INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)

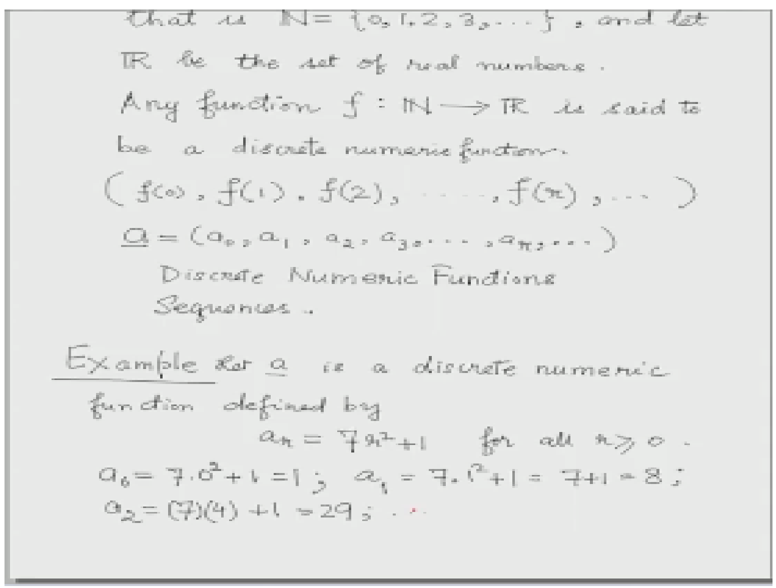
Discrete Mathematics

Module-09
Discrete numeric functions
Lecture-01
Discrete numeric functions

With
Dr. Sugata Gangopadhyay
Department of Mathematics
IIT Roorkee

In today is lecture we will be discussing discrete numeric functions.

(Refer Slide Time: 00:40)



A discrete numeric function is a function from the set of natural numbers, which consists of 0 1 2 and so on to the set of real numbers. Let us write the definition somewhat formally let capital N be the set of natural numbers that is N = 0 1 2 3 and so on and let R be the set of real numbers. Any function from f N →R is said to be a discrete numeric function is said to be a discrete numeric function.

In practice we do not write discreet numeric functions, as usual functions what we do instead is that we write them as sequences. So let us start with the function f what we can do, is that we can evaluate it at all possible points of the domain that is the set of natural numbers and write the sequence of the results. So when I evaluate f at 0 I get F 0 then I evaluate it at 1 so I get F 1 then I get evaluated at 2 so I get F 2 and so on I go on and somewhere much later I will evaluate it at some R, so I will write F R and go on like that.

And we can put this sequence inside the first brackets to delineate their beginning and end, it is to be understood that these are essentially infinite sequences. Then somebody may ask me that since it is infinite why we are writing like that like this one. Now the answer is that although they are infinite I should have a way of obtaining the expression for the r^{th} element in finite time, no matter how large the element R is or how long we take to compute the value of and a value of a Fr.

The most important fact is that the computation time has to be finite no matter how large, now again in practice we have a more convenient way of writing a discrete numeric function. We will denote a discrete numeric function / a symbol, let us say a which will be typically a lowercase letter in the αbet, so ABCD but when in print we will be writing boldface lowercase a boldface lowercase B and so on and when we are writing / hand.

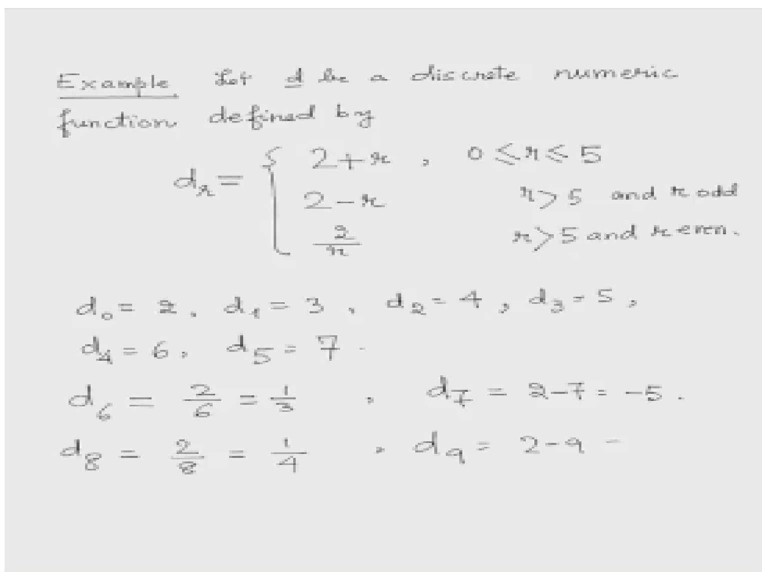
Then we will write a and put a underline to specify that this is a discreet numeric function. Now this a is a sequence whose elements are given as a 0, then a_1, then a_2, then a_3 and so on, the general element is given as a_r and so on and we close the bracket. Now we shall be doing this for other letters as well like lowercase B boldface or B with an underline and so on. So these are discreet numeric functions for us and in other contexts these are also known as sequences.

So we could also say that a is a sequence of numbers a_0 a_1 a_2 the general term is given as a_R, now let us look at a look at an example. Consider the sequence a let now this is something that we will be doing typically over and over again in this lecture, we will say that suppose a is a discrete numeric function and it is defined / a_R =let us say an expression in R and then we will write for all $R \geq 0$ well.

We implicitly understand that a_r is the r^{th} element of a and R starts from 0, so we have elements a_0 a_1 a_2 a_3 and so on a_r and so on and r will keep on increasing over the set of natural numbers. So suppose I would like to know the first few elements of this discrete numeric sequence get numeric function, then I take the first element that is a_0 and that is 7 times $0^2 + 1$ which = 1, then the second element that is a_1 this is 7 times $1^2 + 1$ which = 7 + 1 which = 8, then the third element a_2 is 7 times 2^2 that is 4 + 1 and this = 29.

And we can keep on evaluating like this at this point we note that discrete numeric functions may not be defined / a single formula, it is also possible that these functions are different are defined / different formulas over different regions in the domain. So let us take an example now here I denote the discrete numeric function / D.

(Refer Slide Time: 12:48)

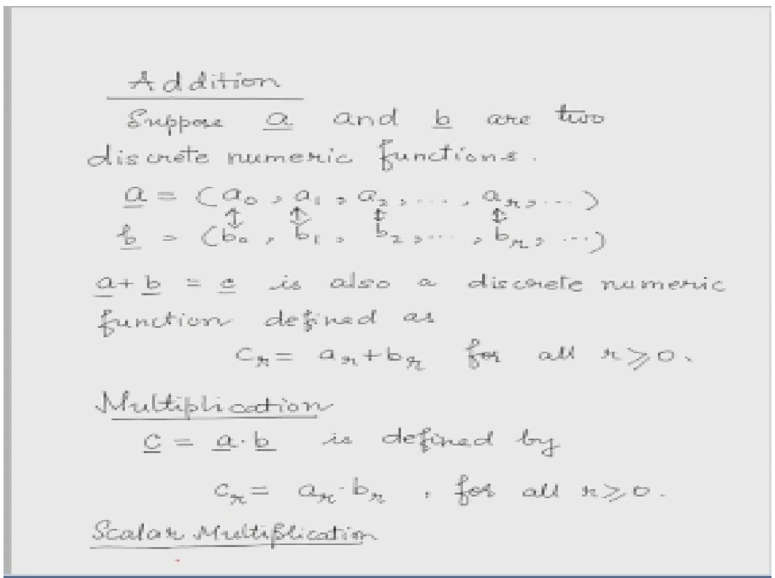


Let d underline discreet numeric function defined / d_r = now see $d_r = 2 + r$ $0 \leq R \leq 5$ dr = 2 - R for R > 5 and r_{odd} and dr = 2 / R for R > 5 and R even. So we have got three regions one region is between 0 & 5 inclusive of 0 & 5 and in which dr is 2 + r if you would like to compute dr for these r and it can start with d_0 which gives me 2 and then d 1 which gives me 3 then d 2 gives us 4 d 3 gives us 5, d 4 gives us 6 and d 6 sorry d 5 which gives us 7.

And we have to stop here because we have come to R = 5 when R is > 5 and our is even, so that is the case now d6, 6 is even and > 5 then it is 2/6 so it is 1/3rd and d_7 this is 2 - 7 so it is - 5 d_8 this is 2/8 so we will have 1 /4 and d_9 this is 2 - 9 so it = - 7, in this way we can compute first few elements of the discrete numeric function that we have defined. Now we will we will come to manipulation of discrete numeric functions.

What we see here is that we have several discreet numeric functions, then we can add multiply or subtract them with one another and also, we can have the ideas of multiplying some scalars to these discrete numeric functions / scalars we mean real numbers. We will also see that we can do some other operations on these discrete numeric functions but first we start with addition.

(Refer Slide Time: 17:43)

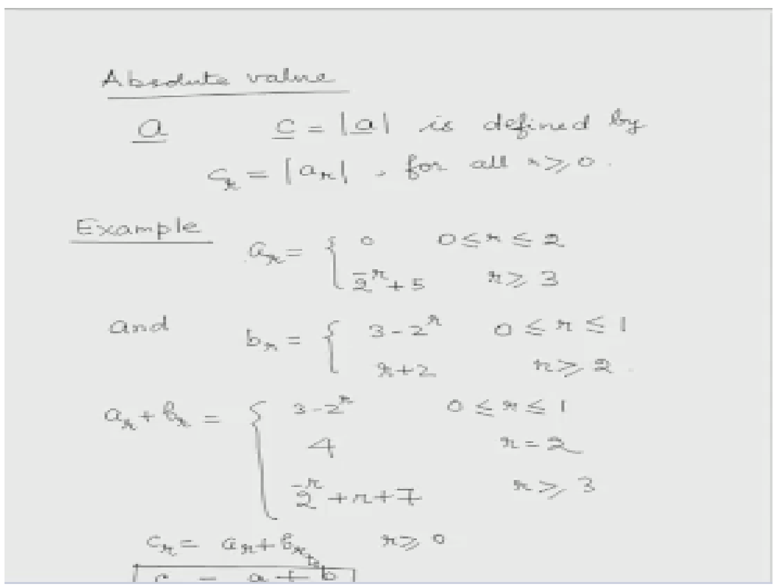


Suppose a and b are two discreet numeric functions, so a is given / a0 a1 a2 so on up to a $_R$ and onward b is given / b0 b1 b2, so on br and onward a + b =c is also a discrete numeric function defined as $C_R = a_R + b_R$ for all r $\geq$ 0, thus in short if we have two discreet numeric functions and we add them element wise, that is add the first element with the first element, second element with the second element, third with the third and in general r^{th} with the r^{th} then we obtain another discreet numeric function which is called the sum of the first two discreet numeric functions.

Next we define multiplication this multiplication is also term wise, so I write in short c = a. b is defined / $c_r = a_r \,.\, b_r$ for all are $\geq$ 0. So this is the multiplication next we define scalar multiplication, now / a scalar we will mean any element from the set real set of real numbers, so that is any element in our is called scalar. So let α be an element in α is called a scalar α times a = C is a discreet numeric function defined / C are =α times a_r for all are ≥ 0.

So this means that if we take a real number and multiply each element of the discreet numeric function / that scalar number and obtain a descriptive Eric function then that discreet numeric function that we obtain is called a scalar multiplication of the original function / the chosen real number. We can also define absolute value of a discrete numeric function.

(Refer Slide Time: 24:00)

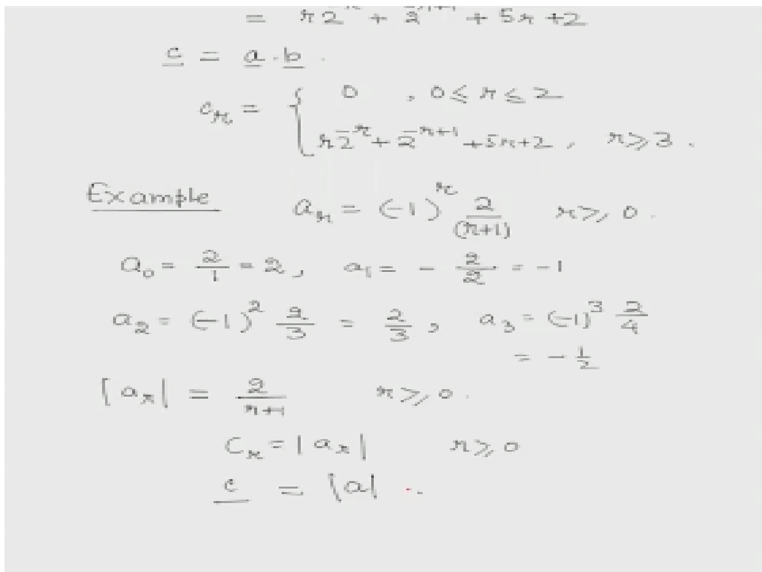


Here take a discrete numeric function a bar see =absolute value of a bar is defined / C_R = absolute value of ar for all r ≥ 0, now we go over to some examples, example we take two discreet numeric functions one is given / a_r=0 when 0 ≤ r ≤2 and 2^{-r} + 5 when r is ≥ 3 and b_r this = 3 - 2^{r} for 0 ≤r ≤1 and r + 2 for R ≥ 2. Now suppose we are interested in checking the sum of these two discreet numeric functions, we have to compute the value of a_r + b_r for all are now a_r + b_r.

When r =0 and 1 gives us the value 3 - 2^{r} this is when r is between 0 & 1 now when r = 2 then air is still 0 but br is r + 2 butr = 2 so we get 2 + 2 which = 4 for r ≥ 3 this the sum will be $^{-r}$ + r

+ 7 so ar + br if we write as C_R then the corresponding sequence see is the sum of the functions a and b as I have told before I will be using the word sequence and discrete numeric functions interchangeably from time to time. Now if you would like to know the product of these two.

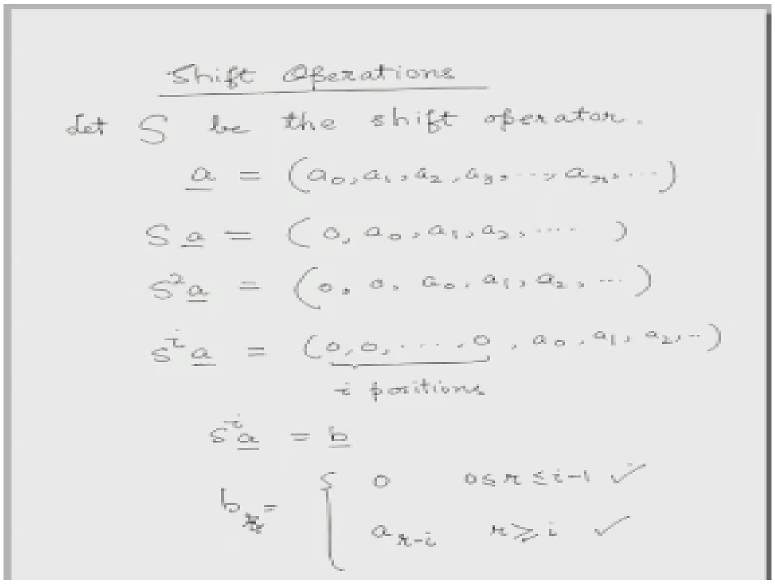
(Refer Slide Time: 28:57)



Numeric functions we will note that ar is 0 for r =0 1 & 2 so the product a_r x b_r is going to be 0 for r $\geq$ 0 and $\leq$2 and then when r is $\geq$ 3 the product ar dot b r = 2 $^{-r}$ + 5 x r + 2 which gives us r x 2 $^{-r}$ + 2 $^{-r}$ + 1 + 5 r + 2 so in short c is a . b where C $_R$ is defined as 0 when r is between 0 and 2 r 2 $^{-r}$ + 2 $^{-r}$ + 1 + 5 r + 2 for r $\geq$ 3. The next example is on absolute values for this we consider the discreet numeric function a r given / - 1 ^{r} x 2 / let us say r + 1 for r $\geq$ 0.

Now see if I put r =0 so a 0 is 2 / 1 = 2 / 1 so it is 2 a 1 = - of 2 / 2 which is - 1 then a 2 is - 1 raised to the power 2 2 / 3 so I have got 2 / 3 and a 3 = - 1 ^{3} 2 / 4 so it is - of 1/2 and so on now the absolute value of ar is simply 2 / r + 1 for all r $\geq$ 0 and if we define C r =absolute value of Ar for r $\geq$ 0 then the discreet numeric function c is called the absolute value of a.

(Refer Slide Time: 33:04)

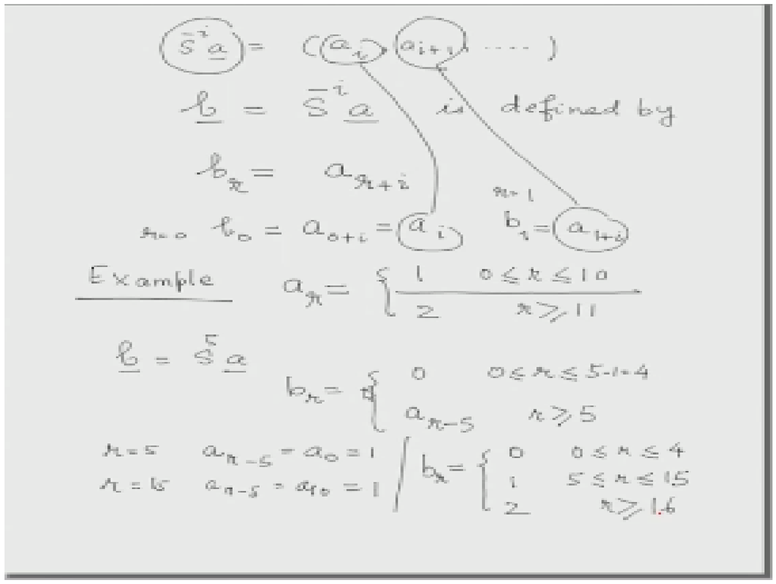


So we have defined addition multiplication scalar multiplication and then absolute value operation on discrete numeric functions. Now we will define some other operations on discrete numeric functions which are called shift operations, now we will be defined denoting a shift operation / s the question is what does it do? We take a discrete numeric function a which is given / a 0, a 1, a 2, a 3, so on then we have a r and so on if I apply s on a I get a discreet numeric function which is called sa and which is simply 0, a 0, a 1, a 2 and so on.

If I apply s again I get a^{2} a which is a discreet numeric function given / 0 0 a 0, a 1, a 2, and so on so the net effect is that s shifts the whole sequence of the values of this kid numeric function / one position is a square shift / two positions and so on so I can think about s^{i} a is 0 0 so on up to 0 I positions and then I have got a 0 a 1 a 2 and so on.

So if you would like to write down formally then we will write that s^{I} a is a discreet numeric function let us say B such that b_i = sorry not bi but should write br =0 for how many positions when r is >=0 and $\leq$I and then b r = a r - I for r $\geq$ I here I have instead of i we will have i - 1 because 0 to i - 1 gives me i positions, so we could also call it a right shift operation in the same manner we can define a left shift operation which we will be writing in this way.

(Refer Slide Time: 38:04)



We take s and then we write s inverse so this is s inverse does something just opposite to s, when it gets applied to a then we get a 1, a 2, a 3 and so on if I apply s^{-2} to a then I will get a 2, a 3 and so on and similarly. So if I apply s^{-I} to a then my sequence will start from ai + 1 and so on so we will write that the discreet numeric function be corresponding to s^{-I} of a is defined / bi =a of our Here I am sorry it will be br =a r + i.

So let us check what happens, so suppose I take b0, b0 what is it is a 0 + i is a I, so this is the first element of s^{-I} a and indeed it is so these are equal then if i take r =this is for r =0 if I take r =1 then I have got B 1 which = a 1 + I which is again same as the second element, so like this we will get all the elements of the shifted sequence and of course we will lose the first I elements. Now let us look at some examples of this shifting operation, you now we take a discrete numeric function a r given / 1 if 0 ≤r ≤10 and 2 if r is ≥ 11 and suppose we would like to know the discrete numeric function be given / s^{-5} of a.

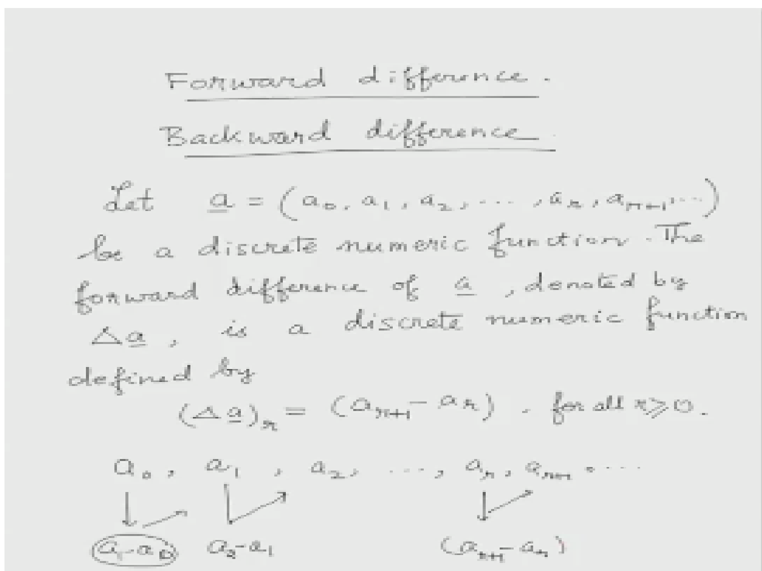
Then as I have defined earlier be r = 0 for 0 ≤r ≤I - 1 so it is 5 - 1 which is 4 and then it is a r - 5 for r ≥ 5, now we have to see what happens to a r – 5. So if r =5 a r - 5 is a 0 which = 1, now if r =15, a r - 5 = a 10 which = 1. Here we have to note that according to the definition of ar first 11 values of a r is 1 and that is corresponding to r = 5, 2 r =15 and when r is >=16 it is going to be 2. So we can write br more explicitly as 0 when 0 ≤r ≤4 is 1 when r is ≥ 5 and ≤15 and 2 when r is ≥ 16.

So these are the ways to write, now if somebody tells me to take the other way round like take probably s $^{-7}$ of a, let us see whether we can solve that problem we recall again our discreet numeric function which is a r.

Given / one when $0 \le r \le 10$ and 2 when $11 \le r$ that is $r \ge 11$, now suppose somebody tells me to find out s - 7 of a it is of course a discreet numeric function be defined / br which = a r+ 7 for $r \ge 0$, so what happens in this case suppose r =0 then we get a 7. Now what is a 7 a 7 is 1 r =1 we get a 8, a it is 1 r =2 we get a 9 a 9 =1 r =3 we get a 10 a 10 =1 r =4 we get a 11 which = 2 and so on we will keep on getting 2 after this.

So I can write br as 1 for $0 \le r \le 3$ and 2 when r >=4, so this is a sequence shifted / 7 $^{-s}$ to the $^{-7}$ now we come to a pair of very typical operations on discrete numeric functions the first one is called the forward difference.

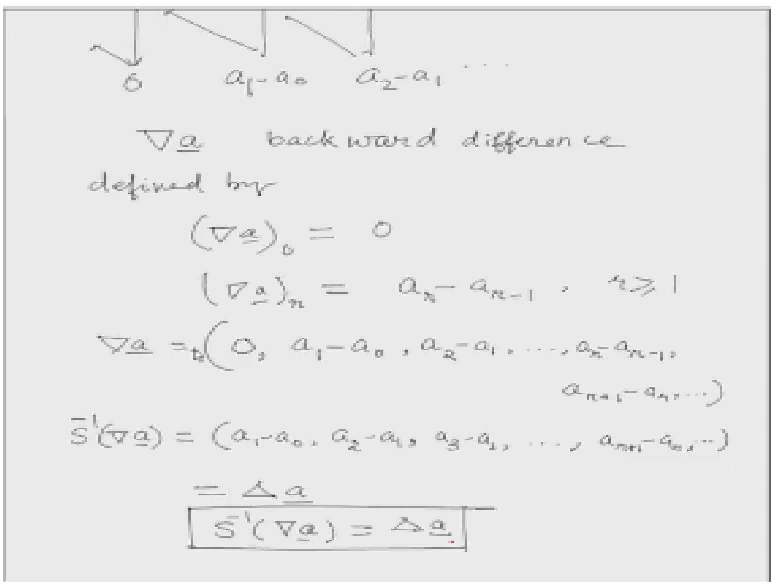
(Refer Slide Time: 47:45)



Forward difference the second one is called backward difference, so for the beginning let us start with the forward difference let a be a discrete numeric sequence be a discrete numeric function or a sequence whatever, the forward difference is a discrete numeric function defined / the forward difference of a denoted / capital Δ of a is a discrete numeric function defined / the rth element of this function is a r + 1 - ar and that is why it is called the forward difference and this is for all are $\ge$ 0.

Why forward? The reason is that if you check the sequence values a 0, a 1, a 2 so on a r then a r + 1 and so on so come to the first position I am looking forward I am looking forward and checking, how much increment happens in the forward direction. So I am writing a $_1$ - a $_0$ in the first place in the second position again I am looking forward I am writing a 2 - a 1 and at the r_S position I am again looking forward and I am writing a r + 1 - Ar that is why it is called the forward difference. The backward difference is when we look backward, so it is like this.

(Refer Slide Time: 51:33)

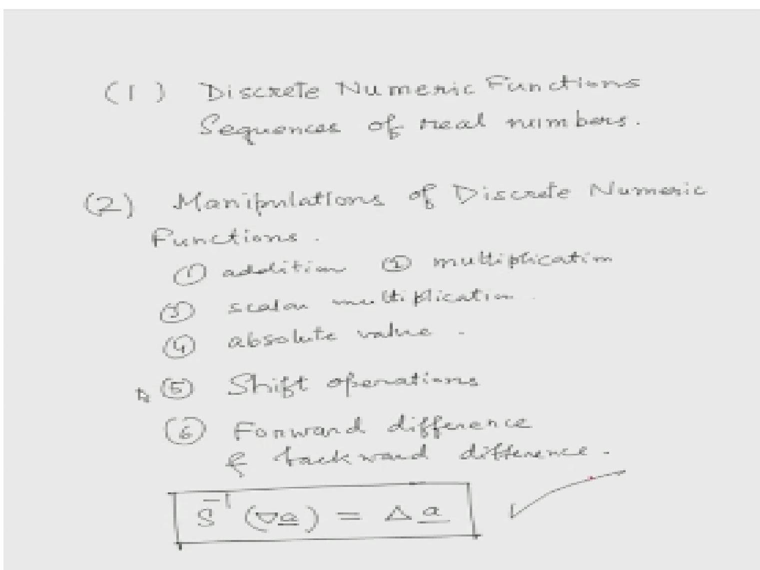


We again start with a discreet numeric function a start from a0, a1, a2, ar, a r + 1 and we insert a r - 1 before ar and so on because we have to look backward. Now when I come to the first position come to the first position I look backward there is nothing I write 0, when I come to the second position I look backward I find a 0, so I write a 1 - a 0 when I come to the 3rd position that is a position corresponding to a2 I look backward I see a 1 so I write the difference a 1 - a 2 and so on.

So here we will write the backward difference as this and it is defined / 0-8 position it is 0 and from one onward it is ar - a r - 1 for r $\geq$ 1 this is the backward difference. So if we start with a with a forward difference sorry, if we start with a backward difference the sequence will look like this is a backward difference. The first one is 0 then the second one is a 1 - a 0 the third one is a 2 - a 1 we go on like this then we find ar - ar - 1 and after that if we venture forward we will find a ar + 1 - ar and so on.

If we apply s inverse on this s inverse will shift one step to the left I will get the sequence a 1 - a 0 in the z-row at position a 2 - a 1 in the first position a 3 - a 2, in the second position and a ar + 1 - ar in the r^{th} position and so on and just a brief recall of what we define just before in the name of forward difference, we see that this discrete numeric function is nothing but the forward difference so this is nothing but this. Therefore we have a nice relationship between forward difference and backward difference as this.

(Refer Slide Time: 56:43)



In this lecture we have first discussed the definition of discrete numeric functions, we have also shown the connection between discrete numeric functions and sequences of real numbers, and second we have studied some manipulations of discreet numeric functions. We have checked addition, multiplication, scalar multiplication and taking absolute value then we have checked shift operations.

Which are going to be very important in the context of discrete numeric functions shift operations and lastly we have checked the forward difference and backward difference and a relationship, which connects the forward difference with the backward difference through the shift operation which is this is one left shift of the backward difference gives me forward difference, this is all for this lecture thank you.

Educational Technology Cell
Indian Institute of Technology Roorkee

Production for NPTEL
Ministry of Human Resource Development
Government of India

For Further Details **Contact**

Coordinate, Educational Technology Cell
Indian Institute of Technology Roorkee
Hoorkee-24/667
Email:etcell@iitr.ernet.in,etcell.iitrke@gmail.
Website: www.nptel.iim.ac.in

Acknowledgement
Prof pradipta Banerji
Director,IIT Roorkee

Subject Expert & Script
Dr.Sugata Gangopadhyay
Dept of Mathematics
IIT Roorkee

Production Team
Neetesh Kumar
Jitender Kumar
Pankaj Saini
Meenakshi Chauhan

Camera
Sarath Koovery
Younus Salim

Online Editing
Jithin.k

Graphics
Binoy.V.P

NPTEL Coordinator
Prof.Bikash Mohanty

An Educational Technology Cell
IIT Roorkee Production

WANT TO SEE MORE LIKE THIS
SUBSCRIBE