INDIAN INSTITUTE OF TECHNOLOGY ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY ENHANCED LEARNING (NPTEL)

Discrete Mathematics

Module-05
Graph theory
Lecture-02
Isomorphism and sub graphs

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In today's lecture we will start with isomorphism of graphs.

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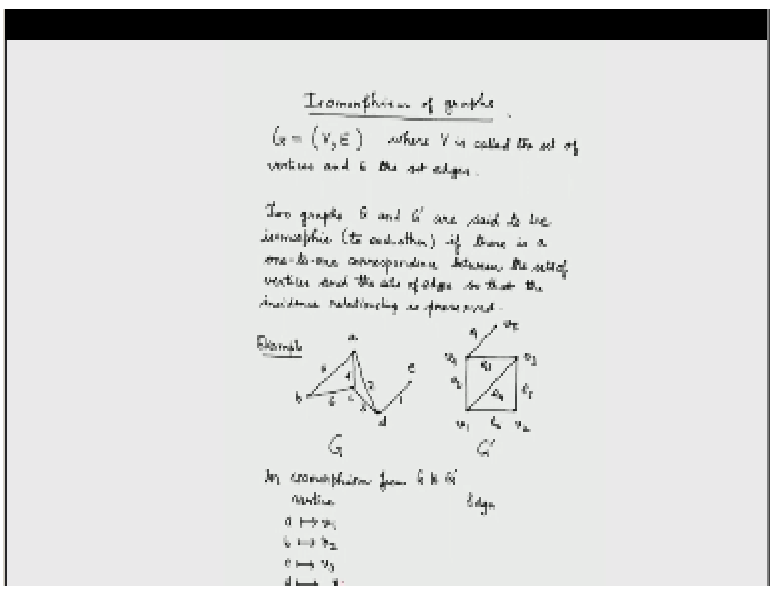

Now we have already seen that a graph G is an ordered pair of sets V and E where V is called the set of vertices and E the set of edges now we have also seen what we mean by vertices and edges roughly speaking V contains some points and E contains pair of those points not necessarily all the pair is and each pair corresponds to an edge and it may so happen that there may be repetition of these pairs.

So that within two vertices there may be more than one edge more than one edges as well as there may be self loops now sometimes what happens is that we draw two graphs but there may be a one-to-one on two mapping from the set of vertices of one graph to the set of vertices of the other such that the adjacency of the vertices are maintained and in those cases we say that the two graphs are isomorphic.

Now we go for the formal definition it says that two graphs G & G - are said to be isomorphic well of course to each other if there is a one-to-one correspondence between the set of vertices and that between the sets of vertices and the sets of us and sets of edges so that the incidence relationship is preserved now let us look at an example now we have drawn two graphs let us call this graph G and this graph G Prime.

Now apparently there are different graphs but I will now give a one-to-one on two correspondence from the set of vertices of G to the set of vertices of G Prime and a one-to-one correspondence one-to-one onto correspondence to from the set of edges of G to the set of edges of G Prime and we can check that the incidence relationship is preserved or in another way the adjacent C's are preserved that if two vertices are adjacent in G.

They will become adjacent in G Prime and the corresponding edges also will be mapped to the corresponding edges so I will give the mapping so isomorphism of G to G prime so one side I ride one-to-one on two maps between the set of vertices and the other side the 1 2 1 and 1/2 mark maps between the set of edges so between the set of vertices we have a map to v 1 B map to v 2 C map to v3 d map to v4 and E map to v 5 and among the set of edges one map to e 1 to map to e 2 3 map 2 e 34 map to e 4 and 5 map to e5.

Now let us look at the first few correspondences so a is mapped to v 1 so that means a is going over here and B is mapped to v 2that means that B is going to this point now a and B are adjacent to each other in the graph G now let us look what happens in graph G Prime here we see that V 1 is adjacent to V 2 and what about the edge which is connecting a and B the corresponding edge is 5.

And here we see the corresponding edge is a 5 and according to our already defined correspondence we see that 5 maps to e5 that means according to this map a is going to V 1B is going to be 2 and a B the edge AV that is labeled by 5 in the graph G is getting mapped to the edge E 5 and which is the edge between v 1 and v 2 and this should happen with each vertices each vertex and each edge if it happens.

Then we say that we have an isomorphism it is needless to say that finding out isomorphism between two graphs is a very difficult problem and we do not have complete answer to this so that means that if I am given two graphs in general I do not have a quick or efficient way of deciding in general whether these two graphs are isomorphic or not but we can always hope to get some partial results depending on certain properties of the graphs.

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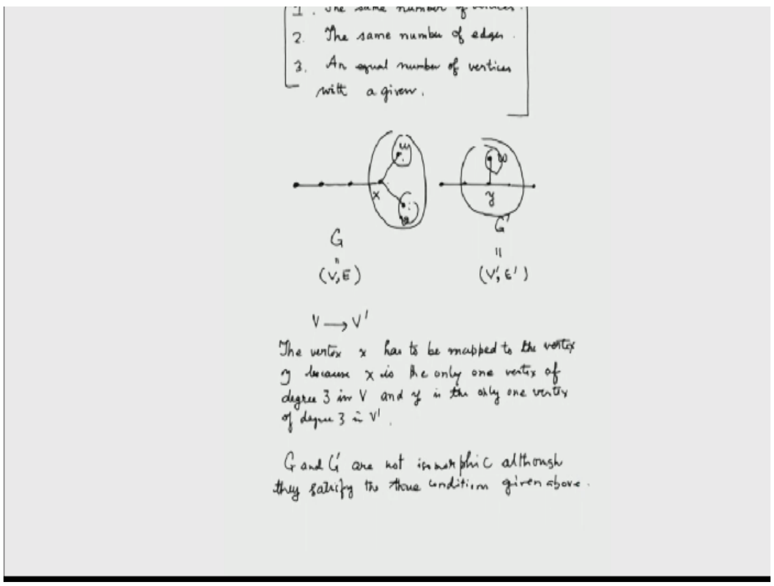

Now let us look at some partial results partial results on decision of isomorphism of graphs now what we will do here is that we list down three rules or three conditions that to isomorphic graphs must have we will of course so, so later with an example that even if these conditions are satisfied it does not mean that the graphs are isomorphic what we can say is that if two graphs are not isomorphic then sorry what we can say that if these conditions are not satisfied by these two graph any two graphs.

Then they cannot be isomorphic I repeat again we will give three conditions now what we will claim and what is very evident is that if there are two graphs for which these conditions are not satisfied then they cannot be isomorphic however we will give examples and show that there are graphs which satisfy these properties and still they are not isomorphic now let us look at the properties all right.

So if we have two isomorphic graphs and then they must have same number of vertices and they must have the same number of edges three and equal number of vertices with a given degree now it is not difficult to directly see that these conditions are absolutely necessary for two graphs to be isomorphic that means that if two graphs are isomorphic of course these things must happen so if these things do not happen then they are not isomorphic.

But it is quite possible that these things do happen but still the graphs are not isomorphic let me give an example now we have a graph which we denote by G and another graph we denote by G Prime now G has six vertices G prime also has six vertices now if you look at the edges G has four edges G Prime sorry G has five edges and G prime also has five edges now if we look at the degrees now there are vertices with of degree one.

And in G there are three vertices of degree 1 in G prime there are three vertices of degree one and about degree two in G there are two vertices of degree two and in G prime there are two vertices of degree two and in G there is exactly one vertex with degree one sorry exactly one vertex with degree 3 and in G Prime there is exactly one vertex with degree three now what we claim is that these two graphs are not isomorphic.

So we have to show that they are not isomorphic although it is now clear that if we look at these three conditions G & G prime satisfy all these three conditions now if we are trying to build an isomorphism suppose we write G as V, E and G prime as V prime, E Prime we have to develop a one-to-one correspondence between V and V Prime now it is obvious that the vertices of the same degrees will be mapped to vertices of the same degree.

Because otherwise it is not possible to preserve the adjacency or incidence relationship so the vertex X the vertex X has to be mapped to the vertex why because X is only one vertex of degree three in V and y is the only one vertex of degree three in V Prime now if we look at this region and this region we see that from X two vertices are connected which are of degree one but from y only one vertex is connected which is of degree one.

Therefore there is a structural difference between these two graphs if you think a little more we will see that no matter how we arrange our permutations or so to say the mapping between V and V – I will never be able to map all the vertices in such a way that the incidence relationship is preserved that the reason is that as I as I said that this there are two vertices of degree one getting connected to the degree three vertex.

And here there is only one vertex of degree one getting connected to the degree three vertex therefore G & G dash are not G and G prime are not isomorphic although the satisfied they satisfy the three conditions given above although they satisfy the three conditions given above.

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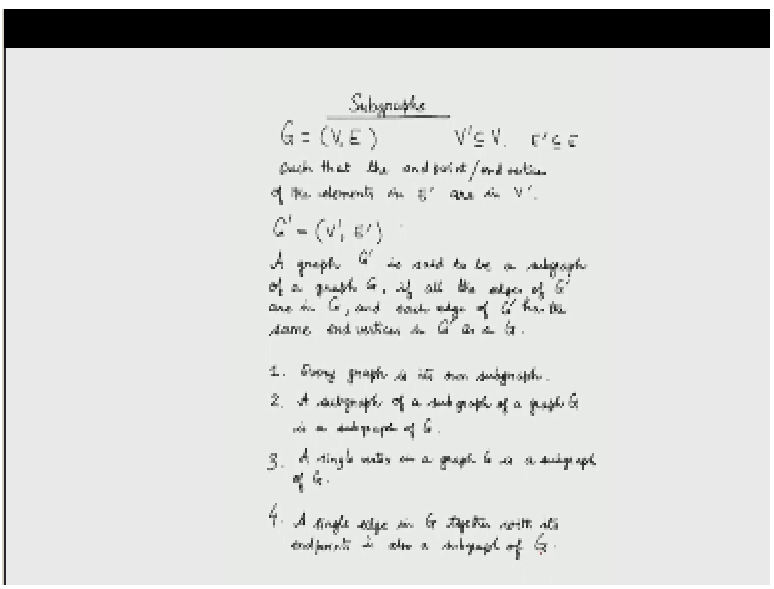

Next we move on to sub-graphs if we have a graph let us denote it by G = V, e we can always think of a subset of V well let us call it V prime which is a subset of V and we can consider a subset of E such that the end points of the that subset let us name it a prime belongs to V prime such that the endpoints slash end vertices of the elements in a prime are in V Prime then the resulting graph which is in some way a smaller graph than g-getting derived from G is called a sub graph of G.

Let me write the definition more systematically a graph G prime is said to be a sub graph of a graph gee if all the edges of G prime are in G and each edge of G Prime has the same end points or end vertices in G prime as in G so this is in a formal way of defining a sub graph but we can always remember what I have told in the beginning that a sub graph will have less than possibly a lesser number of vertices.

And edges we will pick up the edges from the set of edges of G which has got endpoints in that subset of vertices now we have got some observations related to a sub graph one every graph is its own sub graph well this is of course obvious then to a sub graph of a sub graph of a graph G is a sub graph of G this too is something that goes without saying and third a single vertex in a graph G is a sub graph of G that is also direct from the definition.

And for its single edge in je together with its endpoints is also a sub graph of g we see that these are more or less straight forward now we will move on to certain special types of sub graphs which are important in several applications of graph theory.

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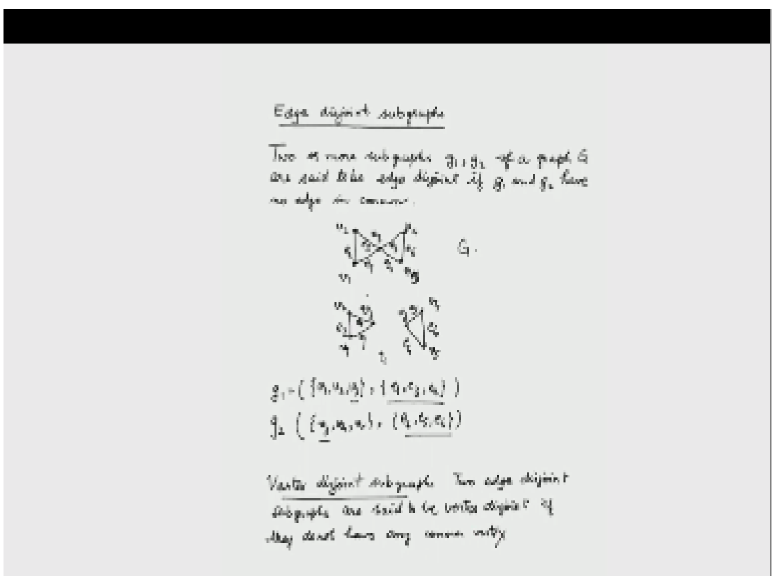

Now we come to edge disjoint sub graphs edged is joint sub graphs two or more sub-graphs g1 g2 of a graph G are said to be edge disjoint if g 1 and g 2 have no edge in common now let us look at an example suppose we have a graph like this so let us label the vertices v1 v2 v3 v4and v5 and label the edges even e 2 e 3e 4 e 5 and e 6.

So let us call this graph as G now let us look at a sub graph of this type v3e for known is not if all over here sov3 this is the vertex v 3 e3 is over here even is over here v 1 v 2 and v 2and another sub graph v 3 v 4 here this is v 5 so this is v 5 like this and we have E 4 E 5 + E 6.

Now suppose I write given as v1 v2 v3 and even III e 2 and g 2 as v3 v4 v5 and e 4 e 5 + e 6 then g 1 and g2our edge disjoint sub graphs see that this is the set of edges for G1 and this is a set of edges for g 2 and there is no common edge but it does not mean that they do not have common vertices we see that the vertex V 3appears in both the sets of vertices of the sub-graphs now we come to vertex disjoint sub graphs to edge disjoint sub graphs are said to be vertex disjoint if they do not have any common vertex.

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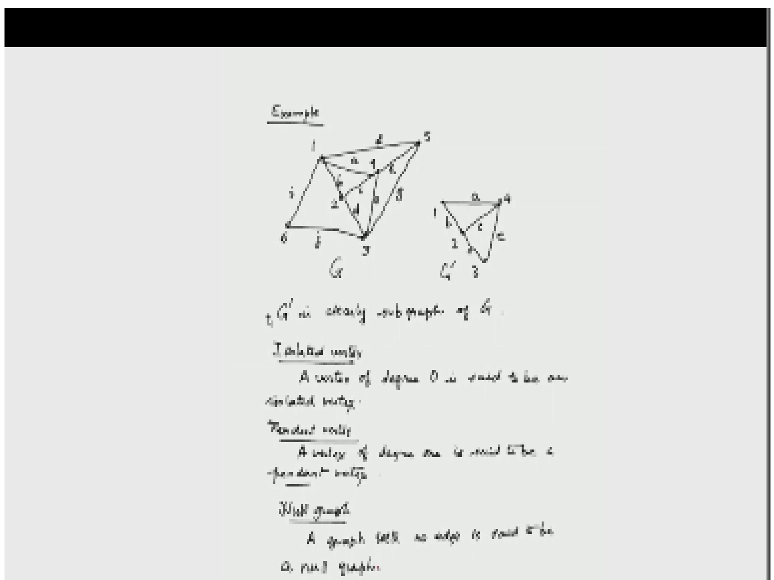

Now let us look at an example of a sub graph of a graph let us consider this graph where the vertices are labeled by 1 2 3 4 5 6 and the edges are labeled as ABCDEFG now let us consider another graph now this is of course a sub graph of the original graph let us name it G and let us name it g prime G prime is clearly a sub graph of G before I end today is lecture I will introduce few more terminologies one is an isolated vertex a vertex of degree zero is said to be an isolated vertex.

Then a pendant vertex a pendent vertex and in null graph a graph with no edge is said to be a null graph now for isolated vertex an example may be like this suppose I have a graph over here a vertex like this suppose this is v 1 v2 v 3 v 4 and the vertex is over here which is called let us say v 5 this v 5 is will called an isolated vertex and a pendent vertex will be a vertex which has got only 1 degree.

So possibly like this v1 v2 v3 v4 and v5 disband on vertex and a null graph will be graphs of this type which has got only vertices but the set of edges is a null set and then there is an idea of regular graphs a graph in which each vertex has same degree is called a regular graph and if this degree is let us say D then it will be called a D regular graph.

Now we think of again the isomorphism problem here we will see that it is it is very, very difficult to solve the isomorphism problem for 2d regular graphs whatever with the value of D which has the same number of vertices and the same number of edges with this I end today's talk thank you.

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