Approximate Reasoning using Fuzzy Set Theory
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Lecture - 54
Monotonicity of an SBR

Hello and welcome to the last of the lectures in this week 11 of the course titled Approximate Reasoning using Fuzzy Set Theory a course offered over the NPTEL platform. In this lecture we will discuss the Monotonicity of a Similarity Based Reasoning scheme.

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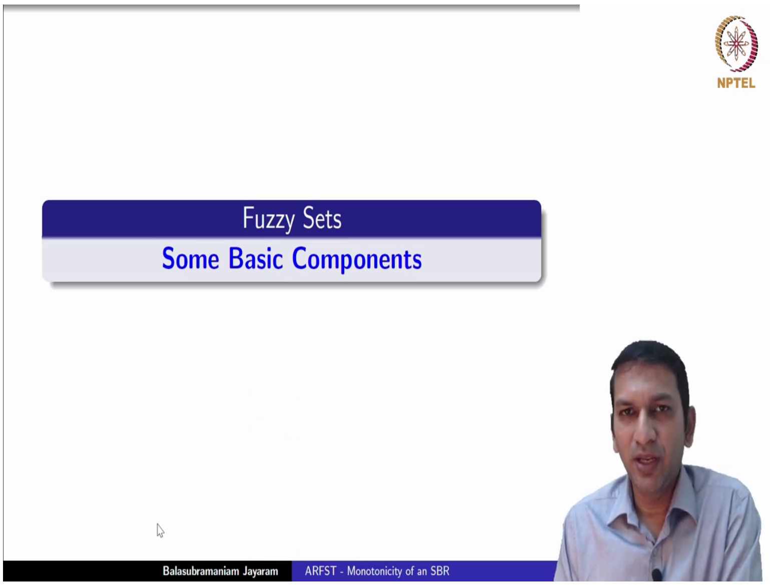

Let us begin by recalling some of the basic components of fuzzy sets that will come into play in this lecture too.

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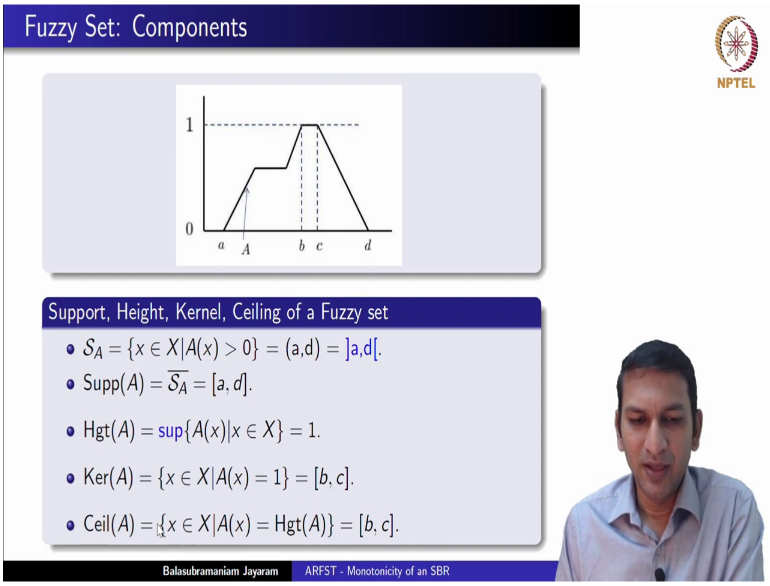

We know given a fuzzy set we could talk about it is support, height, the kernel or the ceiling of this fuzzy set..

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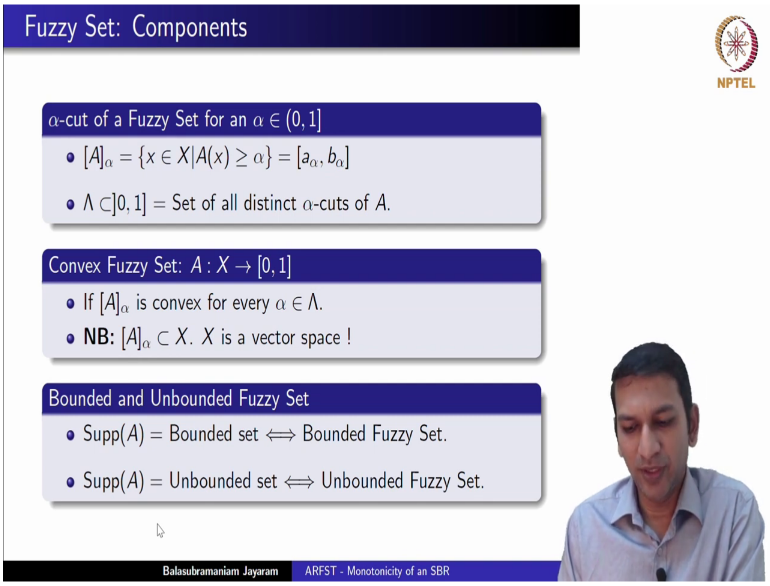

Given a fuzzy set we know what an alpha cut is, it is essentially all those elements of x the underlying domain whose membership value is greater than or equal to alpha.

A fuzzy set is said to be convex if every alpha cut of the fuzzy set is convex. Note that alpha cuts are actually subsets of the underlying domain. So, we want the underlying domain to be a space in which convexity is defined essentially we take it as a vector space.

We say a fuzzy set is bounded if its support is bounded otherwise it is unbounded.

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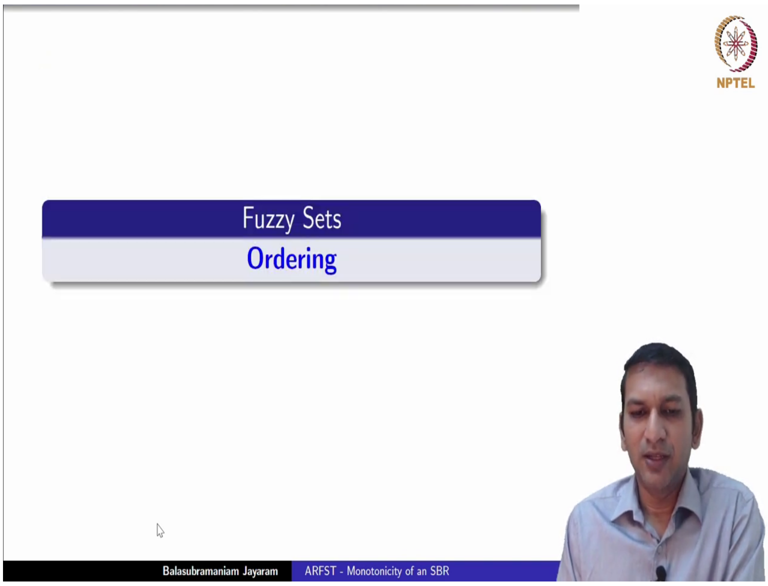

Now, we have seen two different types of orderings between fuzzy sets.

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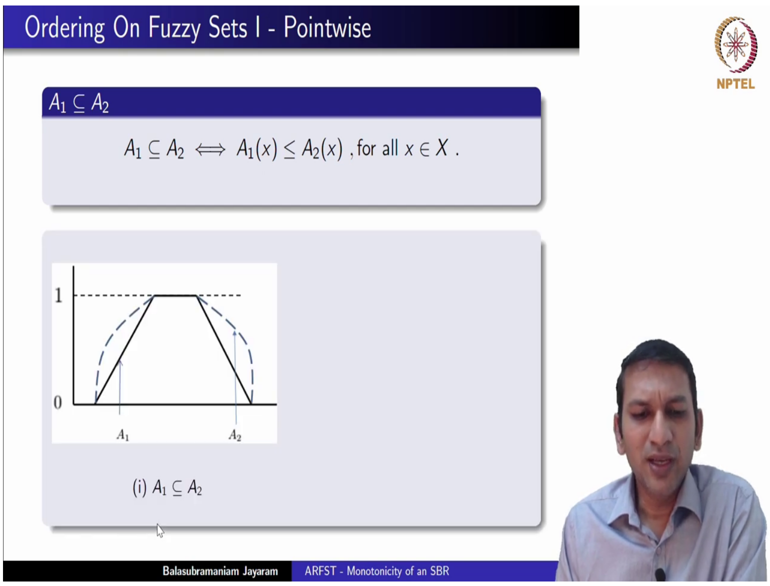

One is the point wise ordering we say that A_1 is ordered with respect to point twice ordering A_1 is contained in A_2 if and only if its membership values are smaller than the other one. So, A_1 is containing A_2 A_1 of x is less than or equal to A_2 of x for every x in X.

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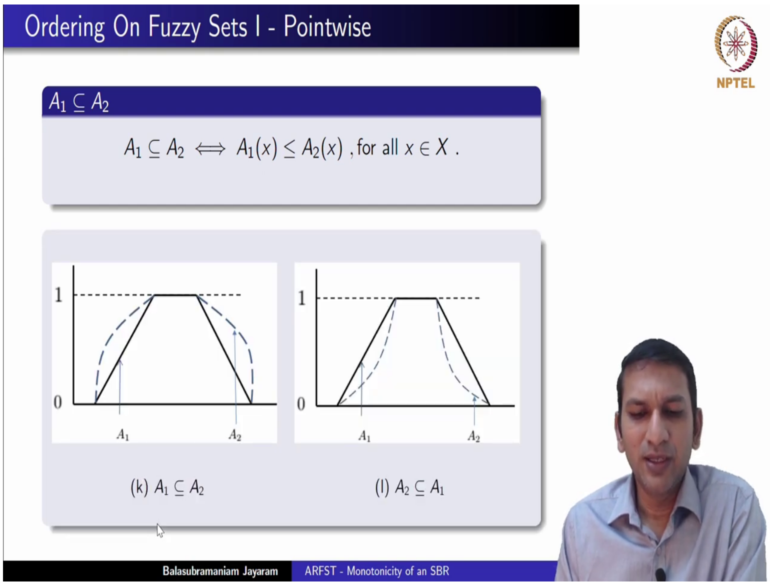

So, clearly if you look at these two pictures, we see that in one case A_1 is contained in A_2 the other case A_2 is contained in A_1.

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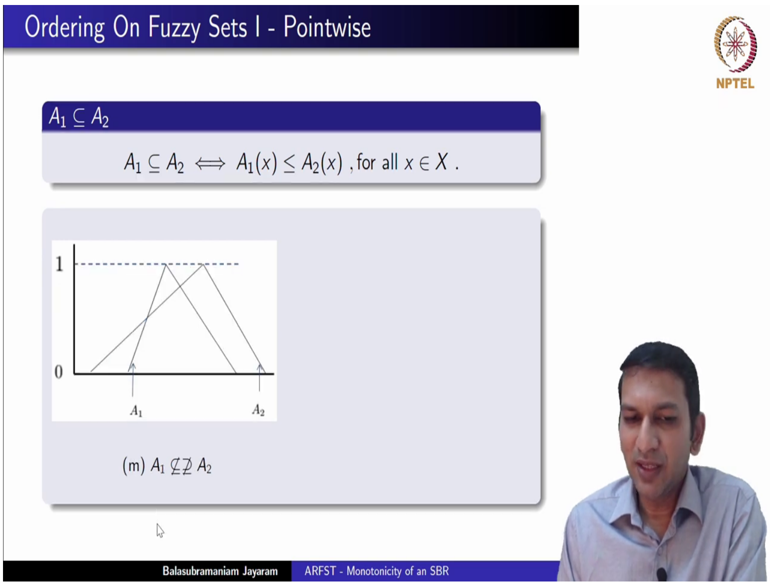

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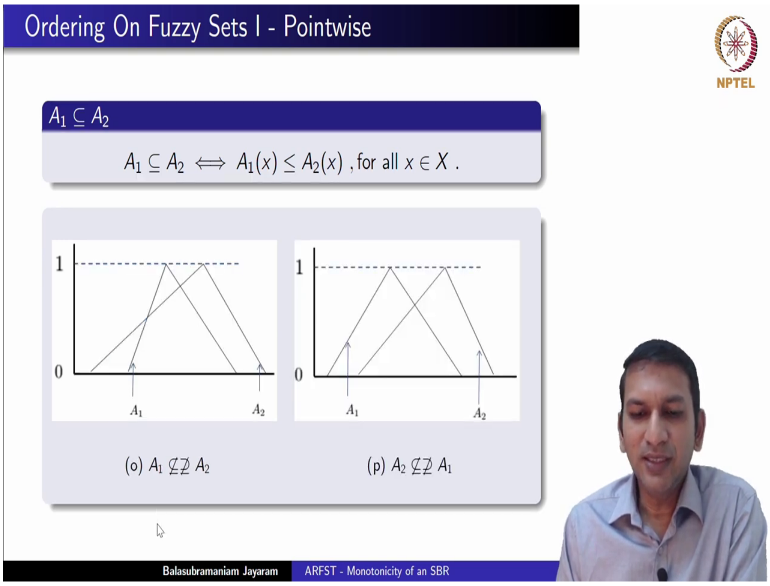

And we have seen that often these kind of fuzzy sets are not orderable with respect to this point wise order.

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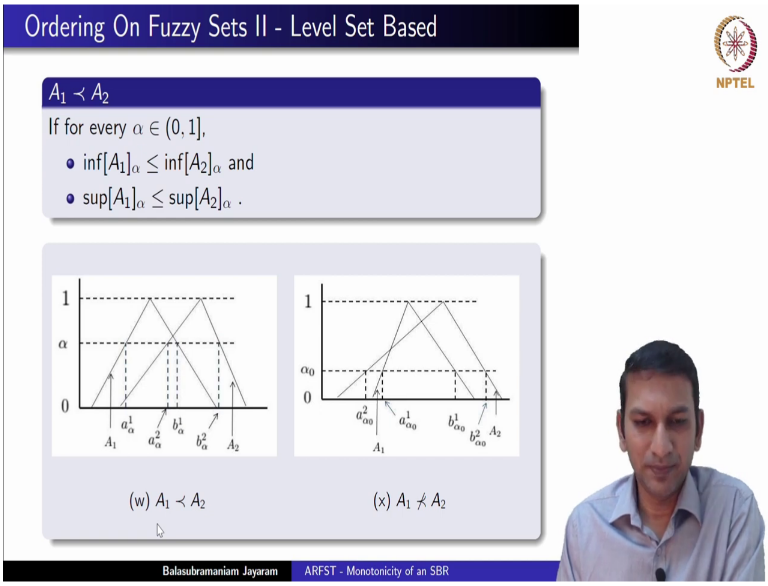

This led us to discuss level set based ordering wherein what we took was for each alpha cut we looked at the corresponding alpha cuts and we ordered them based on the intervals there. We have seen this that the set of the pair of sets on the left hand side to your screen they are orderable with respect to this level set based ordering while the ones on the right they are not.

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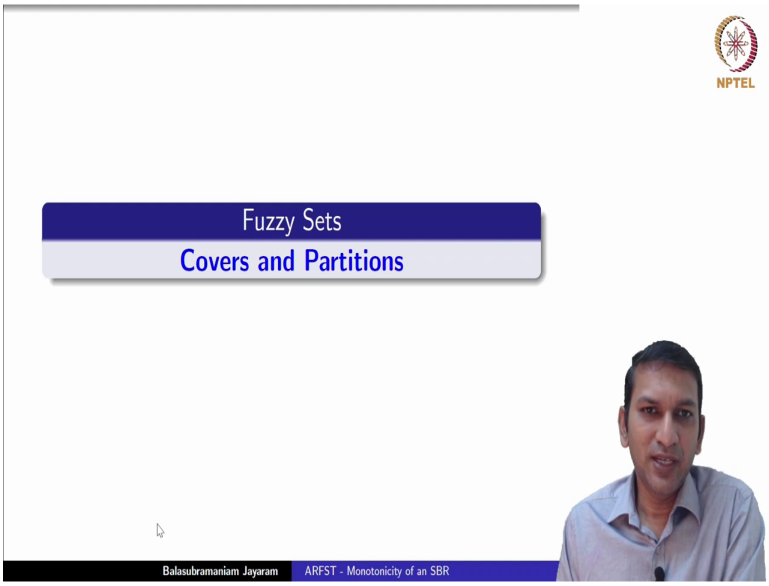

So, we know what fuzzy covers and partitions, we will introduce one more in this lecture.

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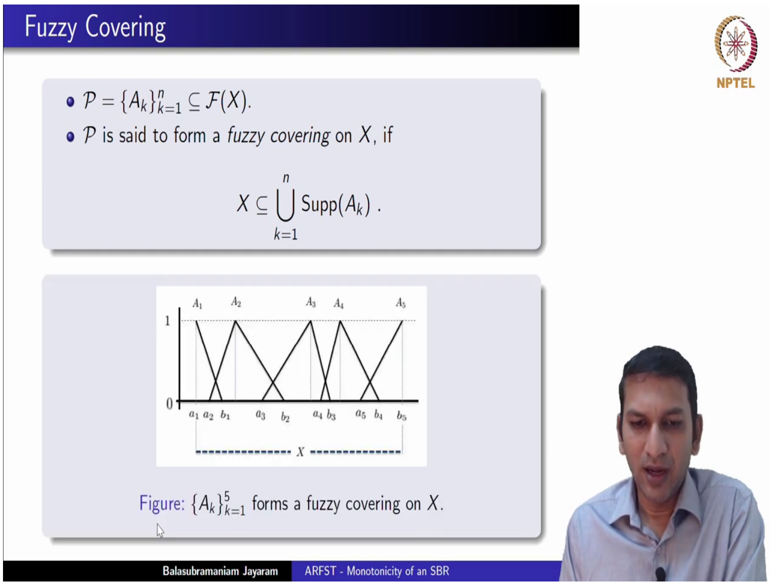

So, by a fuzzy covering we mean a collection of fuzzy sets over X such that the support the union of support of these pieces that we picked up in the set they actually contain the set etcetera.

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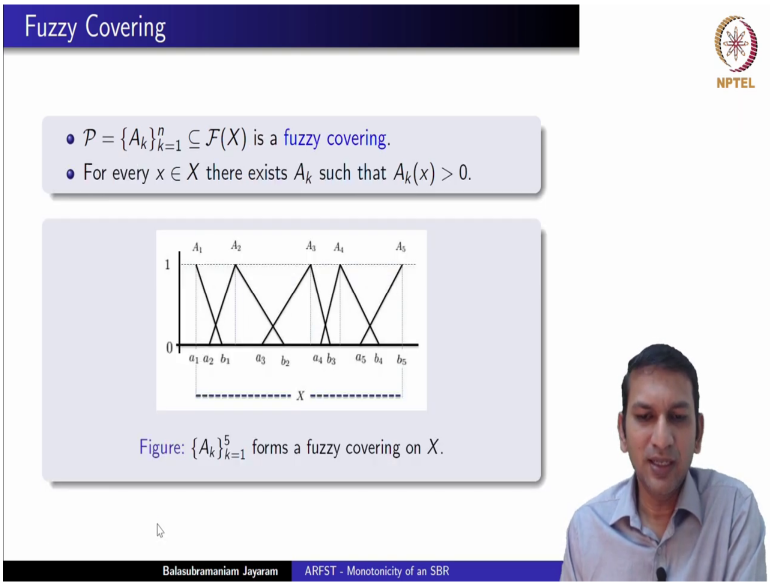

So, the union of supports of these fuzzy sets contains X another way to look at it is that for every x in X there exists sum A k in this collection to whom x belongs to some non zero membership degree, this is just a simple basic fuzzy covering.

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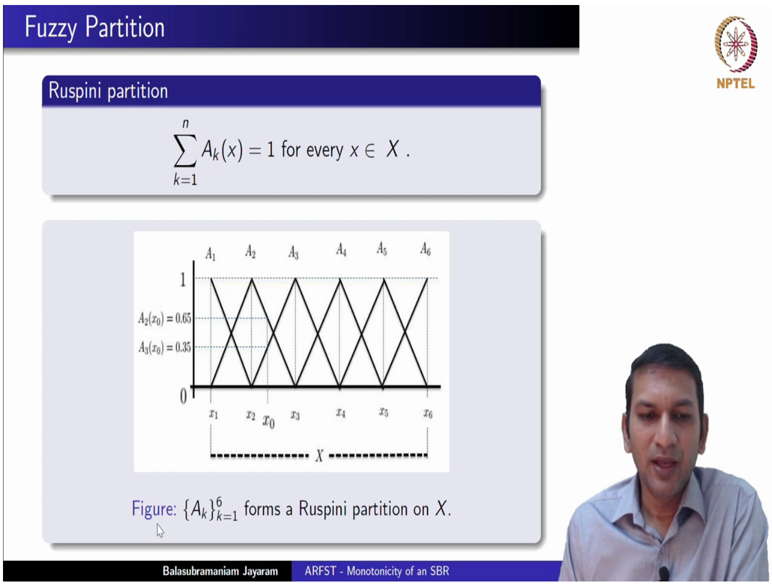

We also defined what is a fuzzy partition, one particular type of partition is called the Ruspini partition. Here what we expect is that for any x the sum of its membership values to every piece in the partition the collection it adds up to 1. So, we have seen this figure.

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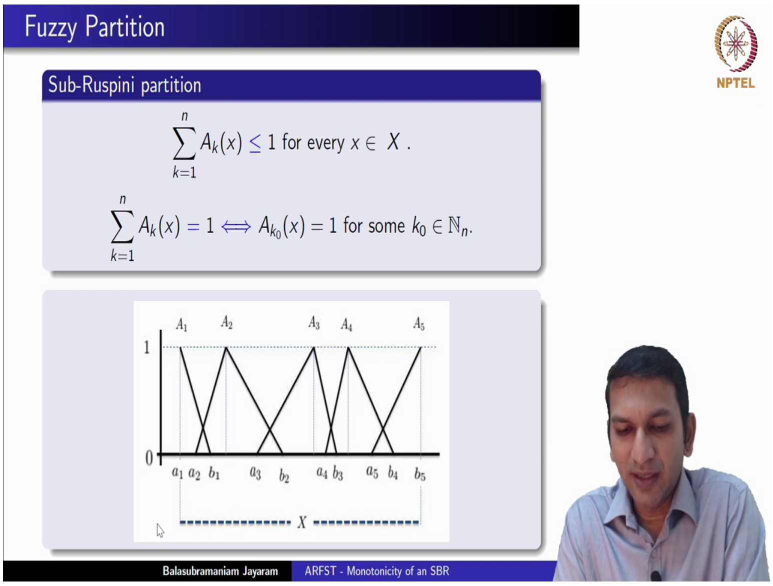

In this lecture we will introduce what is called a Sub-Ruspini partition what is this? In the Ruspini case we wanted the sigma over k A k of x is equal to 1 for every x. Here we allow it to be less than or equal to 1; that means, the sum of the membership degrees to the pieces in the partition need not added to 1. However, if it does add up to 1 then it can add up to 1 only if that x is a point of normality for some element in the partition. Now how does it look like?.

So, look at this. This is exactly the set you know collection of sets which we saw as fuzzy covering because the union of it supports contains x. However, it is also a sub Ruspini partition, clearly it is not a Ruspini partition because if you take a value here then this it belongs only to A_2 and not to degree 1 which means so, it does not satisfy the spinning condition.

However, it is a sub Ruspini partition why do you say this if you add up for any x if you add up the corresponding membership values of the sets to which it belongs of the fuzzy sets to which it belongs in the partition.

We see clearly it is less than or equal to 1 and not only that only at points of normality there are no intersection which means the sum actually is equal to 1 only at those nodes. Other points the sum does not the sum of membership degrees is not equal to 1, clearly in this case among the fuzzy sets that we see A_1, A_2, A_3, A_4, A_5 at any point of time only two of them are intersecting on the space. So, for any point x it at most belongs to two fuzzy sets and clearly you see that some of those membership degrees is not equal to 1. Where it is equal to

1, is only at these points of normality. So, it is clearly sub Ruspini partition we will see that these kind of partitions are useful when we want to discuss the monotonicity of an SBR scheme.

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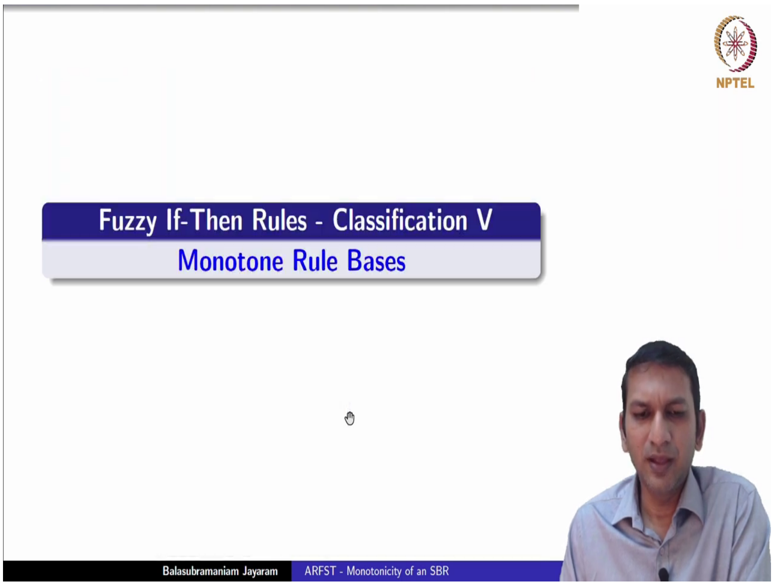

Well, we have seen another type of classification of rule bases.

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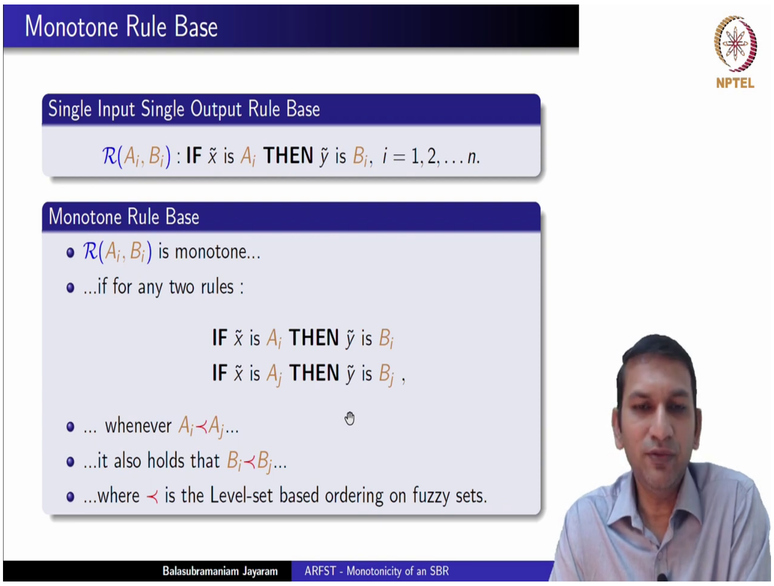

We say that given a single input single output rule base, N of them we say it is monotone if you pick any two rules A i implies B i and A j implies B j we say it is monotone if whenever

A i is ordered A i is less than A j then it should also hold that B i is less than B j where this is the level set based ordering on fuzzy sets.

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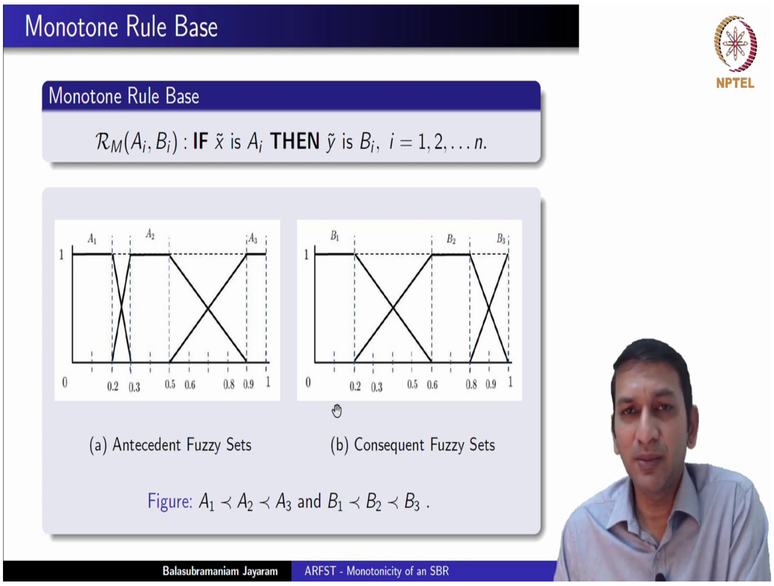

Now, if you consider these three fuzzy sets A_1 A_2 A_3 and B_1 B_2 B_3. They are clearly orderable with respect to the level set based ordering alpha cut based ordering. And so, if you take them as fuzzy sets on x and these are fuzzy sets on y then we could consider them as antecedents and them as the corresponding consequence and if you make such three rules then it will be a monotone rule base. Because clearly A_1 is less than A_2 is less than A_3 similarly B_1 is less than B_2 less than B_3 it is again with respect to the alpha cut based order.

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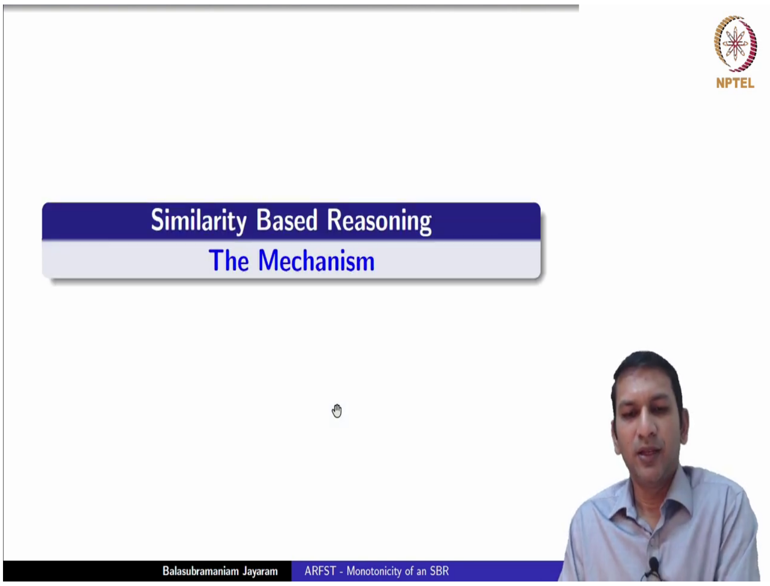

Now, we want to discuss a similarity based reasoning, a quick recap of the mechanism what do we have once again we have a SISO rule base.

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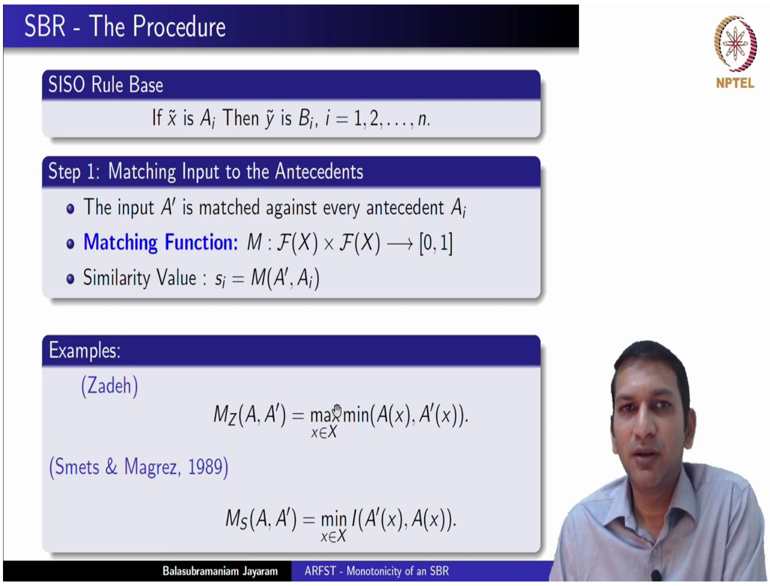

We are given an A dash which is a fuzzy set on x what we do is we use a matching function to match this A dash to each of the antecedents A i and obtain similarity values s i s 1 s 2 so on to s n and these are the two important or commonly used matching functions in the literature.

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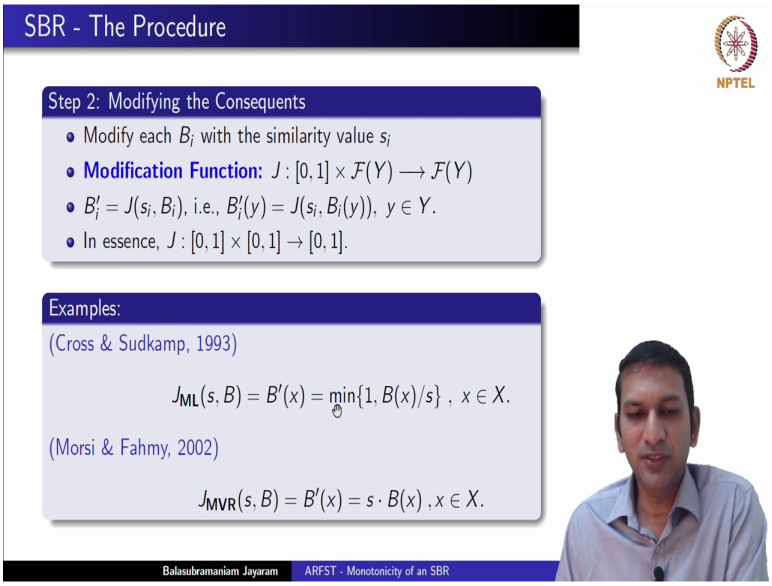

The next step is we take each of these similarity values s i and modify the corresponding consequence B i. And towards this we use a modification function chain and we have seen that even though J is from 0 over cross F of Y implies F of Y because B i is are fuzzy sets of y these are modified into fuzzy sets again on y. So, it is a J is a function from 0 1 cross F of Y to F of Y.

Essentially, since we are dealing only on the membership functions working with the membership values it is clear that J can also be considered as simply a binary operation on 0, 1 to 0, 1 these are some often used modification functions.

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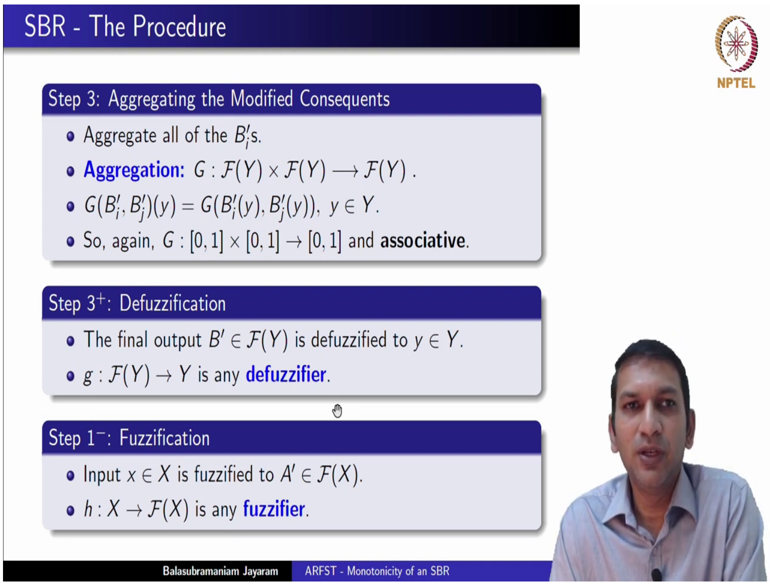

In the third step we take all these modified consequence B_1 dash, B_2 dash so on to B n dash and we aggregate them. Once again it is like collection of fuzzy sets on Y we aggregate them to obtain a fuzzy set on Y. However, we have seen essentially we are only playing or operating on the membership values. So, G can also be thought of as simply a binary function on 0, 1 to 0, 1 and if it is associative it can easily be extended to NRA to an NRA function.

We have seen that often we there is a need to defuzzified not limited as the obtained output fuzzy set, but defuzzified; that means, obtain a value y on the underlying domain Y. Similarly, often we are given only the x from the domain input domain X we need to suitably fuzzify it to a dash. So, that it can be fed into a fuzzy inference system. So, these are two operations defuzzification operation and a fuzzification operation. So, totally there are these five steps right.

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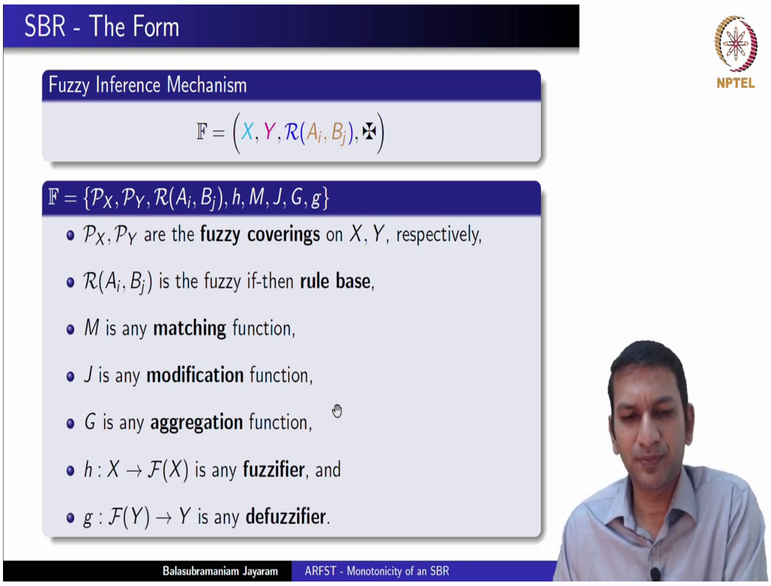

In terms of the form we know that there are these many parameters or degrees of freedom that we have P x and P y are the coverings on fuzzy coverings on X and Y this they give rise to the rule base. Where the A i is come from P x and B i is come from P y they form the fuzzy if then rule base, M is any matching function, J is any modification function, G is an aggregation function, h is the fuzzifier and g denotes the defuzzifier.

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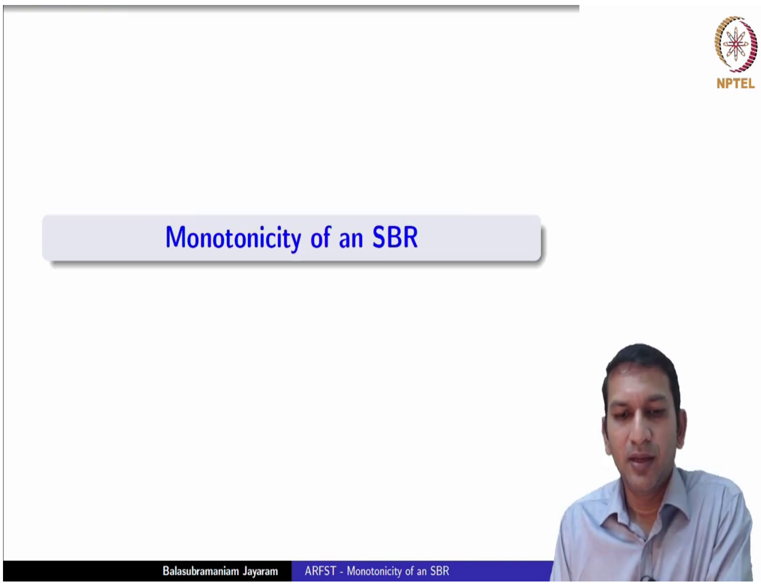

Let us now discuss the monotonicity of an SBR inference scheme.

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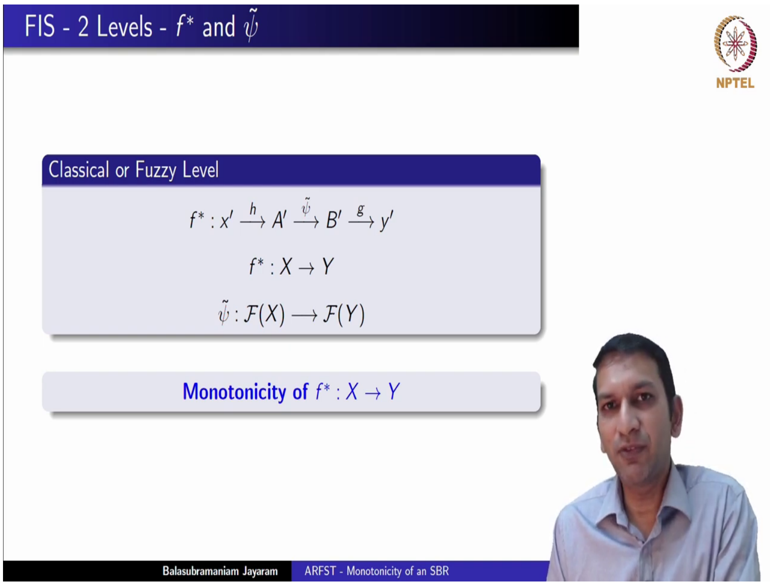

We have seen that a fuzzy inference system can be discussed at two levels one at the classical level as a function from X to Y. If you consider f star to be the system function of an f i s given an x dash from x we could apply a fuzzifier a fuzzification process obtain an A dash and map it using the psi tilde which is essentially a function from F of X to F of Y. So, we obtain a B dash and then defuzzifier using a defuzzifier g down to the output domain y.

So, f star could be looked at as a mapping from X to Y of course, psi tilde is a mapping from F of X to F of Y, once again we will discuss monotonicity of SBR inference scheme also only at the level of classical sets; that means, looking at f star as a mapping from X to Y.

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Now, we have seen in the case of discussing SBR inference schemes with respect to their interpolativity or continuity or robustness that it almost always entitled that some conditions or restrictions should be put on the matching function. In that context we have introduced a few consistency properties on matching function M, we will introduce a few more in this in our quest towards discussing monotonicity of an SBR.

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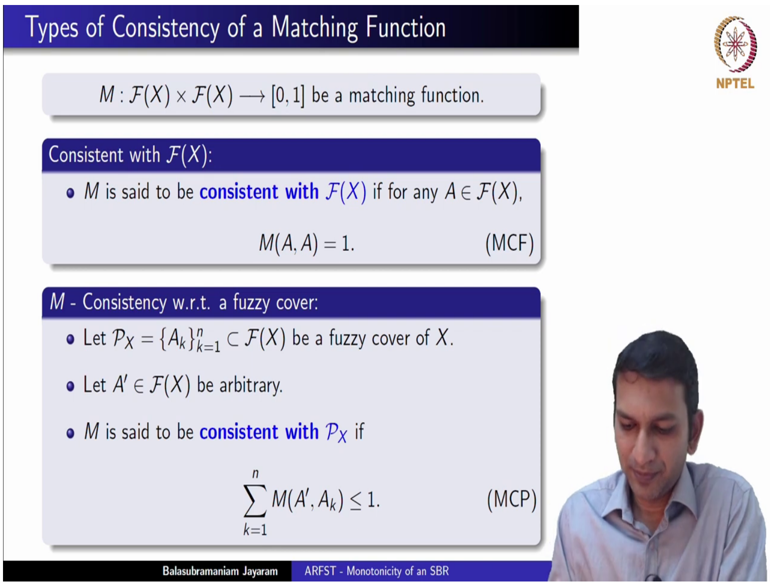

Given an M which is a function from F of X cross F of X to 0, 1 we say it is consistent with the set of fuzzy set that we are considering if it simply ensures that for identical inputs if a pair of identical inputs is given then it the similarity value should be 1; that means, M of A, A should be equal to 1.

Given a fuzzy covering P of x P x which is essentially a collection of fuzzy sets on x which covers the domain X; that means, the union of supports of these fuzzy sets contains X, we say M is consistent with respect to this P x if for any A dash that we get from F of X this inequality is held. What does it say when we compare A dash with A k for each of the pieces in the partition the fuzzy covering that we considered. The sum of the matching values the similarity values should not exceed that of 1.

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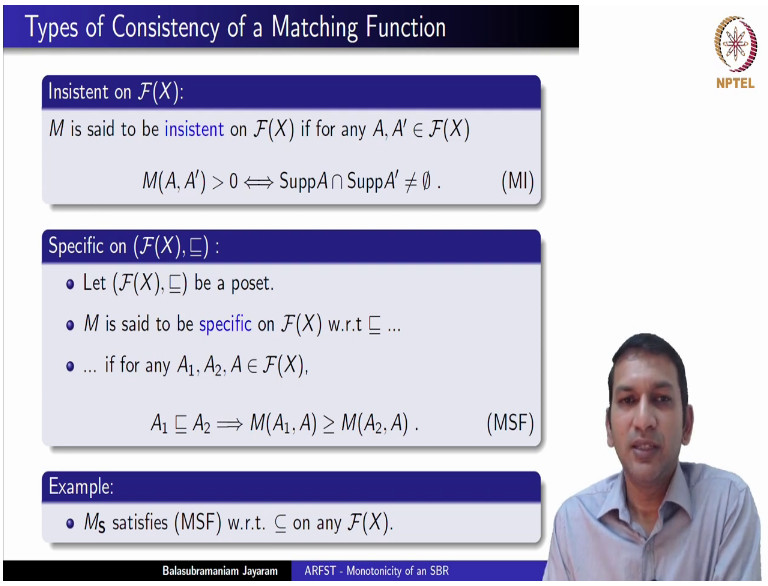

This another condition that we could insist on M which is called insistent, insistence condition on M is said to be insistent on F of X if for any A, A dash coming from F of X the similarity value between A, A dash is greater than 0 only if the supports of A and A dash actually intersect.

Whenever they intersect it should be greater than 0 and if it is great the similarity value of A and A dash is greater than 0 as calculated by the matching function M then it should imply that their supports corresponding supports are in fact, intersecting.

Given F of X we could consider many orders on it. So, we say a matching function M is specific with respect to the ordering that we consider F of X to begin with we consider a poset we know that on F of X a suitable subclasses of F of X also you can consider many orderings which will make it a partially ordered set we say M is set specific on F of X with respect to the order that we consider.

If for any three fuzzy sets A_1, A_2, A that will take whenever A_1 is smaller than A_2 with respect to the ordering that we consider, then M of A_1, A should be greater than or equal to M of A_2 A. So, here we are ensuring that the more specific asset is the more similar it will be to a given fuzzy setting. So, somehow it retains or preserves specificity.

As an example if you consider this (Refer Time: 14:21) matching function it can be seen that in fact, it is it satisfies this MSF with respect to the point wise ordering or any F of X that you can consider.

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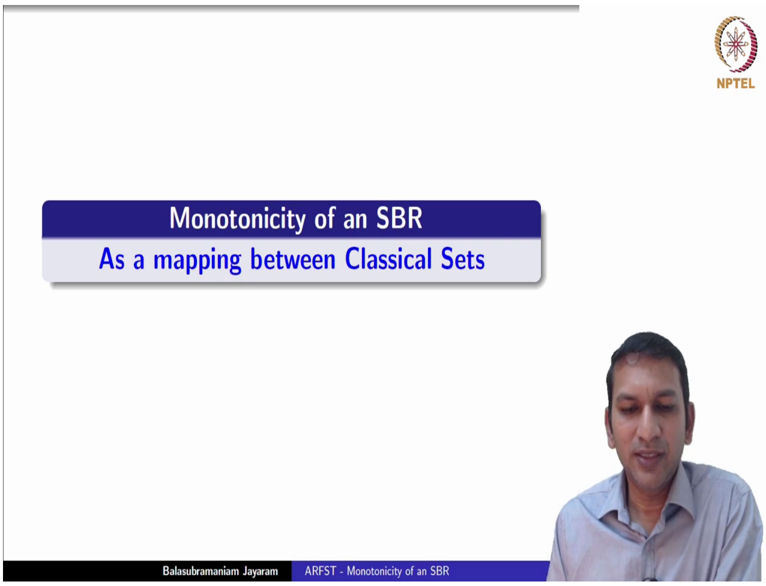

Now, let us come to discussing the monotonicity of an SBR as a mapping between classical cells.

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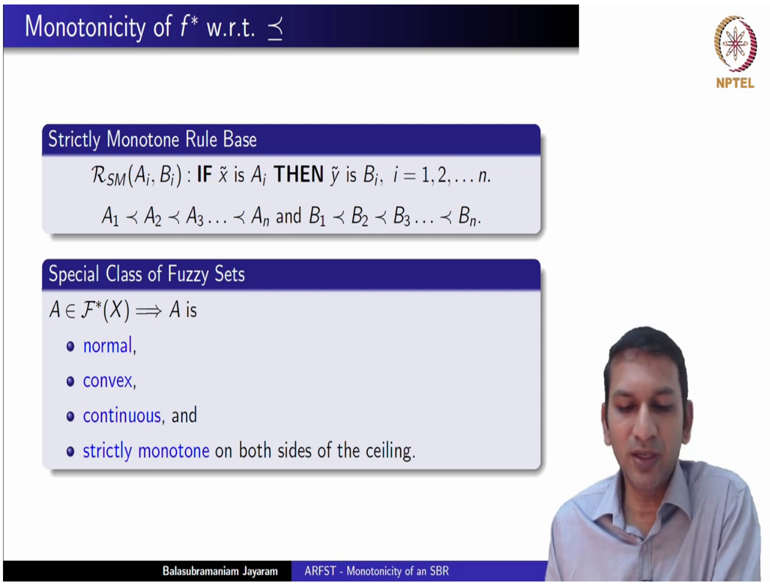

So; that means, we are looking at f star and discussing f star the monotonicity of it with respect to the alpha cut based order. So, once again we start with a strictly monotone rule base; that means, we have the antecedents ordered in a particular way with respect to the alpha cut based ordering and also the corresponding consequence are ordered in the same fashion.

We consider a special class of fuzzy sets which we denote as F star of X and A belongs here means A is normal, convex, continuous and it is strictly monotone on both sides of the ceiling and since it is normal; that means, it is strictly monotone on both sides of the corner well.

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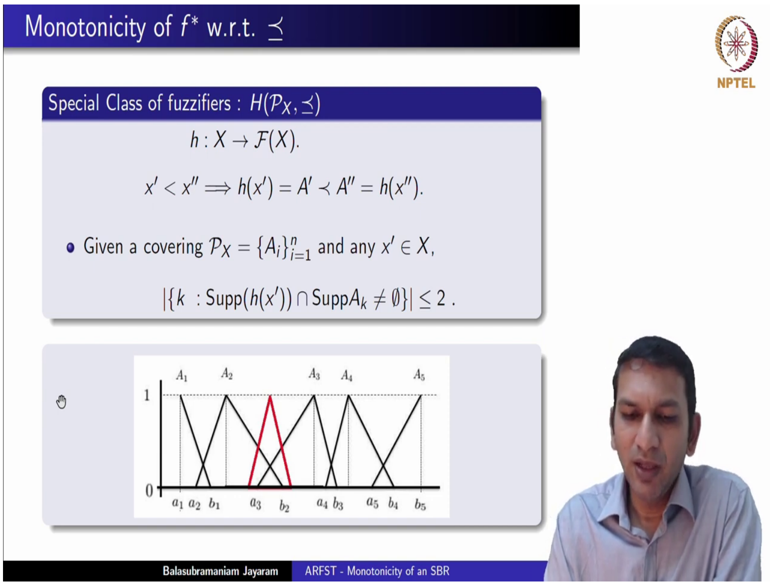

We also want to introduce a special class of fuzzifiers. We know a fuzzifier h is a mapping from X to F of X, we want that this fuzzifier satisfies the following two properties whenever x dash is smaller than x double dash then the fuzzified output that we get from x dash which let us call it A dash this should be orderable with respect to the alpha cut based ordering with A double dash which is the fuzzified output that we get for x double dash.

So, A dash and A double dash are the fuzzifierd versions of x dash and x double dash if x dash is smaller than x double dash then we want that the corresponding fuzzified versions A dash and A double dash should be orderable with respect to the level set based ordering of course, with respect within the same ordering fashion. We should preserve the monotonicity of the ordering.

And if this P x is an covering that is given to us let it consist of n such fuzzy sets we say that this h with respect to this P x also satisfies another property. So, take an x dash and construct it is fuzzifier set A dash what we want is that given this covering look at this formula it says that it can intersect some fuzzy sets in this collection P x. But intersect in the sense of the support of this A dash h of x dash it intersects at most two fuzzy sets the supports of two fuzzy sets in the collection here.

So, essentially it says that given a covering which is collection of fuzzy sets and an x dash the fuzzified version of x dash should utmost intersect with two fuzzy sets from this covering from this collection. That means the support of the fuzzified set should intersect at most support of two fuzzy sets in this collection.

Let us look at an example we have seen this as a collection of 5 fuzzy sets as just a fuzzy covering we see clearly it is a sub Ruspini partition that is why we have seen this collection of fuzzy sets. And we can come up with a fuzzification such that the fuzzified set intersects at most 2 from this collection for instance if we consider this as if x dash then this is one fuzzification which is also normal which intersects at most two such fuzzy sets in this partition it is not really very difficult.

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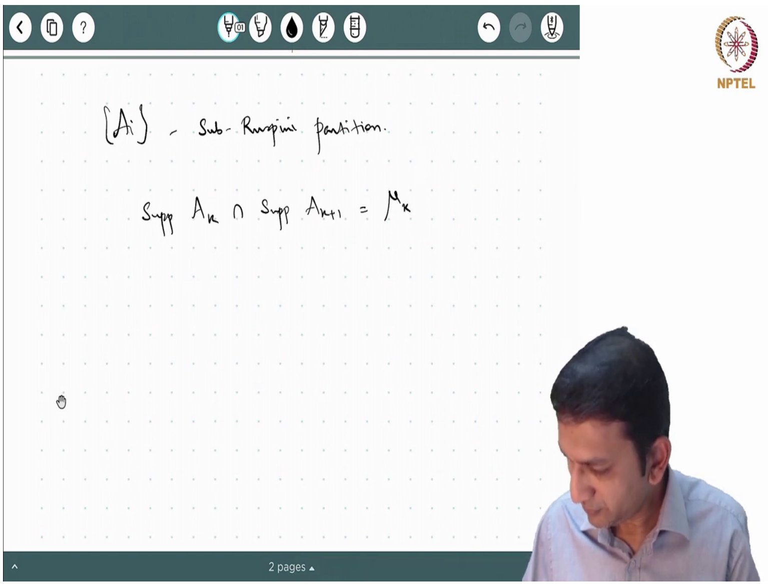

For instance, if you are given the A ''s which form a sub Ruspini partition. So, now, if we consider support of we know that because these are normal also we know that support of A k can only intersect support of A k plus 1 and A k minus 1. So, now, let us consider this to be mu k.

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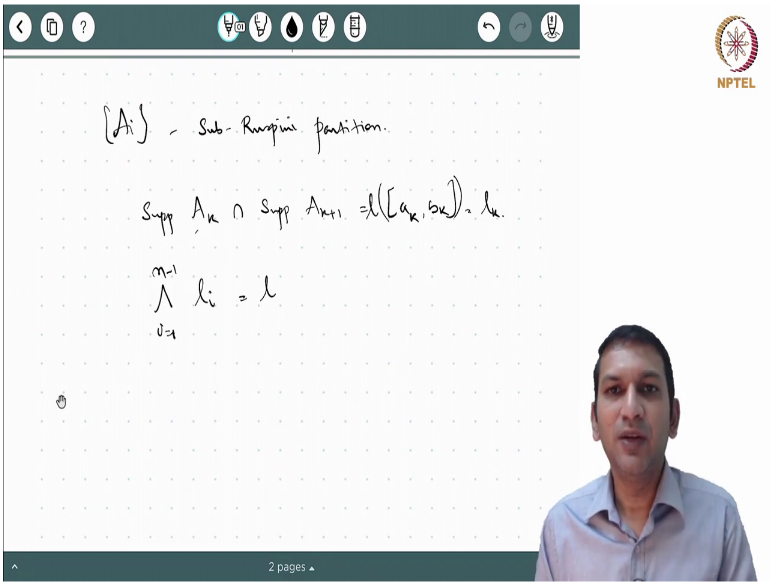

Some set or for the moment let us just take this to be a k here a k b k (Refer Time: 19:17) some b k. So, essentially for every one of them you are going to get this, all we need to do is find the length of this let us call it l k. Now, infinite more i is equal to 1 to n of l i's if you consider this n minus 1 perhaps plus will go to that, if you take this as the corresponding l then if you ensure that the fuzzifier does not fuzzify a given point to a fuzzy set whose support is bigger than this then such a fuzzifier can be considered.

So, given the partition based on the partition or the fuzzy covering P x we could always come up with such fuzzifier such fuzzifier do exist.

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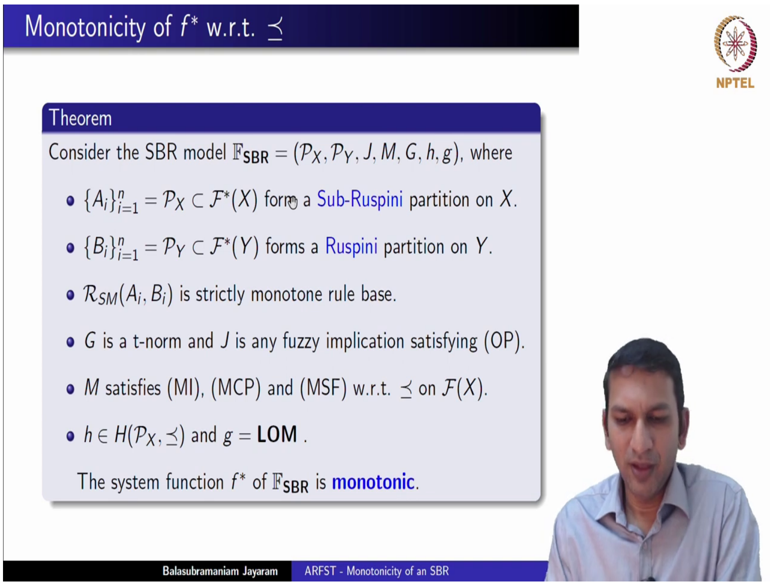

Now, we are in a position to state the main result of this lecture which deals with monotonicity of f star with respect to the alpha cut based order.

So, let us consider such a model of SBR where A i's form the of from the antecedents they form P x and they fallows sub Ruspini partition on X. Note that these are all coming from F star of X; that means, each A i is normal convex continuous and it is strictly monotone on either side of the kernel.

The B ''s are the consequence they form P y. However they form a Ruspini partition on Y and the once again they actually come from F star of Y; that means, on that domain these are essentially normal convex continuous and strictly monotone on either side of the kernel and we have these consequence and antecedents they are orderable and with respect to the alpha cut based ordering. And they actually form a strictly monotone rule base.

Now, let us start specifying what the other functions are J, M, G, h, and g. We take G to be a t-norm and J is any fuzzy implication satisfying OP. M satisfies MI; that means, it is insistent on F of X it satisfies MCP; that means, it is with respect to P x it is essentially insisting on being consistent. And it also has specificity with respect to the alpha cut ordering on overall on F of X.

Now, we pick a fuzzifier h such that it has those two properties that we saw before that is if given x dash is smaller than x double dash we know that the corresponding fuzzified outputs

A dash and A double dash they are orderable accordingly with respect to the alpha cut based ordering. And also when we take P of x and any such fuzzified output A dash intersects at most two fuzzy sets from this connection.

And for the defuzzifier g we take the least of maximum. So, essentially if the output fuzzy set B dash is normal; that means, it has kernel then it is a list of the kernel the smallest or the point at which the output B dash takes us first point of normality. Well, if you consider this as the operations and has the collection of fuzzy sets with these properties then it can be shown the system function f star of F SBR the model that we are considering is in fact, monotonic.

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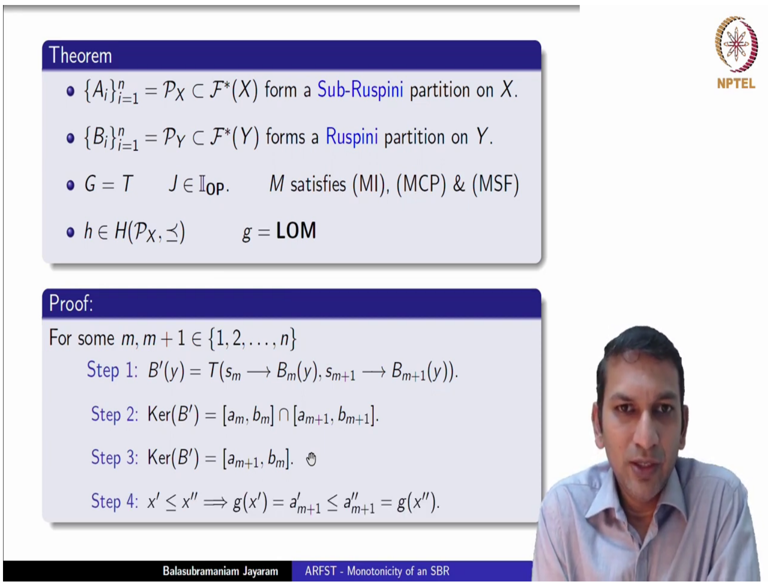

Once again we will keep the important parts on the screen. So, that A i is from a Sub-Ruspini partition on X, B i is actually form a Ruspini partition on X. G is a t-norm, J comes from fuzzy implication that satisfied OP and M satisfies these three properties insistence consistent with respect to P x and also a specific on the overall space of fuzzy sets.

And h is a fuzzifier which has those two properties comes from the special class of fuzzifiers and g is the least of maximum defuzzifier.

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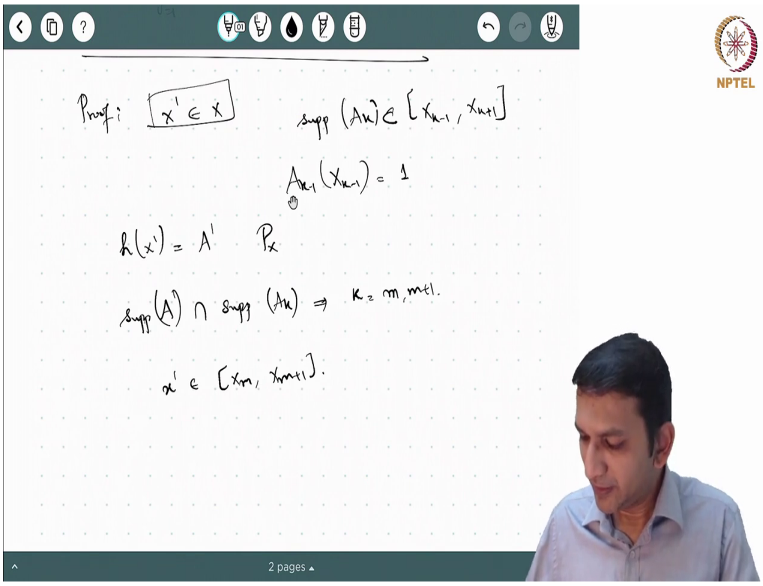

Yeah, let us look at the proof of this to begin with let us assume that we have x dash coming from x and now let us look at support of an A k as the interval x k minus 1 comma x has been contained in this because remember we are considering a sub Ruspini partition if x k minus 1 is the point of normality of A k minus 1 that is our usual notation convention.

So; that means, support of A k will be contained in this perhaps strictly contained this should be set. So, that they do not intercept. So, clearly the supports do not intersect because you are looking at supports do intersect, but they will not overlap with the closure will not intersect with the kernel of another fuzzy set.

Well, so, now given this x dash we are going to apply h of x dash to obtain an A dash note that given this P x we know that A dash the support of A dash if it intersects with support of A k then this implies that there are only two fellows k is equal to some m, m plus 1 m m plus 1 now once again we are given x dash element of x clearly it has to fall in one of these intervals.

So, let us assume that x dash in fact, falls in this interval X m X m plus 1, as in the case that we discussed when we discussed the proof of monotonicity of an FRI let us assume that x dash belongs to X m X m plus 1. Now, the first step here proof proceeds more or less along the lines of proving the monotonicity of an FRI, but of course, there are few differences.

In this case we specifically consider a sub Ruspini because if you had Ruspini then such a fuzzifier is not possible because at the points of normality you cannot have a fuzzified output which only intersects two fuzzy sets in the partition P x. Hence, we consider a sub Ruspini partition, further if you consider responding partition and a singleton fuzzification, then more or less it essentially goes down to the case of discussing an FRI is it a CRI or BKS depending on the J that we consider.

Hence, just to ensure there is some amount of generality we are discussing a situation a context a setting where we have a sub Ruspini partition on A i's and we are considering a different kind of fuzzifier, of course h can also be a singleton fazzification because then it will not intersect more than two position sets in this context, because the P x is found in that way it has it is a sub Ruspini partition and each of the fuzzy sets is normal, convex, continuous and strictly monotone on either side of the curve.

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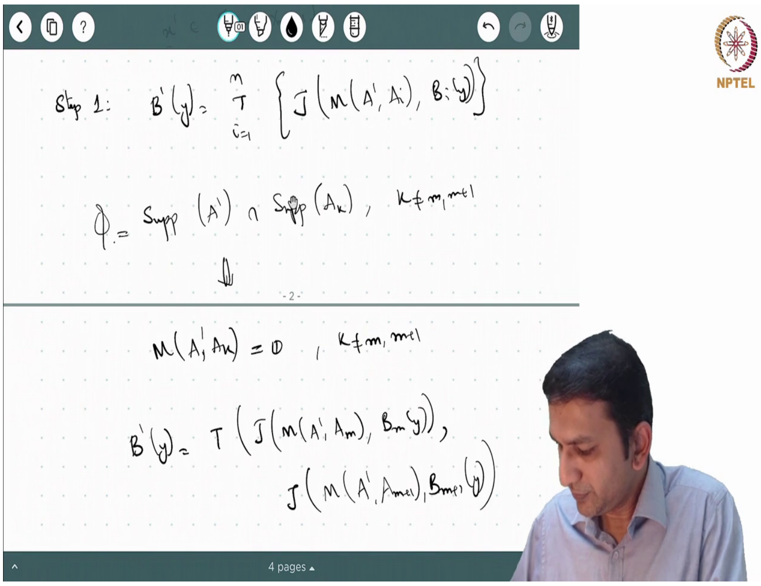

So, the first step is to show the output B dash is essentially this. So, we know that B dash of y is in fact, G i is equal to 1 to n. Now, G is T for us J is an implication M of A dash comma A i comma B i of y. Note that by an assumption on h A dash intersects only two of them let us assume since the x dash comes from X M X m plus 1 it is clear.

Then A dash when we fuzzify it when this x dash is fuzzify it to A dash it is going to intersect only the fuzzy sets k m m plus 1. Now; that means, support of A dash intersection support of A k when k is not equal to m, m plus 1 this is in fact, equal to phi.

Which means this implies M of A dash A k for k not equal to m comma m plus 1 this is equal to 0 because we have taken an M which is insistent on F of X; that means, unless the supports intersect the matching values are going to be 0. So, in essence what we have now is this B dash of y reduces to only considering T of J of M A dash A m comma B m of y and J of M A dash A m plus 1 comma B m plus 1 of y.

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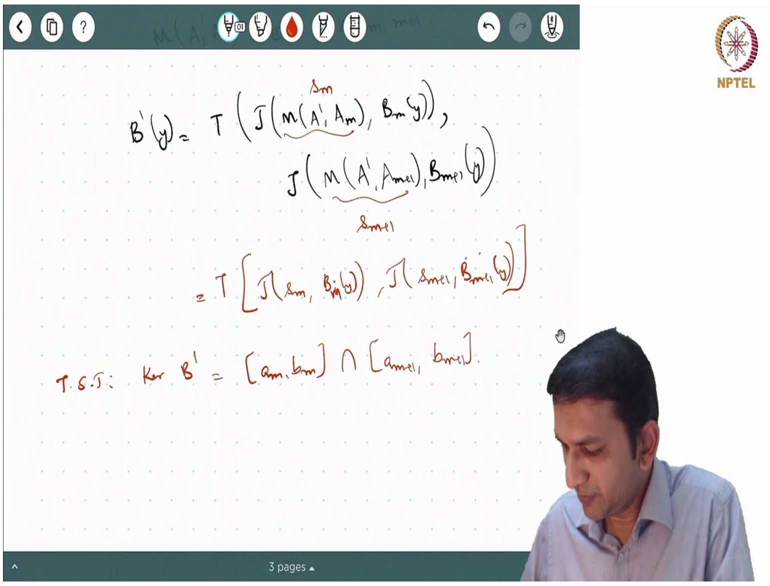

Once again this is the similarity value let us write it as s m and this similarity value let us write it as s m plus 1. So, what we are left with is B of J of s m comma B i of y comma J of s m plus 1 comma B i y B m B m plus 1 of y. So, this is what we have here in terms of J which is an implication written in terms of they have.

Now, the next step is to find the kernel. So, does this B dash have a kernal is this B dash normal, once again let us work this out (Refer Time :29:57) that kernel of B dash is in fact, that exists some values a m b m a m plus 1 b m plus 1 that is happens.

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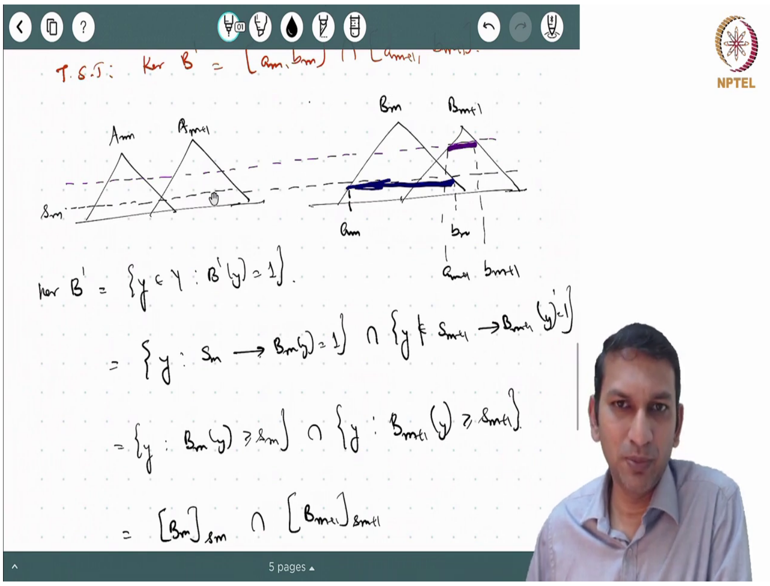

This what we need to show, note that as in the case that we have seen we have a ''s which do not form a Ruspini partition just for illustration purposes let us take triangular fuzzy sets we know that B m and B m plus 1 they come from P y which does form a Ruspini partition on y over Y.

So, if we take s m now let us take to the this to be s m and let us take this to be s m plus 1 we see that once again we can workout now, what we want is the kernel of B dash now this is clearly set of all y element of y such that B dash of y is equal to 1. Once again T is a T norm; that means, both of these should be equal to 1 is nothing but set up for y such that s m implies B m of y is equal to 1 and the other one intersection y such that s m plus 1 implies b m plus 1 of y is equal to 1.

We know that implication J that we are considering s an OP implication; that means, this is set of all y such that B m of y is greater than or equal to s m intersection set of all y such that B m plus 1 of y is greater than or equal to s m plus 1. So, essentially what we are looking at is B m the alpha cut at s m intersection the alpha cut at B m plus 1 value s m plus 1, once again let us color this. So, that is s m plus 1 then we are looking at this step we are at (Refer Time: 32:31) this is let us (Refer Time: 32:40).

So, clearly as in the previous case with respect to a m this to be b m and this to be a m plus 1 and this to be b m plus 1. So, clearly because of continuity of B ''s we know that if it is continuous strictly monotone it is on to there on the domain which means for every value of s m and s m plus 1 there will exist some value on y which you are calling it as a m b m name plus 1 b m plus 1 such that these happen.

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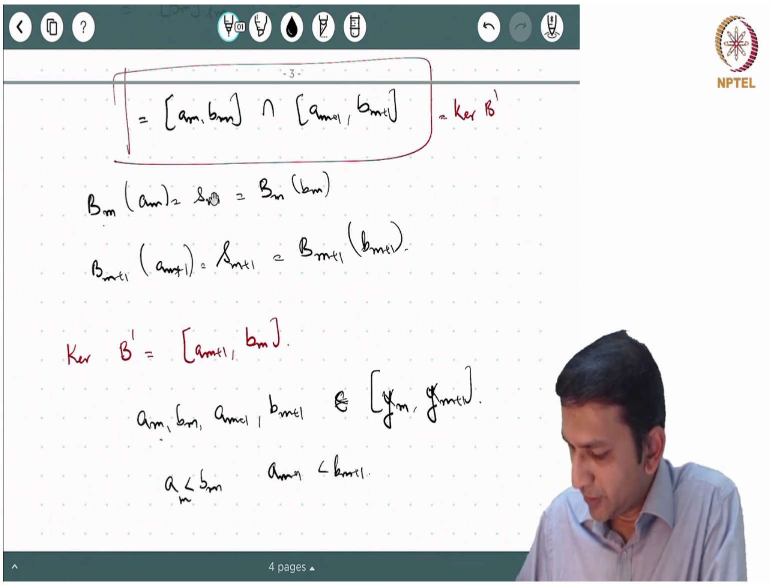

So, this is essentially a m comma b m intersection a m plus 1 comma b m plus 1, note that for complete for just to be more specific B m of a m is equal to s m this also equal to B m of b m and B m plus 1 of a m plus 1 is actually s m plus 1 which is also equal to B m plus 1 b m plus 1.

So, we have obtained what we wanted to get the kernel of t is in fact, intersection of these two. Of course, we need to show that this kernel is in fact, non-empty; that means, there is some intersection available that is a third step. Let us show the kernel of B dash is in fact, a m plus 1 comma b m.

Now, first thing notice ok that we are talking about a m and b m being acted upon by b m and a m plus 1 and b m plus 1 being acted upon by b m plus 1; that means, all of these a m, b m, a m plus 1 and b m plus 1 they actually belong in x m x m plus 1. So, this much is here y m y m plus 1 because we are looking at fuzzy sets on the y.

Now, we will so; that means, it is clear that a m is smaller than b m and a m plus 1 is smaller than b m plus 1. If we can show that a m plus 1 is smaller than b m then clearly because these are intervals we are looking at convex fuzzy sets. So, that is why every alpha cut is an interval on r because that is what we are considering as the domain x is subset of r.

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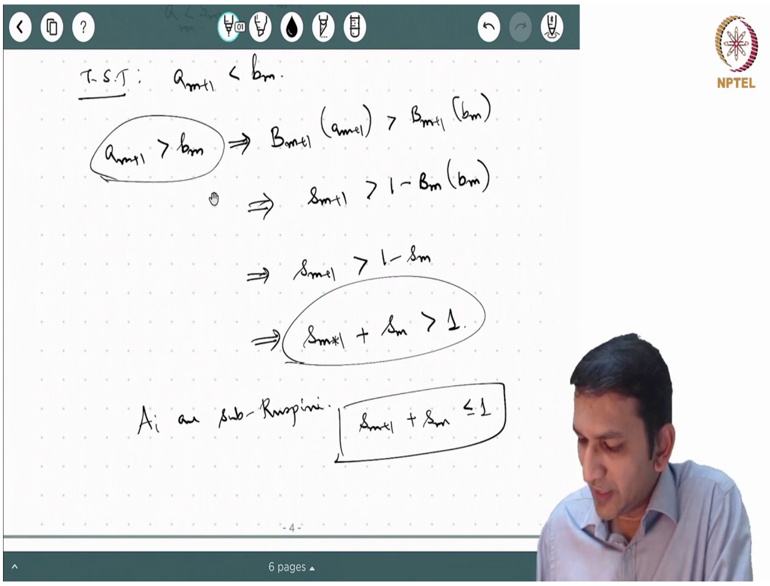

So, it is clear all we need to show is that a m plus 1 is less than 0 and clearly the kernel of b dash will be this. On the contrary let us assume that a m plus 1 is in fact, greater than b m plus 1 now this implies if you apply on this domain y m y m plus 1 we know that b m plus 1 attains its normality at only y m plus 1. So; that means, b m is strictly monotone and increasing over this interval over its support and since we have these following in this interval if you pick b m plus 1 of a m plus 1 this is greater than b m plus 1 of b m.

Note that, b m plus 1 a m plus 1 is in fact, s m plus 1 is greater than now we know that on the output space y, b is indeed from a respecting partition which means b m plus 1 can also be written as 1 minus B m of b m because it is b m b m plus 1 that will be intersecting over this interval y m y m plus 1, but what is b m of b m, it is in fact s m.

So; that means, this implies s m plus 1 is greater than 1 minus s m or that s m plus 1 plus s m is greater than 1, but note that this is a contradiction. Why is it a contradiction because we know that A i's are sub Ruspini so; that means, the matching values when we give this corresponding similarity values have at most been 1.

You know this because it is sub Ruspini when you give this it cannot be greater than 1. So, which means a m plus 1 b m is in fact, not true.

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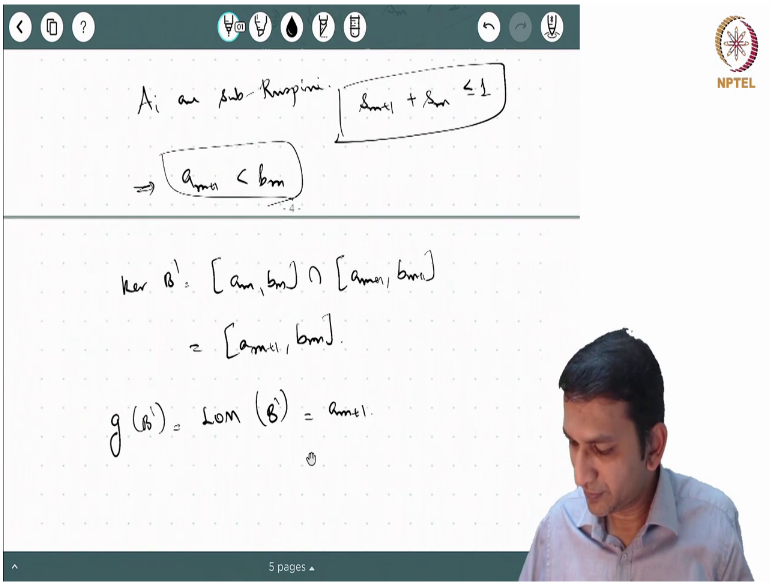

This implies a m plus 1 is less than b m and from there we get the kernel of B dash which was a m b m intersection a m 1 plus 1 b m plus 1 is in fact, a m plus 1 comma b m.

So, we have successfully shown that kernel of B dash is non-empty and it is in fact, given as a m plus 1 now b. Now, when you applying g on B dash it is list of it is list of maximum on B dash which is essentially a m plus 1. So, finally, we need to show that x when you are given two inputs x dash in x double dash if they are ordered monotone, x dash is smaller than x double dash we need to show g of x dash is in fact, smaller than or equal to g of x double dash.

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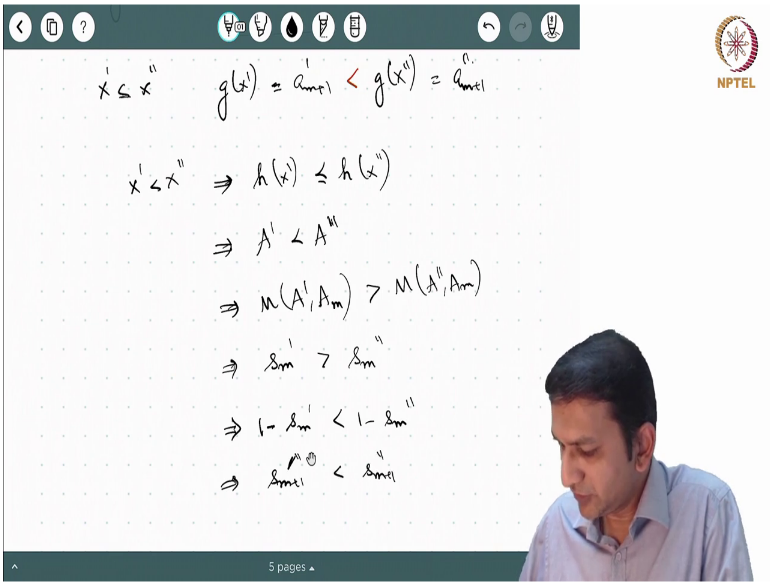

So, let us show this note that x dash is given to be less than x double dash we know that from the above g of x dash will be some a m plus 1 dash and g of x double dash will be some a m plus 1 double dash all we need to show is that this is in fact, smaller this is what we need to show.

Let us see how to show this, we know that x dash is less than x double dash (Refer Time: 40:01) less than, now if you consider h of x dash then we know that by the kind of fuzzifiers that we are considering the corresponding outputs A dash A double dash they are in fact, orderable with respect to the alpha cut based model.

So; that means, we have A dash is orderable with respect to this now this means M of A dash any A any particular A m is greater than M of A double dash A m, but m of A dash A m is nothing but s m dash is greater than s m double dash, once you have s m dash is greater than s m double dash. Note that on the B i's we do have these are similarity values we do have partition which means this is essentially a s m dash which is s m plus 1 double dash, dash is less than s m plus 1 double dash because of Ruspini partition we have this 1 minus this.

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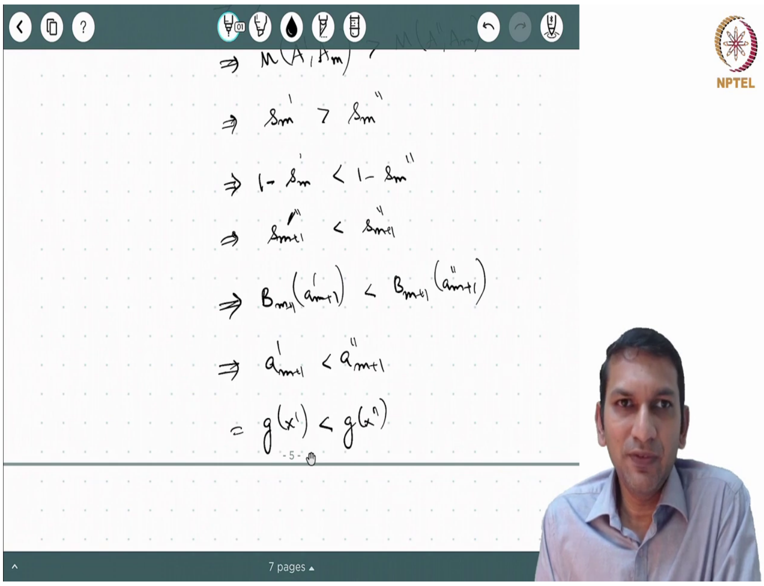

Now, this implies, now once again these are similarity values s m plus 1 dash is essentially let us go back here (Refer Time: 41:44) s m plus 1 dash is B m plus 1 of a m plus 1 dash to this less than B m plus 1 dash a m this B m plus 1 a m plus 1 a m plus 1 dash.

Now, remember B m plus 1 on this interval is strictly increasing which means the corresponding a dash m plus 1 is less than a double dash m plus 1, but this is essentially g of x dash this is g of x double dash which means this is smaller than this. So, all we have made use of are the properties of h, M and the fact that J has OP and on the antecedent side we have a sub Ruspini partition.

And M is in some sense compatible consistent with respect to P x and also the fact that on the consequence B i we actually have a Ruspini partition. So, this shows that whenever x dash is less than or equal to x double dash we also have g of x dash is less than or equal to g of x double dash.

So, this completes the proof and what we have shown now is with respect to the alpha cut based ordering we in fact, do have monotonicity of an SBR under the condition that we have seen, when you choose the different components of the SBR model properly appropriately we can ensure monotonicity of the corresponding system function as a function from X to Y with this we will wind up with the discussion on monotonicity of FIS.

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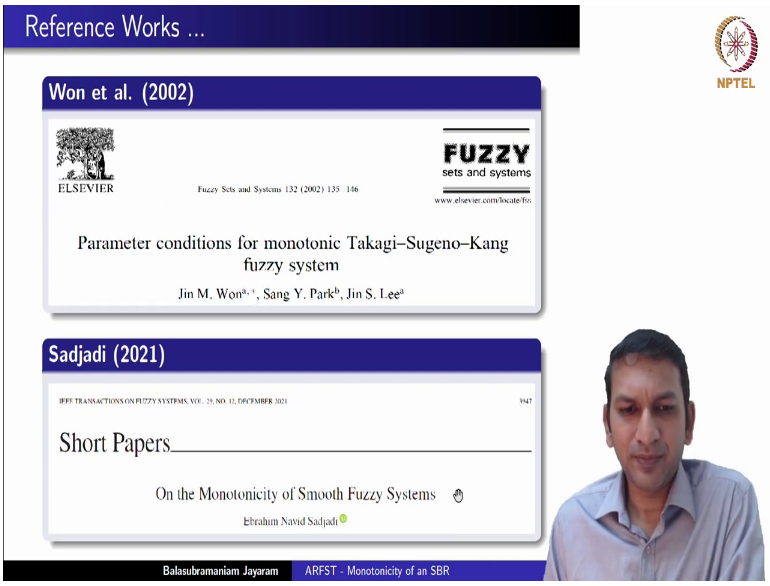

A few reference works should be highlighted, there are there is quite a bit of literature discussing monotonicity of similarity based reasoning schemes especially the TSK fuzzy systems.

So, this is one of the works that you can look into, the works are also quite distinct in their approaches the one that we have taken now in this lecture is to look at it in generality and fix the conditions on the components of the SBR model which will ensure monotonicity.

However, the works do differ for instance here like the conditions on the parameters are specified here in this work similar to ours. Here they assumed smooth fuzzy systems and then they are discussing monotonicity of those it is quite a recent paper and so, you know it is clear that the interests in monotonicity of a fuzzy set systems quite hot and alike.

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In this paper quite interestingly they discussed multi input fuzzy rule based models whereas, in the lectures we only consider single input single output rule basis and this at another paper we discusses monotonicity of fuzzy inference method especially the TS inference method.

As was mentioned these are only some sample works there exists many more works this will only highlighted for the different approaches that they have taken for instance the paper will Tay and Lim they actually are looking at root interpolation techniques to make sure that the multi input the system that they get is in fact, monotonic.

Well, with this we come to the end of discussing monotonicity of a fuzzy inference system.

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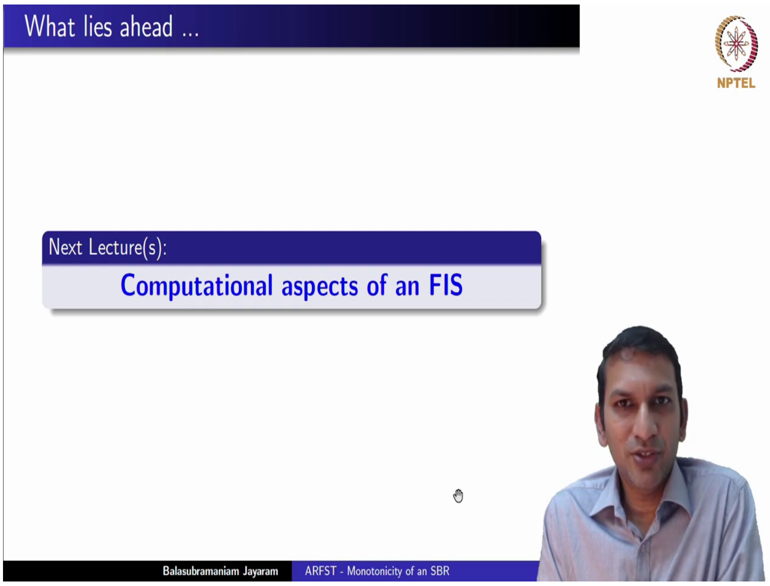

What lies ahead in the next week, week 12 which also will happen to be the last of the week for the lecture series that we are going through on this course title Approximate Reasoning using Fuzzy Set Theory. We will look at the computational aspects of fuzzy inference system.

So, far in the last few weeks we have discussed some desirable properties of a fuzzy inference system that of interpolativity, continuity, robustness and monotonicity, but in some sense all of them deal with what could perhaps be categorized as the correctness of a fuzzy inference of a system, we have not seen the computational aspects of it

We have seen that a lot of the properties thankfully coming from resituated lattices; they helped us in discussing and proving many results. However, if you recall these are in the form of either equations or inequalities these equations and inequalities involving either the minimum of the meat or the joint operation of the lattice and the T norm or the implication coming from it, they are also available for other classes of fuzzy implications.

In the last few weeks we have looked at using these properties towards justifying the correctness of the inference system that we are considering, but in the next week of lectures we will look at these functional equations and functional inequalities for which classes they are available outside of this class of R implications some of them will discuss and also their impact on the computational aspects of fuzzy inference systems. So, this is what we will take up in the set of lectures in the next week, the last week of this lecture series we will discuss the computational aspects of fuzzy inference systems. Glad, you could join us today for this lecture hope to see you soon in the next lecture.

Thank you all.