INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)

Discrete Mathematics

Module-02
Logic
Lecture-01
Fundamentals of logic

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In today's lecture we will start discussing fundamentals of logic. Now in logic we will be dealing with statements rather than numbers. Now a particular type of statements are called propositions.

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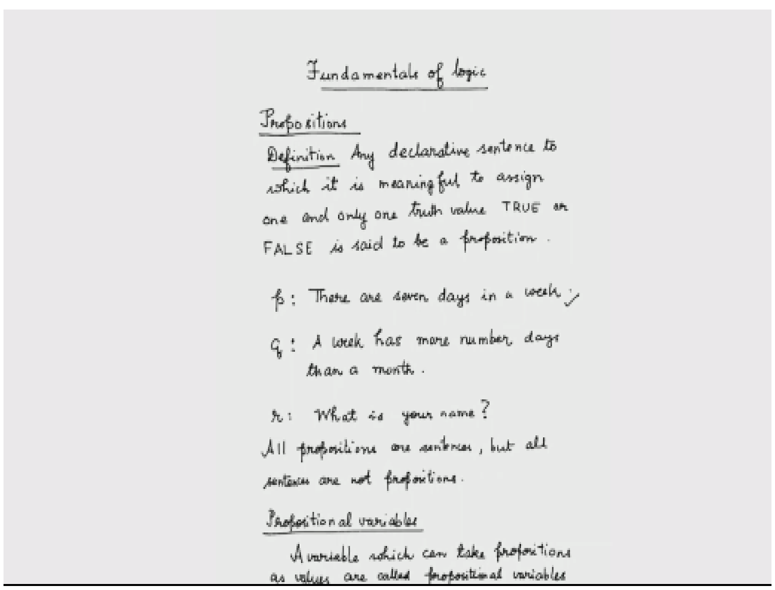

Let us define propositions nor formally any declarative sentence to which it is meaningful to assign one and only one truth value true or false is said to be a proposition. Now we have to be careful that each and every sentence is not a proposition. The key feature of proposition is that it has to be either true or false. And if a proposition is not true and then it is false, and if it not false then it is true.

Let us look at one example of a proposition. We have written down two propositions, so first proposition is denoted by P, and the second proposition is denoted by Q. and the proposition P states that there are seven days in a week. Now this is of course a sentence and this is true. So we know that this cannot be true and false at the same time, and we know that it is meaningful to say that it is true.

Thus, P is a proposition, the next sentence is Q which states that a week has more number of days than a month. Now this sentence is obviously false, but is a proposition because it is a declarative sentence which has a truth value which in this case is false. Let us look at another sentence we denote by R which is an interrogation what is your name? Now this sentence cannot have a truth value, because that is meaningless.

And hence, this sentence is not a proposition. Therefore, we see that all propositions are sentences, but all sentences are not propositions. Another thing that we notice here is hat we can write propositions, we can denote propositions by symbols like P, Q and R. We generalize this fact and introduce propositional variables. A variable which can take propositions as values are called propositional variables.

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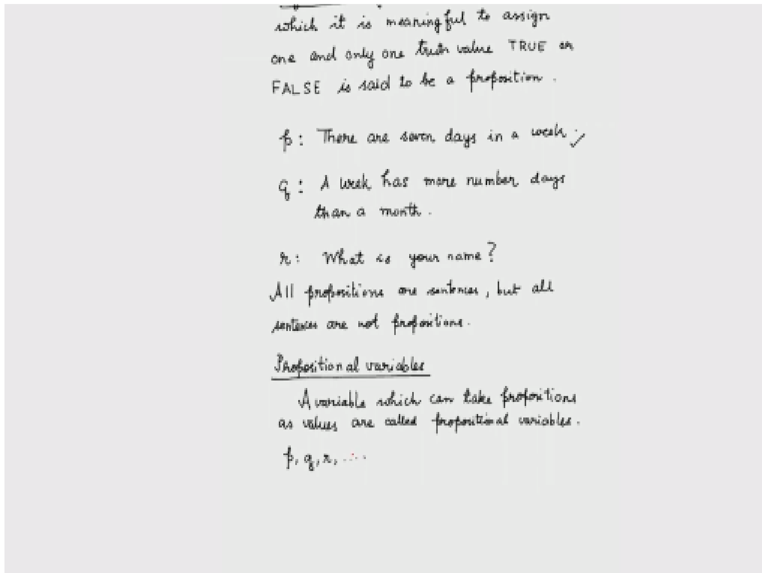

We will denote the propositional variables by small letters P, Q, R and so on. Now if we have some basic propositions we can connect these propositions by using the so called logical connectives and build up compound propositions. The basic connectives are five in number.We will note down these five basic connectives are 5 in number.

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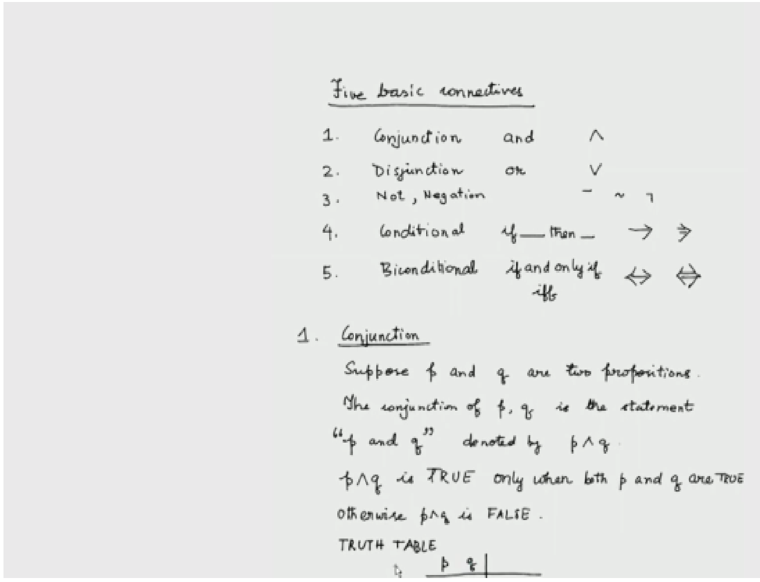

We will note down he this 5 basic connectives the first connective is called conjunction it is also called and denoted by a ^ the 2nd connective is called disjunction it is also called or and denoted by v the 3rd connective is not or negation and it is denoted by either an over line or a ~ or symbol like this prefixed before a propositional variable 4 conditional this is also known as if – then – it is denoted by an arrow like this or sometimes an arrow like this the 5th one is bi conditional this is also known as if and only if or just if with 2 f 'S it is denoted by a both sided arrow or a both sided arrow like this.

Given now one by one check the effect of these connectives the prepositions one conjunction suppose p and q are 2 propositions the conjunction of p, q is the statement p and q if we denote this compound statement or this compound proposition as p ^ q now since which is a proposition we must know definitely when it is true and when it is false p ^ q is true only when both p and q are true.

Otherwise p ^ q is false now we can translate this things to a table which is called a true table and which is very useful in understanding these connectives and more complicated compound propositions if we write the propositional variables p, q and also like the statement p and q the possible values of p, q are FF that is false, false TF, FT and TT.

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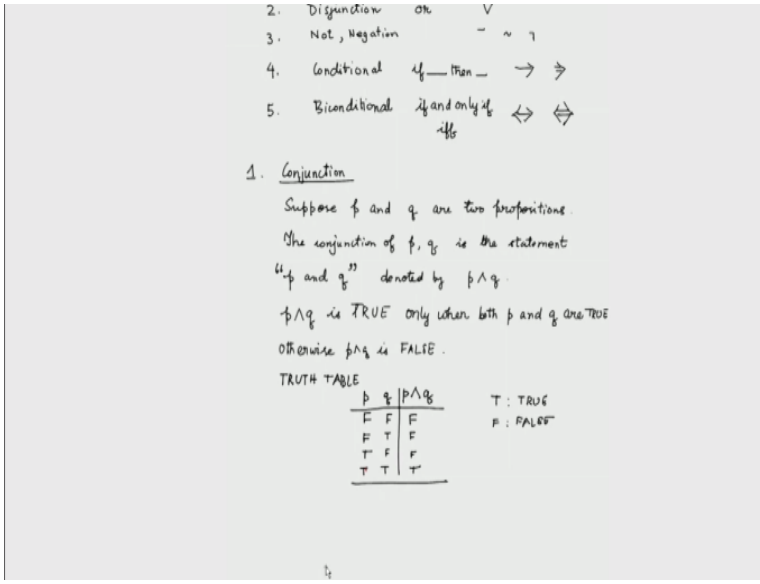

P conjunction q or p and q will have the truth values FF and T here T means true and F means false this table is called a truth table and this specifics the truth values of the compound proposition p and q next we move on disjunction.

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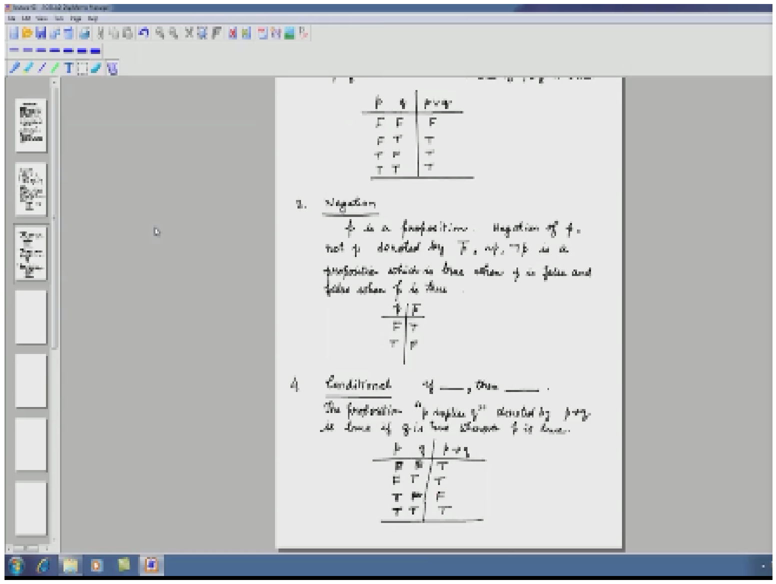

Again we take 2 propositions p and q and p or q is called disjunction of p, q or simply p disjunction q which is denoted by p v q now this statement is true only when at least one of p or q is true, so we write now we go to that truth table of p or q the third connective is called negation this is the invalid apposition that is it involves only one the position and variable suppose p is a preposition negation of p or cp not of p denoted by either go for line or ~p or this is a preposition which is true when p is false and false when p is true, the corresponding truth table will look like this where T false and true and negation of p true when p is false.

And false when p is true next we have conditional now this is called in conditional language as if then, so if something, then something else the preposition p implies q denoted by p arrow q is true if q is true whenever p is true that truth table of p → q is like this, now when p is true and q is true that means that p → q is true now when p is false then I cannot prove that p → q is false because if we have to that p implies q is false we have to show one is stands where p is true but q is false.

Since we cannot prove that p implies q is false if we have the truth value true so in case p is false in both these cases p → q is true whereas if p is true and q is false that means that p → q is not true because p → q forces q to be true when p is true therefore we will write false over here and of course when p is true and q is true p implies this the last conjunction last connective is called by conditional.

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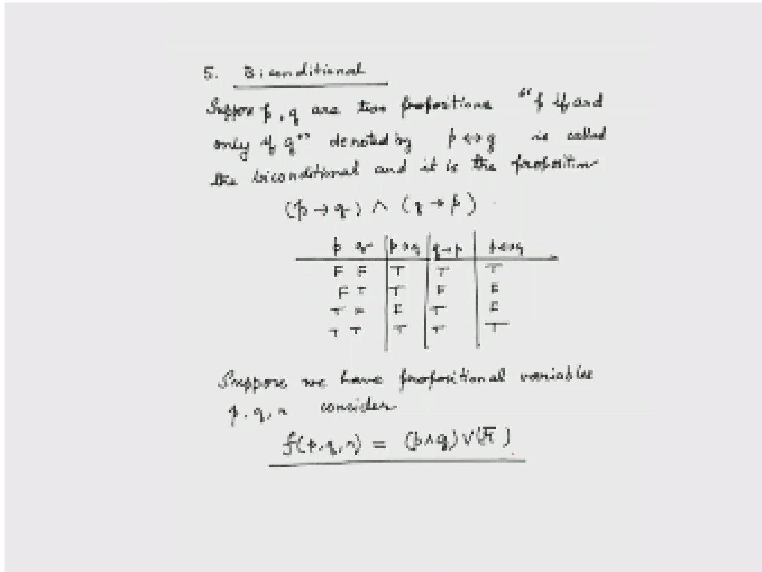

Now if p and q are prepositions p if and only if q denoted by p is called the bio conditional and p if an only q is essentially congestion of p implying q and q implies p, now if we consider the truth table of bio conditional we will have p q taking all the possible values and p implying q is true f and true q implying p is true f q and q and therefore the bio conditional which is congestion of this two prepositions is true f f and t.

Now next we will take preposition are variables and use this logical connectives to build up compound statements or compound prepositions. Now let us see examples of that, suppose we have prepositional variables pqr consider a compound preposition f p q r p and q are not of r, now these types of expressions will be called prepositional functions. We can find out the truth table of these prepositional functions for example for the one that I have written just now we can build up the truth table in this way we write the prepositional variables then we start writing the terms p and q and r0.

Now all the possibility list down all the possible truth values of pqr which are f f f, f f t, f t f, f t t, t f f, t f t, t t f, t t t, now first column at the right hand side is p and q which is true if an only if both p and q are true therefore here it will have f the next row also f then f f f f t t and r0 is t f t f t f t f. now in the last column we will calculate the function f pqr which is equal to p and q are 0 of r this is t f t f t f t and t.

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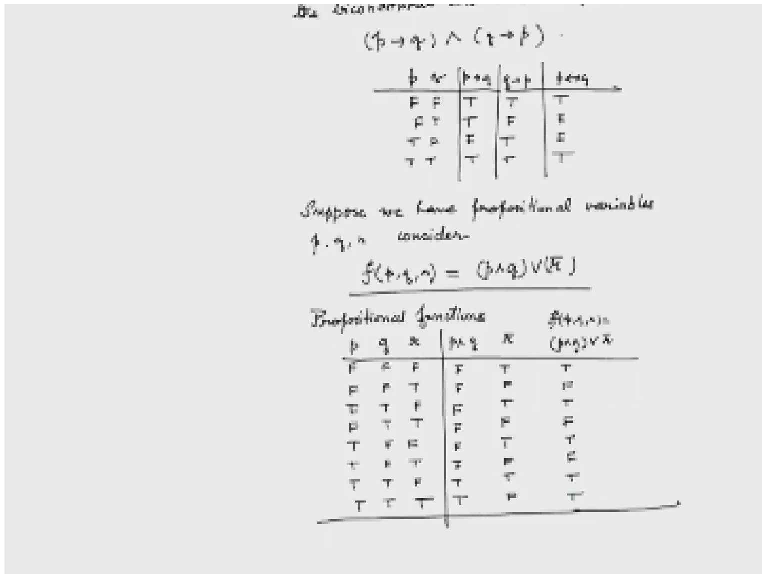

Once we have discussed the prepositional functions which are also called compound statement or compound prepositions then we introduced the idea of equivalence of prepositional functions.

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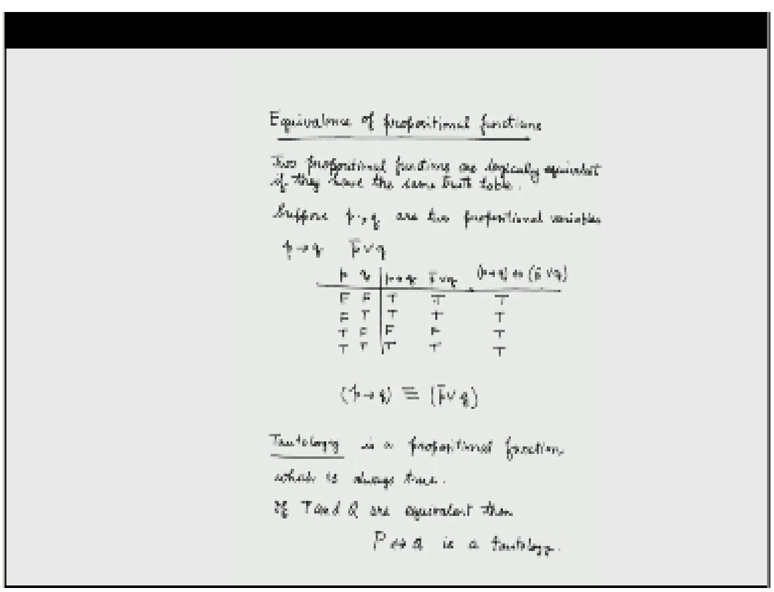

Two proportional are functions are logically equivalent if a have the same truth table to be example we start with two proportional variables p and q and consider two proportional functions pq and not of p or p so $p \rightarrow q$ so we have already seen that it is first row but here not if not p is t so the so r will be t this one is false because lot of p is going to be false and q is false and lastly it is going to be p therefore we see that $p \rightarrow q$.

And both p and q have truth table therefore these two statements are equivalent we have two statements as $p \rightarrow q$ equivalent to three pair of deadlines equivalent not of p and q for that we observe another point let me consider the bi conditional between these two equivalent statements that is $p \rightarrow q$ by conditional not p or q we will see that it is always true now introduce one more notion that is of equivalent.

If there is a proportional function which is true with respective of the values of the original variables involved in it then it is called a tautology from what we have discussed it is not difficult to see if two proportional functions are equivalent then if it for another function by connecting these two proportional functions by conditional and the resulting function is going to be the tautology now if we have a proportional function which is never true and it is called the continuation that usual proportional which are sometimes true sometimes false are called contingents by this we come to an end of this lecture thank you.

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