# Approximate Reasoning using Fuzzy Set Theory

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## Lecture - 39
## Fuzzy Interface Systems - Interpolativity

Hello and welcome to the first of the lectures in this week 8 of the course titled Approximate Reasoning using Fuzzy Set Theory; the course offered over the NPTEL platform.

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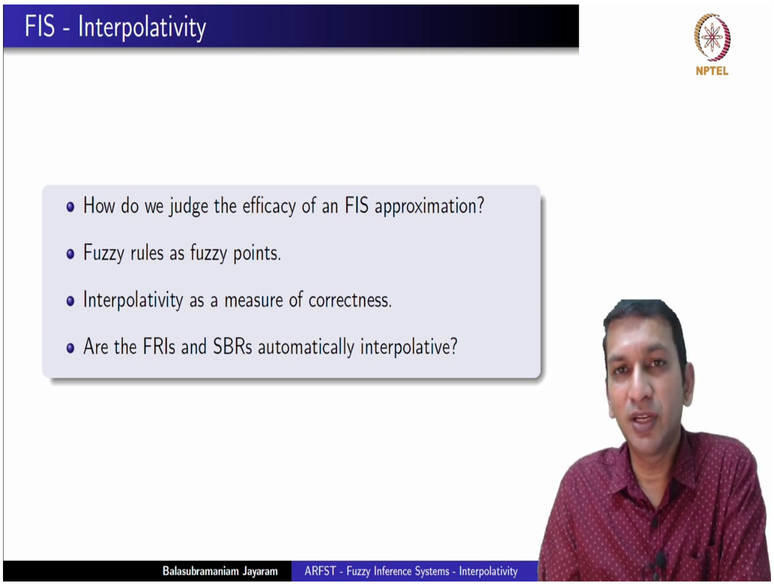


We have seen that a fuzzy inference system approximates a given function or the behavior of the system which has a function underneath a mapping of the inputs to the outputs. We know that a fuzzy inference system approximates this function by covering it with overlapping rule patches.

But, the question now is how do we judge the efficacy of such an approximation? Are we sure that the function that we are trying to capture through the fuzzy inference system is actually close enough to the function that the system inherently has or the way the system inherently behaves? But, one view is if you have to look at fuzzy rules themselves as fuzzy points, then perhaps interpolativity can be thought of as a measure of correctness.

This immediately springs up a question as to whether the two types of fuzzy inference mechanisms that we have seen, namely the fuzzy relational inference and the similarity based raising schemes; whether they are automatically interpolated? In this lecture, we will look at this question with illustrative examples and see if this interpolativity is automatically guaranteed.

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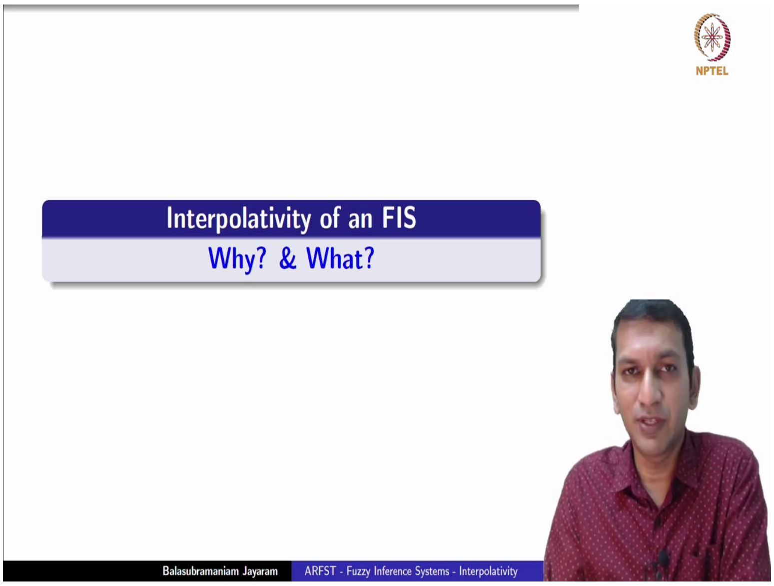


But, first what do we mean by interpolativity of FIS of Fuzzy Interface Systems and why is it required?

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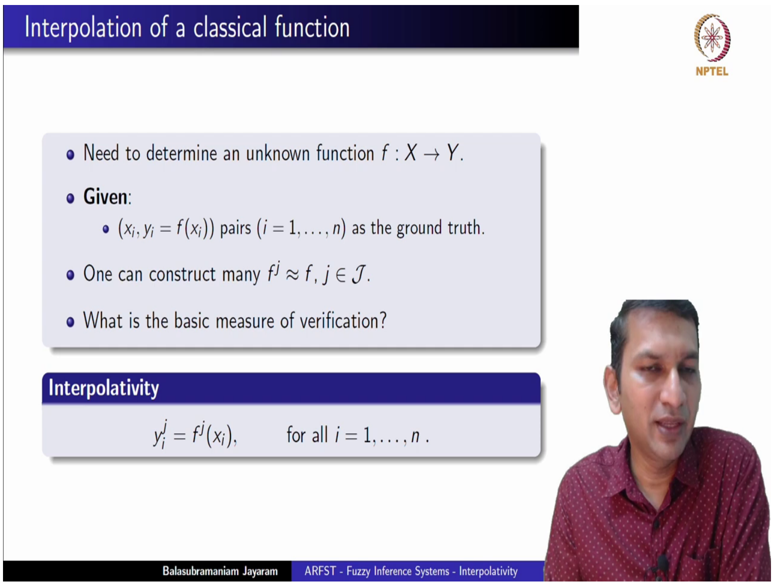

Let us look at interpolation of classical function. What we need to determine is an unknown function f from X to Y. To determine this function, all we are given is a set of n points, pairs of points x i, y i such that y i is a function of x i and this function f is what we are interested in determining. One can construct many functions f j which can approximate this function f.

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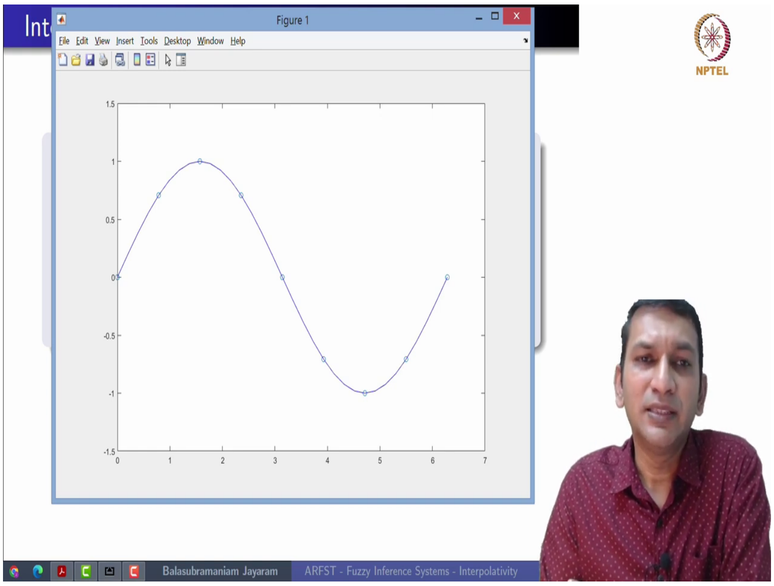


For instance, consider the set of points that are there on the screen, that seen on the screen. Now, it is clear that it has come from the sine function. But, let us assume for the moment we really do not know what is the function which has given into this set of points.

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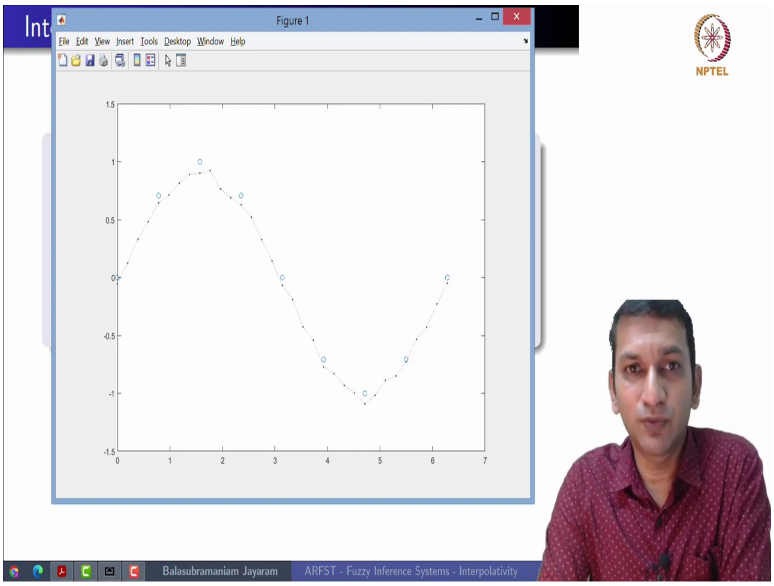

If you were to approximate this function through an algorithm and, if the approximant is given as this function, then I think we would not have too many complaints. Because, this is close enough to whatever points that have been given, the initial ground to that I have been given. So, this can be considered as a good enough approximation of the function that we are doing, but this is not the only function.

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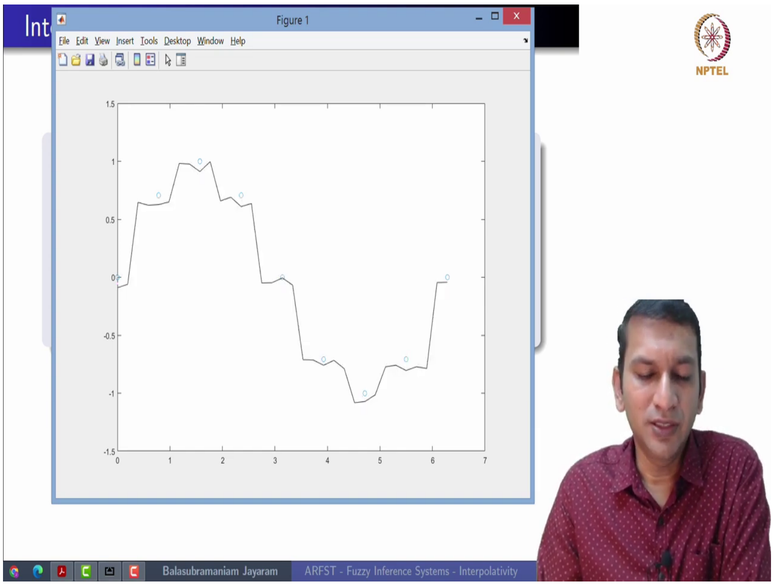


For instance, this could also be thought of as another good approximant of the function that we have in mind. So, given these two functions, if you were to choose perhaps we need to apply other aspects of the function that we are going to get out of the algorithm. Perhaps some continuity properties, smoothness properties, monotonicity properties and so on. But, these are not the only two functions which can be thought of as approximating the function that we have not known.

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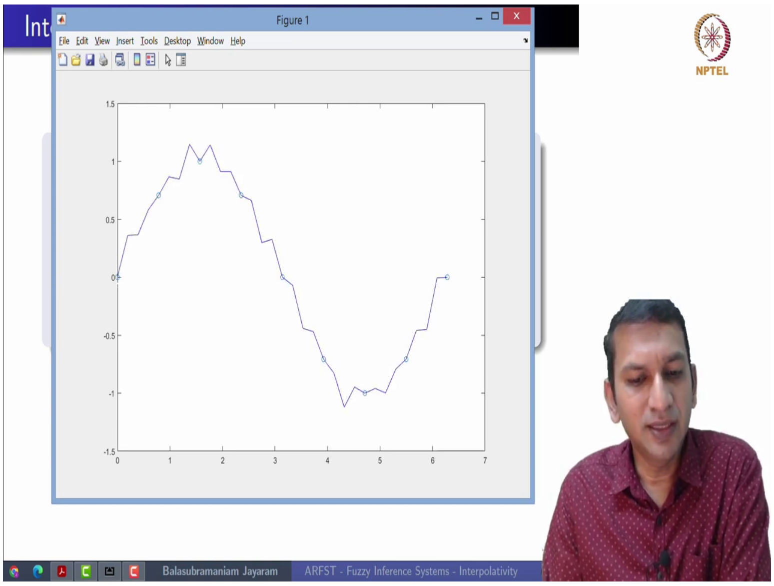


This is yet another function, once again it probably is very jagged not smooth. However, notice that compared to the previous two functions, this function actually passes through each of the points that we have been doing.

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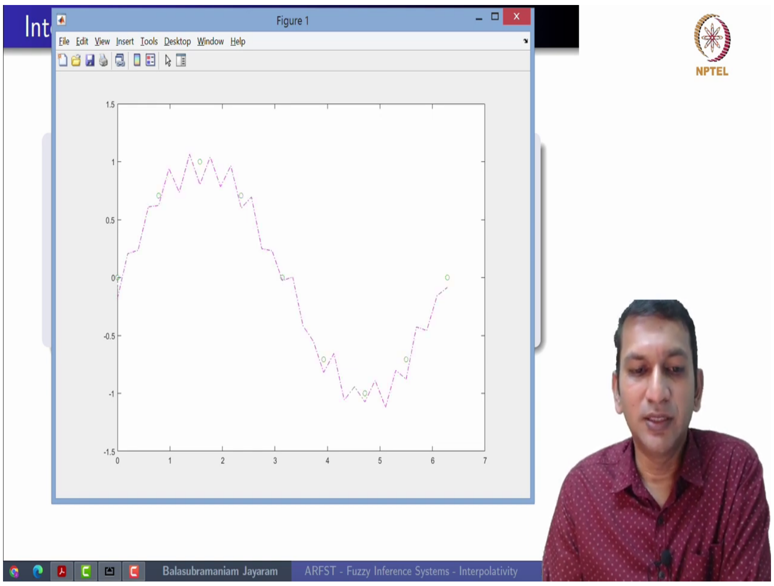


This is another function, perhaps not as jagged as the previous one, but it does not pass through all of you. Now, if you consider this as the original function, then we could say that the previous four function that we have shown, all of them are good enough approximation for this information.

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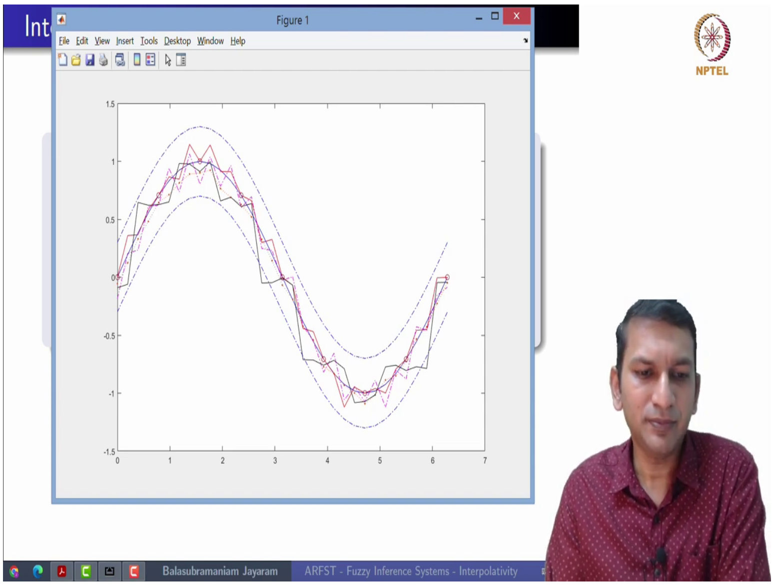


Now, why do we say this? Because, all of these four functions seem to be within a very narrow band of the original function we have in mind. So, you could consider these as good approximations of the function. Well, if you ask them what is the basic measure of verification, among these four what do you choose? So, typically you could put other constraints like that of continuity or smoothness, differentiable considerations or monotonous consideration.

But, typically in the absence of anything else, what normally we go for is that it should be interpolative. What do you mean by interpolativity? So, now if you take this exercise and pick one f j from the set of functions that which we consider as approximating in the given function in our minds, then what we want is y i should actually be equal to f j of x i. So, this is what we would expect.

So, this is interpolativity, whether the given pair of points; pairs of points x and y whether they can be recreated using the function that we have. So, this is essentially interpolativity and it is considered a good measure of a fitness of the function to what we have in our minds.

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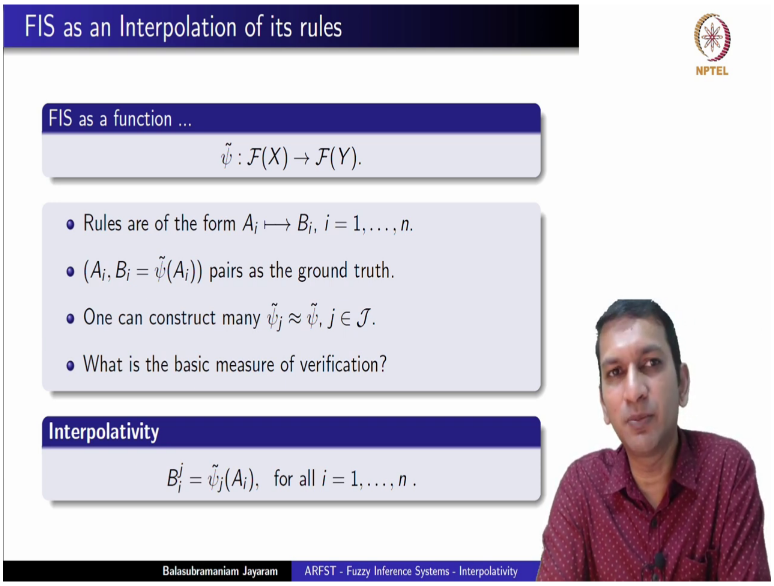


Now, we know that a fuzzy inference system can be thought of as a function phi tilde psi tilde from F(X) to F(Y). Once again, we know that this is this inference mechanism is based on the ground truth of rules that are given to us and rules of the form A i implies B i. Now, you could also consider these as pairs of points A i B i is equal to psi tilde of A i, these pairs as the ground truth that are given to us.

So, what we have in mind is a function of psi tilde which is what we want to approximate and you could think of these rules as pairs of points coming out of this function. So, essentially you are sampling from the graph of this cycle. Now, once again one can construct many such approximants psi j tilde which are close enough to psi tilde. Then, the same question comes what is the basic measure of verification?

Once again you can go with the classical approach and say that all these different functions which are considered to be a good approximation of the original function, we choose one that actually interpolates the given ground truth which means among all the j's we pick that j's as that psi j tilde of A i is actually equal to B i. So, this is how under interpolativity comes into picture, while means while picking one out of the many possible options.

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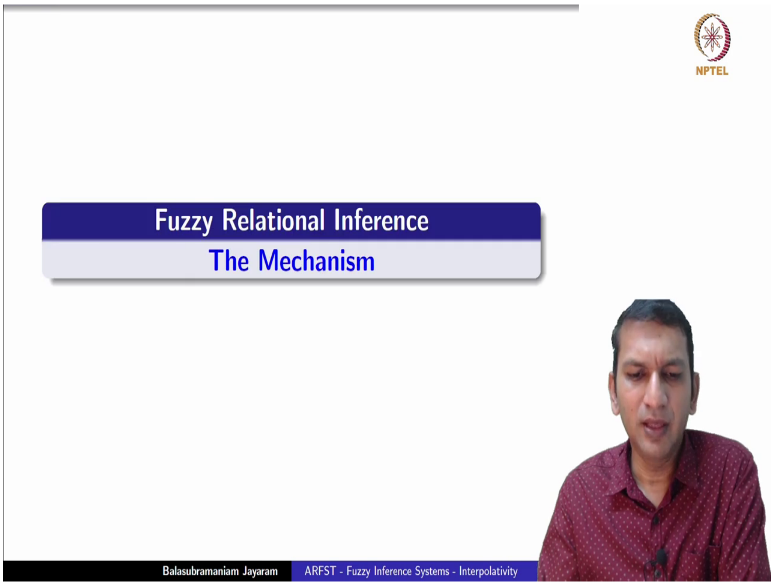


In this lecture, we will restrict ourselves to discussing fuzzy relation inference. So, let us have a quick recap of the mechanism.

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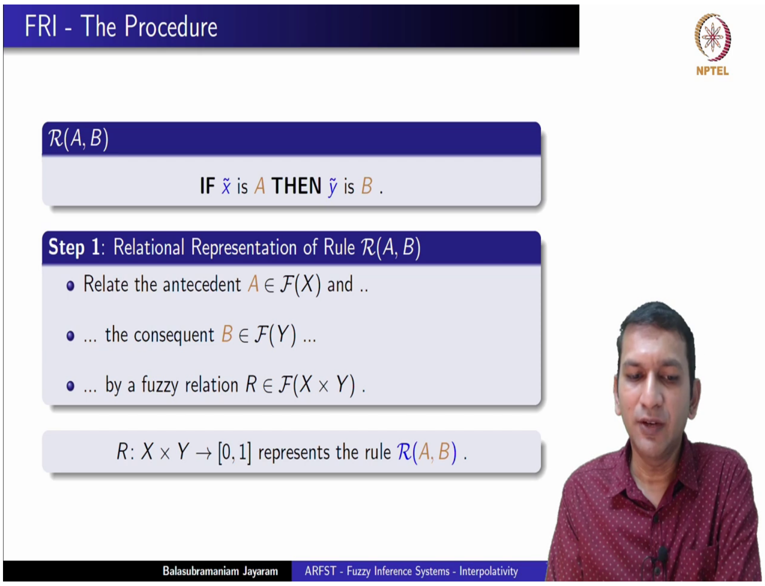


We know that given a rule it is represented as a relation relating the antecedent to the consequent of the rule. And, it is related as a fuzzy relation on X cross Y, to this end we use a function if.

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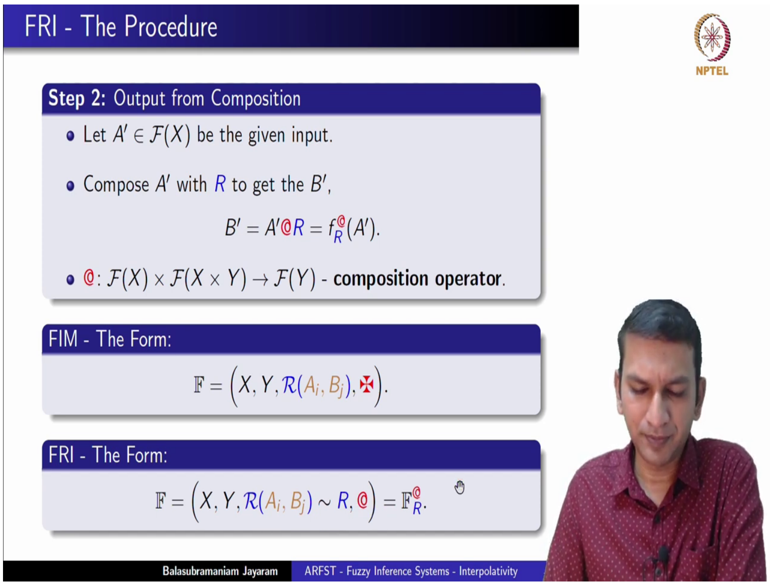


In the second step given an input A dash, we compose it using a composition operator, with this relation which captures the essence of the rule and we obtain an output B dash which is a fuzzy set on y. We have seen the general form of a fuzzy inference mechanism is given by the fuzzy's, the domains of the input and output X and Y, the rule base and the inference operators involved. In the case of an FRI, it is specified essentially by the relation that captures the rules and the composition operator.

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Let us look at some illustrative examples.

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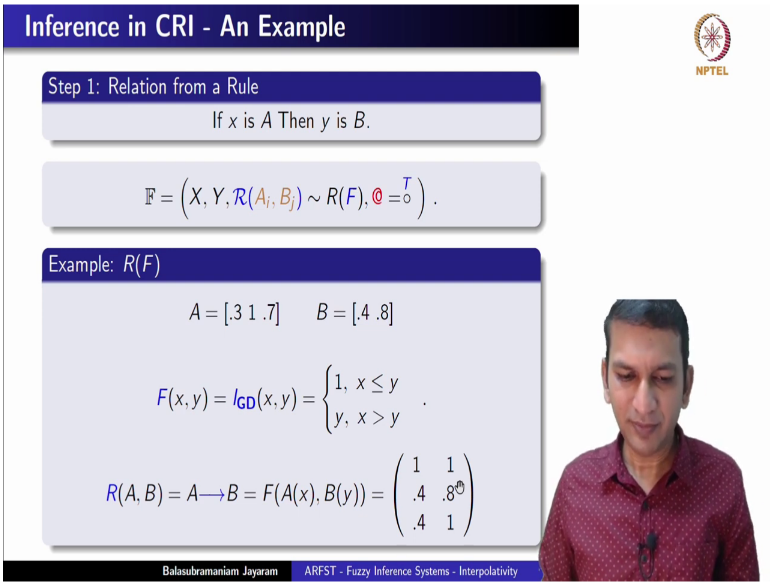


To begin with let us take a single rule and we need to specify only the operation F which creates the relation from this rule and also the composition operator. For the moment, let us look at the CRI and let us fix F to be the Godel implication. And, if A and B are given as follows, R can be obtained like this. Let us work this out.

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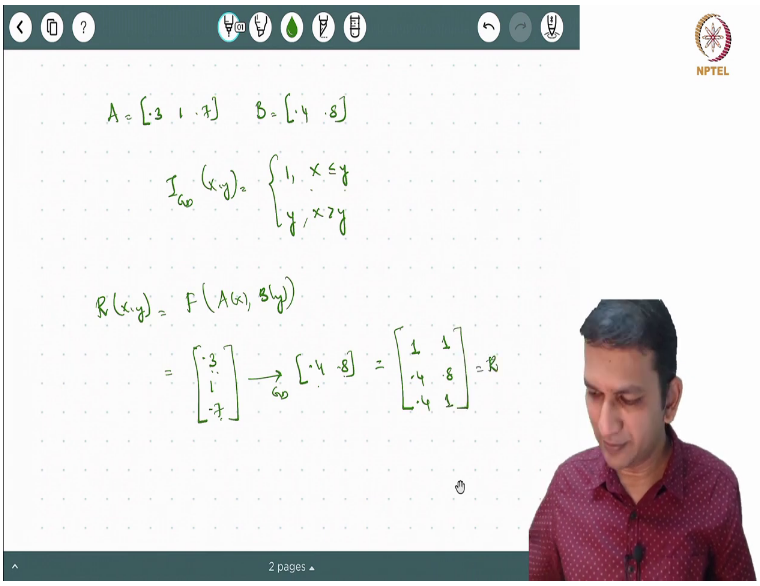


So, what we have are this A 0.3 1 0.7, B is 0.4 0.8. Let me recall Godel implication, 1 if x less than equal to y and y x greater than y. Now, R(x,y) is nothing, but F of A x, comma B y. So, now, this we know is obtained like this, we take 0.3 1 0.7, I am applying Godel implication from 0.4 0.8. So, you see here you are looking at it from the point of view of a (Refer Time: 10:14). So, we take 0.3 and 0.4, apply the Godel implication 0.3 is less than 0.4.

So, you get 1, 0.3 is less than 0.8 ok, you get 1, 1 implies 0.4, it is a neutral implication its 0.4 0.8; 0.7 implies 0.4, it has to be 0.4, 0.7 implies 0.8 its smaller than that because of (Refer Time: 10:35) we get this. So, this is the R we have obtained, in this essentially the R that we have.

So, given A and B, we use the operation if and obtain the relation R that captures this fuzzy rule if x is A and y is B. In this case R is represented like this where A and B are given as follows and F is the Godel implication.

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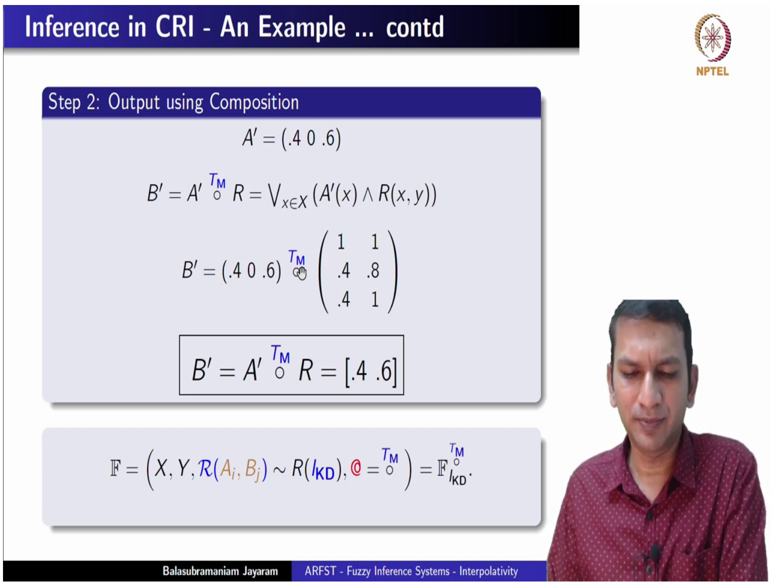


The next step is to compose a given input with this R and obtain the output. We are using the sup min composition here. So, T is the minimum T norm here and it looks like this. So, take the A dash, this is the relation that we have. Then, when we apply the sup min composition. All we are doing is in some sense vector inner product of this row into each of the columns. Of course, inner product is sum of products, but here we are going to do max of min so, 0.4 and 1 is 0.4.

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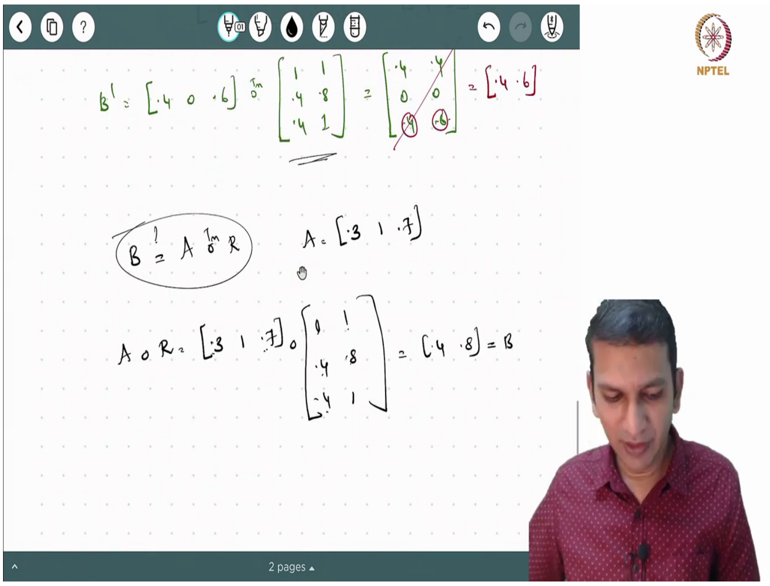


(Refer Time: 11:38) like this ok, (Refer Time: 11:41), can take this ok 0.4 into 1 is 0.40 (Refer Time: 11:52) write this, what we have here it is B dash is 0.4 0 0.6 1 0 4 0.4 0.4 1 0.8 1. So, this will be 0.4 I temporarily write like this 0.4 minus 0.4, 1 is minimum is 0.40 0.4 is 0.6 implies to the 4 0.4 0.4 1 is 0.40 is 0.6 minus 0.6 from along these you need to choose maximum.

So, here it is 0.4 and 0.6. So, what we get is 0.4 0.6. So, this is the; this is the corresponding answer, that is what we are obtained. So, if you look at it from point of view of an FRI, all we have done is fixed the implication of the Godel implication F and then sup T norm min as the composition ok.

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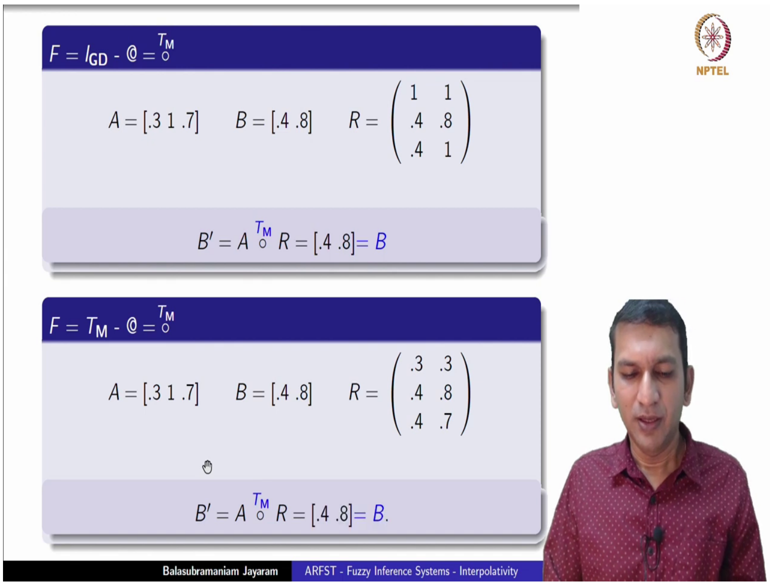


So, now what we will do is we will look at the same rule A and B, this is the relation that we have got. Now, to check for interpolativity means we need to give A as the A dash, the input and find out if you are obtaining B as the B dash. So, now let us do the map (Refer Time: 13:23). So, you want to check is B equal to A sup T m into R, this is the question we have.

So, now, recall A is 0.3 1 and 0.7 and R is (Refer Time: 13:49). Let us come to the A and R 0.3 1 0.7, compose to 1 0.4 0.4 1 0.8 1. So, previous calculation we do it again this way; 0.3 1 is 0.3, 1 0.4 is 0.4 and 7.4 is 0.4. So, now 0.3 0.4 and 0.4, the maximum is 0.4, 0.3 and 1 is 0.3, 1 and 0.8 is 0.8, 0.7 and 1 is 0.7. Among these three values, 0.3 0.8 and 0.7, the maximum is 0.8.

So, this is the B dash of your (Refer Time: 14:29). But, is it actually equal to B? Yes, it is. This is equal to B in fact, what we find here. So, for this particular combination where F is given by the I Godel, Godel implication and we consider sup min composition, we see that B dash is actually B when A dash is A. Now, let us keep the same rule; that means, A and B are same. Let us look at changing the operation F to minimum. So, now obviously, the rule will change.

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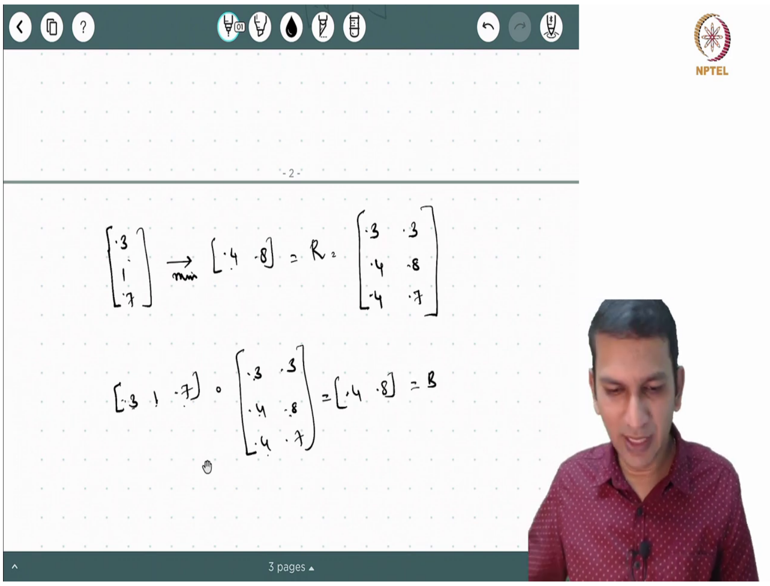

So, now once again we have 0.3 1 0.7, now it is the minimum operation that you are using, B is 0.4 0.8. This will give us the R. So, if you do the map, we are going to take the minimum here 0.3 0.4 is 0.3, 0.3 0.8 is 0.3, 1 0.4 minimum is (Refer Time: 15:34) T norms (Refer Time: 15:35), one is the neutral minimum; 0.7 0.4 is 0.4, 0.7 0.8 is 0.7. So, this is the R, that you will get is exactly what we have.

Now, note that we have used the minimum T norm for F instead of the Godel implication. Now, once again let us do this. We are asking the question, if I compose A with this R; that means, assume A dash is A, would we get same consequent B as the B dash? Let us work it out, A is 0.3 1 0.7 composed with 0.3 0.3 0.4 0.8 0.4 0.7 and see here when we compose this 0.3 0.3 is 0.3, 1 0.4 is 0.4, 0.7 0.4 is 0.4.

So, the maximum of them is along 0.3 0.4 0.4 is 0.4, 0.3 0.3 is 0.3, 1.8 is 0.8, 0.7 0.7 is 0.7. Once again, among these three values 0.3 0.8 and 0.7 maximum is 0.4 0.8, once again we see that it is actually equal to be B. So, we took arbitrary A's and B's, we took arbitrary pairs of functions for F and the composition. In two out of two cases, we have actually managed to show that it is interpolative; that means, given an A as the A dash, we are obtaining B as the B dash. But, the question is will this magic continue?

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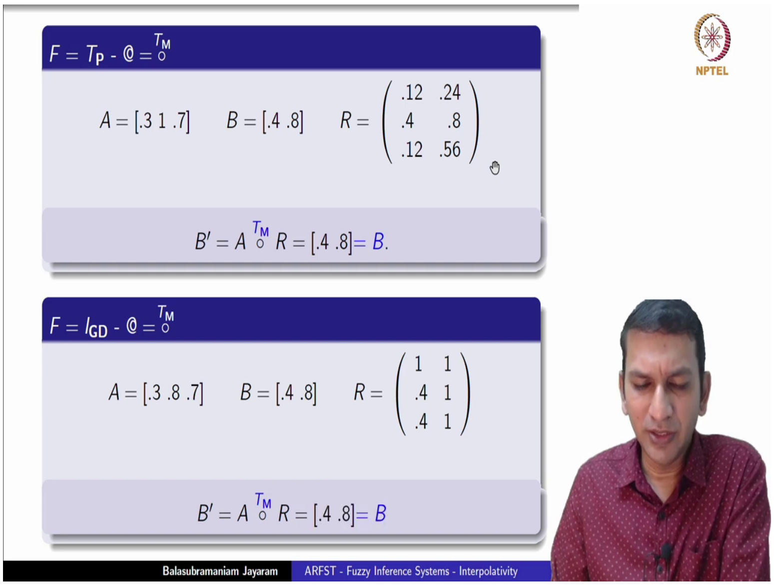


For instance, here we have retained the sup min composition which means we are in the framework of CRI, but you have changed the F to the product T norm. So, which means it is going to actually affect the relation R.

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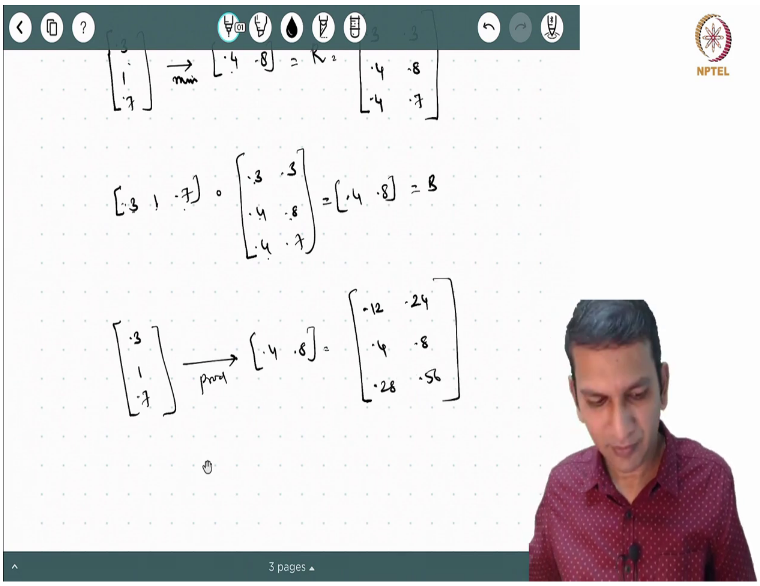


So, let us do the map once again. So, you are going to pick product here 0.4 0.8, once again applying the similar techniques out of product with 0.12 0.24 0.4 0.8 0.28 0.56. This is the relation that we have here. Now, once again we ask the question, if you give A dash to be A, what will be dash B?

You can see from here 0.3 and 0.12 0.12 1 and 0.4 is 0.4, 0.7 and 0.12 is 0.12. Among these three elements, the maximum is 0.4. Similarly, you will see that maximum when you compose, do the operation between this vector, this column here you will get it to be 0.8 which means once again we are getting the output to be B ok. So, third time lucky, let us change it. Keep sup min and take the total implication again.

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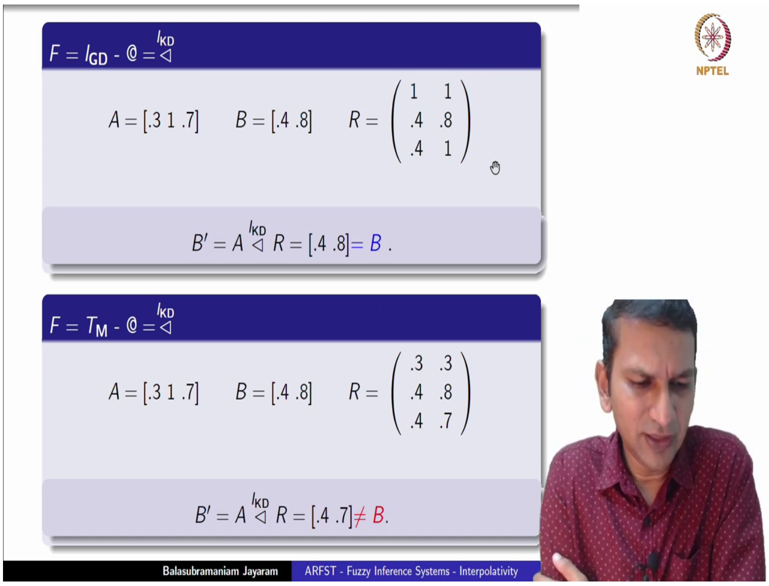


We are getting this. Let us move to the BKS inference. Now, the composition has changed, we have kept the Godel implication. So, the R we obtained will be the same because, F is what is going to determine the R. But, now composition changes to the BKS inference with the implication as the Kleene Dienes implication. So, let us work this out.

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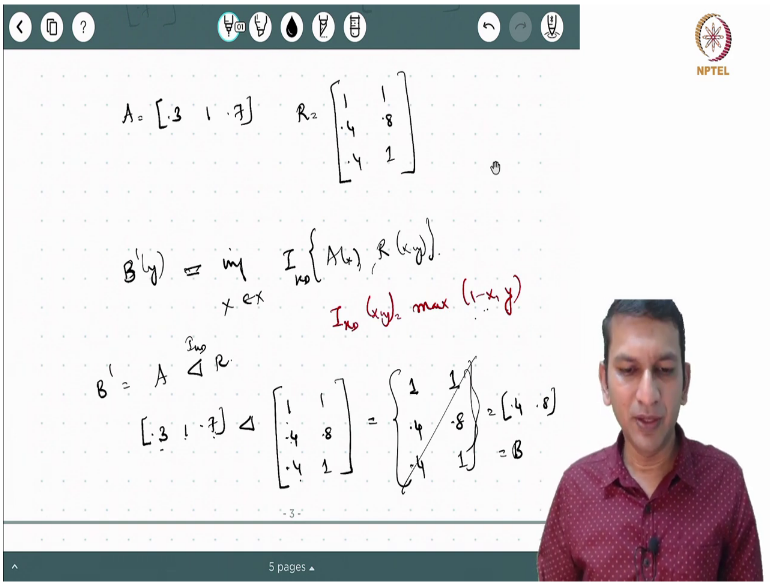


So, we have A as 0.3 1 and 0.7, R is given as 1 0.4 0.4 1 0.8 1. Please note that the (Refer Time: 19:25) BKS inference, what it has infimum on X element of X, implication A of x comma R(x,y). This implication what we are using is the Kleene Dienes implication. So, now, let us keep let us find out B dash is equal to A right AD of R. So, now, the particular form B dash of y B equal to; I can never be calculate because we have the matrices.

So that means, you are taking 0.3 1 0.7 1 1 0.4 0.4 0.8 1. Notice that, we are going to apply the Kleene Dienes implication which is given as follows; maximum of 1 minus x comma y. So, now, if you take 0.3 and 1, then maximum of x 2, we know that it is an implication x comma 1 is 1. So, this will be 1 1 and 0.4 it is a neutral implication, it will be 0.4, 0.7 and 0.4 because x is 0.7, y is 0.4. So, it is maximum of 0.3 comma 0.4, this 0.4. So, this is what we have got.

And, if you take the first this row and this column, if you apply this operation; what we get is 0.3 1 is 1, 1 0.8 is 0.8, 0.7 1 is 1. However, from these two columns what we need to take is the minimum instead of the maximum which means the answer will go is 0.4 0.8. So, this is the B dash that we get, but in fact, it is actually equivalent to B. So, what we have seen is even in the case when we are looking at BKS with F as the Godel implication, we seem to be obtaining interpolativity.

Now, let us keep the BKS inference and change to change the function F which captures the relation to that of a minimum. Of course, now this R is going to change.

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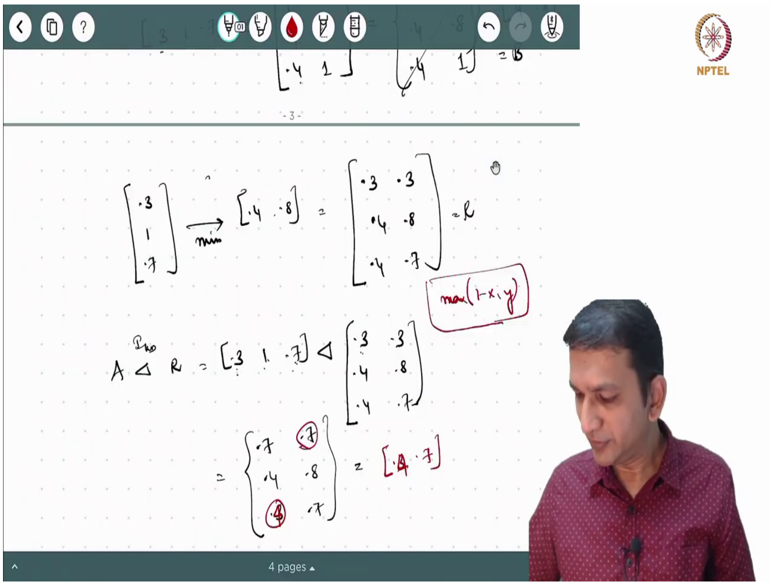


Let us calculate this R 0.3 1 0.7 (Refer Time: 21:59) what. So, what you are going to use is min 0.4 0.8 add a 0.8 0.4 minimum is 0.3 between 0.3 1 0.4 is 0.8 0.4 0.7. This is the R (Refer Time: 22:25). Now, we are going to apply the BKS inference with the Kleene Dienes implication.

So, let us apply A R 0.3 1 0.7, again brackets 0.3 0.4 0.4 0.3 0.8 0.7. Once again by the intermediate step 0.3 0.3, please note the implication is max of 1 minus x comma y is what we applying. So, 0.3 0.3 is actually 0.7 1 0.4 is 0.4 0.7 0.4 is 0.3, 0.3 0.3 is 0.7 1 0.8 is 0.8, 0.7 0.7 is 0.3 and 0.7 which is 0.7. And, remember we are going to pick the list here, because you are looking at the BKS inference which is infimum of all of these.

So, which means from this we will get 0.3 and this will get 0.7. So, output is 0.3 0.7 clearly, sorry this must have 0.4. So, output is 0.4 0.7 which is not equal to B. So, we have seen for the first time using BKS inference that for a particular choice of F that it is not interpolated, if you give A dash A to be A, we are not getting B dash to be B.

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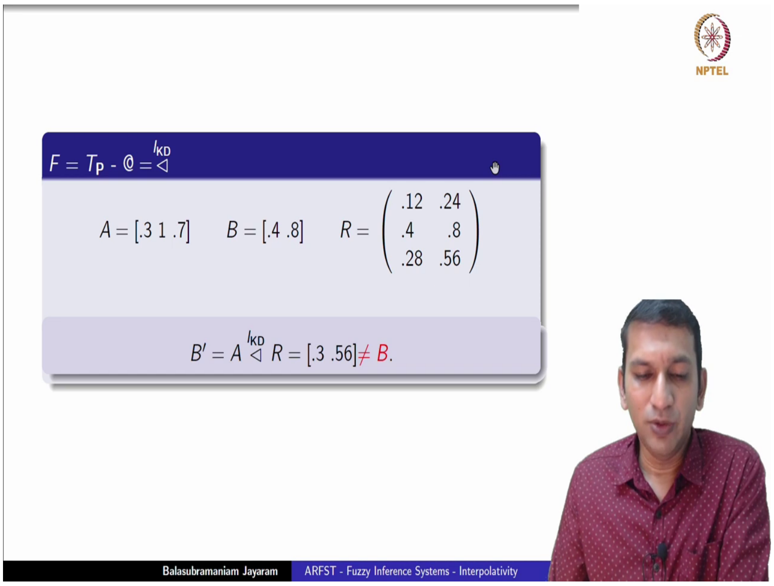


But, will this magic happen only once of not getting into interpolativity? Well, if you consider the BKS inference with the product T norm, let us see what happens. The R is something that we have calculated few minutes earlier, this is the R. If you actually compose A with R, what you would get is this is the value that you would get 0.3 0.56 which is again not equal to B.

So, among the 5 or 6 combinations that we have seen, in the case of CRI it seems to hold, always we have obtained the interpolativity. But, in the case of BKS out of the 3 combination that we have considered, 2 of them do not turn out to be giving us the desired result; that means, they do not turn out to be interpolate. Note that, this is just the case with a single SISO: Single Input Single Output. Of course, what happens when we have multiple rules?

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Recall, there are two inference strategies.

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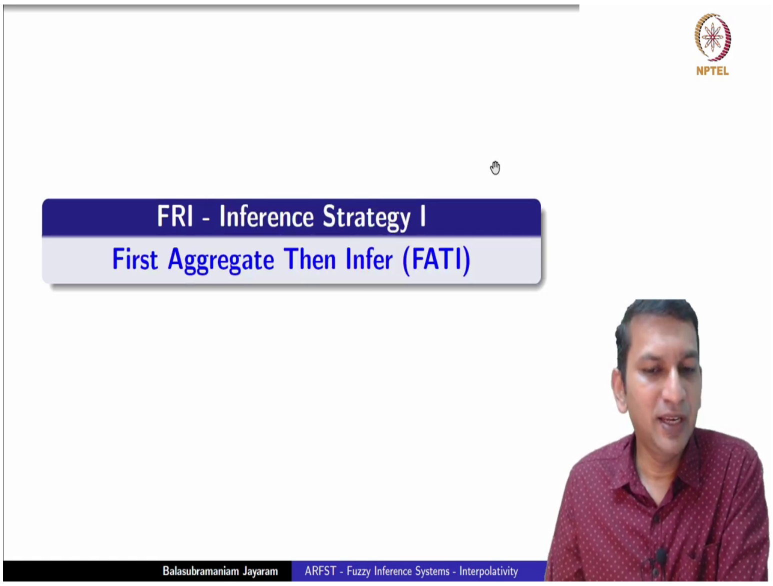

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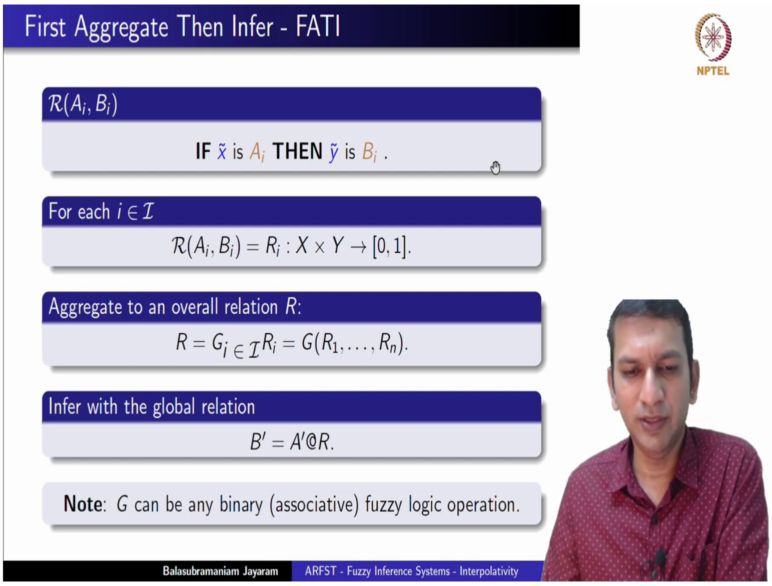


First aggregate then infer. In this case, what do we do? Given a set of rules, we convert each of them to a rule. Then, aggregate all these rules, all these relations into one overall relation R and then do the inference. We have seen typically G is a binary associative fuzzy logic operation.

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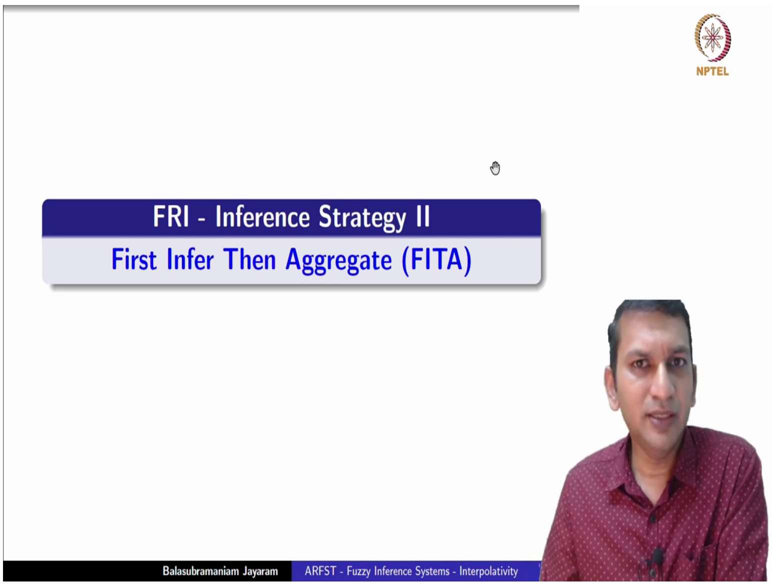


There is also another strategy first infer then aggregate.

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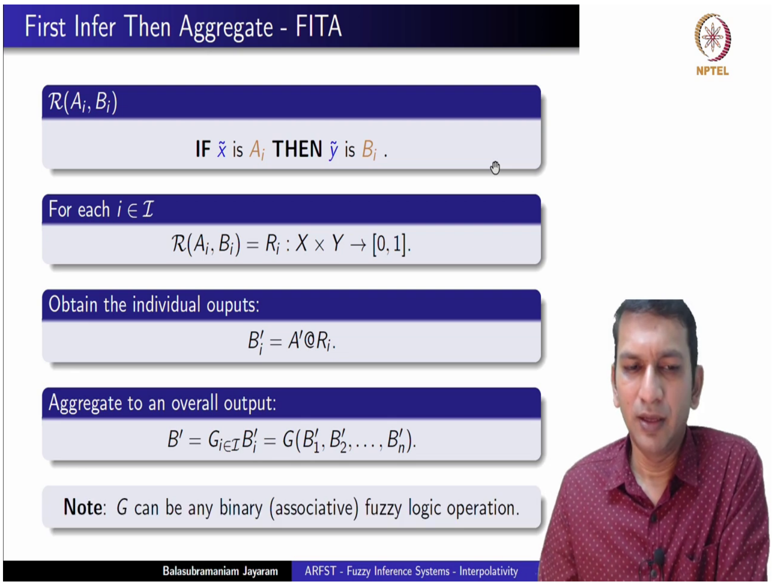


So, given the set of all rules, for each rule we find the relation which represents it. Now, instead of aggregating all the relations at once, we actually use each of these relations and obtain a local output B i dash. It is these B i dashes that we are going to aggregate later on. So, first infer then aggregate and this turns out to be the final overall output obtained from the FIR.

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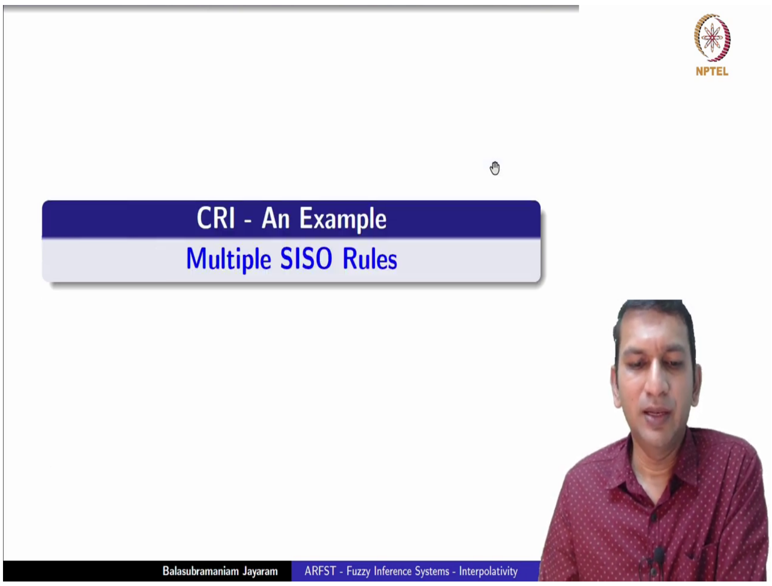


Well, let us once again look at some examples and then we can easily check for whether interpolativity is present or not.

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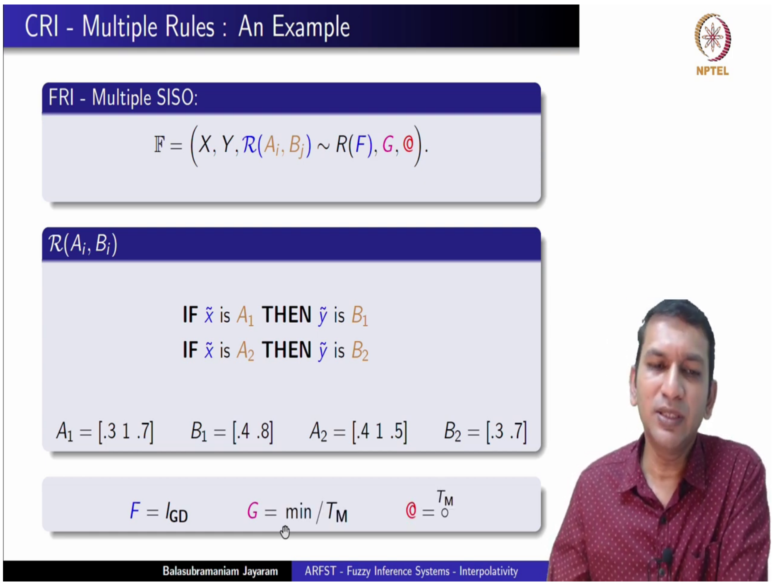


Note that now other than the F that we use to obtain the relation, we also need an aggregation operator that is what is represented as capital G here and of course, the composition. For, the moment let us consider only two rules, A 1 implies B 1 and A 2 implies B 2 and let us take these as the works.

Note that these are exactly the same as A and B earlier just so, that we can save up constant computation time here during the lecture. Of course, A 2 and B 2 are different. For F let us start by considering the Godel implication itself, for G we consider the minimum T norm and let us start with the CRI case and typically the sup min composition.

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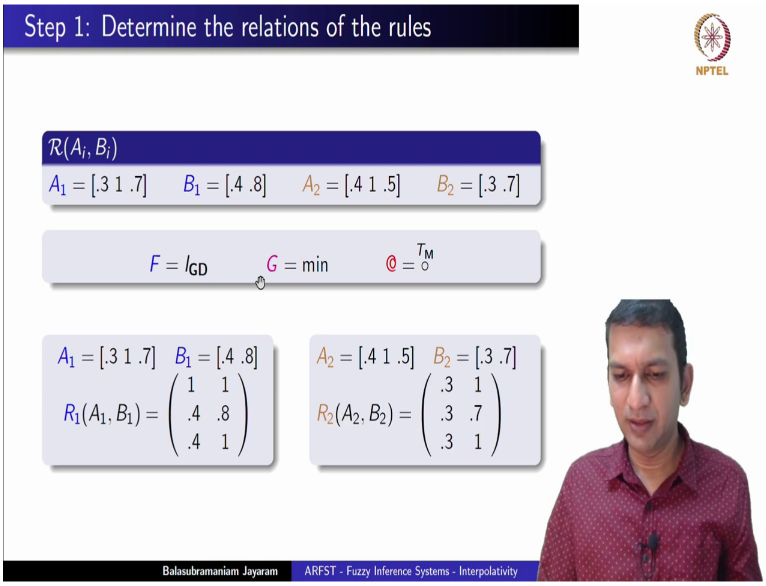


Once again, we need to determine the relations. So, we have seen that if you use this combination I GD and sup min composition, the aggregation does not come into picture yet. A 1 and B 1 are the ones that we have seen earlier as A and B ok. We see that this is the relation that we get ok. If we use the other two perhaps we will just do for this, you will carry over from them.

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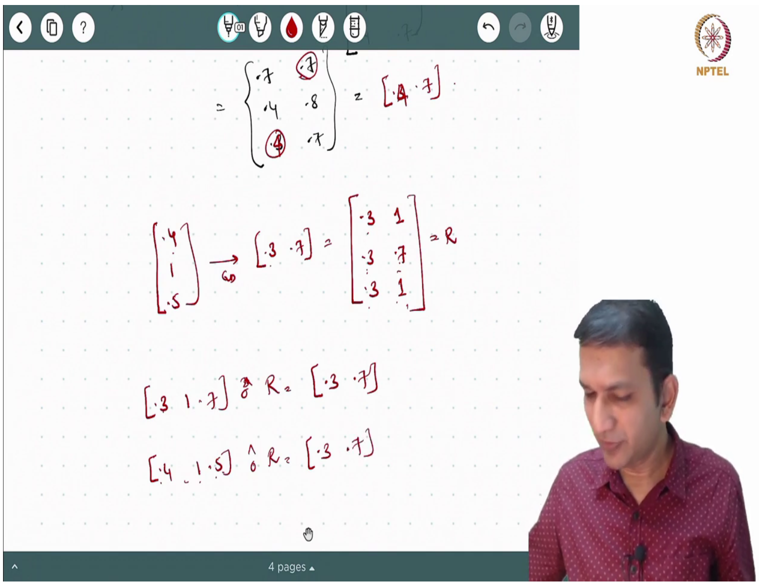


What we have is 0.4 1 0.5 Godel implication, that will be using 0.3 1 0.7. Note that the Godel implication is 1 if x is less than y, if x is greater than. So, 0.4 is greater than 0.3, that will be

0.3, 1 is greater than 0.3, 1 is greater than 0.3, 0.3 is greater than 0.3, 0.4 is smaller than 0.7, we get 1. 1 is greater than 0.7, we will get 0.1, 0.5 smaller than 0.7. So, this is the R that we have obtained. You see that is the R we get.

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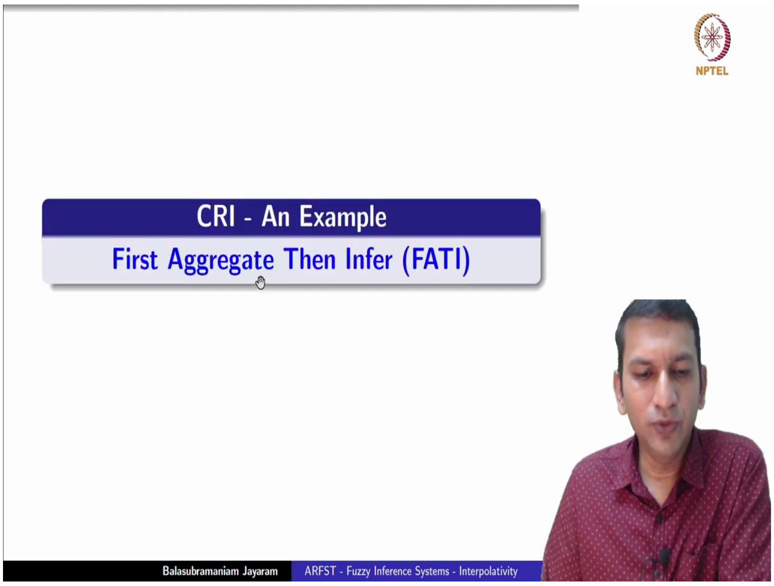


Now, after determining this rules R 1 R 2 there are two ways to do it, FITA or FATI. So, first let us look at how to aggregate and then inference.

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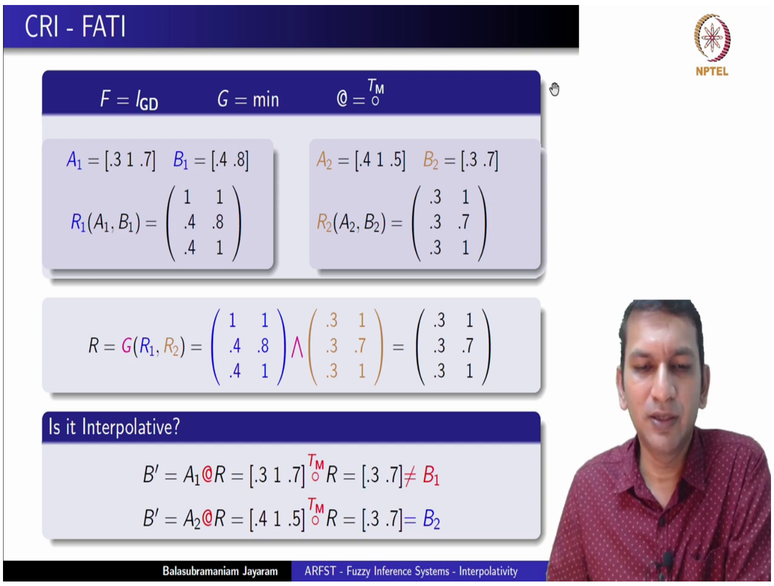

Now, we are not going to do it on MP premise, we want to check for interpolativity. So, what we do in first aggregate? We aggregate the relations. So, this is R 1 and this is R 2. We are using minimum as the aggregation function, clearly it is component wise aggregation. This relation is bigger than this point wise so, we get the relation R.

Now, let us look at whether it is interpolated; that means, once again for this overall relation R, we give A 1 and see whether we obtain B 1. So, let us look at this. We are doing sup min composition. So, what do you want to do is 0.3 1 0.7, composted R here you can easily see that is 0.3 0.3 0.3 is 0.3 0.3 0.7 0.7 is 0.7. So, what you obtain is 0.3 0.7 which is actually not equal to 0 which is because it is 0.4 0.8.

On the other hand to the same relation R, if you give 0.4 1 and 0.5, quickly do it 0.4 1 0.5 sup min R 0.4 0.3 1 0.3 0.5 0.3, 0.4 1 is 0.4 1 0.7 is 0.7, 0.5 1 is 1 0.5, max of all of these is 0.7. So, we get same output 0.3 0.7. But, this time it is actually equal to B. So, you see here interestingly from this overall aggregated relation R what we obtain is for A 1 we do not obtain B 1.

So, it is not interpolative at the point A 1 B 1; however, it is interpolative at the point A 2 B 2. So, now, when we are speaking this language, we are looking at rules as fuzzy points, pairs of fuzzy points. So, it is this function that it captures with these operations, it is not interpolated at A 1 B 1 but, it is interpolated at A 2 B. Now, the question is what if you change this min to some other aggregation?

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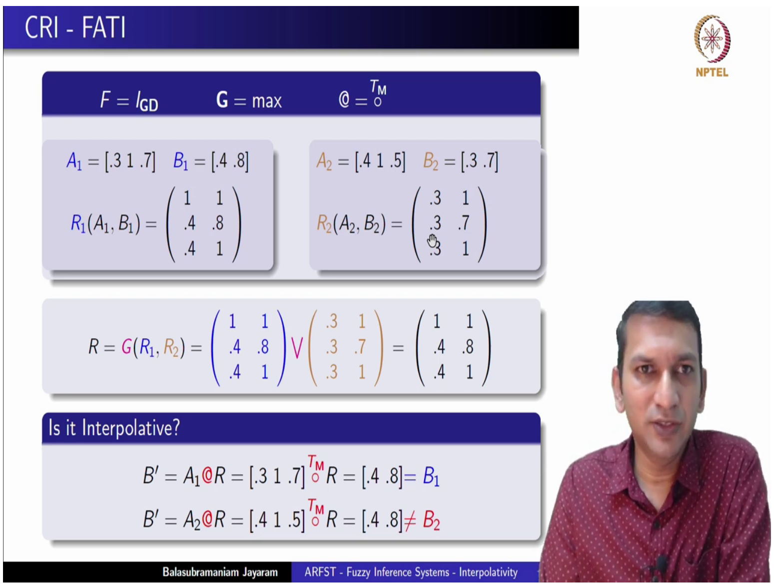

Let us look at G is max, the relations remain same R 1 and R 2, but we need to apply the max operation while combining the relations. So, now, we have seen that earlier itself that this is component wise bigger. So that means, R 1 max R 2 is going to be R 1. Now, if you apply the question of interpolativity here, if you give A 1 to this relation.

It can be easily verified that it actually gives B 1. So, if you consider this relation, this aggregated relation it seems to be interpolative at A 1 B 1. But, what about A 2? If you give A 2, unfortunately it does not give B 2. So, now, you see here by changing the aggregation function, you might make the function to be interpolative at one point, but unfortunately we are losing it at other point. So, now, this is with CRI and FATI.

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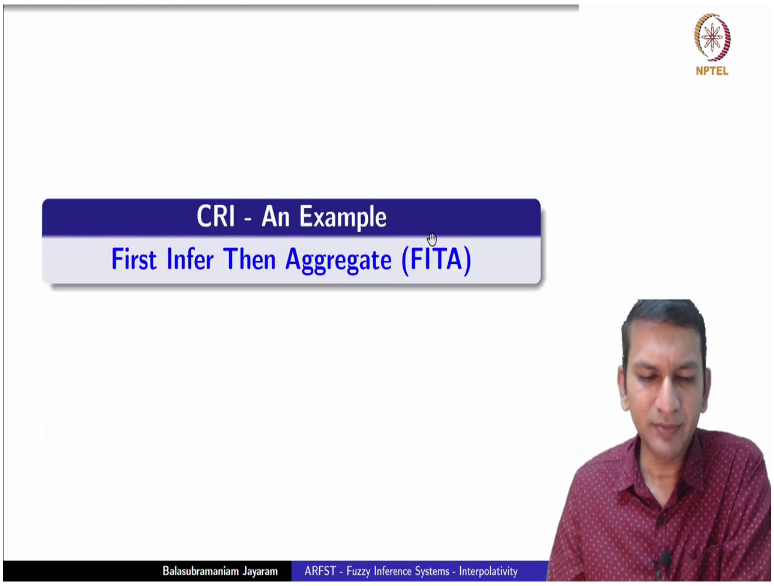


But what happens when we actually use FITA? Let us look at it.

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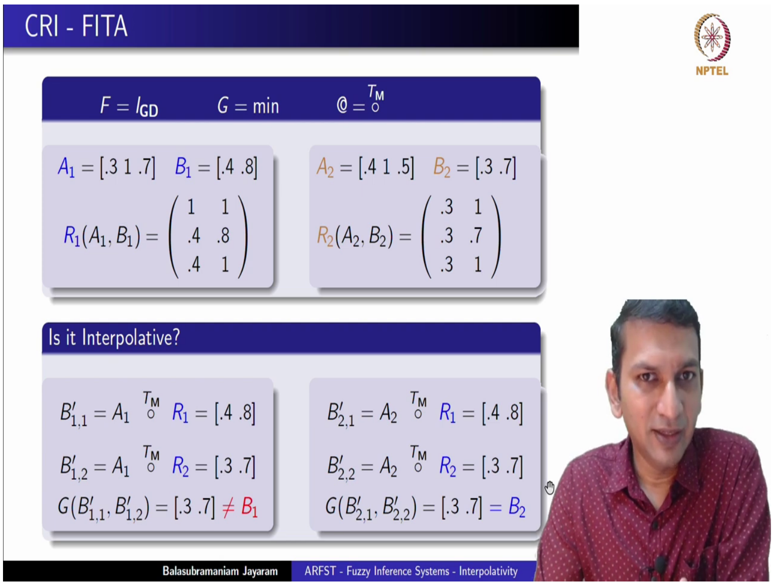


Here, once again the relations are same R 1 R 2, but what we are going to do is we are first going to infer and then aggregate. So, let us infer giving A 1 to R 1. So, we have seen that we would get 0.4 0.8 as B 1, 1 dash. The same A 1 we are also composing with R 2 to obtain the other local output. So, A 1 composed with R 2, but we have seen earlier when we took this relation, it turns out to be 0.3 0.7. And, when G is equal to min, when you combine both of these what you obtain is 0.3 0.7 which is not equal to B 1.

What happens when we actually look at interpolativity at the point A 2 B 2, are we going to get it? So, A 2 composed with R 1 is 0.4 0.8, we have seen that earlier too and A 2 composed with R 2 is in fact, 0.3 0.7. Now, if you apply min here, we see that it is actually equal to 0.3 0.7 which is B 2. So, once again just as in the case of FATI with this set of operations even if you perform FITA, what we say is it is interpolated at the point A 2 B 2, but not interpolated at the point A 1 B 1.

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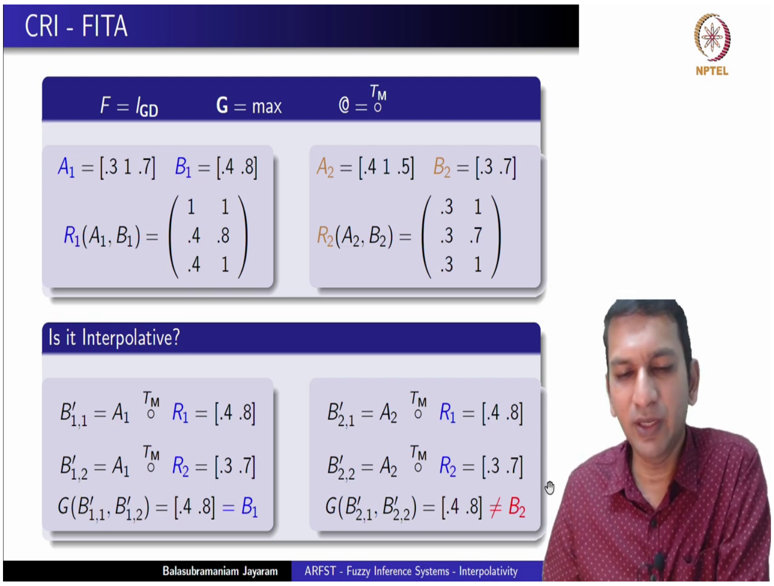


What if you use max? Similar, is the story as you can see with min it was interpolated at A 2 B 2, but not A 1 B 1. If you use max, the rules reverse. It is interpolative at A 1, but not interpolative at A 2.

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Now, let us look at BKS.

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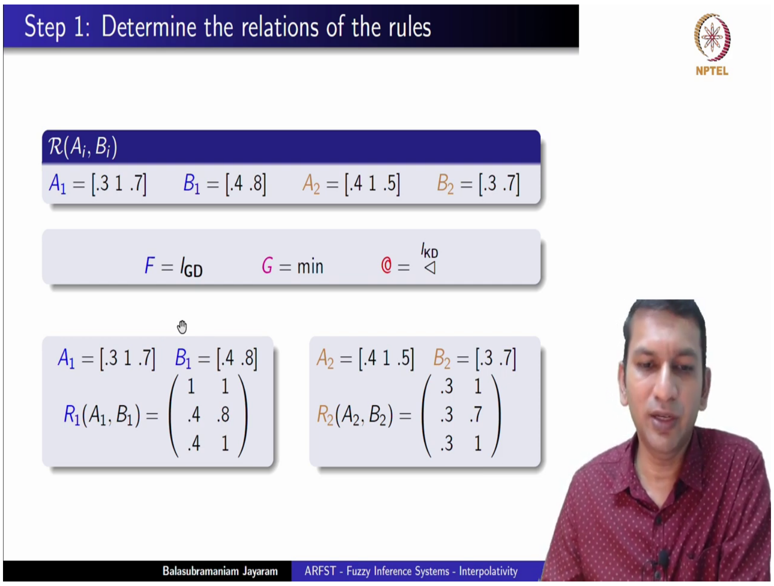


Once again, we determine the relations first. Of course, the operations F and G we keep them the same, Godel implication of the minimum operation for aggregation. However, it change composition. It is clear it is not going to have any effect on the relations obtained because, we are using the Godel implication to obtain the relation and we have seen that this is the relation that we obtain R 1 R 2. Now, given these two relations, let us apply FITA and FATI inference strategies in this manner course of product scheme.

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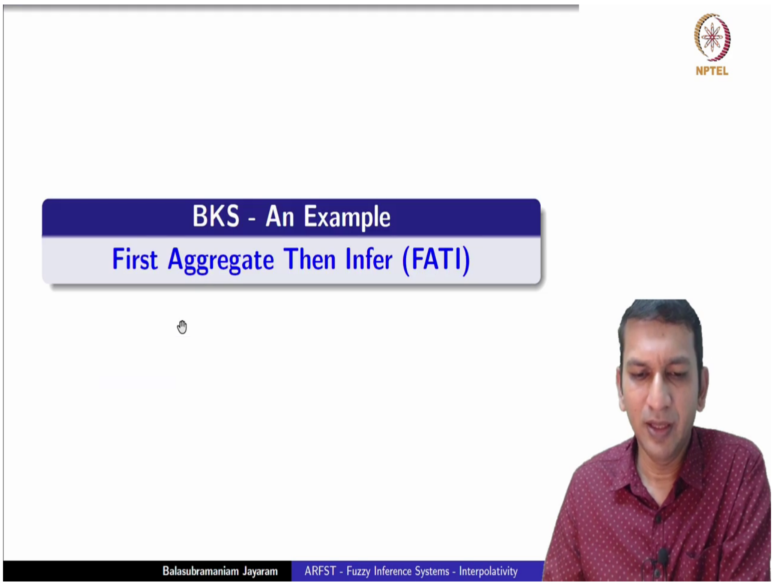

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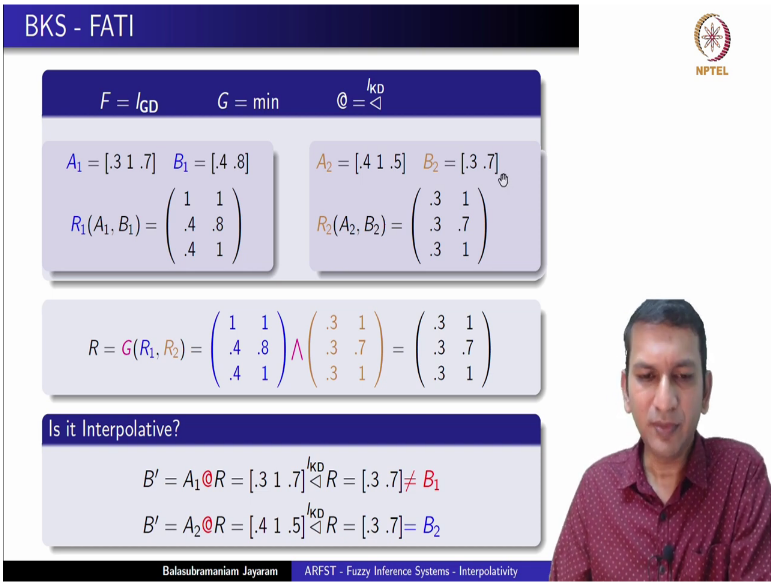


So, let us apply FATI; that means, first we need to aggregate this relations. We have seen that when we aggregate them using min, the R 2 is smaller than R 1 so, you get R 2. Now, let us see whether it is interpreted which means we take A 1 which was earlier A and use the same Kleene Dienes implication on R. Let us look at what happens here.

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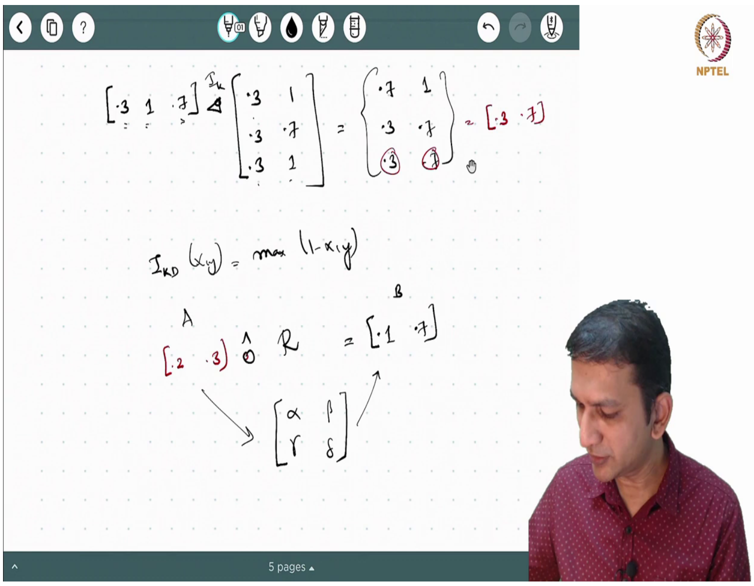


Let us record this. So, what we have is A 1 which is 0.3 1 and 0.7 composed with R 0.3 0.3 0.3 1 0.7 1. Note that we use Kleene Dienes implication I KD of x y max of 1 minus x comma y. So, the intermediate step you take 0.3 and 0.3, it is going to be 0.7, 1 and 0.3 is 0.3, 0.7 and 0.3 7, 0.3 and 1 is 1, 1 and 0.7 is 0.7, 0.7 and 1 is 0.7.

Note that we have calculated I KD of each of these, but we have taken min amount them. So, the min here is clearly the surface. So, the overall output 0.3 0.7 which is not equal to B. Similarly, if we calculate for A 2, we see that it is 0.3 0.7, that it is actually equal to B 2.

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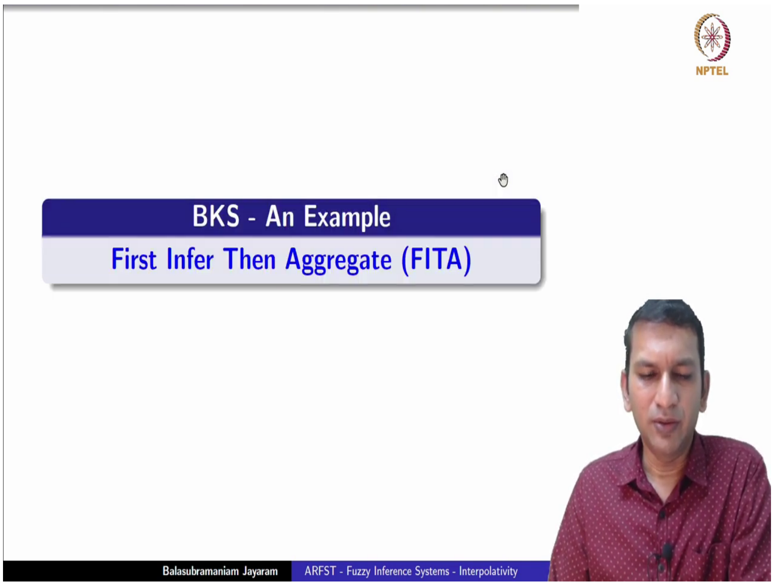


Now, this is what we got from FATI using BKS. We have seen once again it is interpolative at the point A 2 B 2, but not at A 1 B 1.

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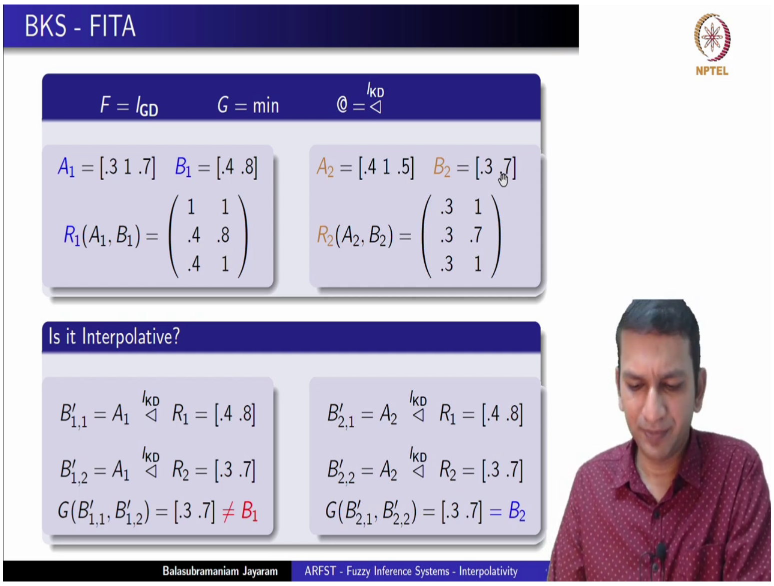


Let us apply the FITA inference strategy, if you ask is it interpolative. So, let us apply A 1, take A 1 and compose it with R 1. These are the values that you get. Once again by taking minimum aggregation and say that A 1 does not lead to B; however, A 2 will lead to B 2. Once again by changing the inference strategy, we have not obtained interpolativity in both the points; still it is not interpolative at the point A 1 B 1.

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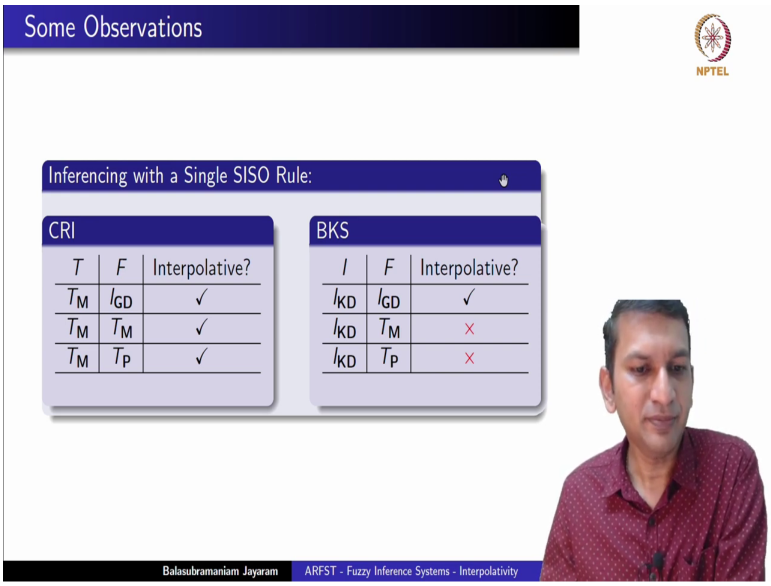


Let us quickly recap the different operations, we have used and whether it is interpolating or not. With a single SISO rule and if you are using CRI, for these two operations taking T M to be the T in the T norm is the composition and Godel implication as the F to obtain the relation, we found that it is interpolated; at least in the example that we have taken.

So, is the case when both of them are minimum T norms and when one is minimum T norm, the other is product; we again obtain that it is interpolative. Once again note that for the rule that we have considered and fuzzy sets A and B that we have considered. However, in BKS what we have seen is only when I and F are implications, one of them being Kleene Dienes and other being Godel, we obtained interpolative.

If you fix I to be the Kleene Dienes implication and vary F to be a minimum T norm or the product norm, we have seen it is not interpolate. This is for a single SISO rule.

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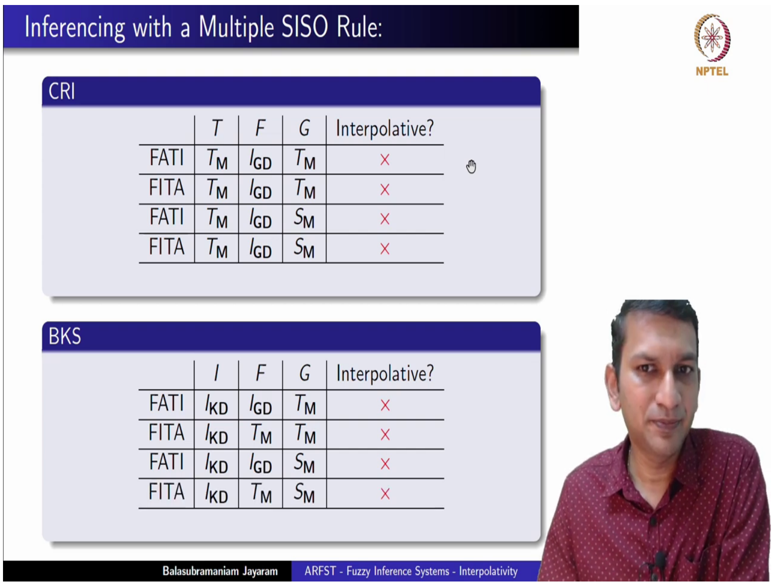


In the case of multiple SISO rule, we have seen that we need to introduce the aggregation of G. Keeping T and F as the combination that we have considered earlier, we have seen that no matter what we do whether it is FATI or FITA inference; whether we use max or min aggregation, we have seen that it is not interpolated.

And, similarly the case with BKS, keeping I and F to be both implications Kleene Dienes and Godel. In the case of single SISO rule, we did obtain that it was interpolated. However, in the case of multiple rules, we see that whether we apply the FATI or FITA strategy with G being the min or max aggregation, we see that it is not interpolative.

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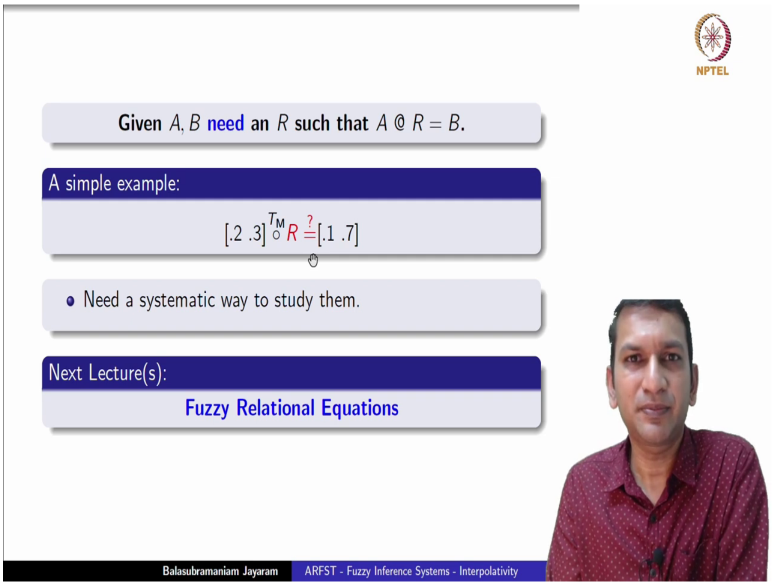


Now, what we need at least in the single SISO rule cases given a fuzzy if then rule A implies B; so, essentially A and B are specified, we need an R. We need a relation such that A composed with R is equal to B. The question is such an R always available? For example, consider this simple example.

So, now, we have 0.2 0.3, we want an R, we want an R such that an in composite using sup min composition, if this is A we want this to be B. So, this is what we are discussing here. But, now how should this R be? Let us call this R to be alpha beta gamma. So, we want to compose A with this to obtain this. Now, when you look at this, what are the values that alpha and gamma can take?

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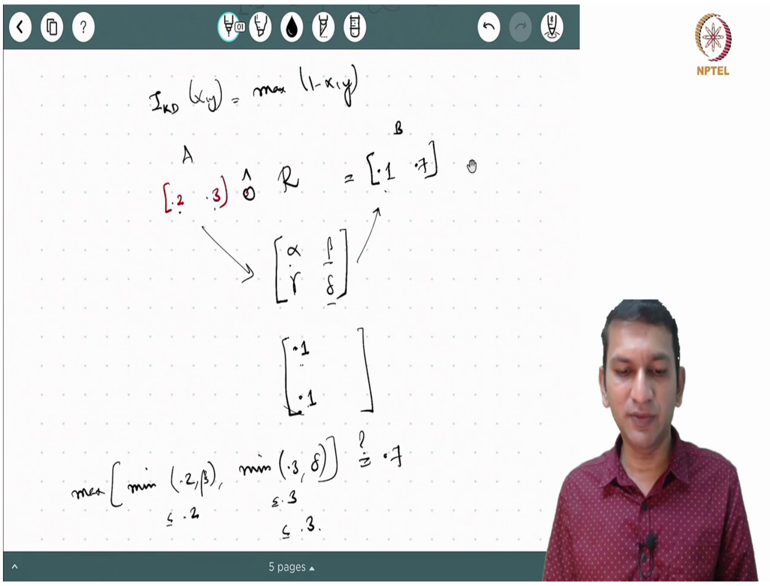

Note that, we are taking minimum of 0.2 1 alpha of 0.3 and gamma and then we are taking the maximum and we want that to be equal to 0.1. So, clearly we need to put alpha and gamma to be 0.1, only then minimum of 0.1 0.1 will be 1, 0.1 0.3 and 0.1 will be 0.1 and maximum among them will be 0.1. Of course, we could also take one of these to be less than 0.1, but can never go beyond 0.1. Now, the question is what can be the values of beta and delta?

Note that, if you take minimum of 0.2 comma beta, minimum of 0.3 comma delta and I take the maximum of them, question is can we get answer? Clearly minimum of 0.3 delta is going to be less than equal to 0.3, minimum of 0.2 comma beta is going to be less than or equal to 0.2, maximum of these two will actually be less than or equal to 0.3 which means we will never be able to get answer.

So, clearly for this A and this B, there does not exist any R such that A sup min composed with R is going to give you the value, the fuzzy set B which is 0.1 0.7. So, now you see we are actually going back to discussing relational equations much like matrix equations, we are going to discuss relational equations. But, to determine R means we need to do a systematic study of them.

This is what we will take up in next few lectures, during this week. We will discuss fuzzy relational equations, because they play an important role while discussing the interpolativity of fuzzy relational inference. So, in this lecture, all we have seen is we have taken a couple of examples and seen whether interpolativity is available automatically.

We have seen that not always, clearly there are examples for certain combination of operations they are not interpoledtive which means we need to look deeper into perhaps the fuzzy sets themselves. The type of fuzzy sets that we can use as antecedents and consequence and also the combination of operations. And, what properties these operations should process so, that we can get interpolativity.

Remember, we have only considered the case of single SISO rule or couple of multiple rules, in the case of multiple rules just two of them. But, you could have many many more of them and so, we need to be careful and devise a mechanism where we handle this multiplicity correctly and efficiently.

So, towards this end in the next lecture, we will look at some sufficiency conditions which can ensure interpolativity, interpolativity when we consider a single SISO rule in both CRI and BKS. Glad that you could join us for this lecture. Hope to see you soon in the next lecture.

Thank you.