INDIAN INSTITUTE OF TECHNOLOGY
ROORKEE

NATIONAL PROGRAMME ON TECHNOLOGY
ENHANCED LEARNING
(NPTEL)

Discrete Mathematics

Module-04
Discrete Probability
Lecture-03
Independent events, Baye's theorem

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So I would like to introduce the idea of independent events two events A and B are independent if and only if probability of B.

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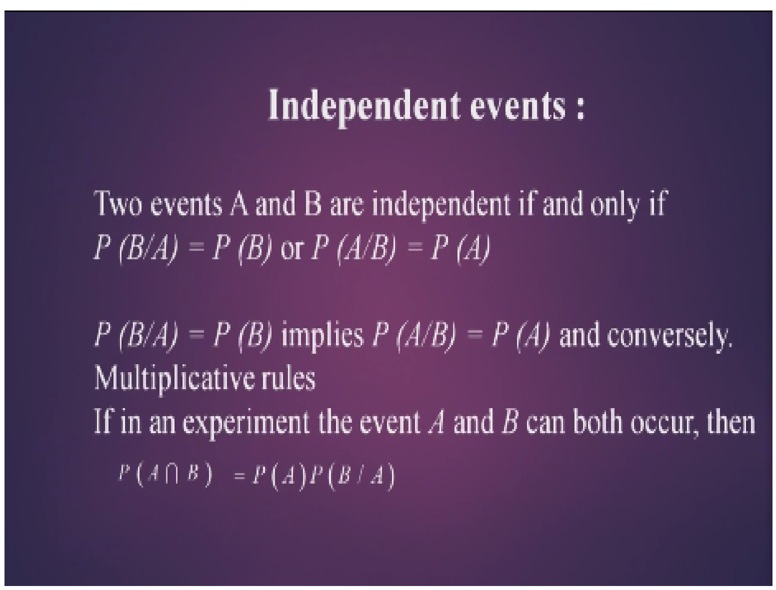

Given a is equal to probability of B now probability of B given a means we are talking about the conditional probability the conditional probability of B given a that is equal to probability of B then we can say that A and B are independent that means whether a has occurred or not there is no effect of this on B, so that is why they are independent or we can say that A and B are independent if probability of a given B is equal to probability of a now probability of B given a is equal to probability of B implies probability of A given B is equal to probability of A and conversely.

So here we can talk about multiplicative rule what is that if in an experiment the event A and B can both occur then probability of A / B will be probability of A into probability of B given A, so this rule is coming from the definition of conditional probability which is already covered because we know that we know that probability of B given A is equal to probability.

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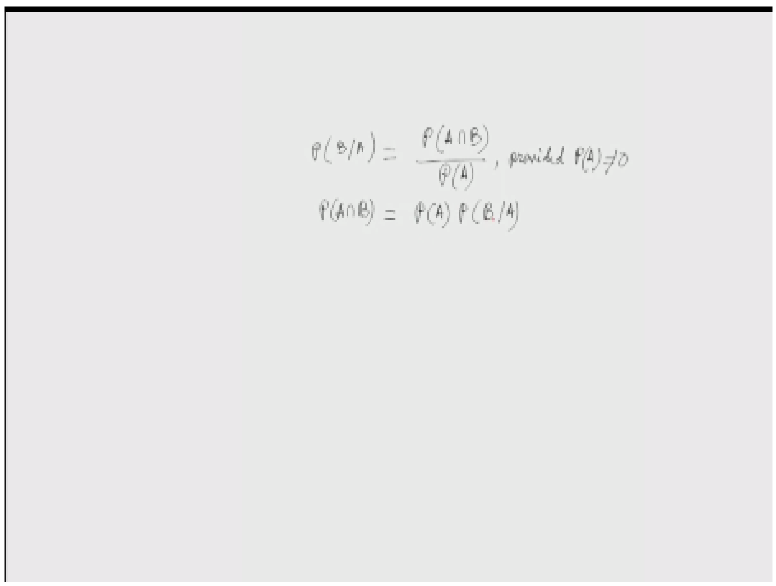

Of A ∩ B divided by probability of A provided probability of A is not equal to 0, so from this we can get probability of A ∩ B is equal to probability of A into probability of B given A what is this probability of A ∩B what does it mean it means that probability what is the probability that both the events A and B will occur what is the probability that A and B both the events will occur, so this is equal to probability that a will occur into probability that B given A the probability of A what is the probability that a will occur in two what is the probability that B will occur given that a has already occurred so this is the meaning of this thus the probability that both A and B occur is equal to.

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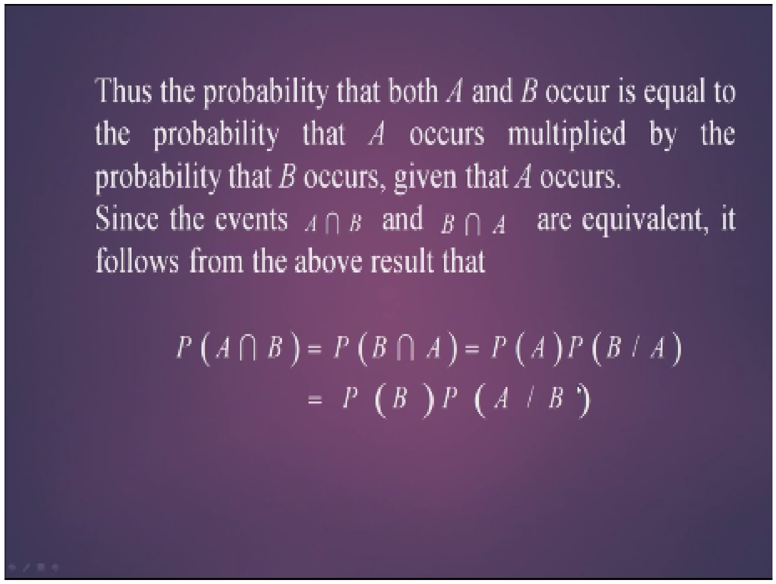

The probability that A occurs multiplied by the probability that B occurs given that a occurs since the events intersection B and B ∩ A are equivalent, it follows from the above result that probability of A ∩ B is equal to probability of B ∩ A which is equal to probability of A into probability of B given A and this is same as probability of B into probability of A given B, so they are all equivalent, next we consider one example suppose that we have a pews box containing 20 fuses of which.

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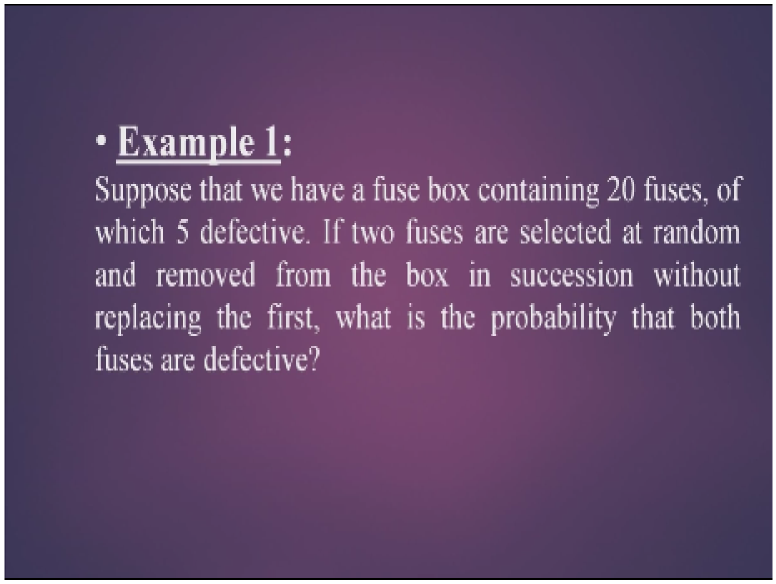

Five are defective if 20 fuses are selected at random and removed from the box in succession without replacing the first what is the probability that both fuses are defective so let us try this example in this way, if a denotes the event that first fuse is defective.

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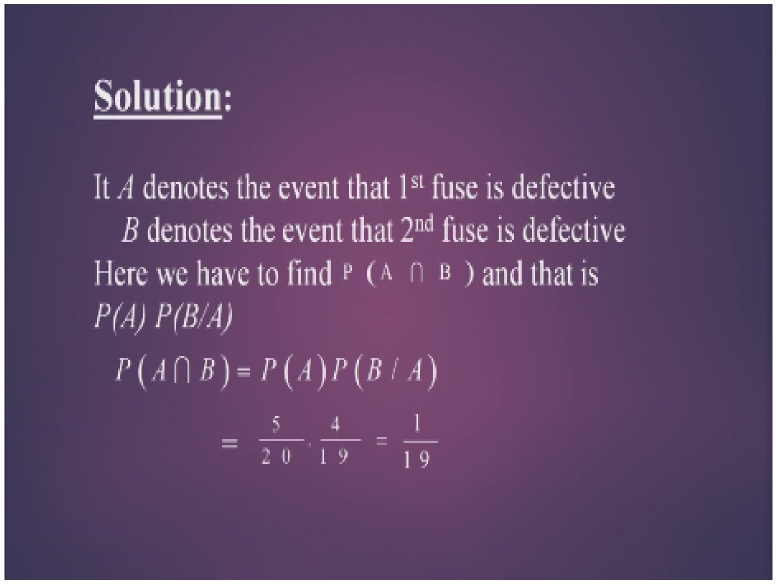

And B denotes the event that second fuse is defective here we have to find the probability of A ∩ B so we have to find the probability that both A and B will occur and that is we know that probability of A into probability of B given A, now what is that first among 20 fuses 5 are defective so according to the problem we know that among 20 fuses 5 fuses are defective, so that is why the first fuse is defective probability of that will be 5 / 20. So probability of A is nothing but the probability that the first fuse is defective so that will be 5 by 20 now once the first fuse is defective it is now taken out.

And kept a side now another fuses randomly chosen from the box so the probability that the second fuse is defective will be that is probability that B given A, so what is the probability that the second fuse is defective given that the first one is already defective, so this probability will be 4 / 90 because one defective fuse is already chosen, so the number of defective fuses will be 4 and there are 19 fuses remain in the box so that is why the probability that B given a will be 4/ 19 and so the resulting probability that is probability that ∩ B will be nothing but 5 by 20 into 4 by 90 which is 1 / 19.

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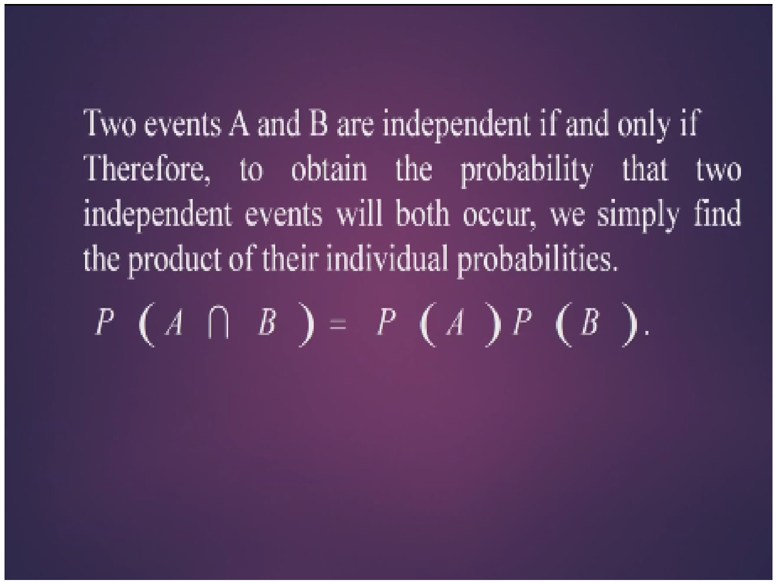

Now two events A and B are independent if and only if probability of A intersection B is equal to P (A) of A into P (B), so this result is coming from the fact that probability of B given A will become, now probability of B so let us see here because we know that probability of A ∩ B is probability of A into P (B), given A so since A and B are independent we know in that case P (B), given a will be probability of B and that is why in this case probability of a intersection B will become probability of a into P (B).

So that is why we can say that two events A and B are independent if and only if P (A) ∩ B is equal to P (A) into probability of B therefore to obtain the probability that two independent events will both occur we simply find the product of their individual probabilities let us take one example here what is the probability of getting two heads in two flips of a balanced coin, now we know that flipping of a balanced coin twice so if we flip it twice they are independent.

So two flips of a balanced coin they are independent and as the coin is a balanced coin the required probability will be ½ into 1/2.

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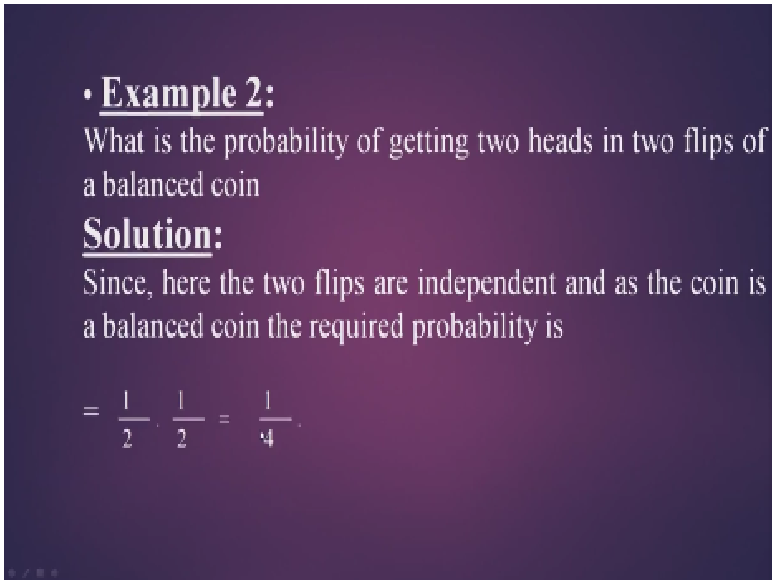

That will be 1 / 4 so this is the probability that first flip will give A into probability that the next one is hit that is also 1/2, so that is why probability of two heads will be ½ into ½ and that will be 1 / 4 because they are independent now suppose there are three events in an experiment event A event B and event C, so in an experiment three events A, B, C can occur so if we have to find the probability that A ∩ B ∩ C that is we have to find the probability that all these three events will occur A B as well as C.

So this will be probability of A into probability of B given a into probability of C given A ∩ B so that means the probability that A will occur into the probability that B will occur given that a has occurred into probability that C will occur given that A and B both have occurred, so how is it coming it is very simple A ∩ B ∩ C now ∩ B can be taken as one event, so I can write it as probability of A ∩ B into probability that C given A ∩ B, so I am taking two events here intersection B and C.

So these two events now if we consider these two events by the definition of conditional probability we can write probability of A ∩ B into probability of C given that A ∩ B has occurred so this can be written as P (A) into P (B) given A into P (C) given A ∩ P.

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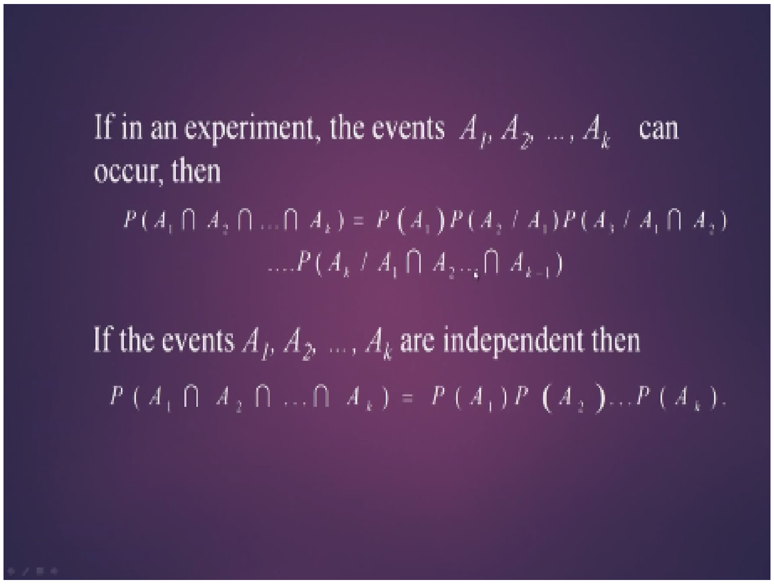

So in this way this will come now if we have more than three events then what should we do if in an experiment the events A 1 A_2 to A K can occur then P (A) 1 ∩ A 2 ∩ so on ∩ A K so ∩ of A1 A2 to 2 A_K, so that I am considering the probability of that event can be written as probability of A1 into probability of A2 given A1 into probability of A3 given that A1 ∩ A2 has occurred dot probability of A_K given A1 ∩ A2 and, so on ∩ Ak – 1 so this result is coming by extending the case which we have already considered that is if there are three events so the way we have done it in the same way.

If we extend number of events then we can get this result very easily, now if the events A1 A2 to Ak are independent then probability of A1 ∩ A2 ∩ whether ∩ A_K is equal to the product of their individual probabilities that is probability of a 1 into probability of a 2 not probability of Ak, so this can be written as probability of A1 ∩ A2 ∩ whether ∩ Ak that will be equal to P(A) 1 into probability of A2 and so on, probability of P(A).

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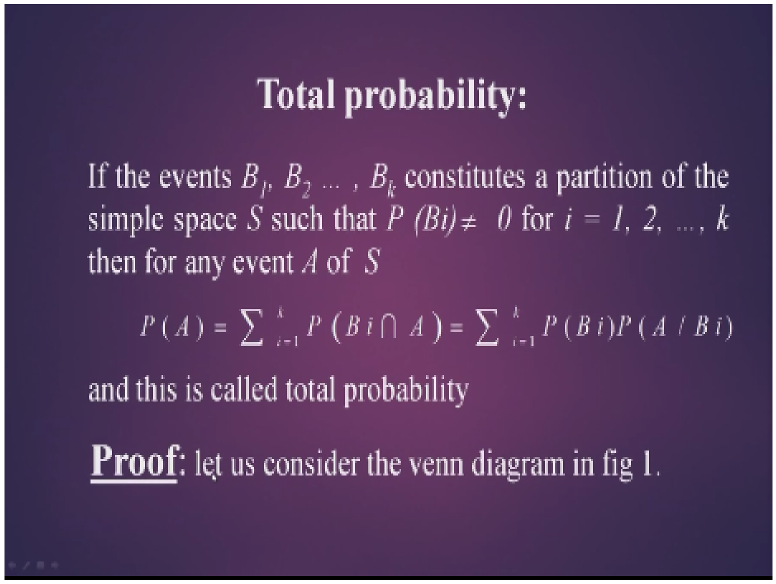

Now let us talk about total probability if the events B1 B2 to Bk constitutes a partition of the sample space is such that probability of Bi not equal to 0 for I equal to 1 to K then for any event A of A probability of A is equal to $\sum$ I equal to 1 to K into P(Bi) $\cap$ A and that is equal to $\sum$ I equal to 1 to K into P(BI) into P(A) given B I and this is called total probability, now let us prove this result for proving this result we consider a Venn diagram partitioning the sample space from the figure.

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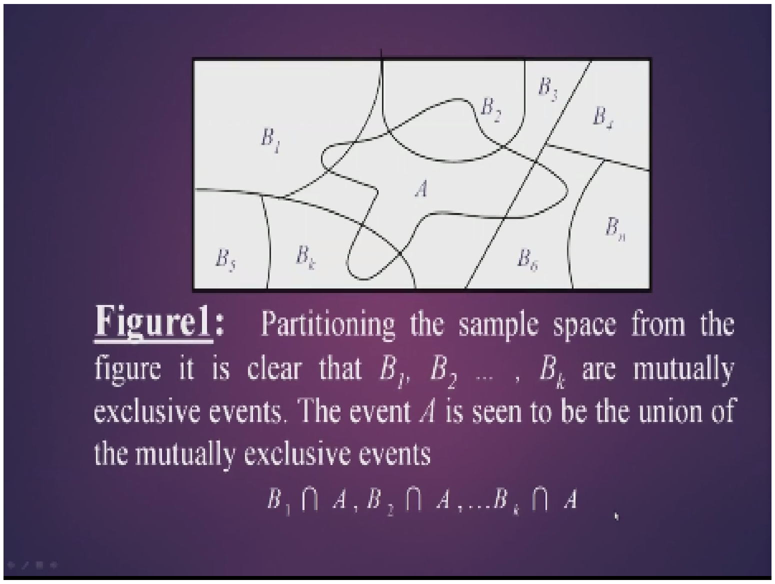

It is clear that B1 B2 to Bk are mutually exclusive events the event A is seen to be the union of the mutually exclusive events B1 ∩ A B2 ∩A up to Bk ∩ A, so this is so A the event a is the union of all this events B1 ∩ AB 2 ∩ A up to Bk ∩E, so that is we can write the event A as B1 ∩ A union B2 ∩ A union law dot in Bk ∩ A.

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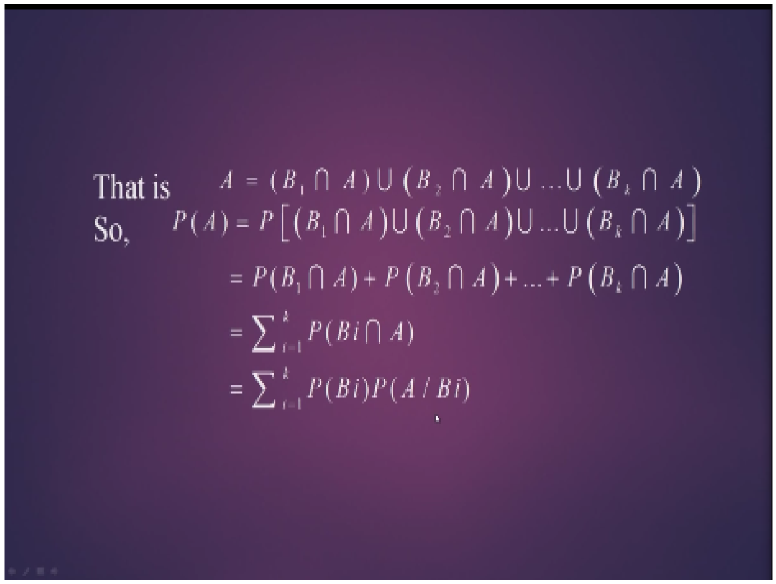

Now since B1 B2 to Bk are mutually exclusive events we can say that B1 ∩ A B2 ∩A up toVk ∩ A are also mutually exclusive and that is why P(A) of A can be written as probability of B1 ∩A plus P(B) 2 ∩ A plus dot plus P(Bk) ∩ A, so this is nothing but the summation I equal to 1 to K of P(BI) ∩ A and this probability can be written in this way that probability of BI into P(A) given BI, so it can be written as the ∑ I equal to 1 to K P(BI) into P(A) given BI, now let us talk about Baye's rule if the events B1 B2 to Bk.

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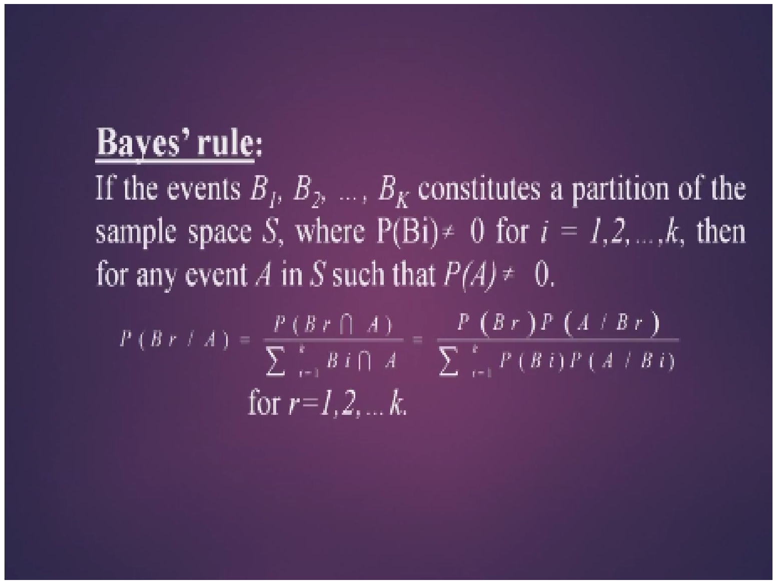

Constitutes a partition of the sample space is where P(BI) not equal to 0 for I equal to 1 To 2k then for any event A in is such that P(A) is not equal to zero then P(B) are given A is equal to P(B) our ∩ A/ ∑ I equal to 1 to k P(BI) ∩ A which is equal to P(B) our into probability of a given BR / ∑ I equal to 1 to k P(BI) into P(A) given BI for R = 1 to k, so this is called the Baye's rule and this Baye's rule is very, very important and it has lot of applications so how to prove this rule by the definition of conditional P(B) are given a will be P(B) R ∩ A / P(A).

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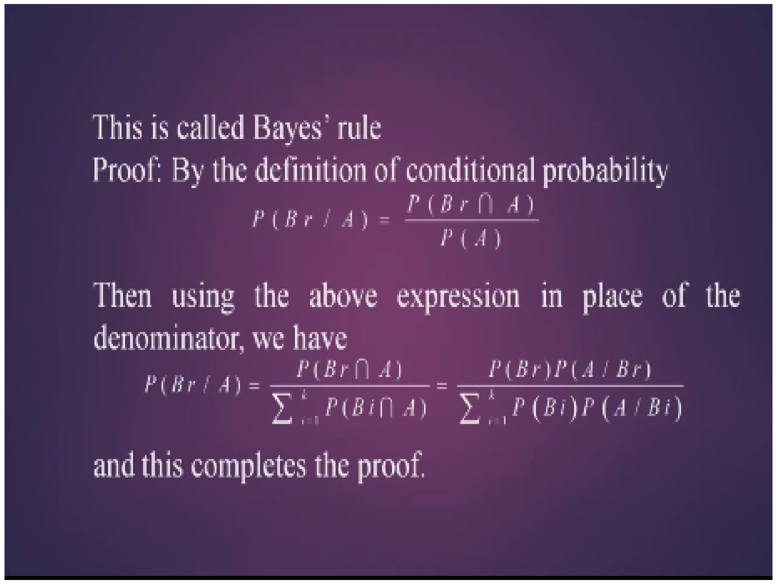

Then using the above result that is the result of total probability we can find the expression in place of the denominator, so P(B) are given a will become P(B) R ∩ A /∑ I = 1 to k probability of BI ∩A which is equal to probability of Br into probability of a given Br /∑ I = 1 to k P(BI) into P(A) given Bi and this completes the proof, so in this way we are getting the Baye's rule and what is this actually so if in some experiment it there are some steps so we are doing something in the first step and doing something in the second step like this.

So and so if it is asked in the other way that is the final event we know final event will occur that is given then what is the probability that before that it has occurred, so this kind of question is asked so in that case we can use Baye's rule and we can find the probability of the conditional probability that that final event has occurred then before this event had occurred, so this kind of question can be tackled by Baye's rule, so let us consider one example in this context one bag contains 4 white balls and 3 black balls.

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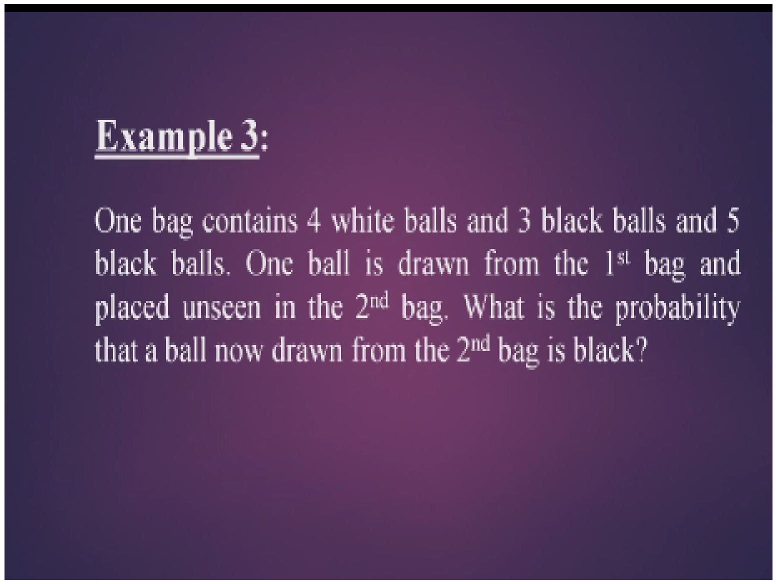

And another bag contains 3 white balls and 5 black balls one ball is drawn from the first bag and placed unseen in the second bag what is the probability that a ball now drawn from the second bag is black, now we have chosen randomly a ball from the first bag and we have kept it at the second back and then we are choosing a ball randomly from the second bag, now we have to find the probability that if what is the probability that this ball drawn from the second bag is black, so let us try this solution in this way W denotes the event that a white ball is transferred from bag one.

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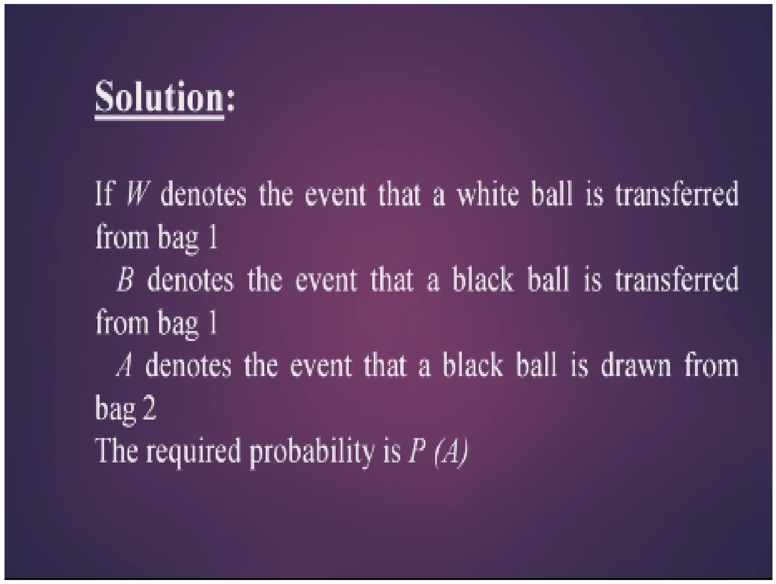

B denotes the event that a black ball is transferred from bag one, so these two events W and B they are mutually exclusive either W will occur or B will occur a denotes the event that a black ball is drawn from back to, now the required probability is P(A) and what is that P(A) can be written as P(W) ∩ A union the ∩ A.

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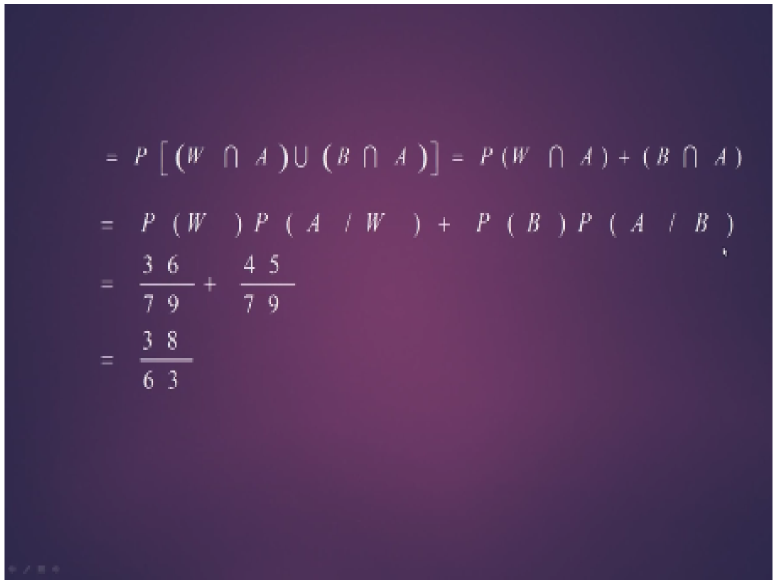

And since W and B are mutually exclusive events W ∩ A and B ∩ A are also mutually exclusive and that is why this can be written as P(W) ∩ A plus P(B) ∩ A, so this can be written as P(W) into P(A) given W + P(B) into probability of a given B, now let us try this in this way that we know that first bag.

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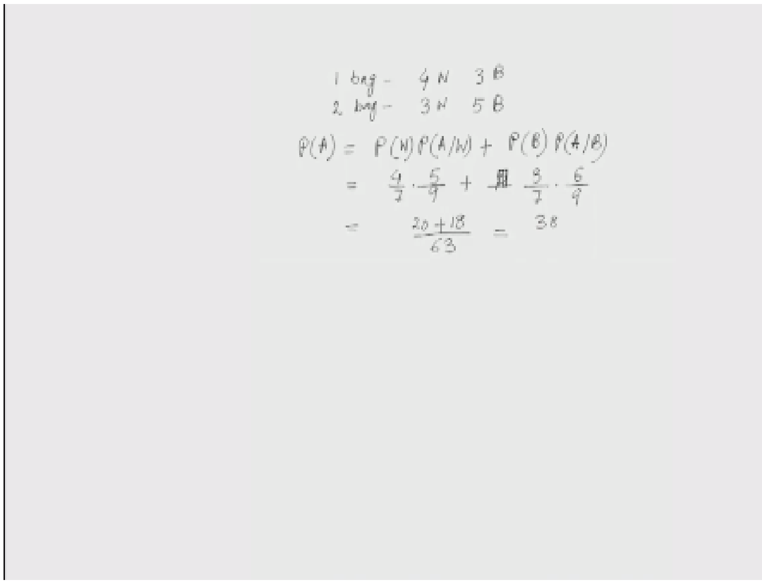

Contains four white and 3 black balls second bag contains 3 white and 5 black balls now we have to find probability of a which is P(W) into P(A) given W + P(B) into P(A) given B, so now P(A) given W, so we have to find the probability that if a white ball is transferred from bag one two back two then what is the probability that a black ball will be chosen from back to so that probability and this is the P(W) is nothing but the probability that a white ball is chosen from bag one so that will be 4/7.

Okay because they are a 4 white balls in bag one among seven balls so 4 / 7 into now if a white ball is transferred from bag 1 2 back to in back to there will be 9 balls after this transfer and number of black balls will remain same because a white ball is transferred from back bag one two back to, so this probability will be 5 / 9 + the probability that a black ball is chosen from back to, so that will happen with probability 5 by sorry it will be 3/7 because there are 3 black balls in bag one among seven balls.

So it will be 3 / 7 in 2 and this conditional probability is nothing but that if black ball is transferred from bag one two back two then what is the probability that from back to black ball will be chosen and that will become 6/9 because, now there are six black balls in back to and total number of balls after this transfer is 9 that is why this probability will be 6/9, so now we can calculate this probability that is 20 + 18 / 63, so it will be 38 / 63.

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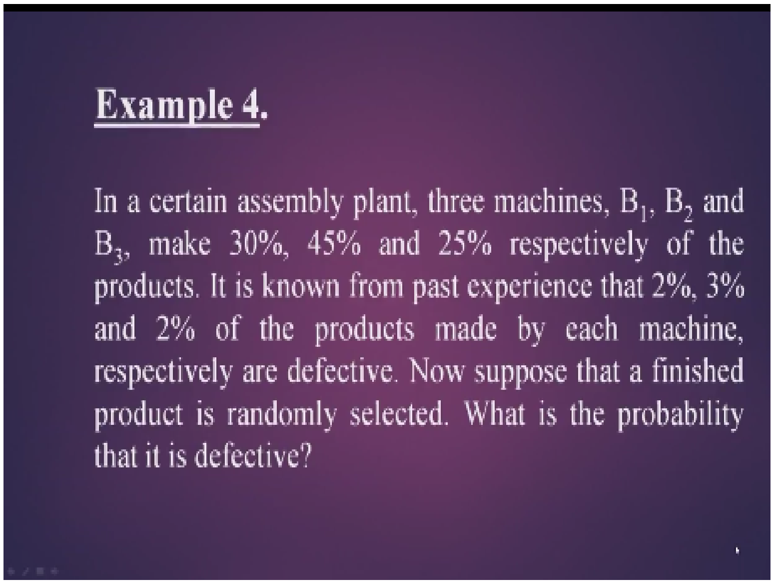

In a certain assembly plant three machines B1 B2 and B3 make 30% 45% and 25% respectively of the products it is known from past experience that 2 % 3% and 2 % of the products made by each machine respectively are defective, now suppose that a finished product is randomly selected what is the probability that it is defective, so let us try this solution in this way the event a represents the product is defective.

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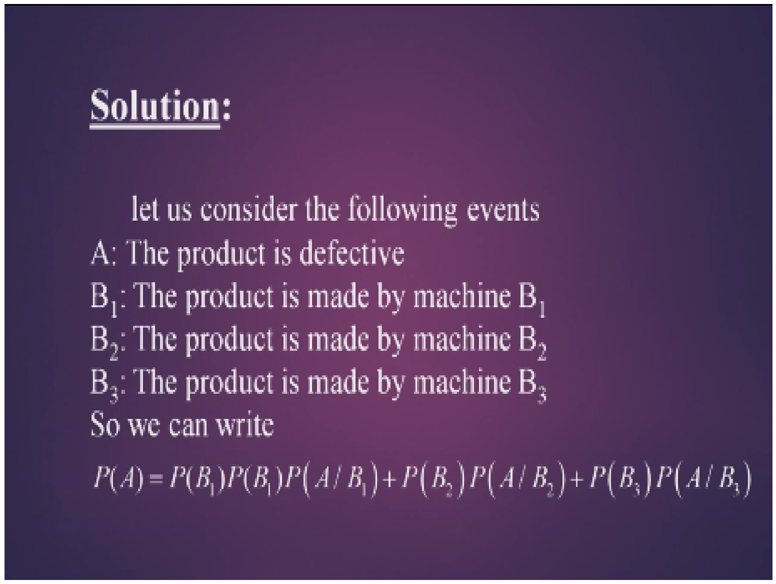

B1 is the event that the product is meant by machine B1 B2 the product is made by machine B2 B3 is the event that the product is made by machine B3, so we can write probability of a that is probability of B1 into P(A) given B1 plus P(B) 2 into P(A) given B2 plus P(B) 3 into probability of A given B3 because B1 B2 B3 these three events are mutually exclusive, so P(A) can be written as P(A).

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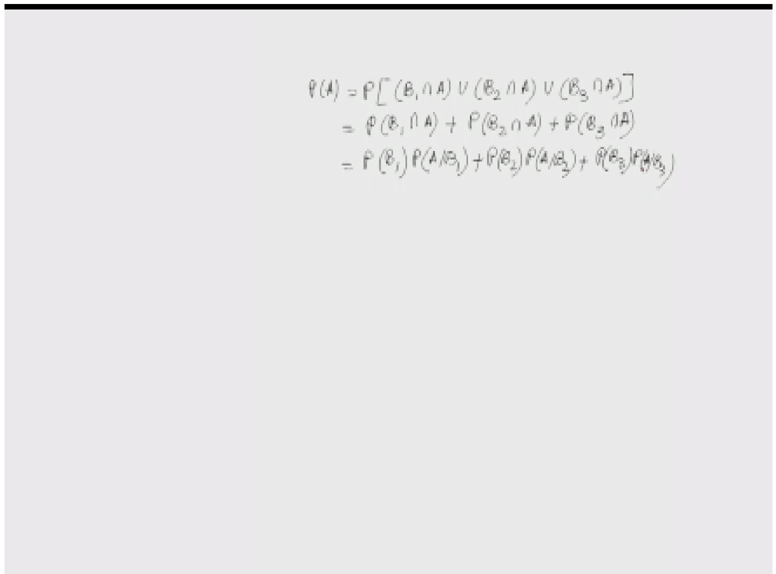

That is P(B) 1 ∩ A union B2 ∩ A union B3 ∩ A since B1 B2 and B3 are mutually exclusive B1 ∩ A B2 ∩ A and B3 ∩ A are also mutually exclusive and that is why this probability can be written as P(B) 1 ∩ A + P(B) 2 ∩ A + P(B) 3 ∩ A and this can be written as P(B) 1 into P(A) given B1 plus P(B) 2 into P(A) given B 2 + P(B) 3 into P(A) given B3, so that is what is here and this problem can be represented.

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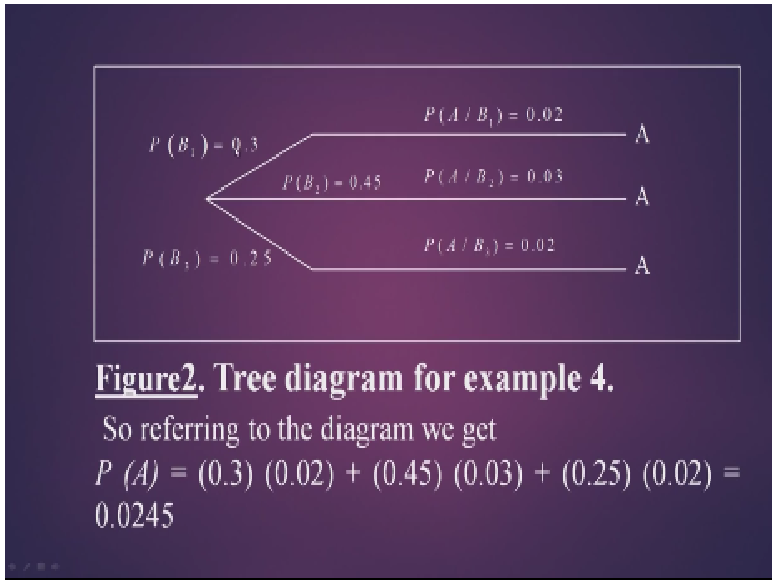

By this tree diagram also if we have this kind of diagram see P(B1) is 0.3 so it has 3 band branches P(B) 1 P(B2) P(B3) these 3 possibilities are there either B B1 will occur or b2 will occur or b3 will occur so probability of b1 now from this we can go to P(A) given B 1 which is 0.02 in this way if we go along this line we will get P(B2) that is 0.45 and from this we can go to P(A) given B2 which is 0 point 0 through 03 and along this path we have probability of B 3 which is 0.25 and from here we can go to P(A) given B3 which is 0.02.

So from each of this path we are going to a so this represents the problem here so probability of a will be 0.3 into 0.02 + 0.45 in to 0.03 + 0.25 into 0.02 and this if we calculate we will get zero 0.2 4 5, now with reference to a previous example if a product were chosen randomly and found to be defective.

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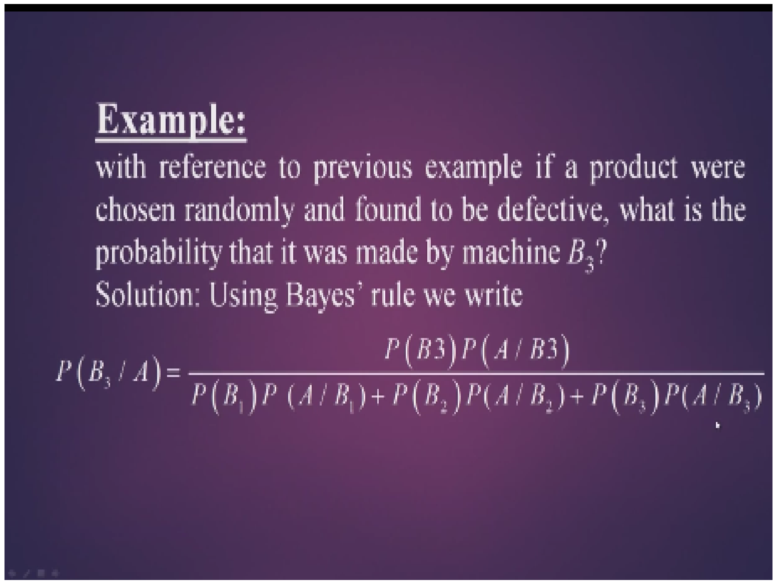

What is the probability that it was made by machine B3 so now we can apply Baye's rule to find this solution using Baye's rule we can write probability of B3 given a is equal to P(B3) into P(A) given B3 / P(B1) into P(A) given B1 + P(B) 2 into P(A) given B2 + P(B) 3 into P(A) given B3.

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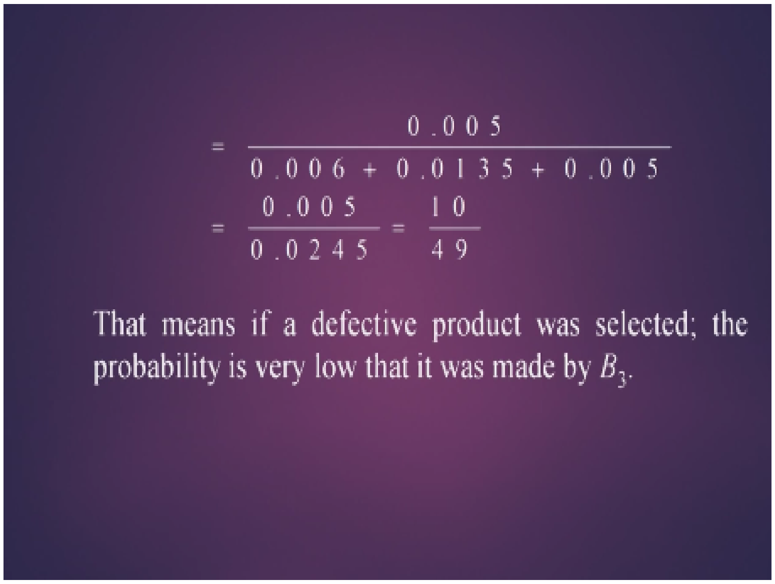

So the values are already obtained so we can substitute these values here so it will be 0.005 /0 6 + 0.0 13 5 + 0.05 which is that the denominator is 0.0245 and the numerator is 0.05, so it will be 10 / 49 so that means if a defective product was selected the probability is very low that it was made by B3 and that is all thank you.

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