see that p and q → p is a tautology third one the third rule is p and T → q therefore q in the tautology form this is p and p → q this is called modous ponens let us check that it is indicate tautology by using a truth table where pq again as before and we have p → q and lastly we have T and p → q and then T and p → q we will use the truth value of pq.

Then p → q is true to false and true T and p → q is false and true and here p and p → q and q it is the implies q is whenever this is false this is true so we have to only check the case mean where p and p →q is true and then q is true this is true therefore the truth value is true and thus we see that modous ponens is indeed a valid inference therefore negation of p → q therefore negation of p this is written in the tautological form as negation of q and p → q → negation of p this is called modous ponens.

This rule is p or q negation of p therefore q in the tautological form this is p or q and negation of p → q this is called disjunction syllogism I am not showing that these rules are indeed tautologies like I have found for the first three cases but using the exactly the same process it can be verify that these are tautologies there are few more common rules of inferences left which I will list right now. The 6th one is p implying q.

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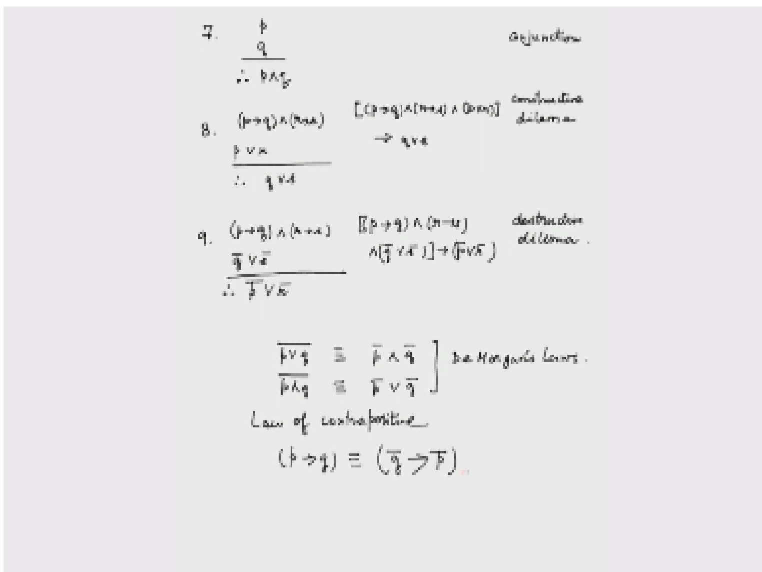

Q implying r therefore p → r in the tautological form this will look like p → q and q → r → p → r this is called hypothetic and syllogism then 7th is p q therefore p and q this is simply conjunction 8 p → q and r implies s p or r therefore q or s this is written as p implies q and r implies s and p or r implies q or s, this is called constructive dilemma. 9 p implies q and r implies s negation of q or negation of s.

Therefore negation of t or negation of r in the tautological form this is called destructive dilemma, and finally there are some equivalence is which are very important one is the de Morgan's law which states that given any two prepositions p and q p or q negation is equivalent to negation of p and negation of q t and q negation of the whole is equivalent to negation of p or negation of q this together are called de Morgan's laws.

And we have another equivalence which is called the law of contra positive which states that p implies q is an equivalent to negation q implies negation of p. once we have seen a long list of valid inferences we will end today's lecture by looking at some faulty inferences, so we move on to some common fallacies.

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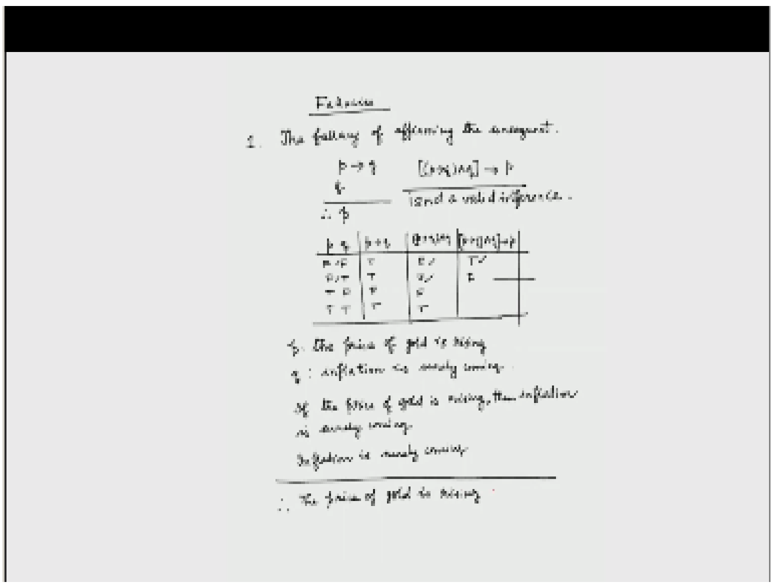

The first fallacy is the fallacy of a firming the consequent which is written as p implying q therefore p let us look at the tautological form and try to build the truth table, this means that p implies q and q this whole implies p now the corresponding truth table will be, now we come to the point where we have problem because in the first line p implies w and q is false and p is false therefore the implication is true.

In the second line p implies q and q is true whereas p is false therefore the implication is false and we stop here because of this we see that the inference that we stated an example of false is like this suppose 3 is the preposition the price of gold is rising and q is the preposition surely coming then p implying q is gold of price is inflation is surely the statement is influent surely coming from these two statements we are tempted to write that therefore the price of gold is rising back what we have learned from the false of the consequent is that this inference is not logical valid this is not a valid inference the next fallacy that we will study is called fallacy of define the incidence .

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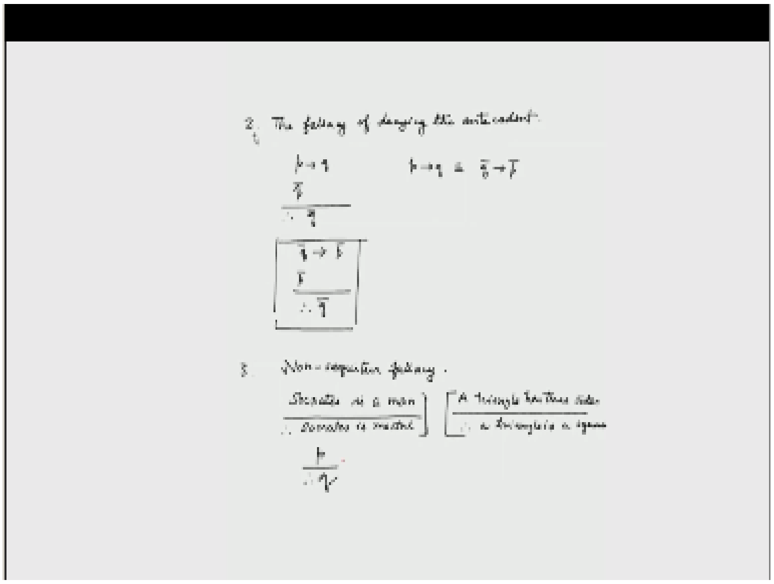

This fallacy is written in this way p implies q negation of p therefore negation of q now we have already stated that p implies q is equivalent to negation of q implies negation of p therefore we can replace this negation of p therefore negation of q and we see that this is beside the previous fallacy that is fallacy of allowing the consequences the third and last fallacy that we will look at this lecture is called non fallacy.

This fallacy is essentially taking two prepositions which are not connected and learning one implies other for example suppose I say that Socrates is a man and on q therefore Socrates is mortal is a fallacy because Socrates is a man and Socrates is mortal they are essentially to be different prepositions here I am not using this prepositions the man is important that something outside is for example all this is reasonable I could have they can have two different prepositions.

Such as a triangle as three sides therefore the triangle is a square so obviously this looks meaningless but they both are same because essentially trying to connect two different prepositions so once if we have fallacy is of this type p therefore q where pq are two different prepositions with this we come to the end of this lecture thank you.

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