Approximate Reasoning using Fuzzy Set Theory

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Lecture - 36
Introduction to Building a Mamdani FIS

Hello and welcome to the 4th of the lectures, in this week 7 of this course titled Approximate Reasoning using Fuzzy Set Theory. A course offered over the NPTEL platform. In this lecture we will see how to build a Mamdani fuzzy inference system to approximate a given function. Towards the end we will take the help of the fuzzy logic toolbox that is available with Matlab.

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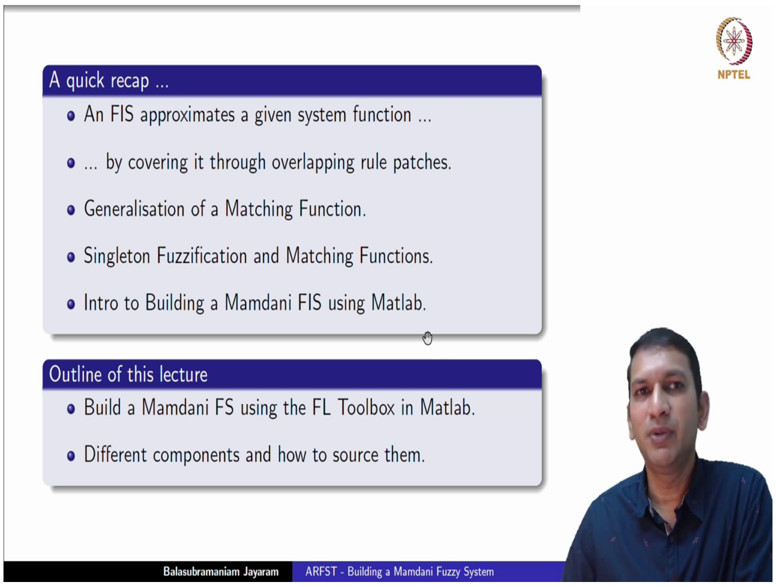

A quick recap ...

- An FIS approximates a given system function ...
- ... by covering it through overlapping rule patches.
- Generalisation of a Matching Function.
- Singleton Fuzzification and Matching Functions.
- Intro to Building a Mamdani FIS using Matlab.

Outline of this lecture

- Build a Mamdani FS using the FL Toolbox in Matlab.
- Different components and how to source them.

A quick recap of the salient points that we should remember when we want to build a fuzzy inference system. We know that a fuzzy inference system approximates a given system function by covering it with overlapping rule patches. We have also seen that matching functions from the given matching functions like Zadeh and that of Smet Magrez, we came up with a generalization of a matching function and it is this matching function that is implemented in the fuzzy logic toolbox that is available with Matlab.

So, we will look at using this for the current lecture. We have also seen that for this class of matching functions in the presence of singleton specification that the similarity value of an input essentially becomes the membership value of that input x naught to the fuzzy set represented by the corresponding antecedent. We have seen in the last lecture a quick intro, general intro to build how what it means to build a Mamdani fuzzy inference system using Matlab, what are the components that we need to look into.

In this lecture we will actually build a Mamdani fuzzy inference system using the fuzzy logic toolbox in Matlab. And we look at each of the components and how to source them, how to find them, how to fine tune them, all of these things we will see by taking two simple functions and trying to build a Mamdani FIS to approximate it.

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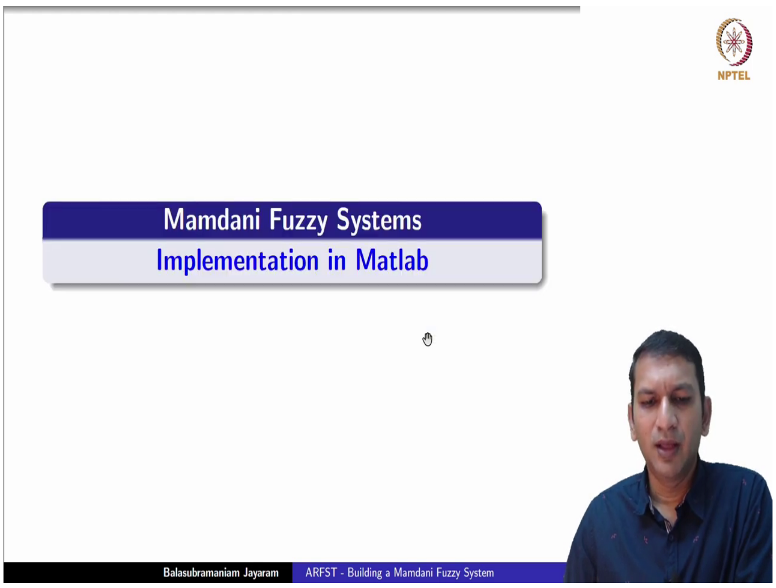

Mamdani Fuzzy Systems

Implementation in Matlab

A quick reminder about the terminology.

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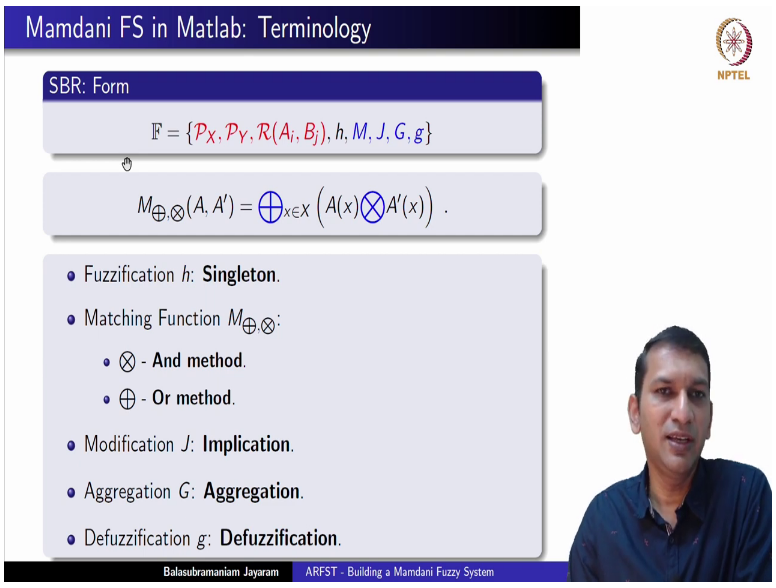

Mamdani FS in Matlab: Terminology

SBR: Form

$$\mathbb{F} = \{\mathcal{P}_X, \mathcal{P}_Y, \mathcal{R}(A_i, B_j), h, M, J, G, g\}$$

$$M_{\oplus,\otimes}(A, A') = \bigoplus_{x \in X} \left(A(x) \bigotimes A'(x) \right) .$$

- Fuzzification h: **Singleton**.
- Matching Function $M_{\oplus,\otimes}$:
  - $\otimes$ - **And method**.
  - $\oplus$ - **Or method**.
- Modification J: **Implication**.
- Aggregation G: **Aggregation**.
- Defuzzification g: **Defuzzification**.

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So, we know that a similarity based reasoning consists of these many parameters of degrees of freedom, P X is the fuzzy covering on X, P Y the fuzzy covering on Y, R of A i, B j it represents the rule base that we have considered, where Ai the antecedents are coming from P X and B j's the consequences are coming from P Y, h is the fuzzifier, M is the matching function, J is the modification function. G is the aggregation of the output, so it is an aggregation function and small g is essentially the de fuzzifier for us.

This is the family of matching function that we will be looking into in this lecture. So, essentially to specify or fix a matching function would mean to essentially fix these two operations, which are typically fuzzy logic connectives and at least the outer operation needs to be associated.

In Matlab the fuzzification is singleton fuzzification by default and the matching function is called the And method and Or method. So, the combination of these two will give rise to the matching function, these are the corresponding inside and outside operations. Modification function is called the implication, aggregation and de fuzzification are known by the same terms.

Now, what is important for us to realize this there are these many degrees of freedom, let us for the moment go with the default fuzzification that is available for us the singleton, which means rest of these 7 degrees of freedom will have to be fixed. Even though it appears like 7, we have to be very careful here because in matching function we again have two degrees of freedom these are the parameters or degrees of freedom highlighted in blue are perhaps easily fixed in Matlab.

And it is easy to fine tune them. Whereas, these which are highlighted in red we need to be extremely careful. Of course, we can fine tune them also, but we need to be very careful and choose intelligently these parameters. What are they? P X, the fuzzy covering on X, fuzzy covering on Y and once we have picked up these coverings, how to relate a fuzzy set from P X to fuzzy set in P Y to relate them as rules. We will see this for two different functions.

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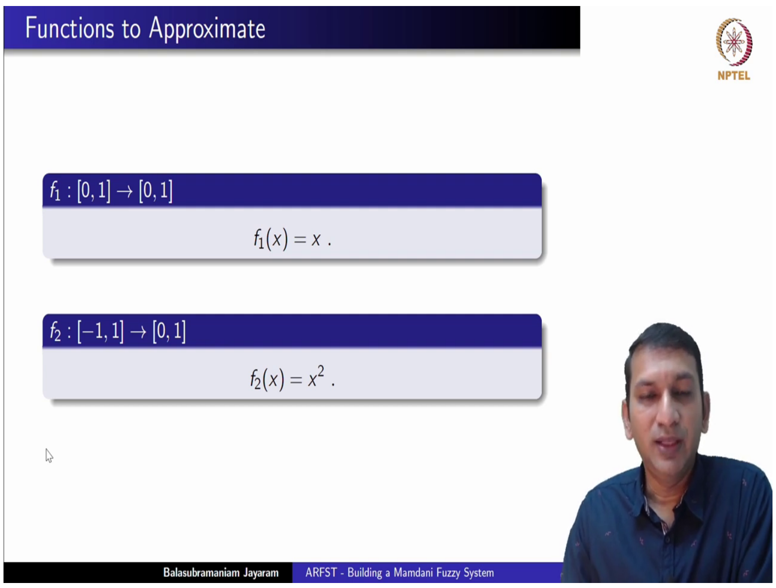

Functions to Approximate

$f_1 : [0,1] \to [0,1]$

$f_1(x) = x$.

$f_2 : [-1,1] \to [0,1]$

$f_2(x) = x^2$.

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The first function that we will consider is as was mentioned earlier too is the identity function, which is a simple linear function.

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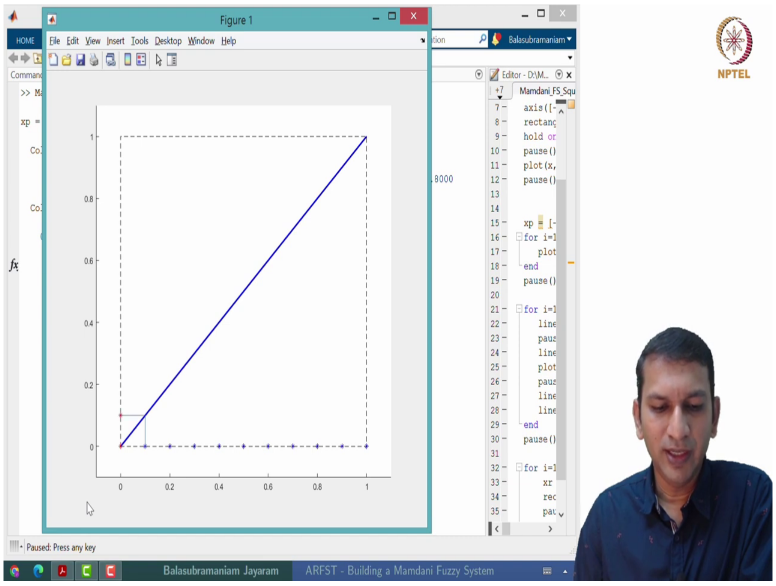
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rectang
hold on
pause()
plot(x,
pause()


xp = [-
for i=1
    plot
end
pause()

for i=1
    line
    paus
    line
    plot
    paus
    line
    line
end
pause()

for i=1
    xr
    rec
    pau
```

Now, let us look at building a Mamdani fuzzy inference system for this identity function. So, we have the domain here, X is from 0 to 1, Y is from 0 to 1. So, essentially the graph of this function is going to be within the square.

This is the graph of this function, the identity function which is essentially the diagonal on this square. The first question is how do we choose P X and P Y, the fuzzy coverings. Now, typically we need to position these fuzzy sets on the domain X and what we do is pick a few points in the domain of X, let us say that we pick arbitrarily these 11 points, if we spaced points from the [0,1] interval, which is our domain X.

And next we can build a fuzzy set around this. So, we have seen that we could build a triangular fuzzy set at each one of these points. So, essentially taking this as the center and defining what should be the length on the left and right side. Now, similarly we also need to look at finding points on the domain Y, which is our range here. Towards this end since we know the function here, all we need to do is for each of these X is that we have chosen let us look at what are the corresponding F OF X i.

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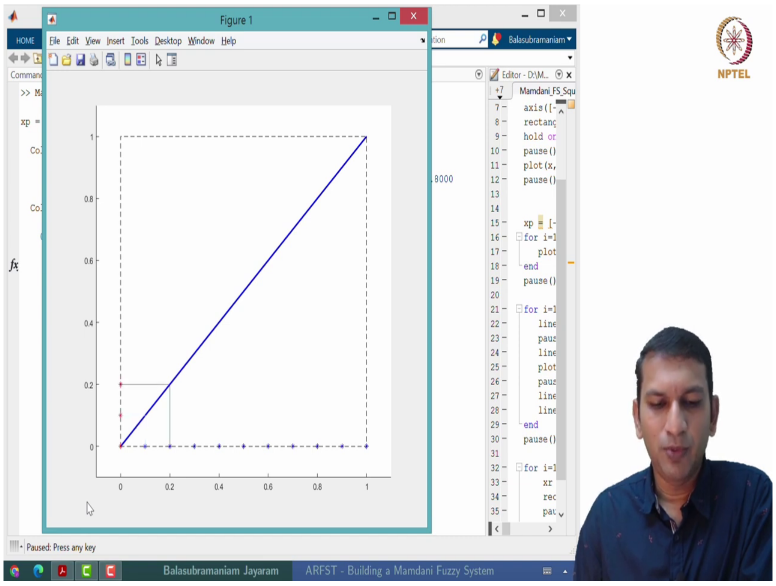
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pause()

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for i=1
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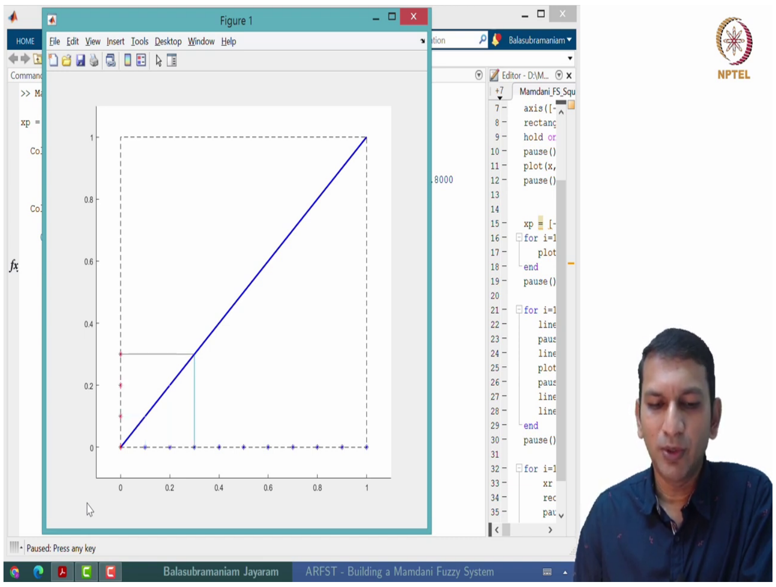
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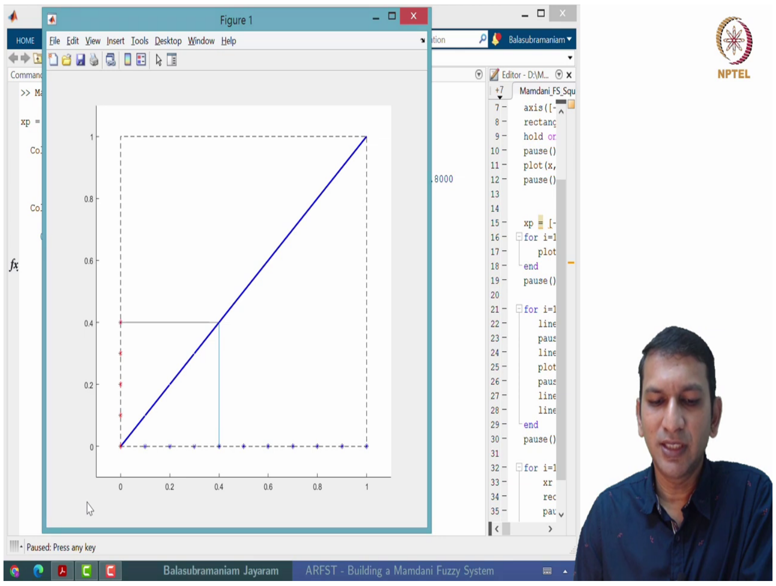
Figure 1
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axis([-
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hold or
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plot(x,
pause()

xp = [-
for i=1
    plot
end
pause()

for i=1
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end
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for i=1
    xr
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```

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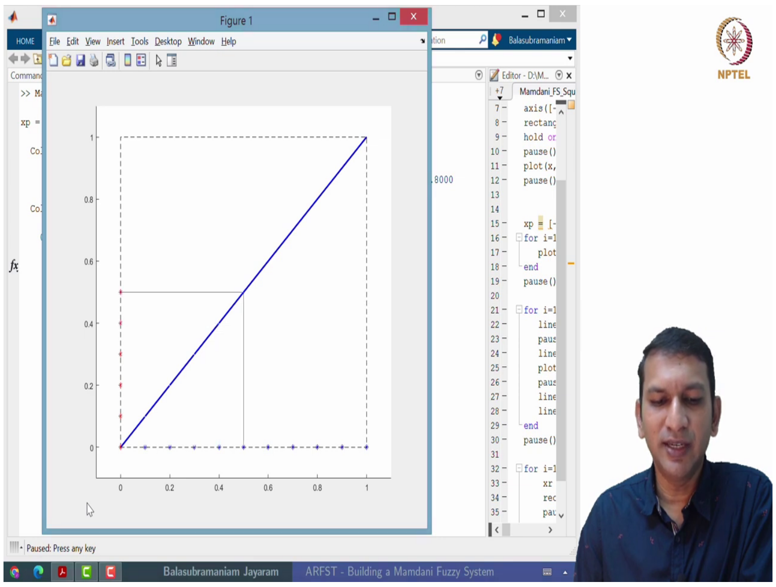
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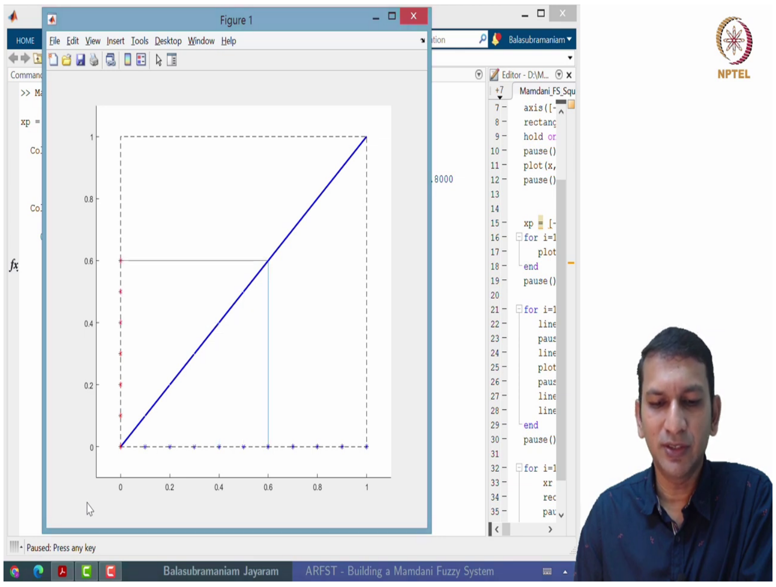

So, now, if you take 0, it is 0, if you take 0.1 it is 0.1, 0.2 it is 0.2 and so on. Because it is essentially the identity function.

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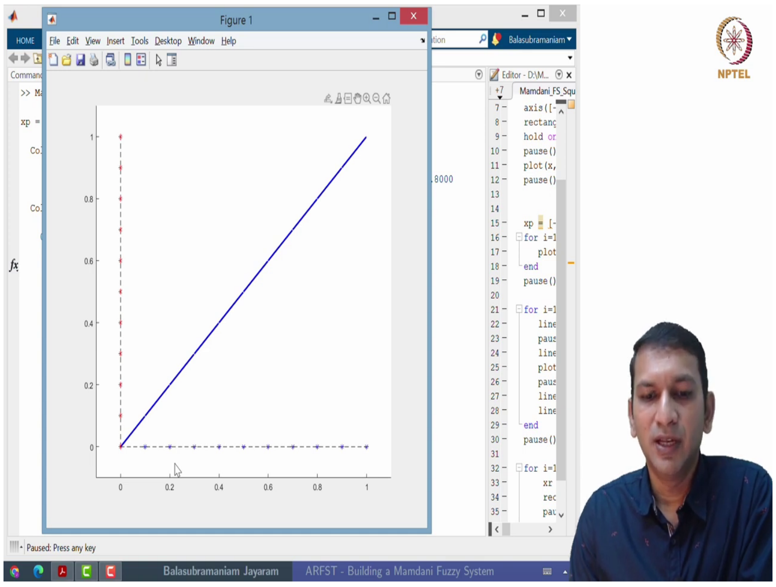

So, now, that we have this let us assume that we are going to take these points as the prototypical points and we are going to build fuzzy sets around them, in such a way they actually form a cover of the domain and the range. The domain of Y and domain of X.

Now, it is clear that if we actually draw triangular fuzzy sets which each of these points in the as the middle, center and extending almost to either to both of the neighbouring points, to the left and the right, then it is clear that they will form a cover. In fact, if you take the triangular fuzzy set, they will in fact, form a spinny partition. Now, what next? once we have this how do we relate the fuzzy sets on x, to fuzzy sets on Y to build a rule base.

Now, this is where the idea of patching the system function with rule patches comes into picture. So, now, one way to do it is let us look at how to actually cover this function with rule patches.

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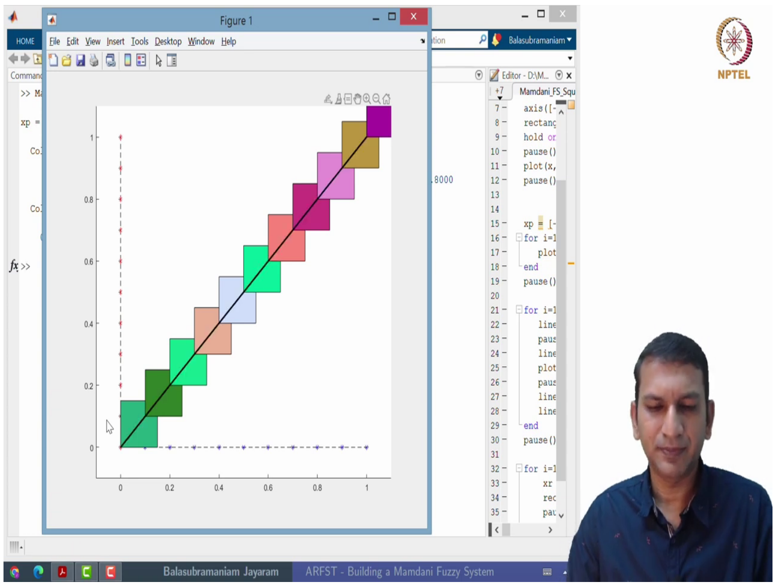
Figure 1
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So, perhaps one way to do it is like this, you will see that this rule patch essentially starts at this point maybe if you call this x 0 and x 1. Starts at x 1 and perhaps goes little more than x 2. And similarly, here starts at y 1 and goes little more than y 2 and this is how the root patches are.

So, it is clearly indicative that we are perhaps looking at supports of these fuzzy sets to actually cover these intervals on both x and y axis. We will use this idea to build the fuzzy system. So, essentially, we are just covering these covering this function using this patches and this gives us an idea as to how we need to form the fuzzy covering on y and also on x.

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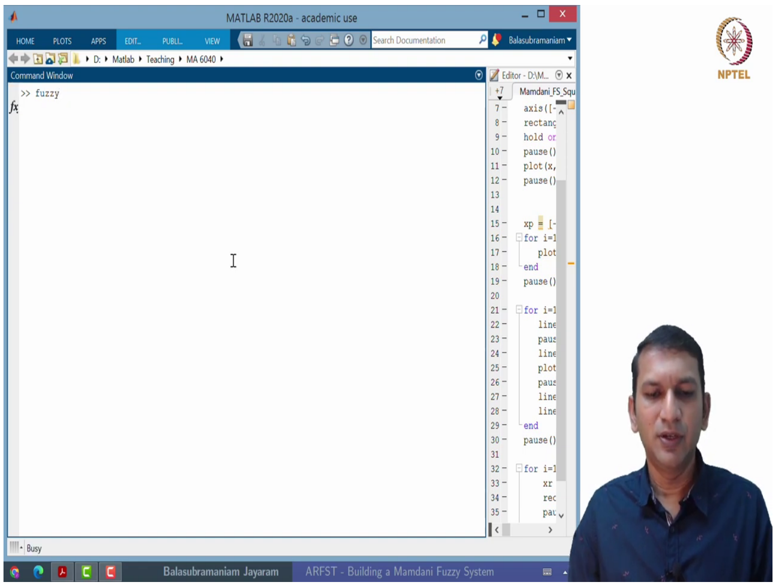
MATLAB R2020a - academic use
HOME PLOTS APPS EDITOR PUBLISH VIEW
Command Window
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```
>> fuzzy
```

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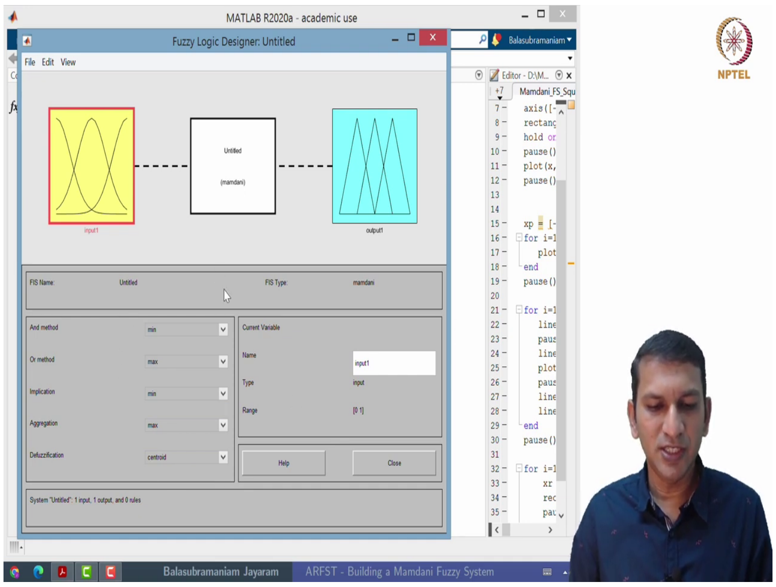
MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
File Edit View
input1
Untitled
(mamdani)
output1
Help
Close
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FIS Name: Untitled FIS Type: mamdani

And method: min
Or method: max
Implication: min
Aggregation: max
Defuzzification: centroid

Current Variable
Name: input1
Type: input
Range: [0 1]

System "Untitled": 1 input, 1 output, and 0 rules

So, if you type in fuzzy, the term fuzzy in MATLAB what you would get is this window, a few things can be seen here. So, these are the parts which specify the different operation that we are going to use. So, as was mentioned this And method and Or OR method together they specify the matching function that we are going to use. If it is min and max, then essentially it is the Zadeh's matching function implication refers to the modification function you can use a minimum operator, which is min here already as given as default.

And for the aggregation there is this max and for defuzzification the default option is centroid. So, essentially with singleton fuzzification as the default and once we fix all these things. So, these two fix M, this fixes J, this fixes the aggregation G and this fixes the de fuzzified small g. Now, what we are left to do with are these three important things P X, P Y and the rule base itself R of A i, B j. So, let us start with that. So now, we need to define what the input membership functions are.

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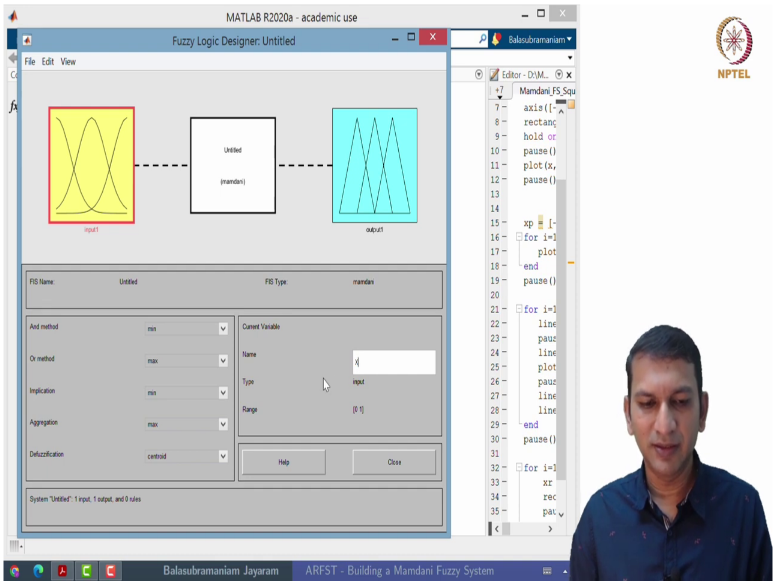
MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
File Edit View
input1
Untitled
(mamdani)
output1
Help
Close
NPTEL
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FIS Name: Untitled
FIS Type: mamdani
And method: min
Or method: max
Implication: min
Aggregation: max
Defuzzification: centroid
Current Variable
Name: x
Type: input
Range: [0 1]
System "Untitled": 1 input, 1 output, and 0 rules

Let us name this as X.

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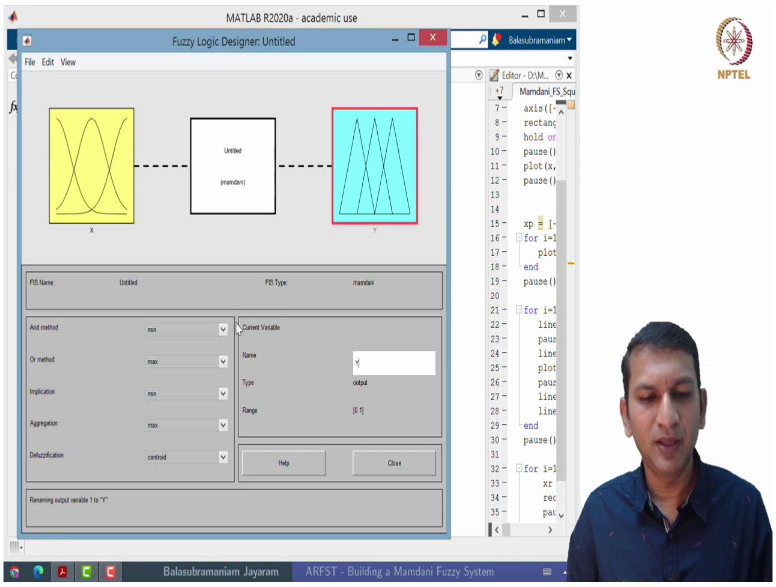
MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
File Edit View
X
Untitled
(mamdani)
Y
Help
Close
NPTEL
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FIS Name: Untitled
FIS Type: mamdani
And method: min
Or method: max
Implication: min
Aggregation: max
Defuzzification: centroid
Current Variable
Name: Y
Type: output
Range: [0 1]
Renaming output variable 1 to "Y"

And let us name this as Y.

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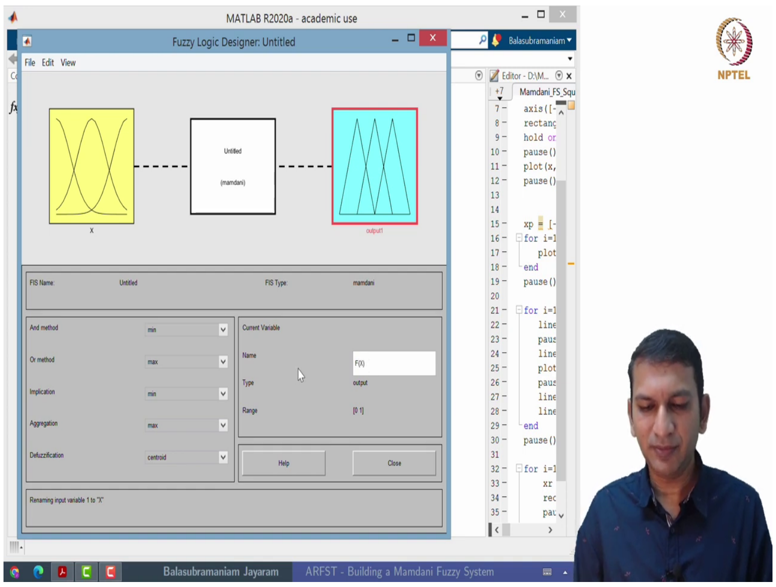
MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
File Edit View
X
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(mamdani)
output1
Help
Close
NPTEL
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FIS Name: Untitled FIS Type: mamdani

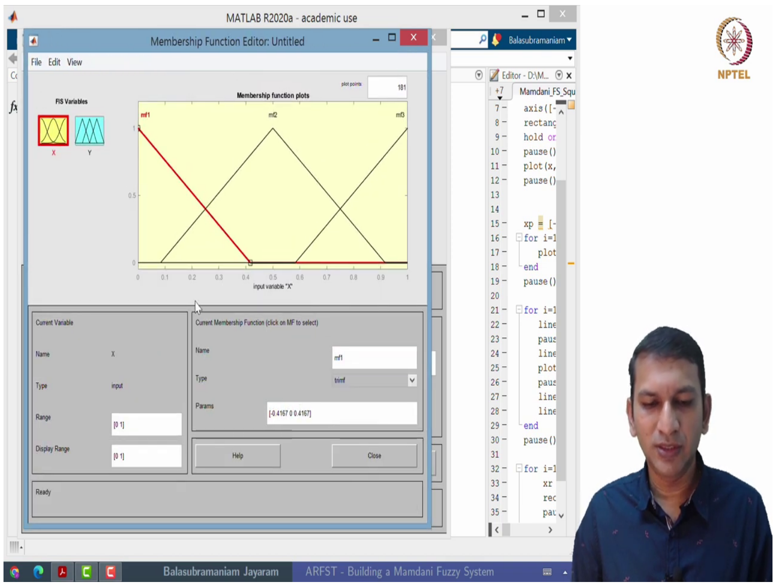
And method: min
Or method: max
Implication: min
Aggregation: max
Defuzzification: centroid

Current Variable
Name: F(X)
Type: output
Range: [0 1]

Renaming input variable 1 to "X"

So, it is a mapping from or maybe F(X), it is a mapping from F to F(X), that is why it is better.

(Refer Slide Time: 10:22)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File Edit View
FIS Variables
X
Y
Membership function plots
plot points 181
mf1
mf2
mf3
input variable "X"
Help
Close
NPTEL
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Current Variable
Name: X
Type: input
Range: [0 1]
Display Range: [0 1]

Current Membership Function (click on MF to select)
Name: mf1
Type: trimf
Params: [-0.4167 0 0.4167]

Ready

So, now if you double click on this you will see that MATLAB, the fuzzy logic toolbox loads three triangular membership functions by default and it takes the domain also to be 0. That is a default thing and as it happens happily this is the domain that we need also.

Now, the question is, how are we going to build a cover for this domain 0. We have got some inkling, we have seen how to do this from the previous figure that we generated that to take 11 points, 0, 0.1, 0.2 so on till 1; 11 equi space points on both the domain and on the co-domain and then build fuzzy sets around that.

Let us build triangular fuzzy sets around this; that means, if you look at it is called mf1, right now the names do not matter to us because we are just only approximating a pure mathematical function. So, let us assume mf1 refers to the first membership function. So, to remain consistent what we will do is we will position this mf1 around the point 0.1. So, now you see here, it is a trapezoidal membership function which means it takes 3 values.

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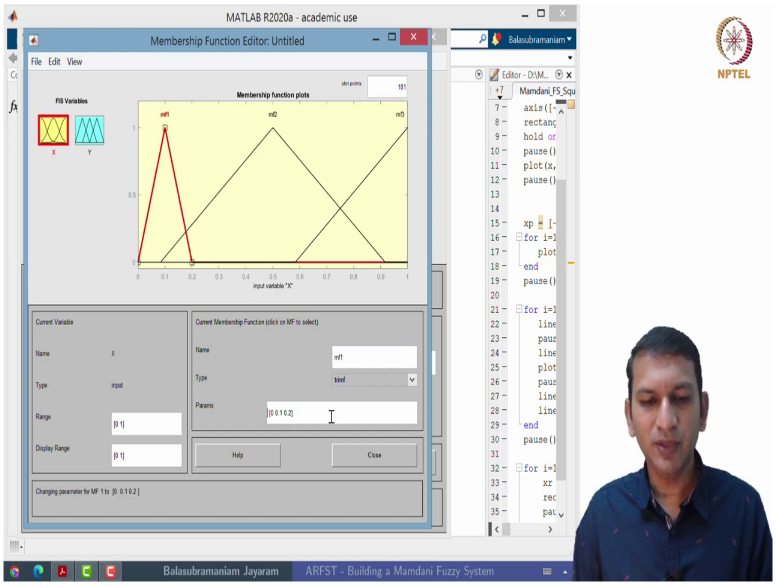
MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points 181
mf1
mf2
mf3
input variable "X"
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Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [0 0.1 0.2]

Help Close

Changing parameter for MF 1 to [0 0.1 0.2]

So, we want to position at around 0.1 and we allow it to extend till the previous 0.0 and the next point which is 0.2. So, you see here immediately it adjust.

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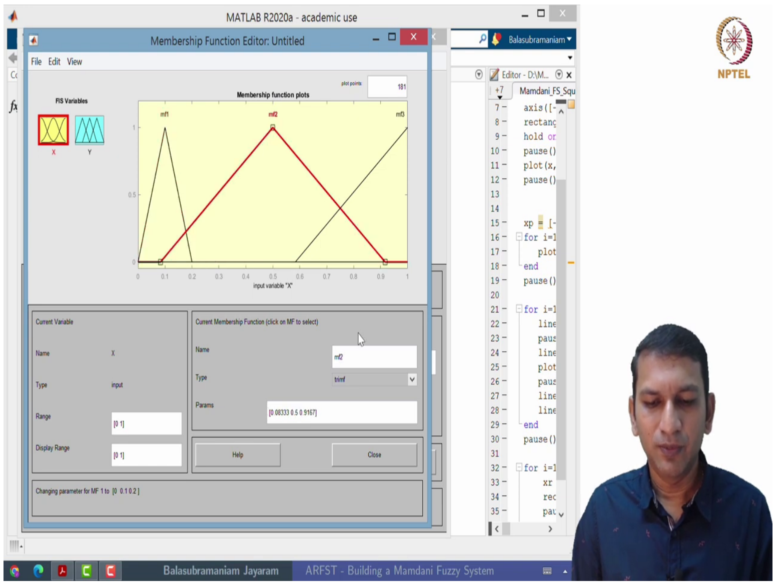
MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
plot points 181
mf1
mf2
mf3
input variable "X"
Help
Close
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Current Variable

Name X

Type input

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

Name mf2

Type trimf

Params [0.08333 0.5 0.9167]

Changing parameter for MF 1 to [0 0.1 0.2]

For mf2 what we will do is, we will do the same.

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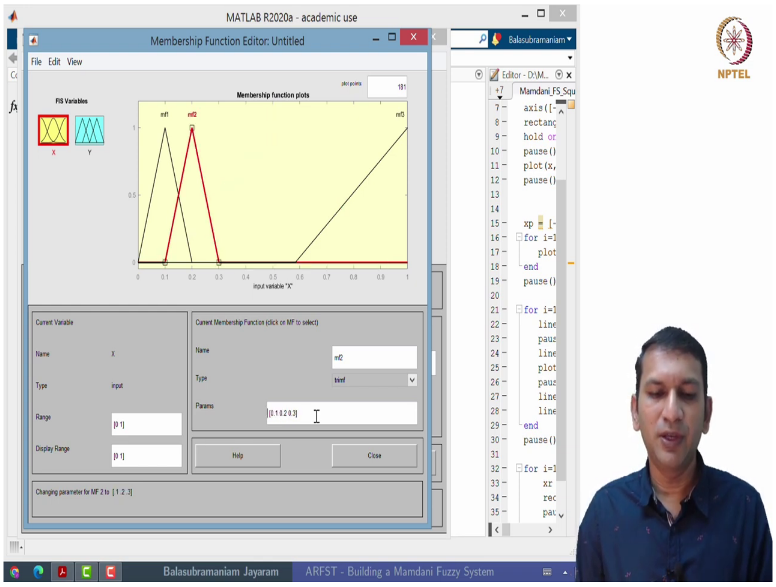
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FIS Variables
Membership function plots
plot points 181
mf1
mf2
mf3
input variable "X"
Help
Close
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Current Variable

Name X

Type input

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

Name mf2

Type trimf

Params [0.1 0.2 0.3]

Changing parameter for MF 2 to [.1 .2 .3]

We will position it from centering it at 0.2 and allow it to, allow its support to be from 0.1 to 0.3.

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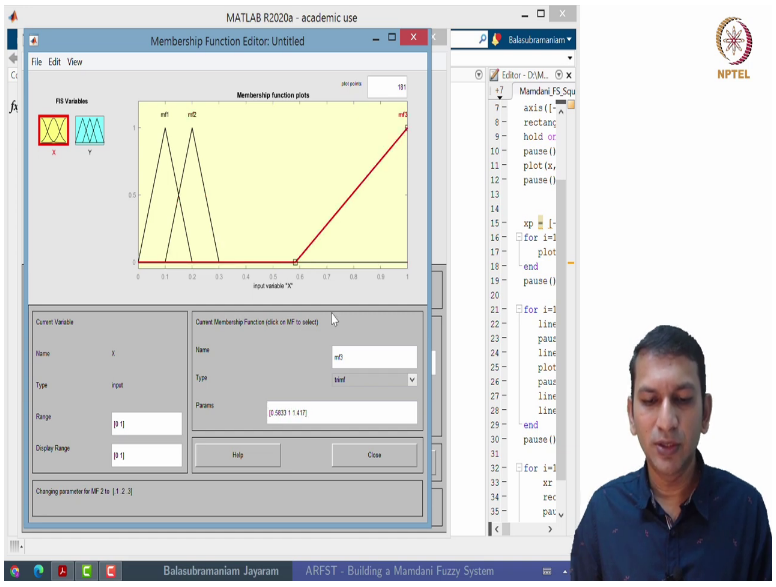
MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
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Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf3
Type trimf
Params [0.5833 1 1.417]

Changing parameter for MF 2 to [.1 .2 .3]

Now, we can do this for every one of these.

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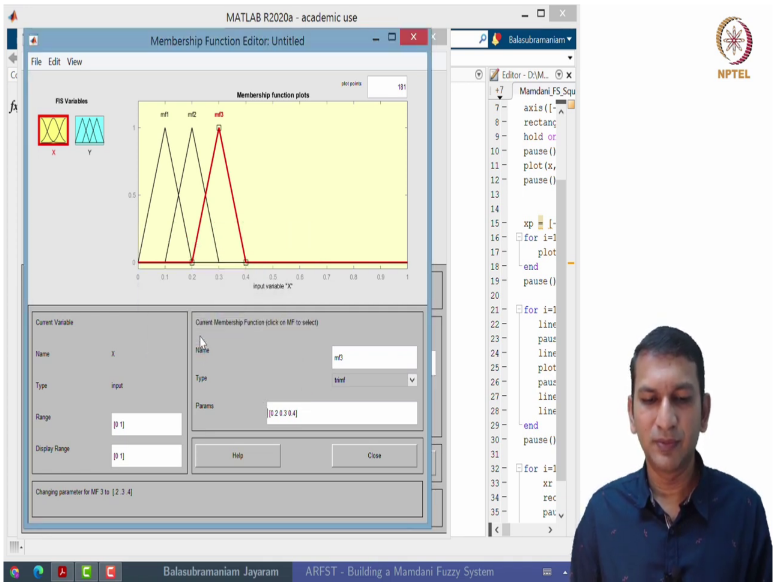
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Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
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Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf3
Type trimf
Params [0.2 0.3 0.4]

Changing parameter for MF 3 to [.2 .3 .4]

Allow me to quickly finish that, 0.2, 0.3, 0.4. So, now, we have these three fuzzy sets on this, clearly they do not form a cover.

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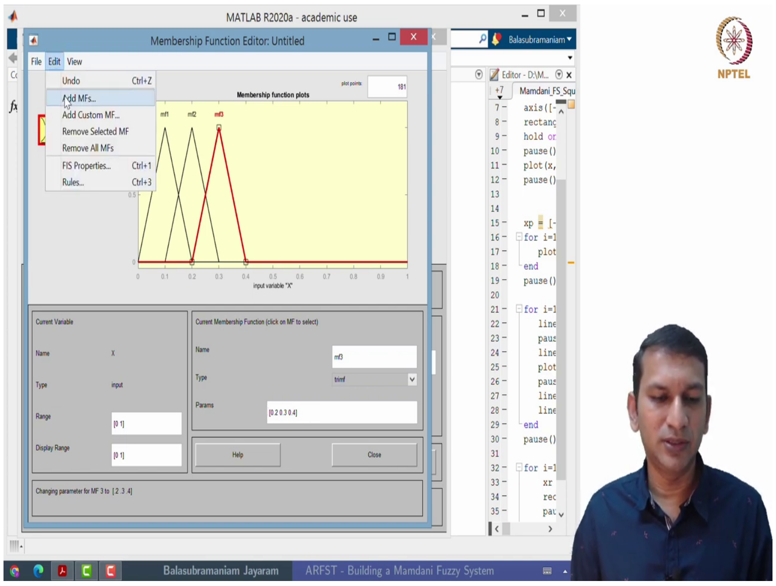

So, we need more fuzzy sets. What do we do?

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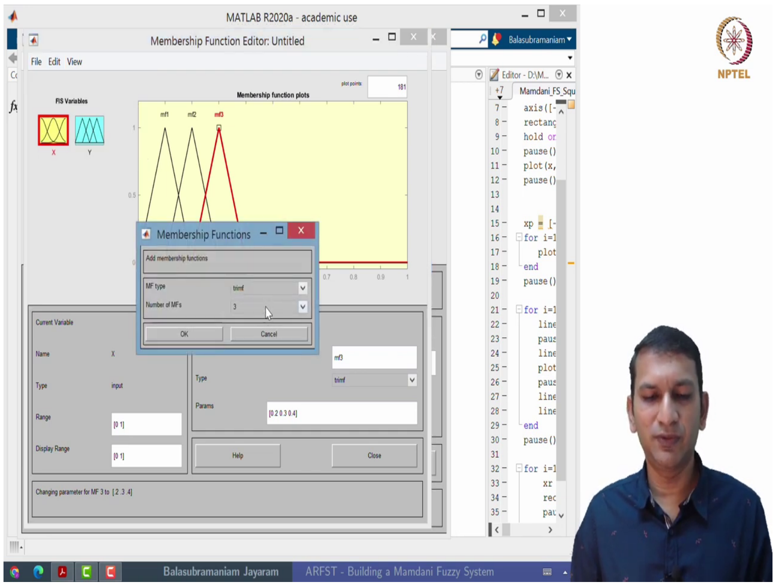
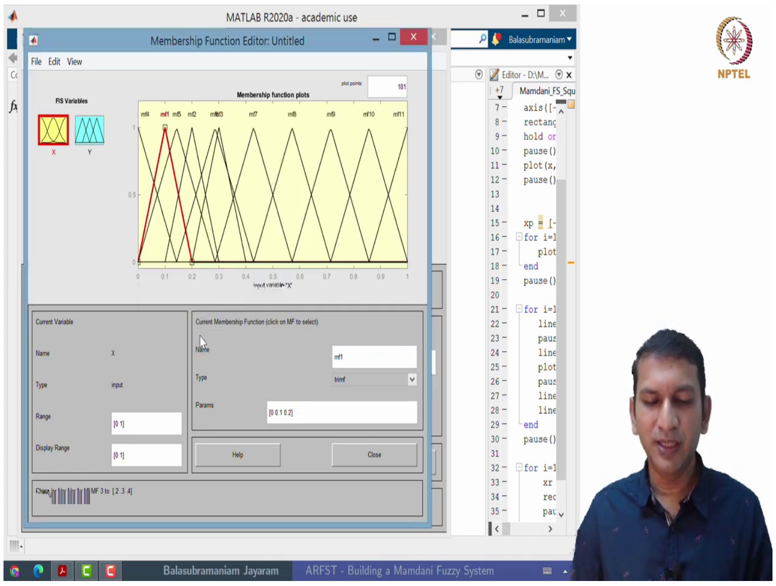


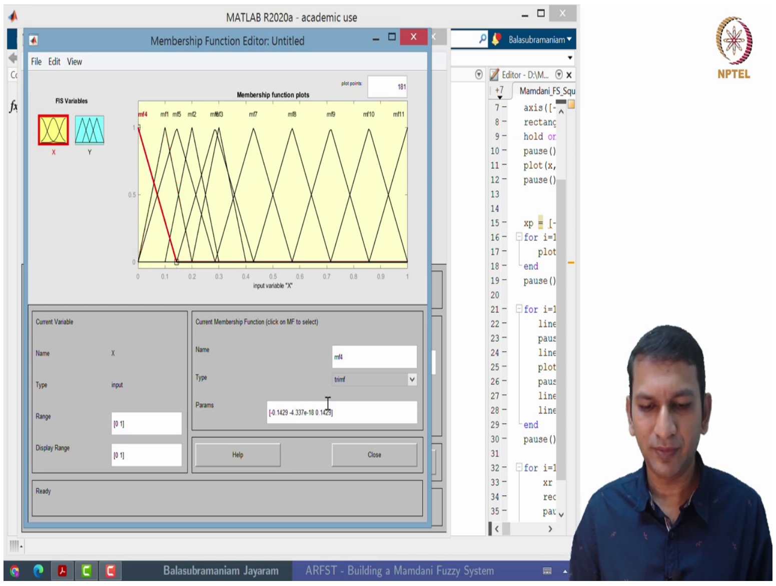
Go to this edit menu, add membership functions. Let us add triangular membership functions.

(Refer Slide Time: 12:33)



We have 3, we need another 8 more and you will see that they are everywhere of course, they are arbitrarily drawn.

(Refer Slide Time: 12:38)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]
Current Membership Function (click on MF to select)
Name mf4
Type trimf
Params [-0.1429 -4.337e-18 0.1429]
Help
Close
Ready
NPTEL
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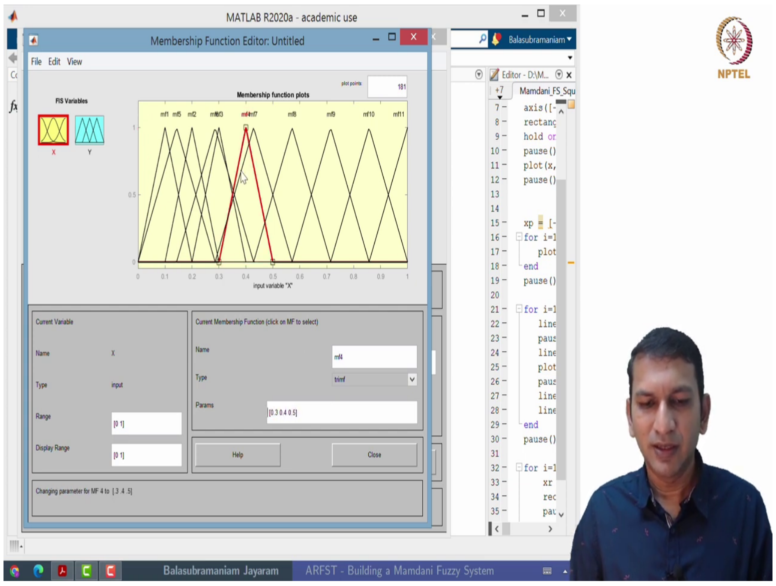
So, let us look at mf4.

(Refer Slide Time: 12:42)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
Balasubramaniam Jayaram
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NPTEL

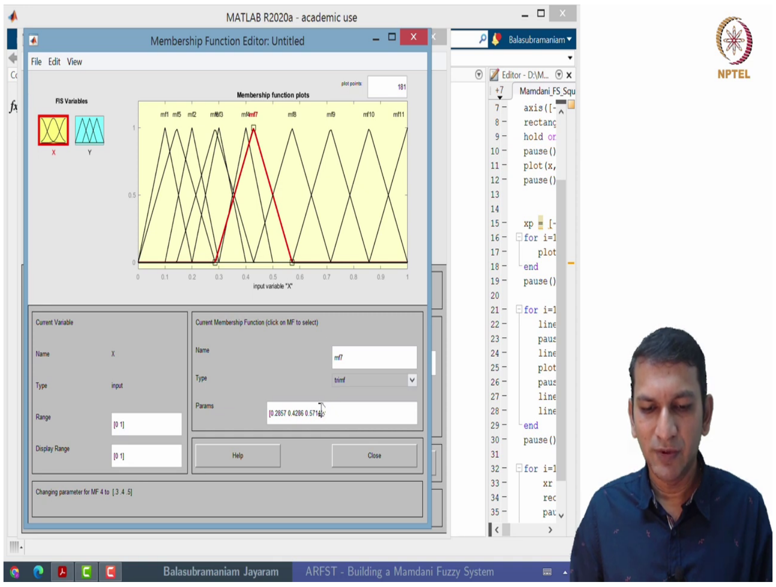
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf4
Type trimf
Params [0.3 0.4 0.5]

Changing parameter for MF 4 to [.3 .4 .5]

And by our convention what we want is we want to position it at 0.4.

(Refer Slide Time: 12:47)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
Balasubramaniam Jayaram
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NPTEL

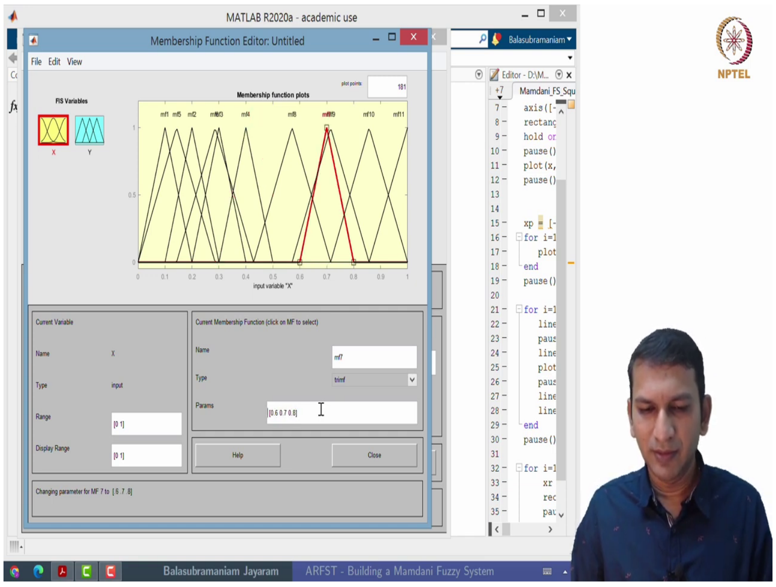
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf7
Type trimf
Params [0.2857 0.4286 0.5714]

Changing parameter for MF 4 to [.3 .4 .5]

Which means; so this is mf7.

(Refer Slide Time: 12:52)


MATLAB R2020a - academic use

Membership Function Editor: Untitled

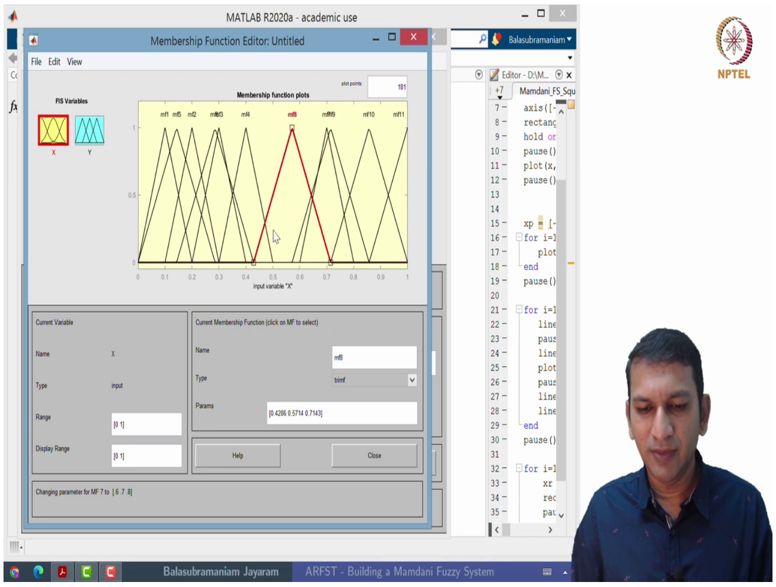
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf7
Type trimf
Params [0.6 0.7 0.8]

Changing parameter for MF 7 to [.6 .7 .8]

So, it makes it easy for us in terms of the labelling. So, it is centered at 0.7.

(Refer Slide Time: 12:58)


MATLAB R2020a - academic use

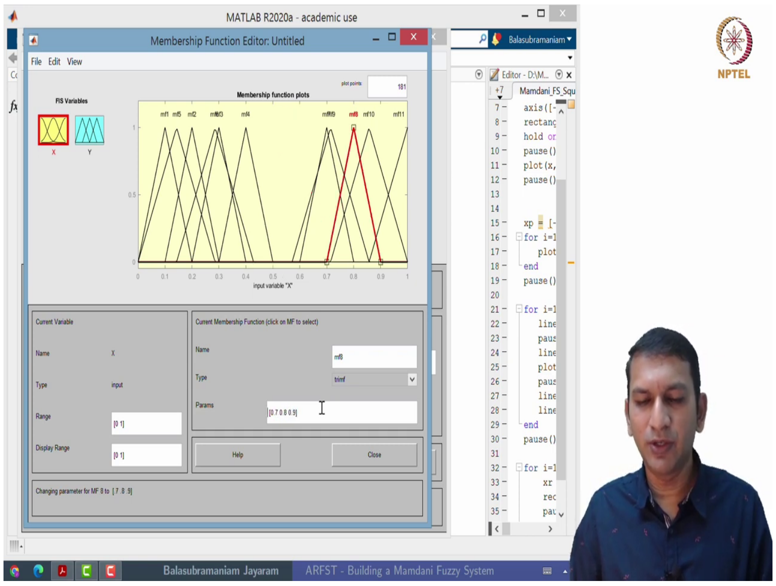
Membership Function Editor: Untitled

Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf8
Type trimf
Params [0.4286 0.5714 0.7143]

Changing parameter for MF 7 to [.6 .7 .8]

(Refer Slide Time: 13:02)


MATLAB R2020a - academic use

Membership Function Editor: Untitled

Current Variable

Name X

Type input

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

Name mf8

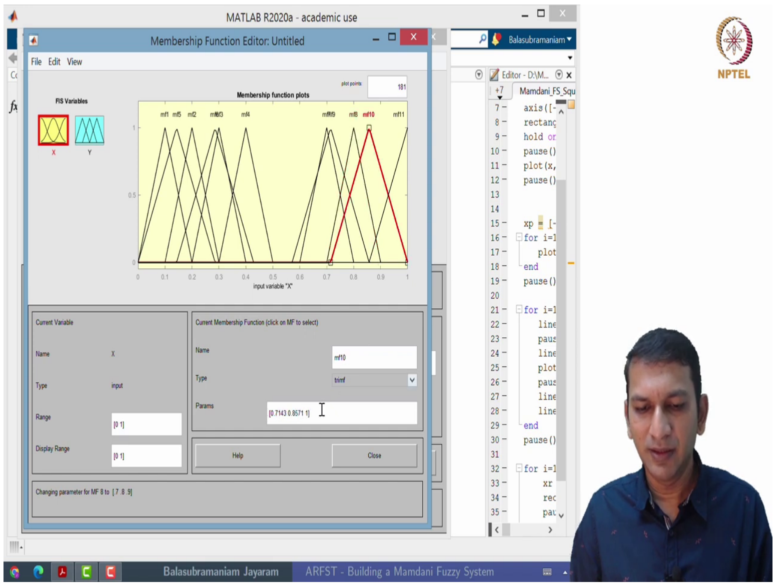
Type trimf

Params [0.7 0.8 0.9]

Changing parameter for MF 8 to [.7 .8 .9]

Mf8; so it will be centered at 0.8. So, it will the support of it will be from 0.7 to 0.9.

(Refer Slide Time: 13:09)


MATLAB R2020a - academic use

Membership Function Editor: Untitled

Current Variable

Name X

Type input

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

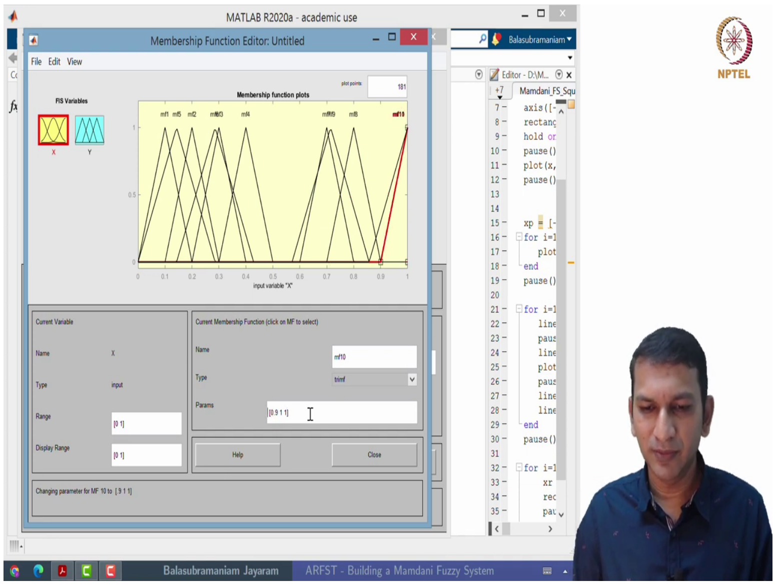
Name mf10

Type trimf

Params [0.7143 0.8571 1]

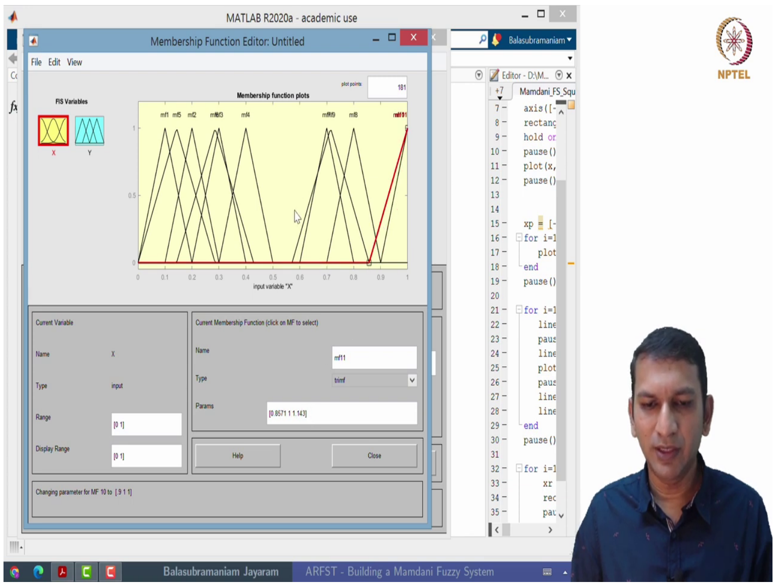
Changing parameter for MF 8 to [.7 .8 .9]

(Refer Slide Time: 13:15)



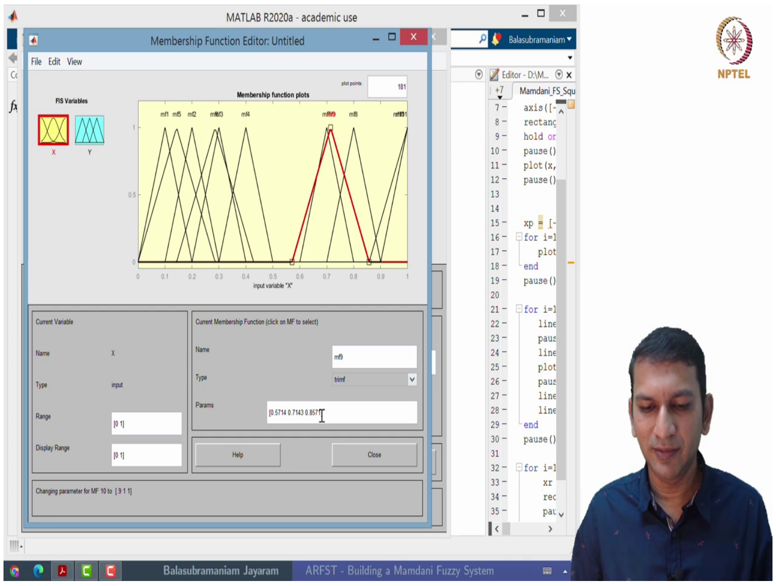
So, we have a mf10, which means it will be positioned at 1. Now, if you want to have a triangular membership function centered at 1, which is essentially the end point then we will be extending outside of the range that we have considered. So, we will saturate it at that and we will keep it as 0.91, this is how it will look like.

(Refer Slide Time: 13:39)



This is mf11, we will come to this in a moment.

(Refer Slide Time: 13:43)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
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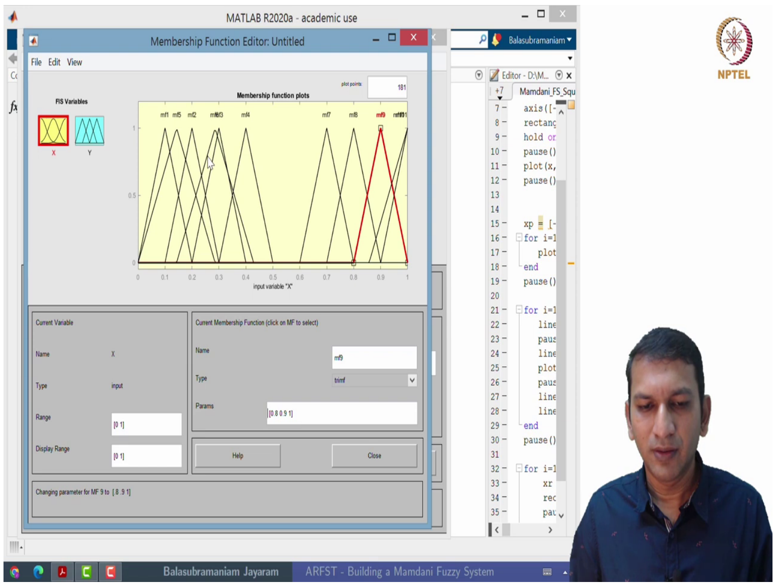
NPTEL

Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf9
Type trimf
Params [0.5714 0.7143 0.8571]

Changing parameter for MF 10 to [.9 1 1]

(Refer Slide Time: 13:47)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
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NPTEL

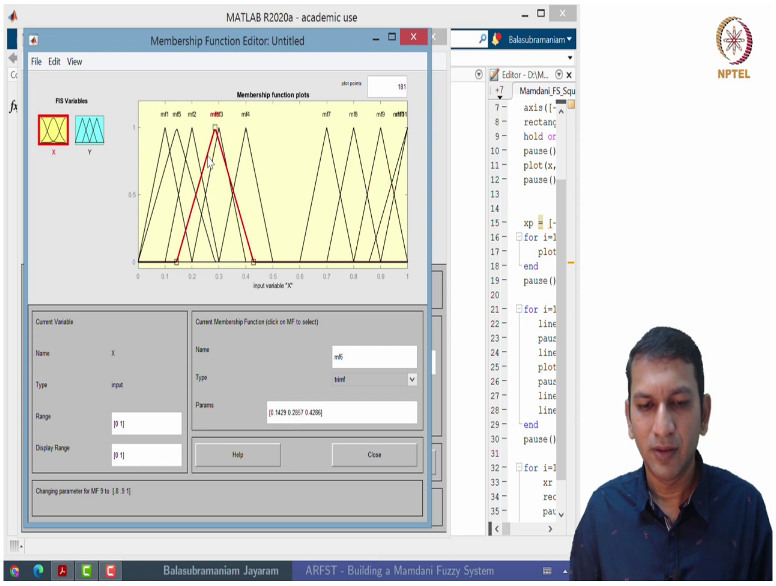
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf9
Type trimf
Params [0.8 0.9 1]

Changing parameter for MF 9 to [.8 .9 1]

Mf9; so it should be centered at 0.9, which means the support of it is.

(Refer Slide Time: 13:55)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
input variable "X"
Help
Close
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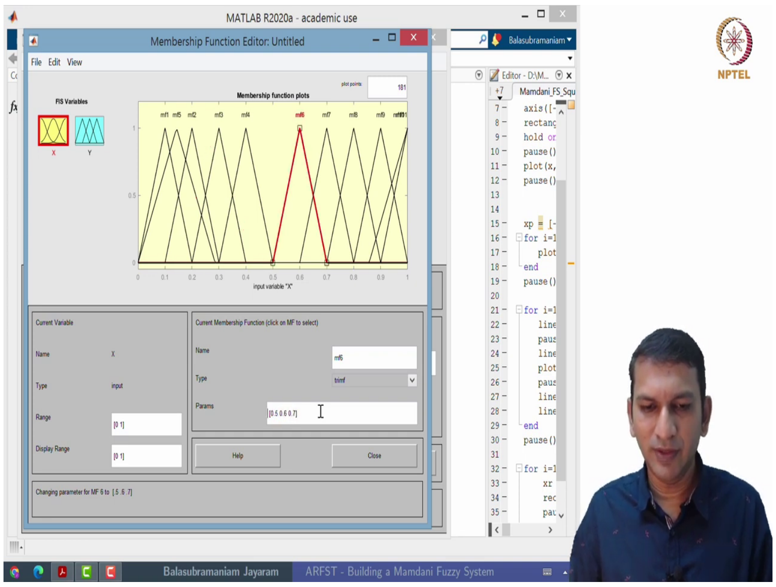
NPTEL

Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf6
Type trimf
Params [0.1429 0.2857 0.4286]

Changing parameter for MF 9 to [.8 .9 1]

(Refer Slide Time: 13:59)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
input variable "X"
Help
Close
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NPTEL

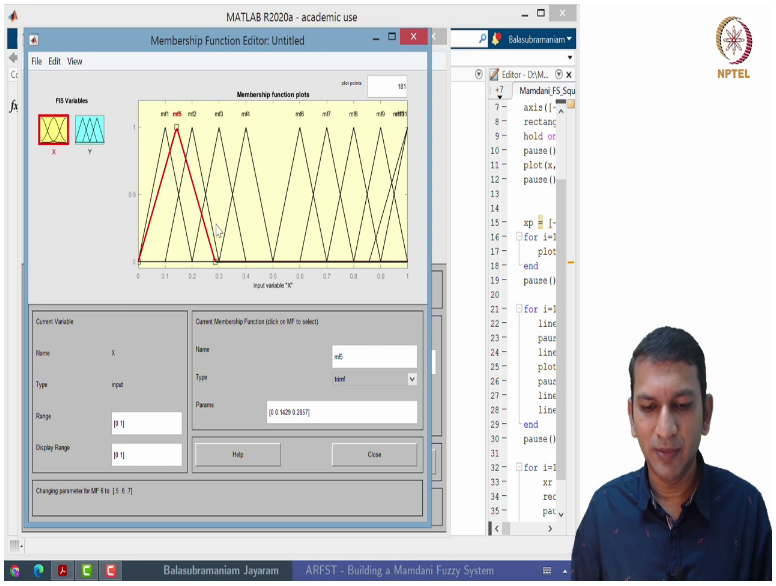
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf6
Type trimf
Params [0.5 0.6 0.7]

Changing parameter for MF 6 to [.5 .6 .7]

This is mf6; so its support will be from 0.5 to 0.7.

(Refer Slide Time: 14:04)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
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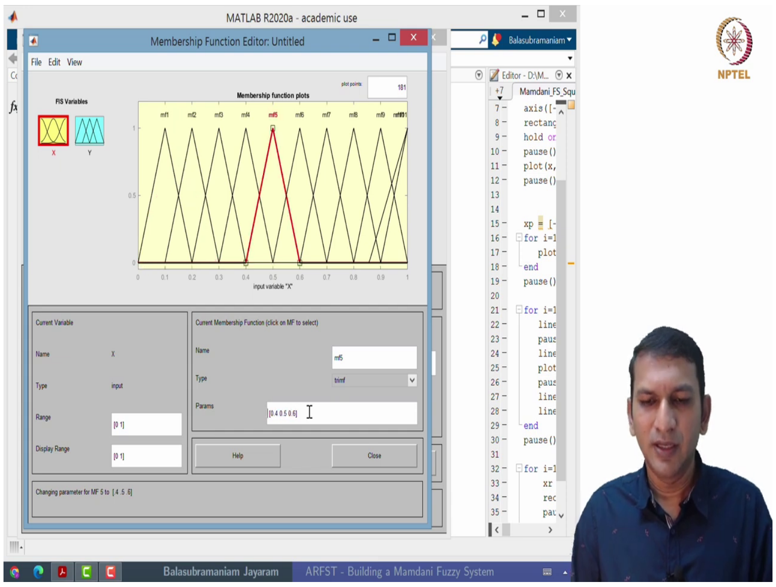
NPTEL

Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf5
Type trimf
Params [0 0.1429 0.2857]

Changing parameter for MF 6 to [.5 .6 .7]

(Refer Slide Time: 14:08)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
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NPTEL

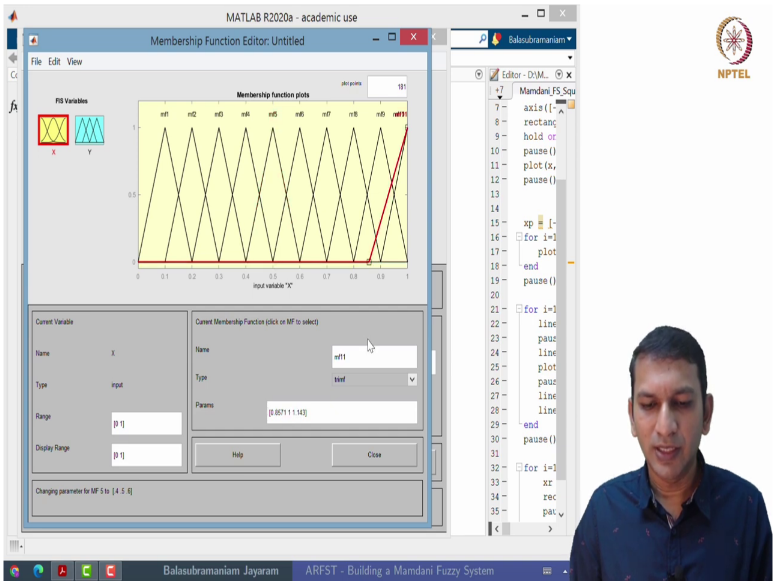
Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf5
Type trimf
Params [0.4 0.5 0.6]

Changing parameter for MF 5 to [.4 .5 .6]

Once we take 5, so support s; we have done this, but we have not defined what it is for 0.

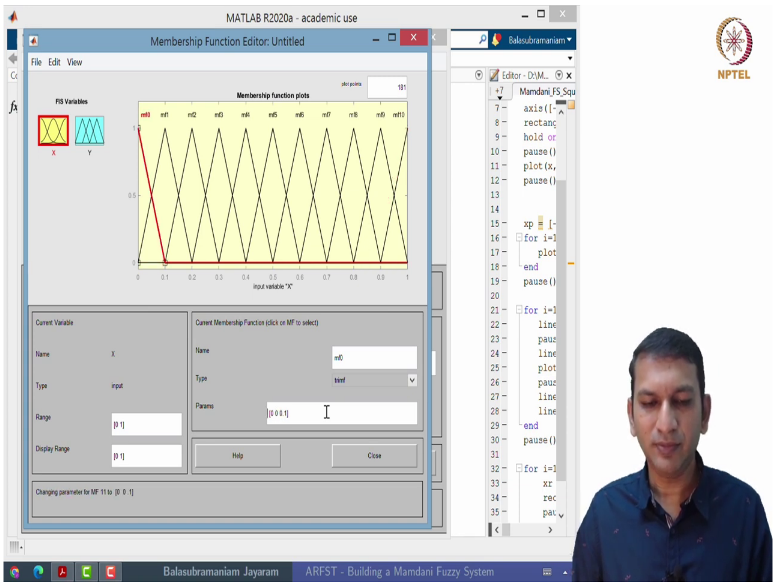
(Refer Slide Time: 14:17)


Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf11
Type trimf
Params [0.8571 1 1.143]

Changing parameter for MF 5 to [.4 .5 .6]

(Refer Slide Time: 14:18)


Current Variable
Name X
Type input
Range [0 1]
Display Range [0 1]

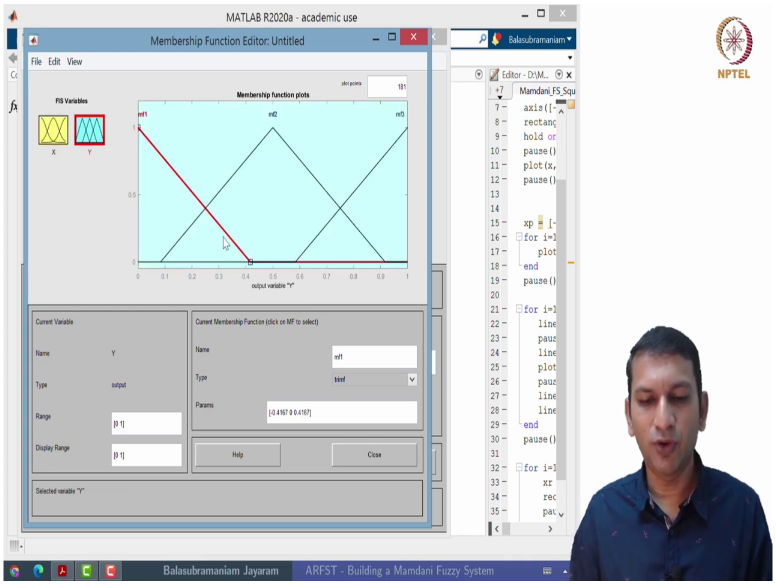
Current Membership Function (click on MF to select)
Name mf0
Type trimf
Params [0 0 0.1]

Changing parameter for MF 11 to [0 0 .1]

So, let us call this mf0 and once again we want to place it at 0, the right end point of the support is point one since we cannot go below 0.

Let us saturate at 0. So, this is how we have actually made the fuzzy covering on the domain X, which is 0, 1 interval you will see that for every point there is some fuzzy set to which it belongs to a non-zero membership value. In fact, it can be easily seen these are linear functions. So, it is also clear that they actually form a re-spinning partition. So, this is the fuzzy covering that we have come up with on the domain.

(Refer Slide Time: 15:03)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
mf1
mf2
mf3
output variable "Y"
Help
Close
NPTEL
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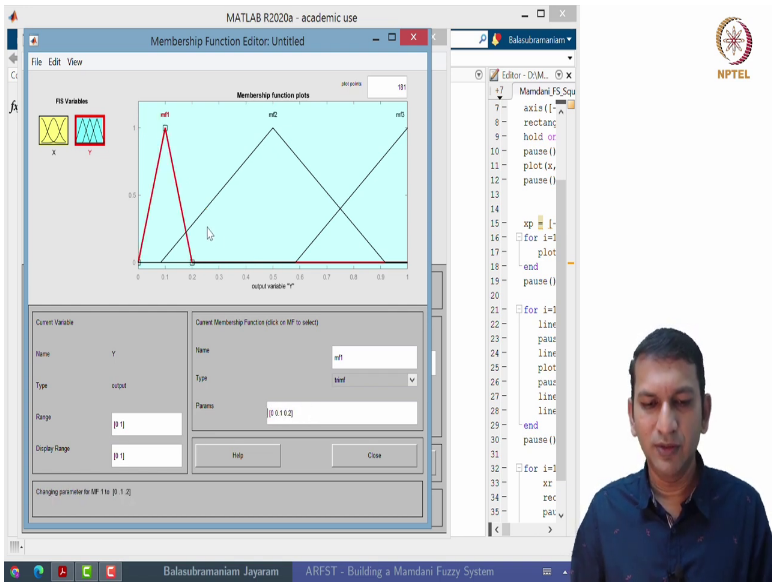
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [-0.4167 0 0.4167]

Selected variable "Y"

Let us do the same also for the output.

(Refer Slide Time: 15:12)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
mf1
mf2
mf3
output variable "Y"
Help
Close
NPTEL
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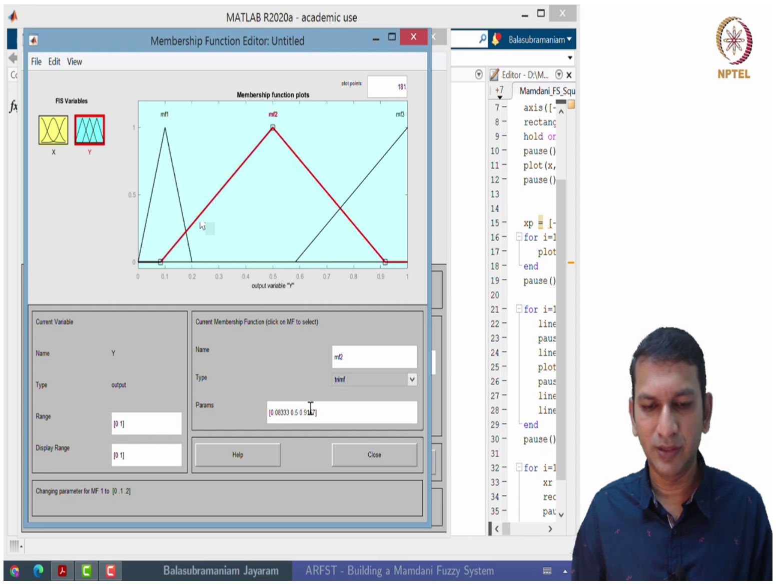
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [0 0.1 0.2]

Changing parameter for MF 1 to [0 .1 .2]

(Refer Slide Time: 15:16)



MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points 181
mf1
mf2
mf3
output variable "Y"
Help
Close
NPTEL
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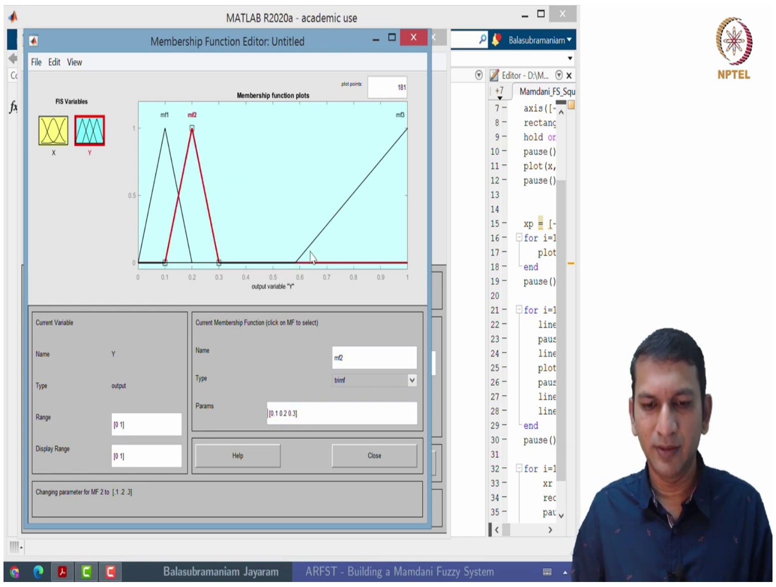
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf2
Type trimf
Params [0.08333 0.5 0.9167]

Changing parameter for MF 1 to [0 .1 .2]

(Refer Slide Time: 15:20)



MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points 181
mf1
mf2
mf3
output variable "Y"
Help
Close
NPTEL
Balasubramaniam Jayaram
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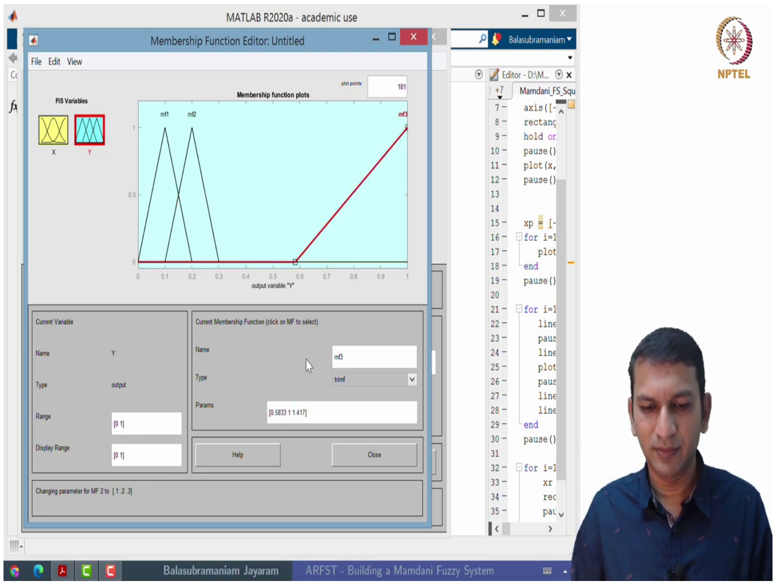
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf2
Type trimf
Params [0.1 0.2 0.3]

Changing parameter for MF 2 to [.1 .2 .3]

(Refer Slide Time: 15:24)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
mf1
mf2
mf3
output variable "Y"
Help
Close
NPTEL
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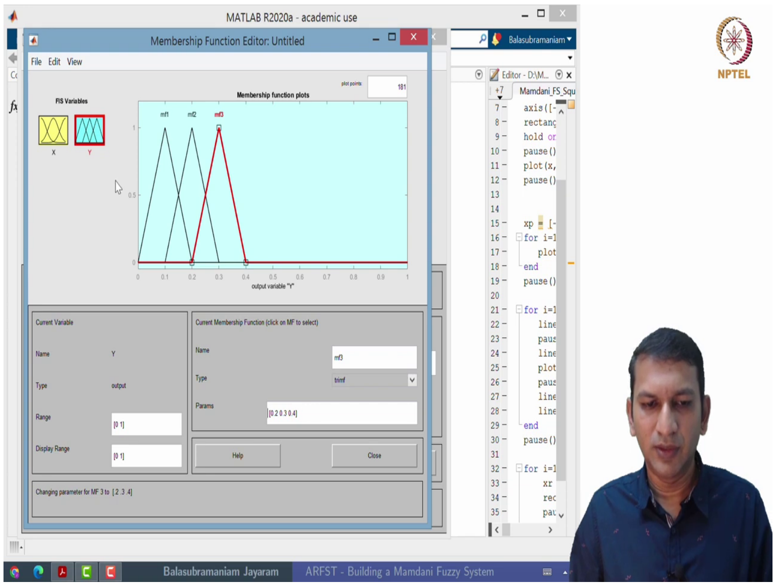
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf3
Type trimf
Params [0.5833 1 1.417]

Changing parameter for MF 2 to [.1 .2 .3]

So, this is 1. So, which means by our convention, we will do this.

(Refer Slide Time: 15:27)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
mf1
mf2
mf3
output variable "Y"
Help
Close
NPTEL
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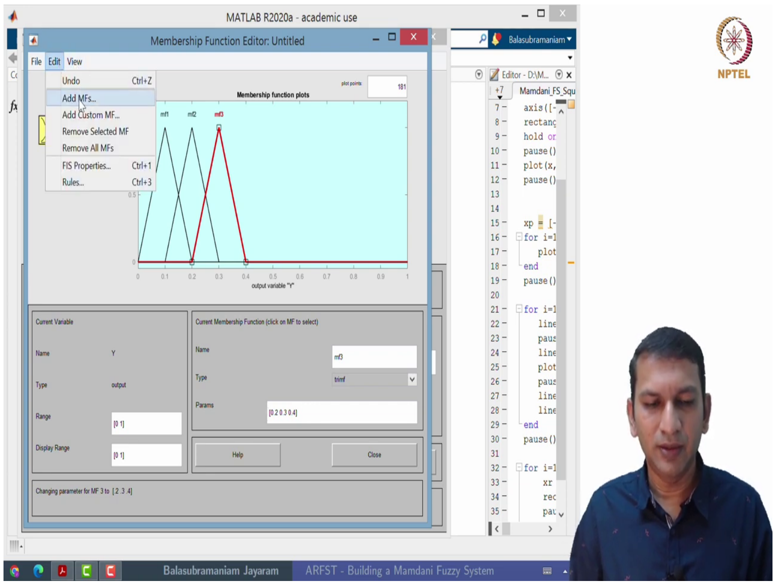
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf3
Type trimf
Params [0.2 0.3 0.4]

Changing parameter for MF 3 to [.2 .3 .4]

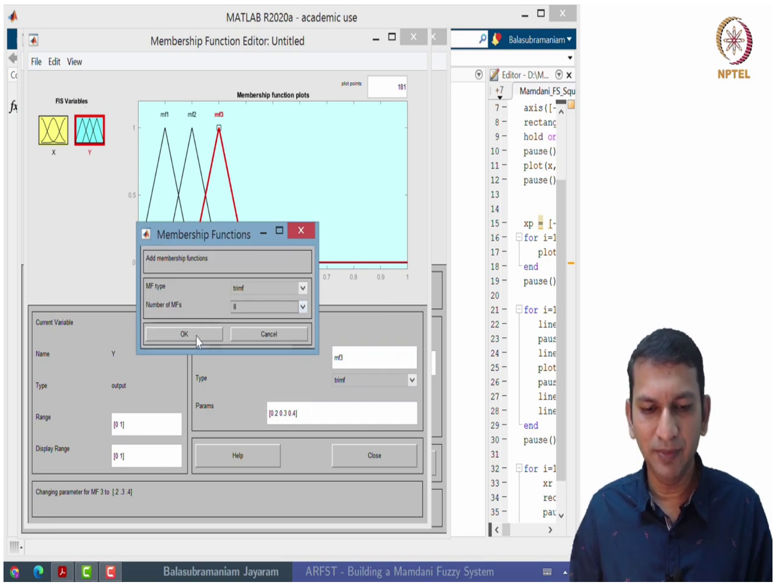
0.1, 0.2, 0.3, 0.2, 0.3, 0.4.

(Refer Slide Time: 15:40)



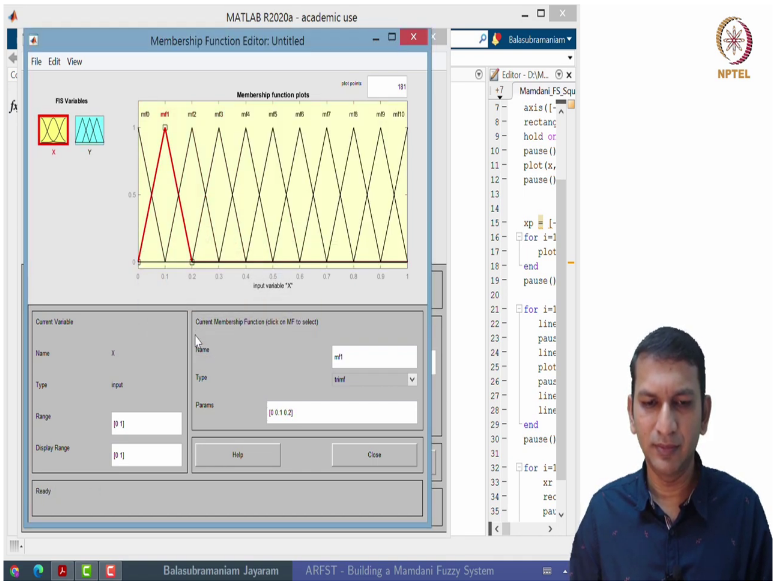
So, now, we have 3, we need to add further MFs membership functions.

(Refer Slide Time: 15:42)



Once again we will add 8 of them and proceed with the same thing.

(Refer Slide Time: 15:49)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
plot points 181
mf0 mf1 mf2 mf3 mf4 mf5 mf6 mf7 mf8 mf9 mf10
input variable "X"
Help
Close
NPTEL
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Current Variable

Name X

Type input

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

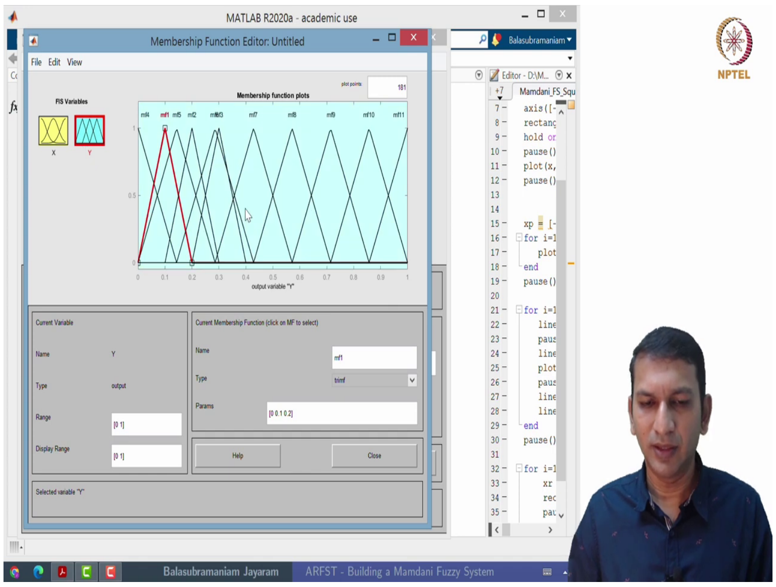
Name mf1

Type trimf

Params [0 0.1 0.2]

Ready

(Refer Slide Time: 15:52)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
plot points 181
mf4 mf1 mf5 mf2 mf6 mf3 mf7 mf8 mf9 mf10 mf11
output variable "Y"
Help
Close
NPTEL
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Current Variable

Name Y

Type output

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

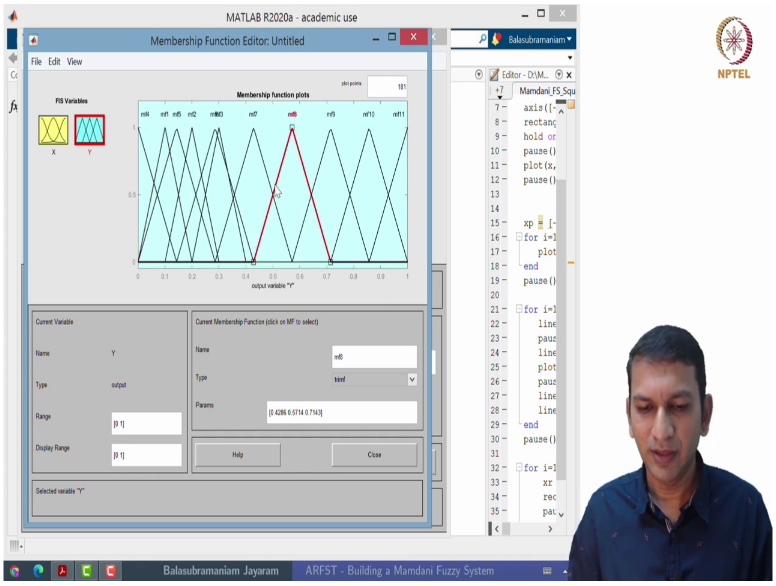
Name mf1

Type trimf

Params [0 0.1 0.2]

Selected variable "Y"

(Refer Slide Time: 15:55)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
output variable "Y"
Help
Close
Balasubramaniam Jayaram
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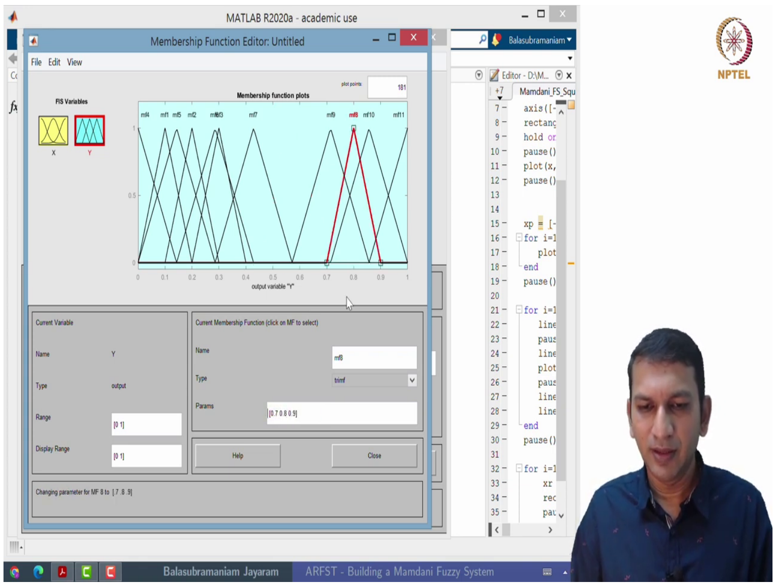
NPTEL

Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf8
Type trimf
Params [0.4286 0.5714 0.7143]

Selected variable "Y"

(Refer Slide Time: 15:59)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
output variable "Y"
Help
Close
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NPTEL

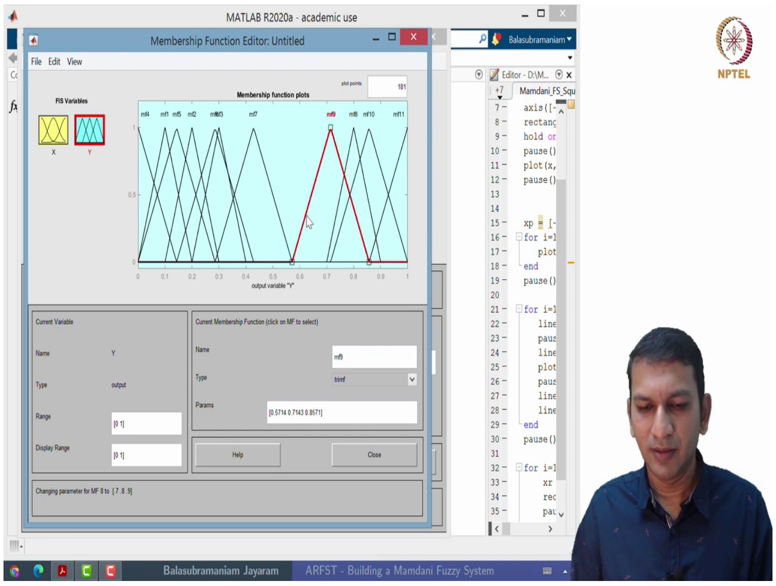
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf8
Type trimf
Params [0.7 0.8 0.9]

Changing parameter for MF 8 to [.7 .8 .9]

So, we start from 8 it is, so that means, it should be positioned at 8, 0.7, 0.8, 0.9.

(Refer Slide Time: 16:03)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
output variable "Y"
Help
Close
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NPTEL

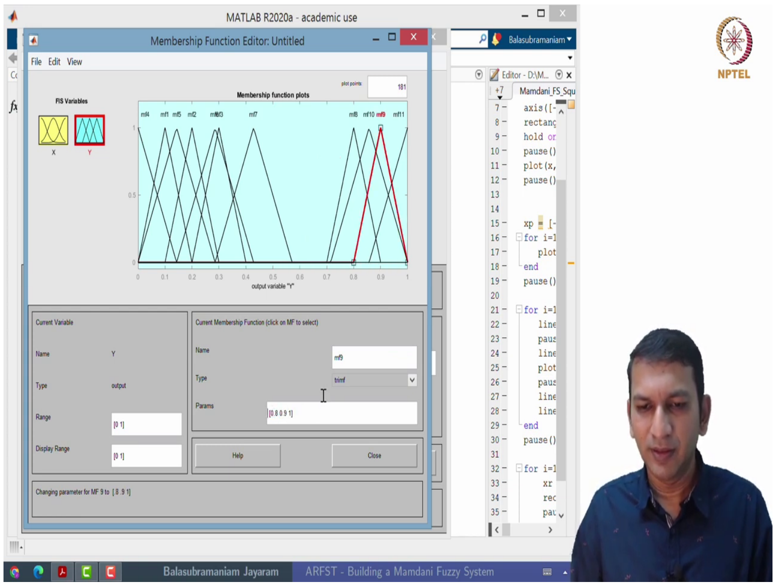
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf9
Type trimf
Params [0.5714 0.7143 0.8571]

Changing parameter for MF 8 to [.7 .8 .9]

9 is 0.8, 0.9 1 (Refer Time: 16:14)

(Refer Slide Time: 16:08)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
output variable "Y"
Help
Close
Balasubramaniam Jayaram
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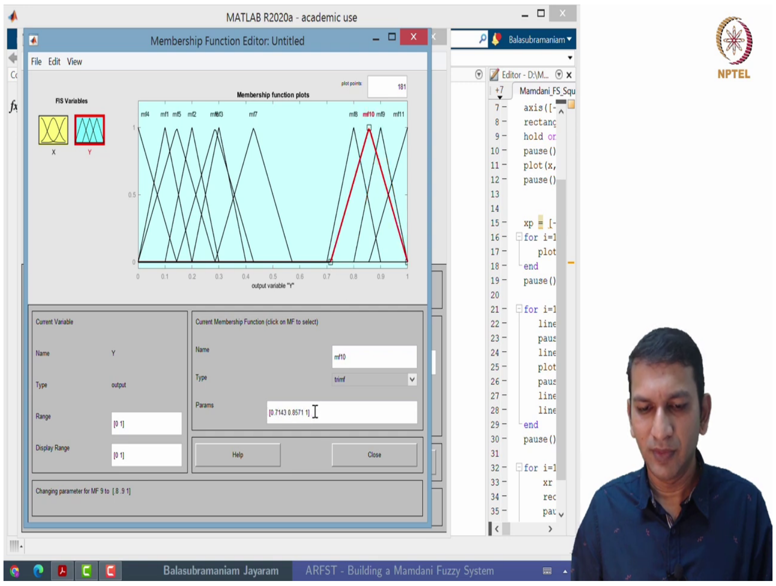
NPTEL

Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf9
Type trimf
Params [0.8 0.9 1]

Changing parameter for MF 9 to [.8 .9 1]

(Refer Slide Time: 16:13)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
output variable "Y"
Help
Close
NPTEL
Balasubramaniam Jayaram
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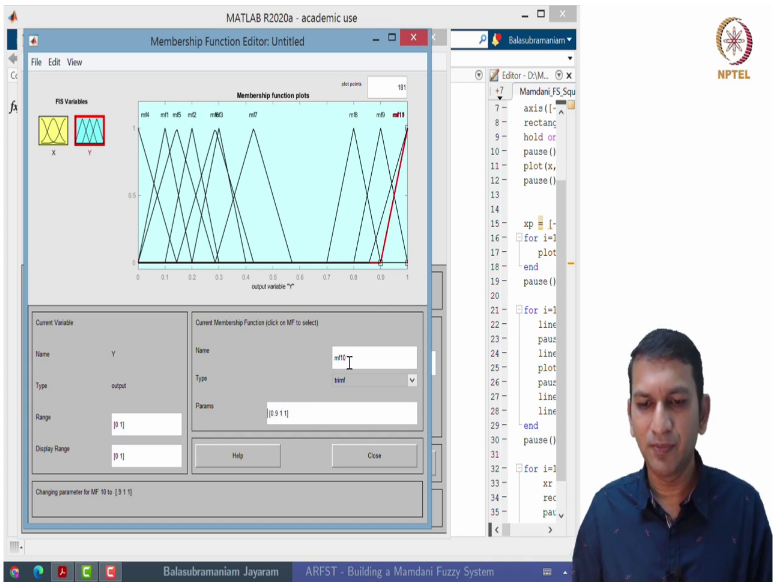
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf10
Type trimf
Params [0.7143 0.8571 1]

Changing parameter for MF 9 to [.8 .9 1]

(Refer Slide Time: 16:17)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
output variable "Y"
Help
Close
NPTEL
Balasubramaniam Jayaram
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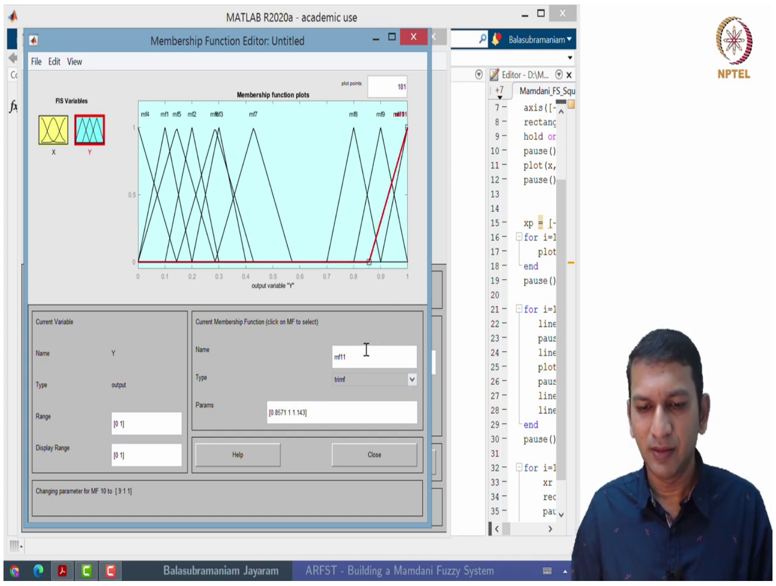
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf10
Type trimf
Params [0.9 1 1]

Changing parameter for MF 10 to [.9 1 1]

(Refer Slide Time: 16:21)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
output variable "Y"
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NPTEL

Current Variable

Name Y

Type output

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

Name mf11

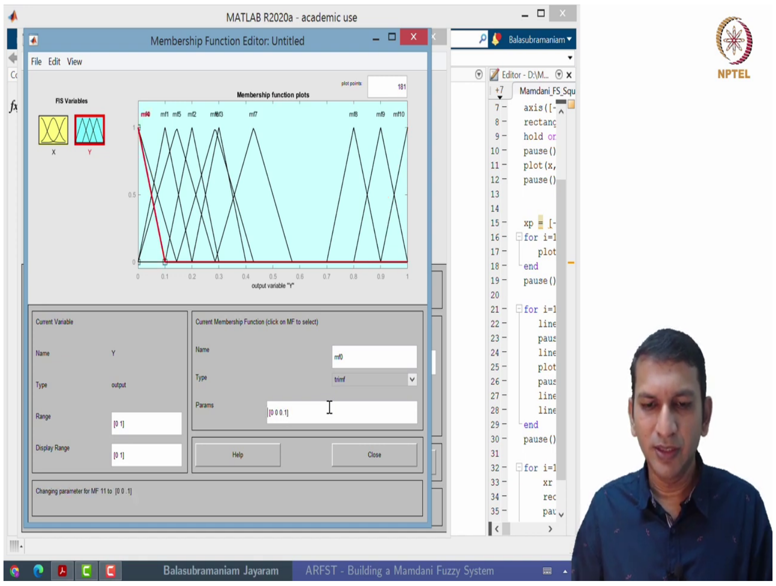
Type trimf

Params [0.8571 1 1.143]

Changing parameter for MF 10 to [.9 1 1]

We have seen that it has to be positioned at 1 and so it saturates at 1, 11 we will rename it as 0.

(Refer Slide Time: 16:23)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
output variable "Y"
Balasubramaniam Jayaram
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NPTEL

Current Variable

Name Y

Type output

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

Name mf0

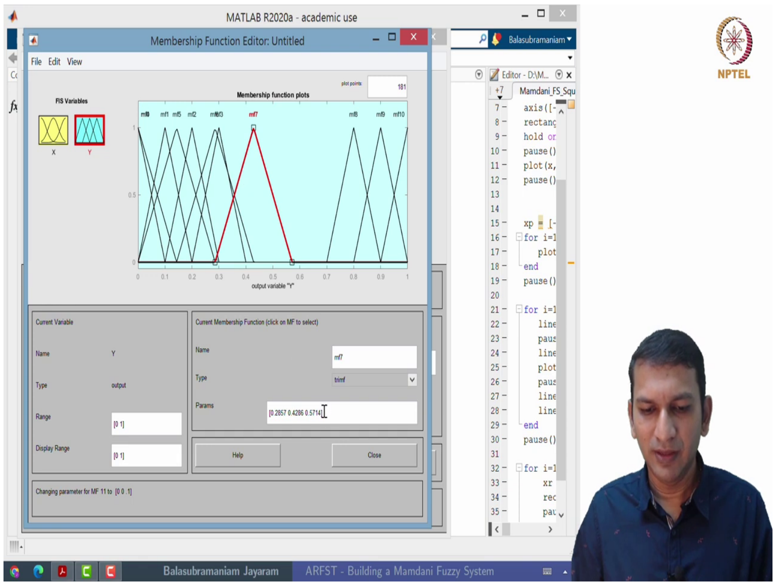
Type trimf

Params [0 0 0.1]

Changing parameter for MF 11 to [0 0 .1]

It is clear again it cannot go below 0.

(Refer Slide Time: 16:34)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
output variable "Y"
Help
Close
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NPTEL

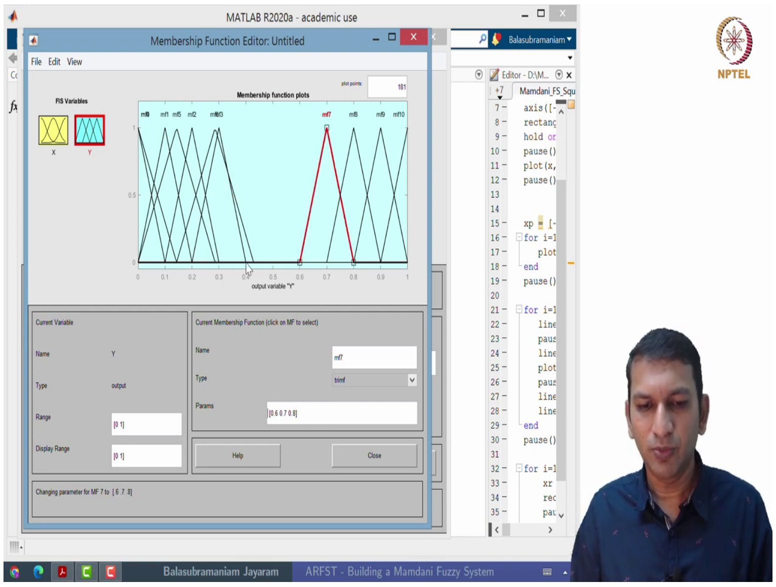
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf7
Type trimf
Params [0.2857 0.4286 0.5714]

Changing parameter for MF 11 to [0 0 .1]

There will be position at 0, but it will be like this.

(Refer Slide Time: 16:36)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points
181
output variable "Y"
Help
Close
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NPTEL

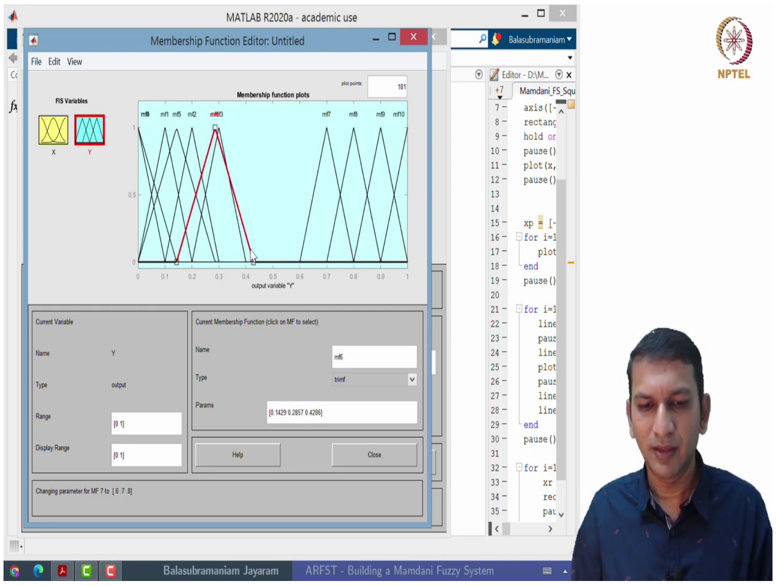
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf7
Type trimf
Params [0.6 0.7 0.8]

Changing parameter for MF 7 to [.6 .7 .8]

Mf7 will be centered at 0.7, its support will be from 0.6 to 0.8 or 6.

(Refer Slide Time: 16:42)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points 181
output variable "Y"
Balasubramaniam Jayaram
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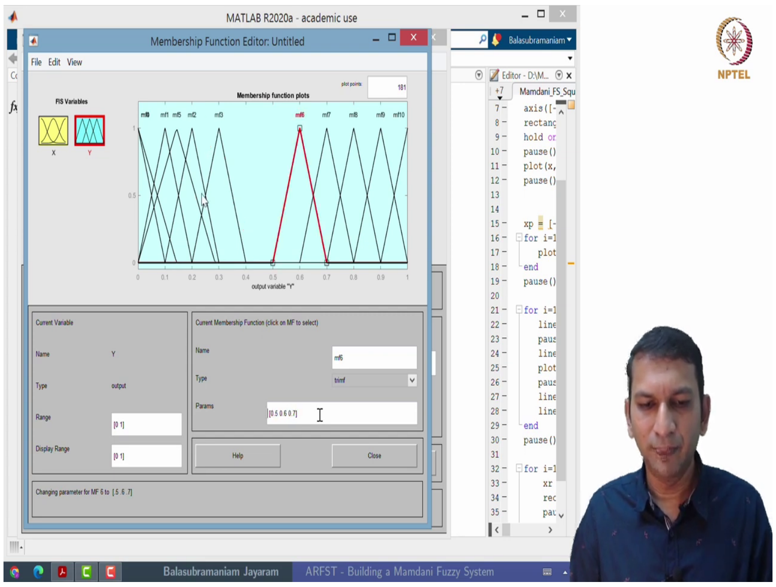
NPTEL

Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf6
Type trimf
Params [0.1429 0.2857 0.4286]

Changing parameter for MF 7 to [.6 .7 .8]

(Refer Slide Time: 16:46)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
X
Y
Membership function plots
plot points 181
output variable "Y"
Balasubramaniam Jayaram
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NPTEL

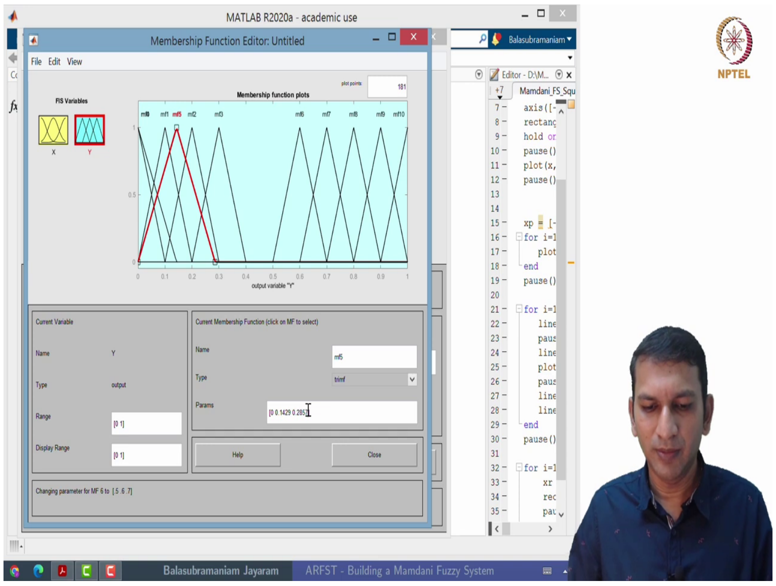
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf6
Type trimf
Params [0.5 0.6 0.7]

Changing parameter for MF 6 to [.5 .6 .7]

It will be centered at 0.6, where support will be from 0.5 to 0.7.

(Refer Slide Time: 16:52)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
plot points
181
output variable "Y"
Help
Close
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NPTEL

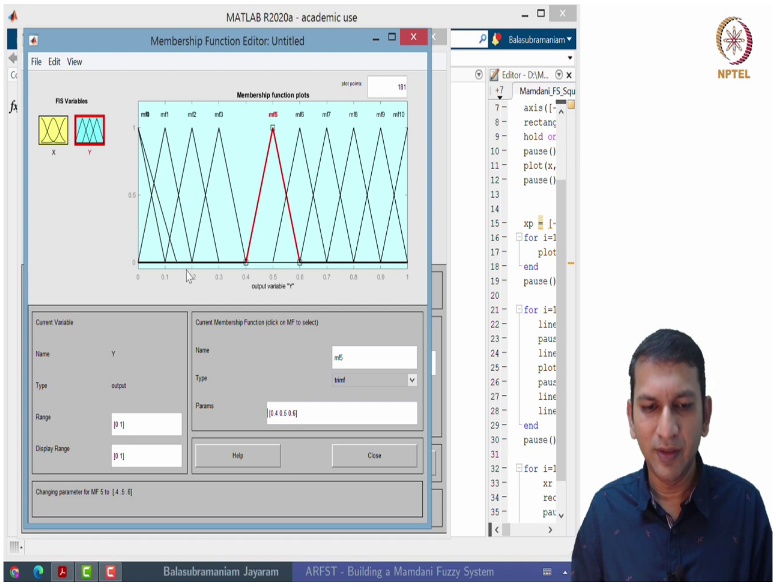
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf5
Type trimf
Params [0 0.1429 0.2857]

Changing parameter for MF 6 to [.5 .6 .7]

What we are left with this mf4, ok.

(Refer Slide Time: 16:57)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
plot points
181
output variable "Y"
Help
Close
Balasubramaniam Jayaram
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NPTEL

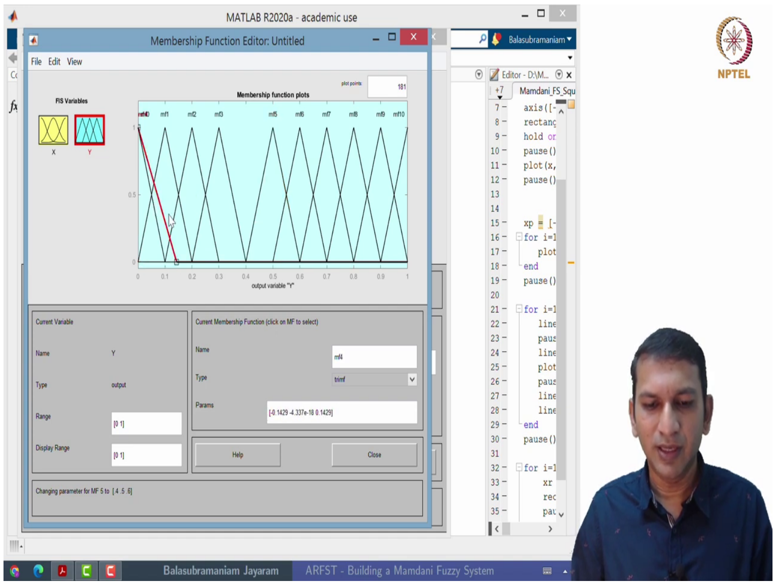
Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf5
Type trimf
Params [0.4 0.5 0.6]

Changing parameter for MF 5 to [.4 .5 .6]

So, this will be centered at 0.5 (Refer Time: 16:58)

(Refer Slide Time: 17:01)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
output variable "Y"
Help
Close
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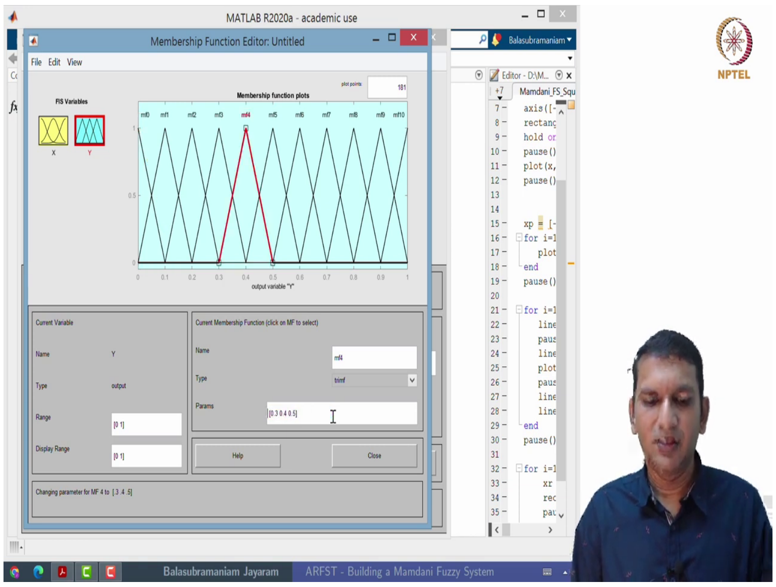
NPTEL

Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf4
Type trimf
Params [-0.1429 -4.337e-18 0.1429]

Changing parameter for MF 5 to [.4 .5 .6]

(Refer Slide Time: 17:05)


MATLAB R2020a - academic use
Membership Function Editor: Untitled
File Edit View
FIS Variables
Membership function plots
output variable "Y"
Help
Close
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NPTEL

Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

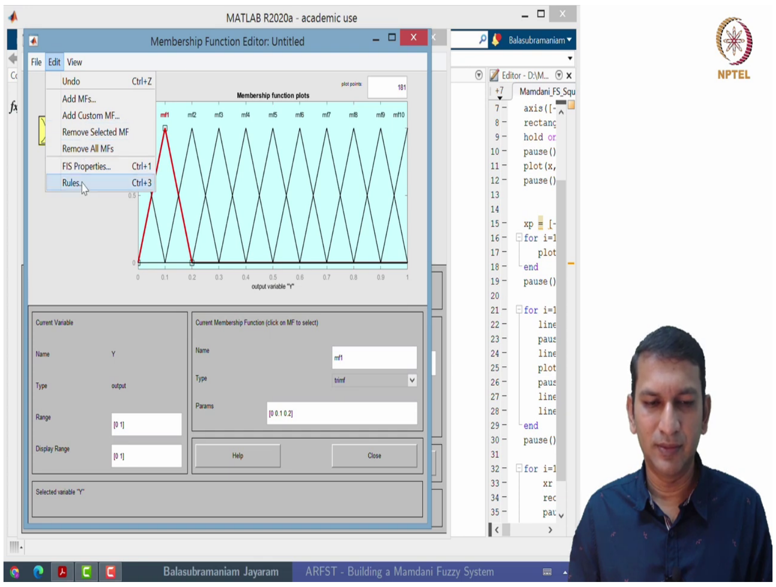
Current Membership Function (click on MF to select)
Name mf4
Type trimf
Params [0.3 0.4 0.5]

Changing parameter for MF 4 to [.3 .4 .5]

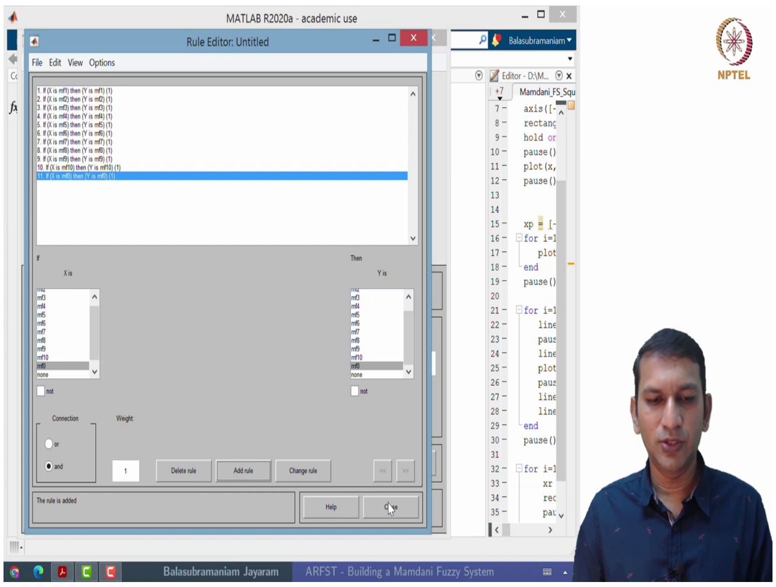
And finally, we have here mf4, it should be centered at 0.4 well. So, we now have all the membership functions that we need; that means, we have come up with a fuzzy covering on X and fuzzy covering on Y. In this case both of them are just the [0,1] intervals.

Now, the question is now we have this P X and P Y, how do we frame the rules. Note that we have seen that it is an identity function. So, essentially what we have in mind is whatever value we give, a dash that should actually come out as b dash. So, the common sensical way to build the rule base is to match the corresponding fuzzy sets from X and Y, from P X and P Y. How do we add rules?

(Refer Slide Time: 17:49)



(Refer Slide Time: 17:53)

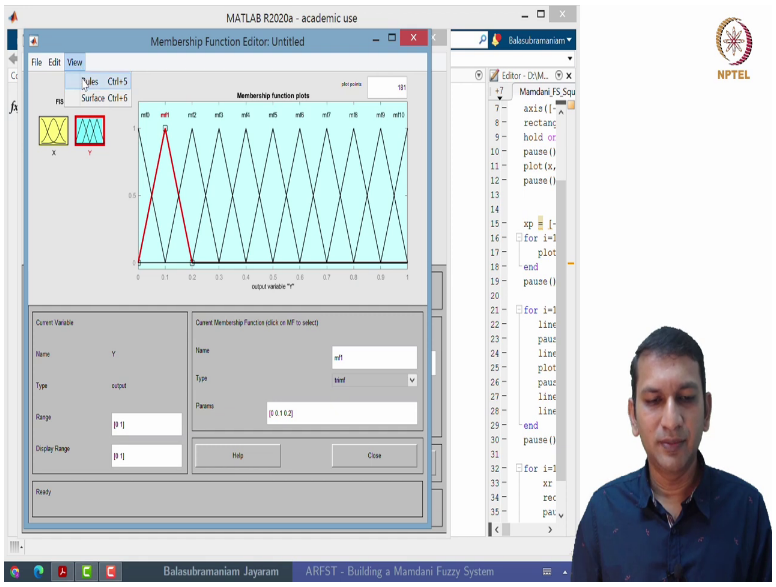


```
1. If (X is mf1) then (Y is mf1) (1)
2. If (X is mf2) then (Y is mf2) (1)
3. If (X is mf3) then (Y is mf3) (1)
4. If (X is mf4) then (Y is mf4) (1)
5. If (X is mf5) then (Y is mf5) (1)
6. If (X is mf6) then (Y is mf6) (1)
7. If (X is mf7) then (Y is mf7) (1)
8. If (X is mf8) then (Y is mf8) (1)
9. If (X is mf9) then (Y is mf9) (1)
10. If (X is mf10) then (Y is mf10) (1)
11. If (X is mf0) then (Y is mf0) (1)
```

So now, fuzzy logic toolbox says here. Now, on this side you have all the rules all the input fuzzy sets listed this side all the output fuzzy sets listed. Now you see here mf1. So, what we want is the fuzzy set centered at 0.1 should also be related to fuzzy set centered at 0.1. So, highlight both of these and say add a rule, next highlight these two and add a rule. So, essentially for the identity function all we need to do is this, makes it easy and of course, we will also see. A next function which is not simple or straight forward, but still not very complex.

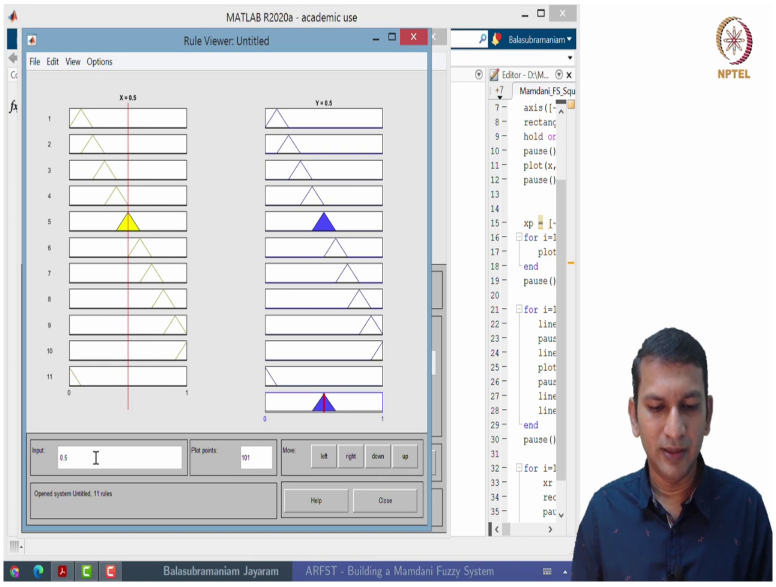
So, I am relating 0.7, the fuzzy set around 0.7 to fuzzy set around 0.7, fuzzy set around 0.8 to fuzzy set around 0.8, fuzzy set around 0.9 fuzzy set around 0.9 and fuzzy set around 1 to 1 and also finally, fuzzy set around 0 to 0. So, these are the 11 rules that we have.

(Refer Slide Time: 19:06)



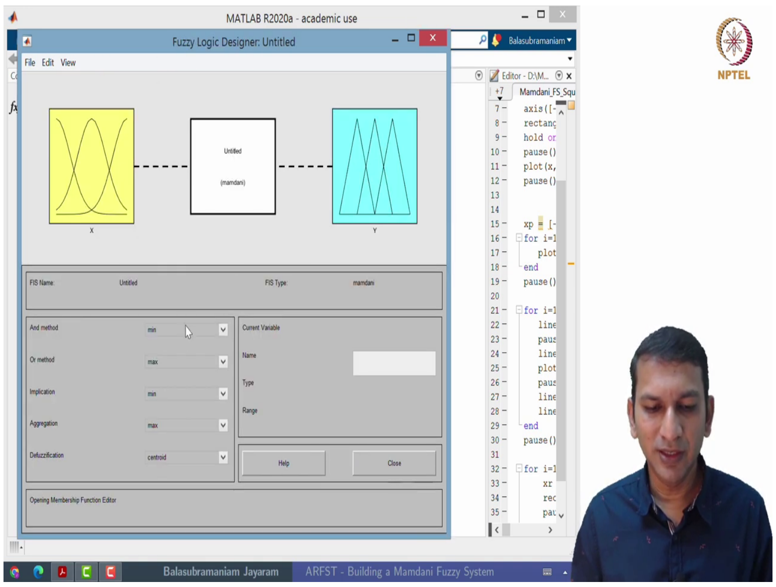
How do they look like?

(Refer Slide Time: 19:09)



This is a very nice visualization. Now; so, this is your input domain this is your output domain and these are the fuzzy sets that we have and these are the rules. So, this is a 1 implies b 1, a 2 implies b 2 so on and so forth.

(Refer Slide Time: 19:30)


MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
File Edit View
X
Untitled
(mamdani)
Y
Help
Close
NPTEL
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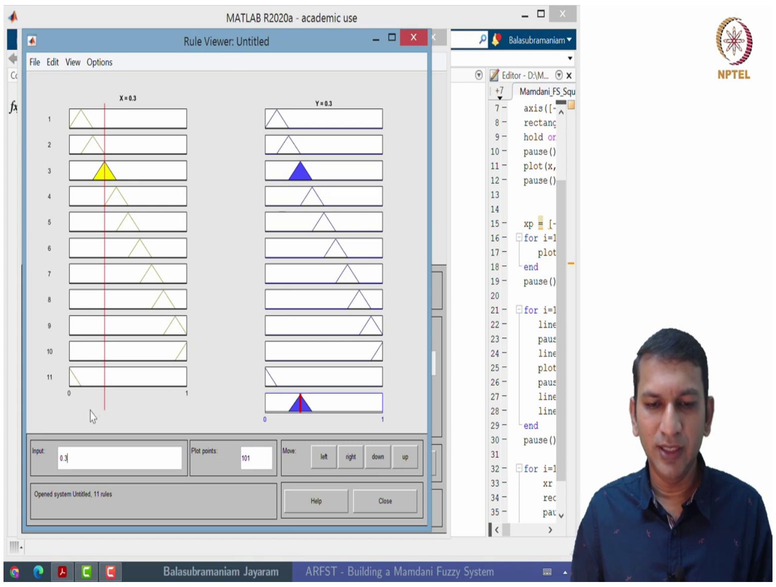
FIS Name: Untitled
FIS Type: mamdani
And method: min
Or method: max
Implication: min
Aggregation: max
Defuzzification: centroid
Current Variable
Name
Type
Range
Opening Membership Function Editor

Now, already we have chosen these operations. So, we are set, we have found the fuzzy covering on X fuzzy covering on Y and we also related them and found the rule base. So, now, when we come here, this is one quick way to give an input and find out what the corresponding output is, when X is equal to 0.5 we get 0.5.

(Refer Slide Time: 19:57)



Input: 0.3
Plot points: 101
Opened system Untitled, 11 rules

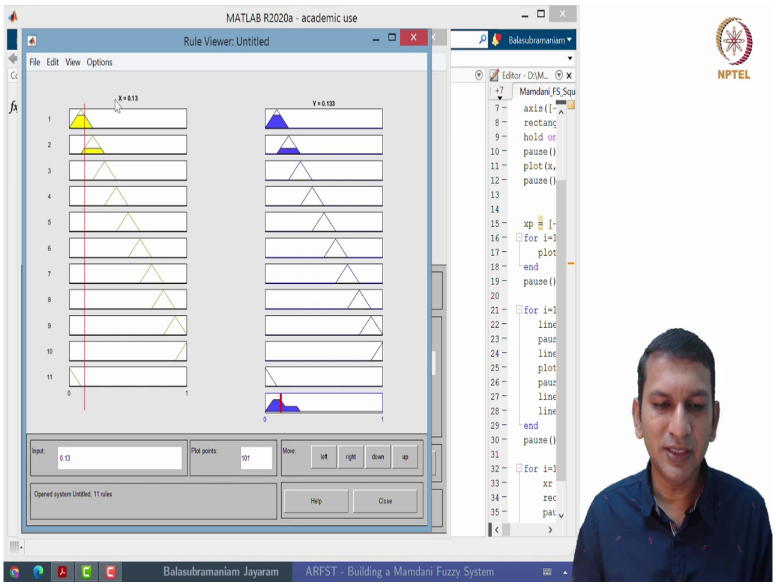
Now, if X is equal to 0.3, you see here Y is also equal to 0.3. So, this is exactly what we wanted, we wanted to approximate the identity function.

Now, if you look a little deeper inside this, what are we doing? Now, this is at 0.5, you can look at the straight line this red line as actually the singleton fuzzification. So, now, what happens is at 0.3 it takes the value 1 in this fuzzy set and nowhere else because we have respinning partition here with triangular membership function.

And so the similarity value is 1 and we are look at the modification, the modification is minimum. So, we are taking this 1 and we are thresholding this membership the consequent membership function here, which is if you call them b 1, b 2, b 3 we are thresholding this b 3.

However, since it is 1 essentially everything comes through and what are we using here? We are using the centroid de fuzzification mechanism, ok. So, next thing that we need to understand is ok for 0.3 and 0.5 it happens, what about other values? Will it happen?

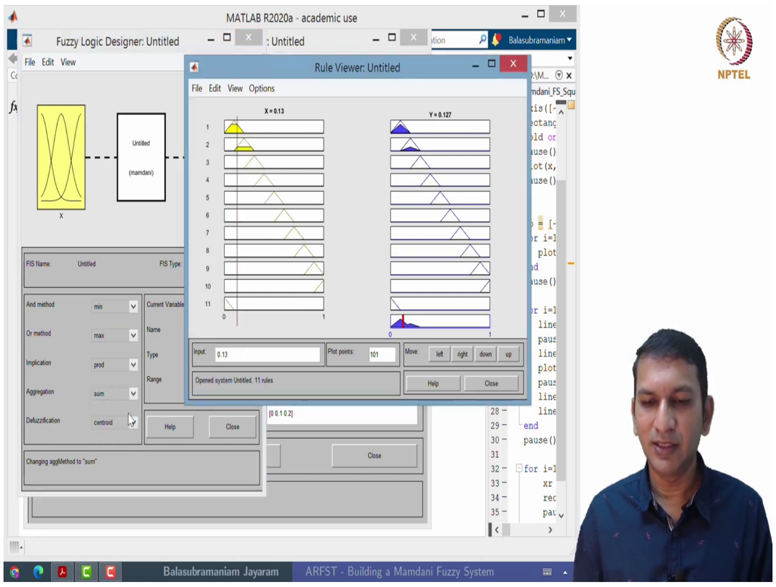
(Refer Slide Time: 21:10)



Yes, we can try for a few of these you see there at X is equal to 1, 0.13, Y is 0.133. So, there is some discrepancy here, but it is approximating that is because when we have this, you see here there are two of these getting highlighted.

So, the similarity value here is taken and it is being threshold here. Similarity, value here is being taken and this is thresholded here and we are using maximum for the aggregation and centroid de fuzzification. Let us see whether by changing this whether we can obtain some better values. So, towards that end I hope you are able to see, let us instead of using min as the modification function, let us try to use the product.

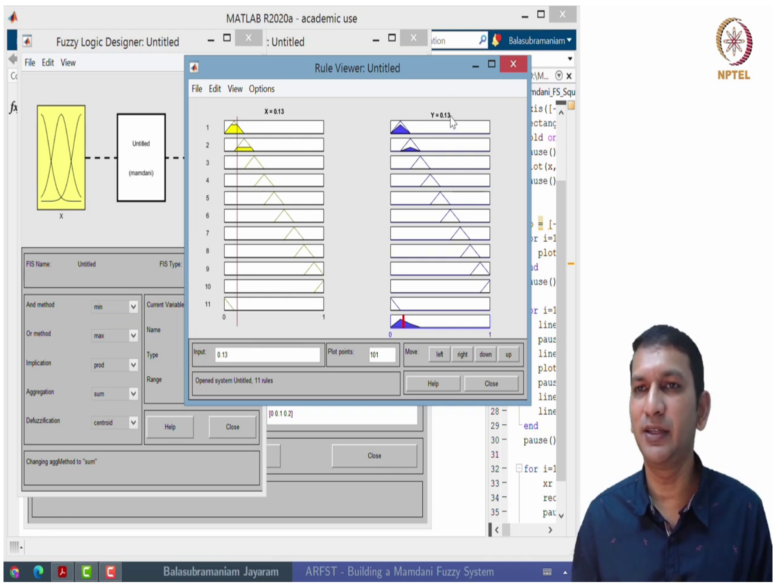
(Refer Slide Time: 22:07)



And you will immediately see while the similarity values have not changed what has changed is the way it is being thresholded.

So, now it is being essentially scaled. And once again after scaling we are still using the maximum aggregation function and the centroid de fuzzification. And you see that in fact, it turns out that the value goes down instead of having 0.133 we are going towards 0.127, it is still not the identity function, but it has actually fallen down.

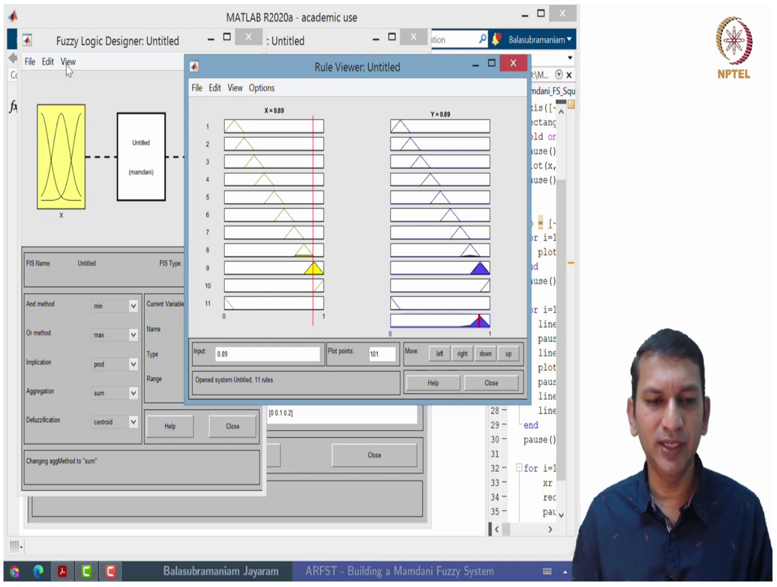
(Refer Slide Time: 22:45)



Now, we can play around here, instead of max, if we take essentially the sum of measure functions you see here with this aggregation we are actually coming up with exact value, what is referred to as sum here is essentially the Lukashevitch t-conorm. So, when you use Lukashevitch t-conorm which is min of 1 comma X plus Y, we see that for 0.13 we essentially get exactly the same identity value.

So, by tweaking these different degrees of freedom by choosing these functions intelligently we can actually approximate a given function. In fact, it has been proven in theory that any continuous function can be approximated using a, there exists of Mamdani fuzzy inference system with a particular choice of operations which will approximate the given function that and that is under consideration ok. So, now here we have given a few values and then we have seen what happens to it.

(Refer Slide Time: 23:46)


MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
Rule Viewer: Untitled
X = 0.89
Y = 0.89
Input: 0.89
Plot points: 101
Opened system Untitled, 11 rules
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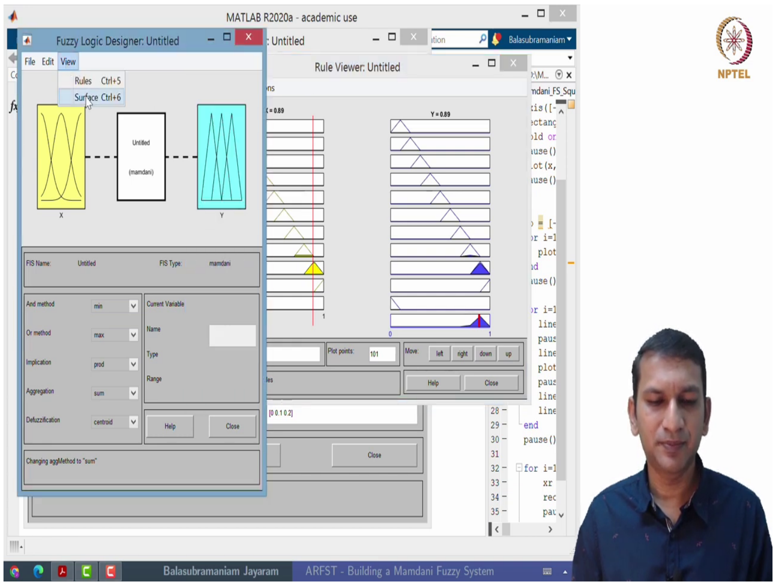
So, 0.89, 0.89, but how long can we go because 0 to 1, there are infinitely many numbers. Is there way to see whether this is the function that it is approximately, what has it understood.

(Refer Slide Time: 24:01)


MATLAB R2020a - academic use
Fuzzy Logic Designer: Untitled
View
Rules Ctrl+5
Surface Ctrl+6
Rule Viewer: Untitled
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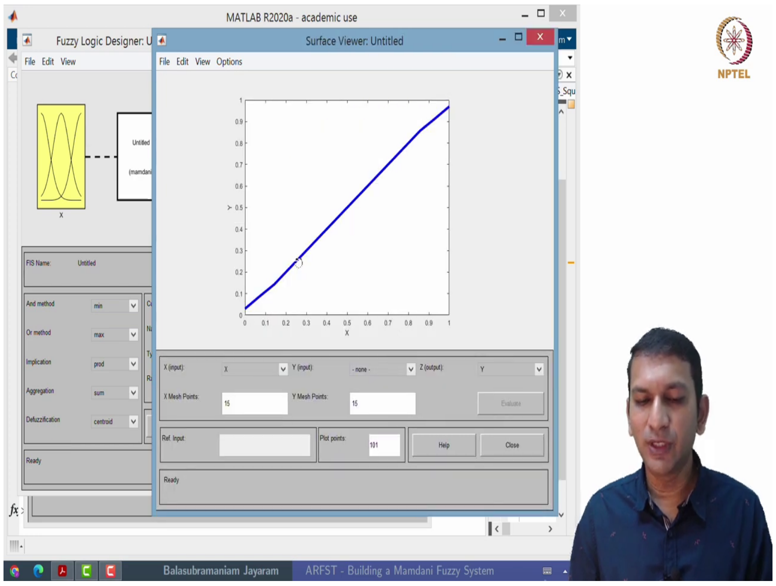
Well there is a way to see it.

(Refer Slide Time: 24:03)



FIS Name: Untitled

And method: min
Or method: max
Implication: prod
Aggregation: sum
Defuzzification: centroid

X (input): X
Y (input): - none -
Z (output): Y
X Mesh Points: 15
Y Mesh Points: 15
Ref. Input:
Plot points: 101

Ready

Now, if you look at this, the graph of this function the de fuzzified values for each of the values that you gave here for X you get a Y and you see here this essentially your identity function. Thus what we get here is essentially the function that we wanted to approximate, which is the identity function.

And along the way we have seen a few things that every degree of freedom that we have can make a difference. While the operations are easy to choose, but once you choose them you see that the overall output is going to change. And what we have to do is intelligently pick the fuzzy coverings on X and Y and intelligently relate the rules if you want to approximate the given function under consideration.

Now, let us look at the second function that we will be approximating. Well, the first function we saw was that of a simple identity function over the 0, 1 interval. Let us look at a different function now, it is from minus 1, 1 to the 0, 1 interval essentially the x square function while the identity function is both linear and monotonic we see that x square is a non-linear function and also not monotonic. Between minus 1 and 0 it is decreasing and 0 on 1, it is increasing.

However, let us apply the same logic that we applied to build a Mamdani fuzzy inference system to approximate this function.

(Refer Slide Time: 25:40)

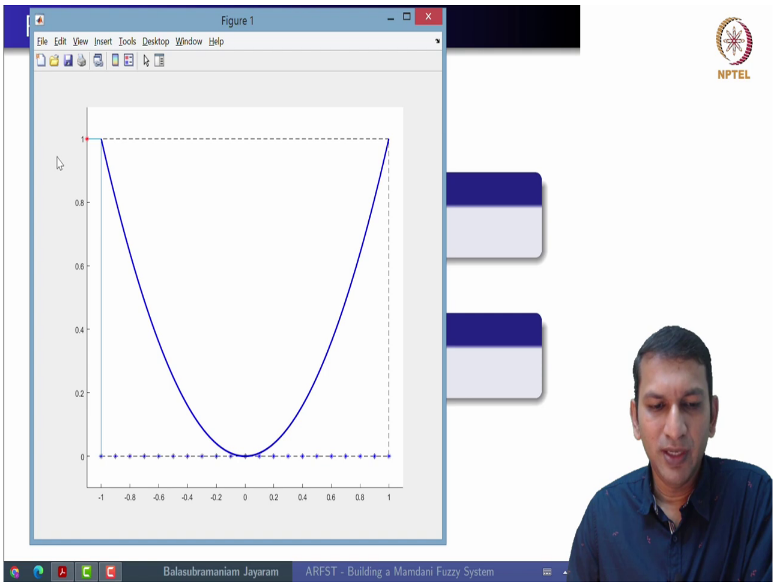


Figure 1
File Edit View Insert Tools Desktop Window Help
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What do we have now this is the domain of x minus 1 to 1 and this is our the domain of Y which is [0,1]. And the function we are considering is the x square function, once again we need to pick points on X and Y. So, that we can build a fuzzy covering around those points. Once again let us pick equi spaced points between minus 1 and 1.

So, there will be 21 of them and as we followed earlier let us find the corresponding F OF X i's to find where to position the fuzzy sets so that we can come up with a fuzzy covering on Y.

(Refer Slide Time: 26:27)

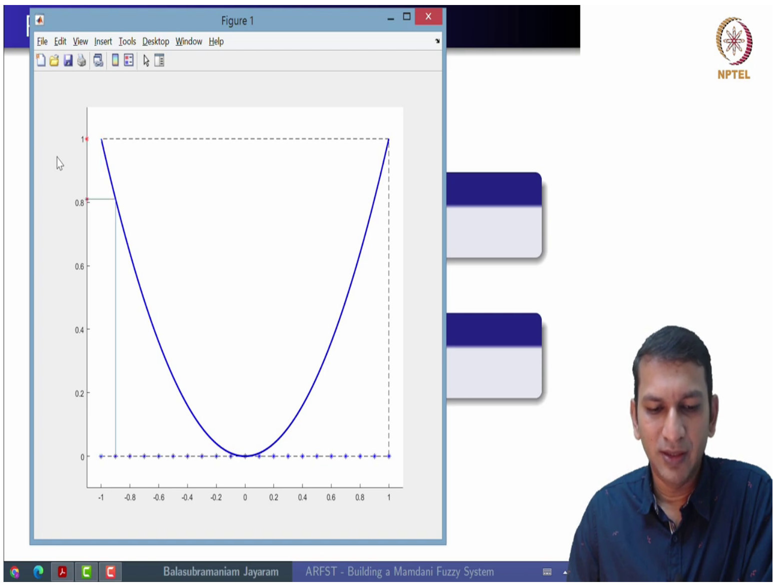

Figure 1
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So, at minus 1 it is 1, at point minus 0.9 it is 0.8 1.

(Refer Slide Time: 26:32)

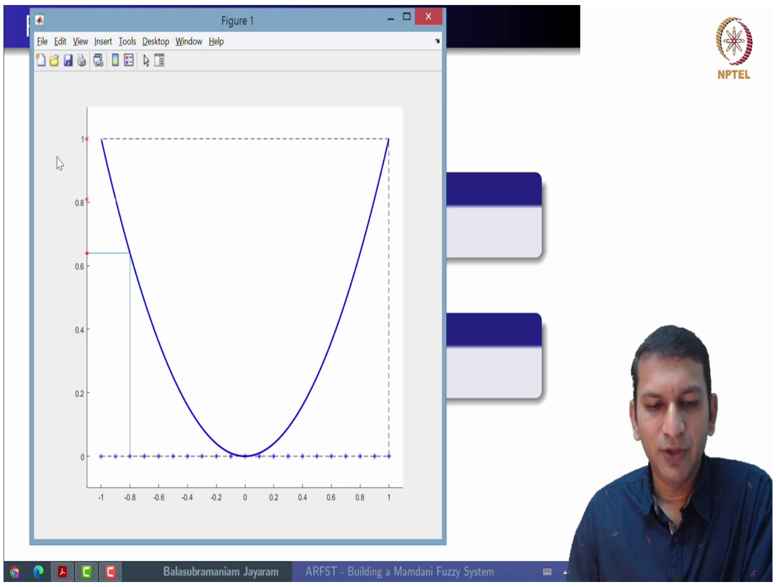
Figure 1
File Edit View Insert Tools Desktop Window Help
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ARFST - Building a Mamdani Fuzzy System

(Refer Slide Time: 26:33)

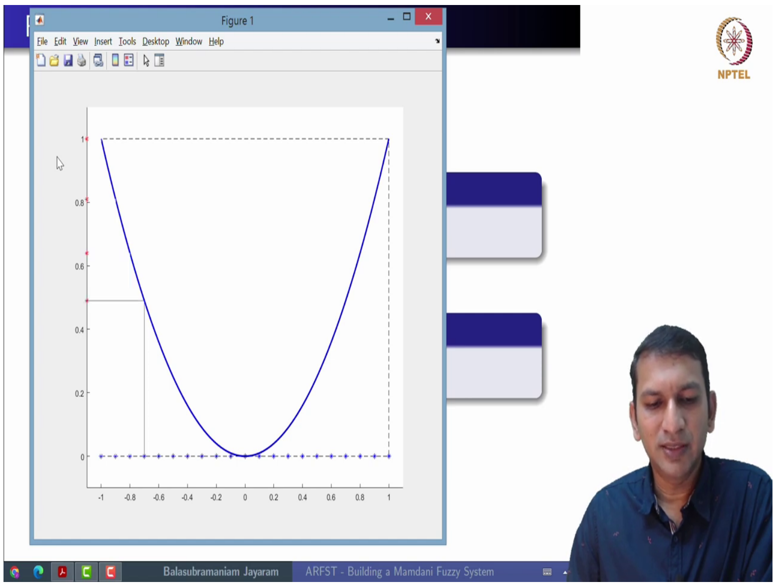
Figure 1
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NPTEL
Balasubramaniam Jayaram
ARFST - Building a Mamdani Fuzzy System

(Refer Slide Time: 26:34)

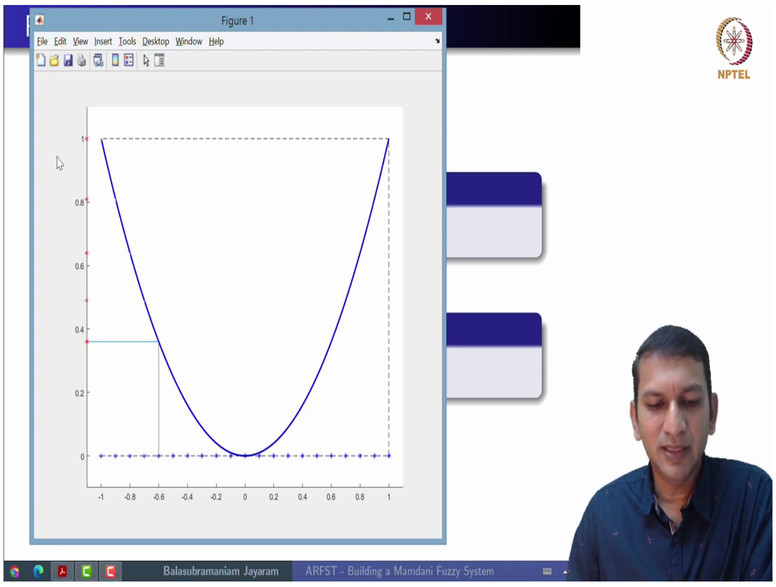
Figure 1
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NPTEL
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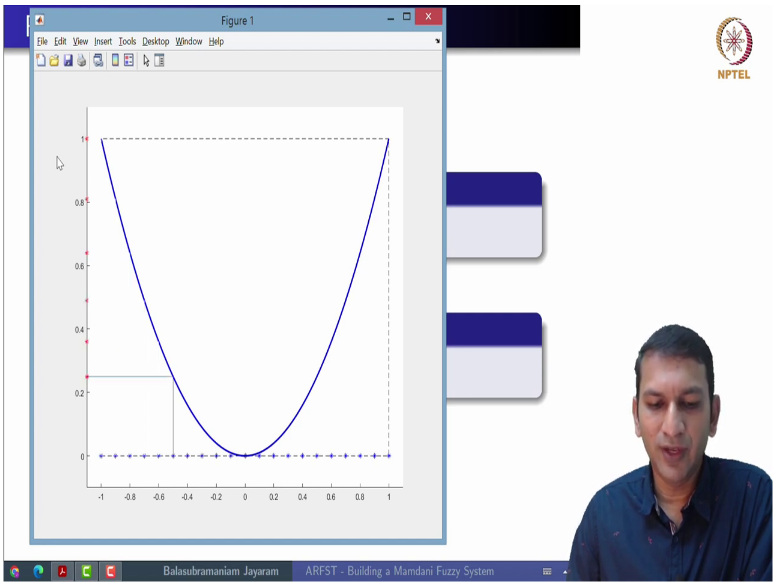
Figure 1
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NPTEL
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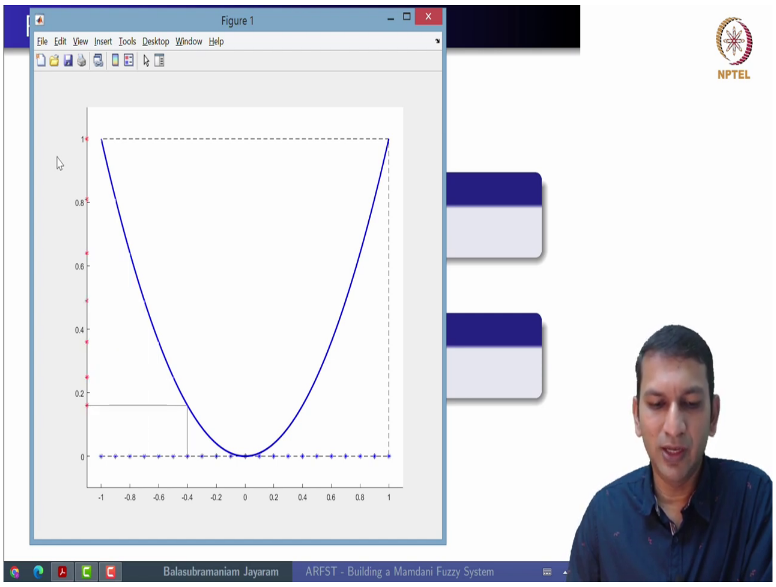
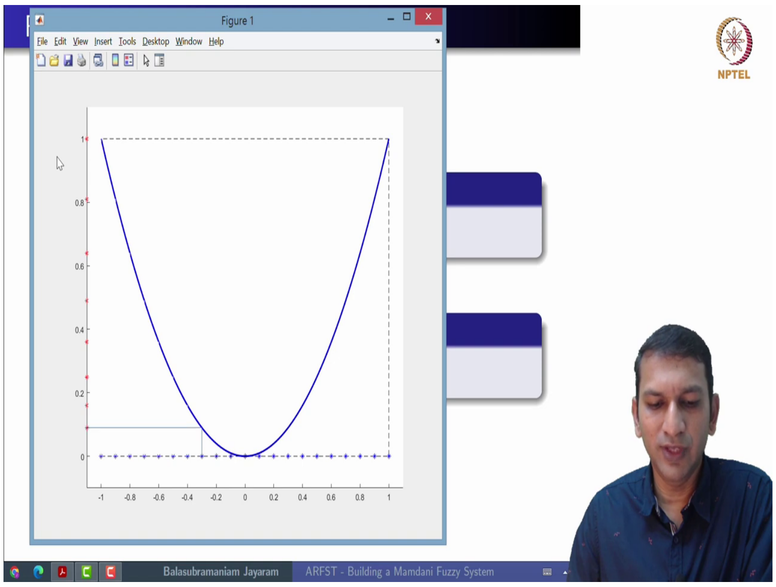
(Refer Slide Time: 26:36)

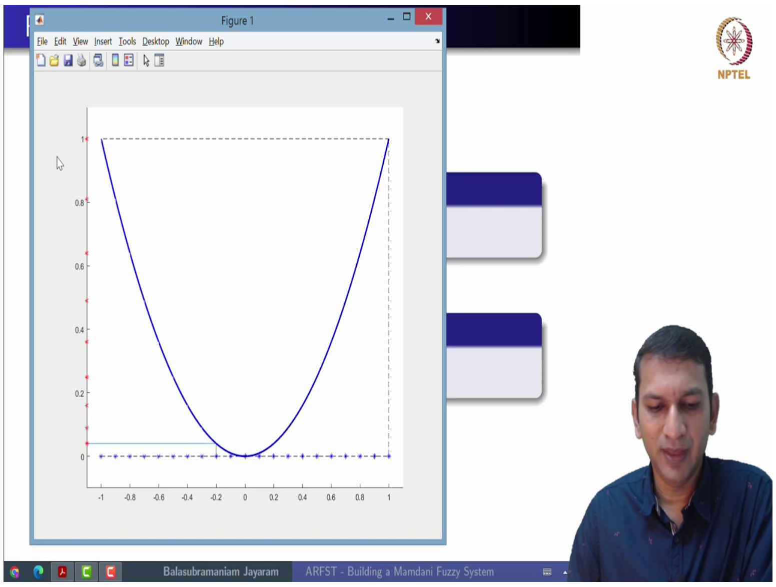


And if you go like this, these are the corresponding F OF X i values for each of the x i.

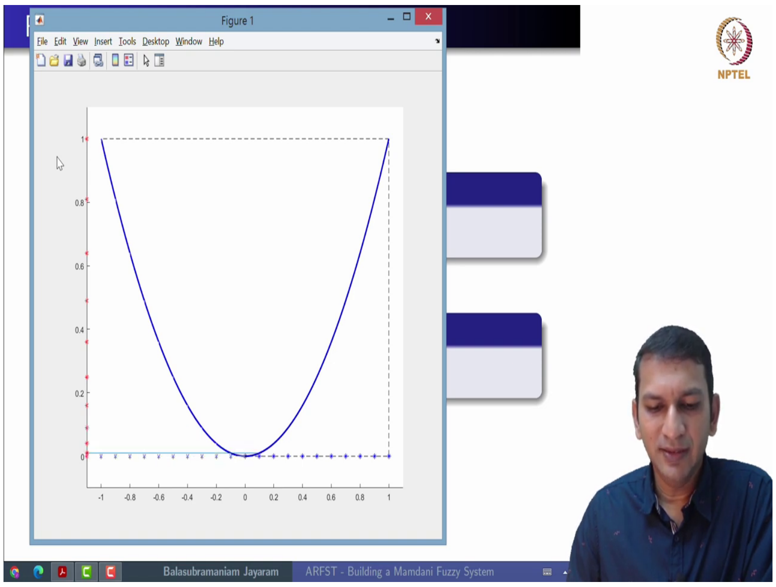
(Refer Slide Time: 26:37)



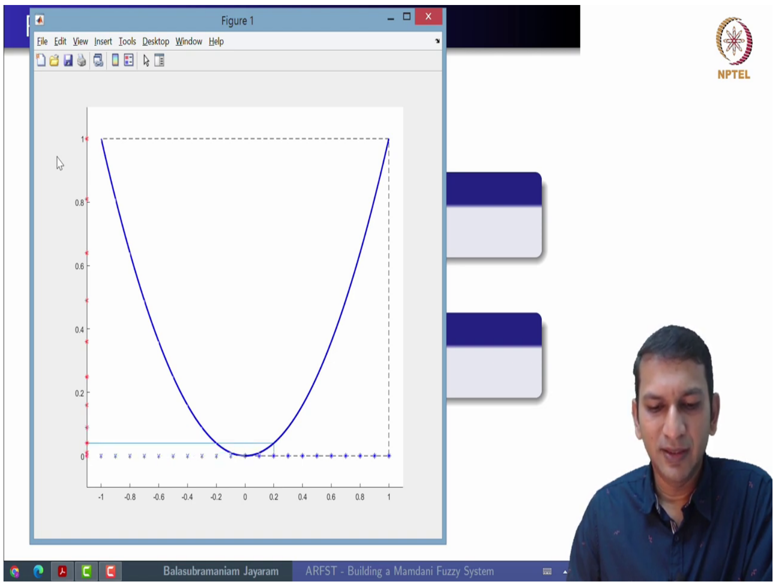
(Refer Slide Time: 26:38)



(Refer Slide Time: 26:40)



(Refer Slide Time: 26:41)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NPTEL
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(Refer Slide Time: 26:41)

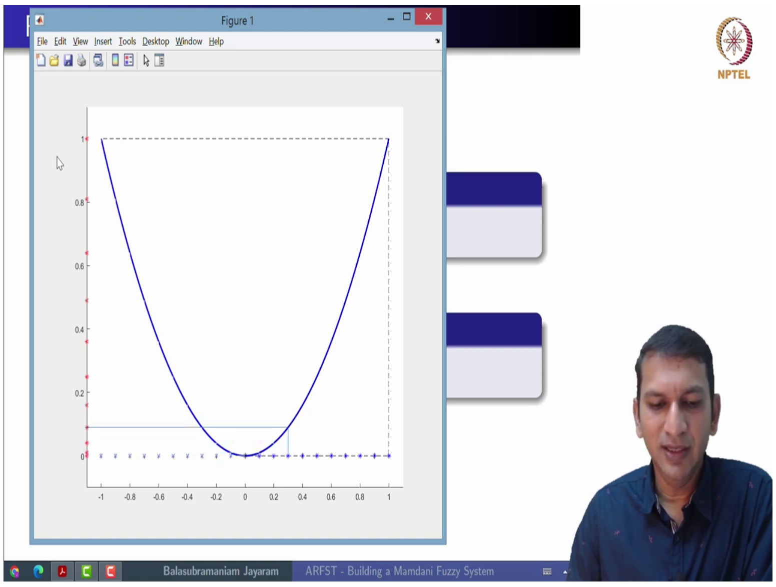
Figure 1
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NPTEL
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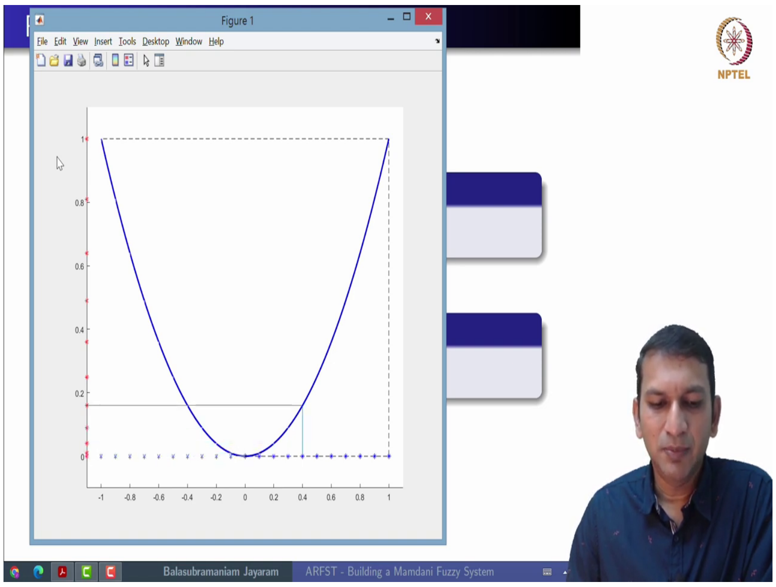

Figure 1
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NPTEL
Balasubramaniam Jayaram
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(Refer Slide Time: 26:42)

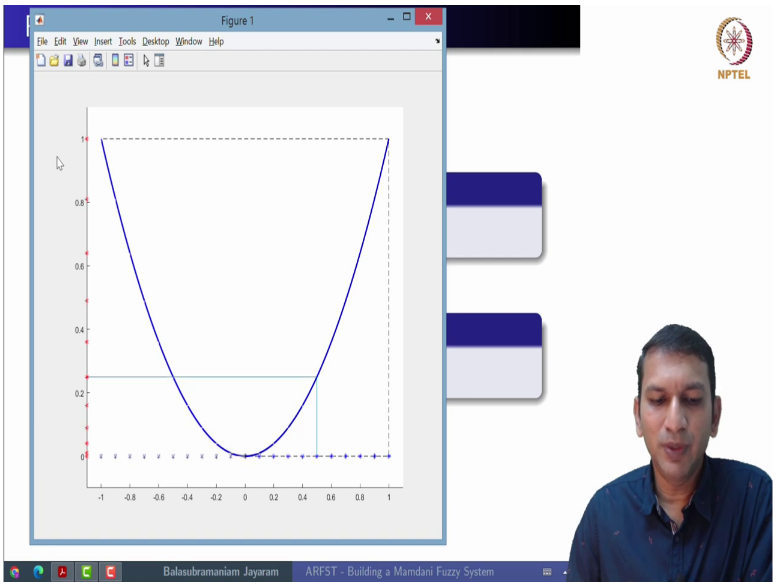

Figure 1
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NPTEL
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(Refer Slide Time: 26:44)

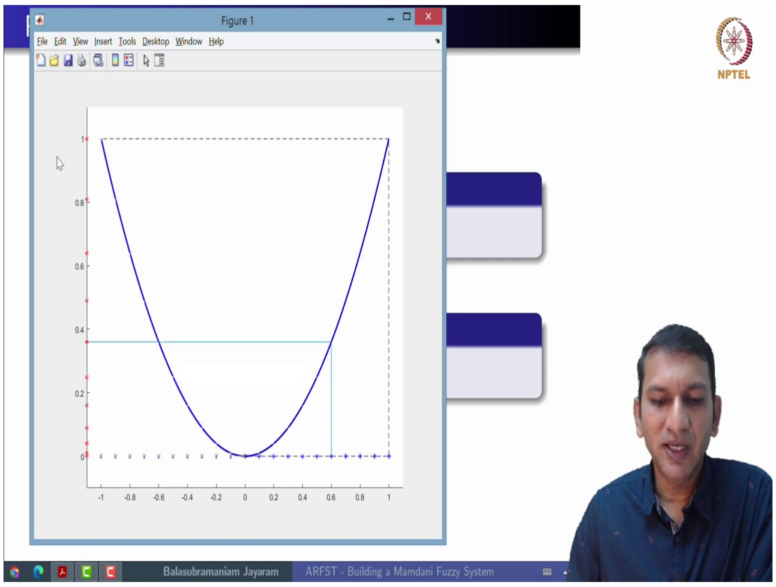
Figure 1
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NPTEL
Balasubramaniam Jayaram
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(Refer Slide Time: 26:44)

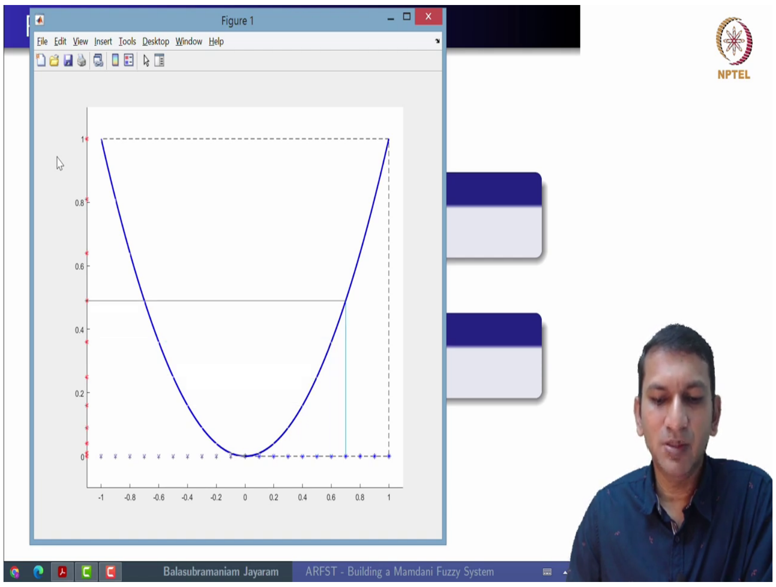
Figure 1
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NPTEL
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(Refer Slide Time: 26:45)

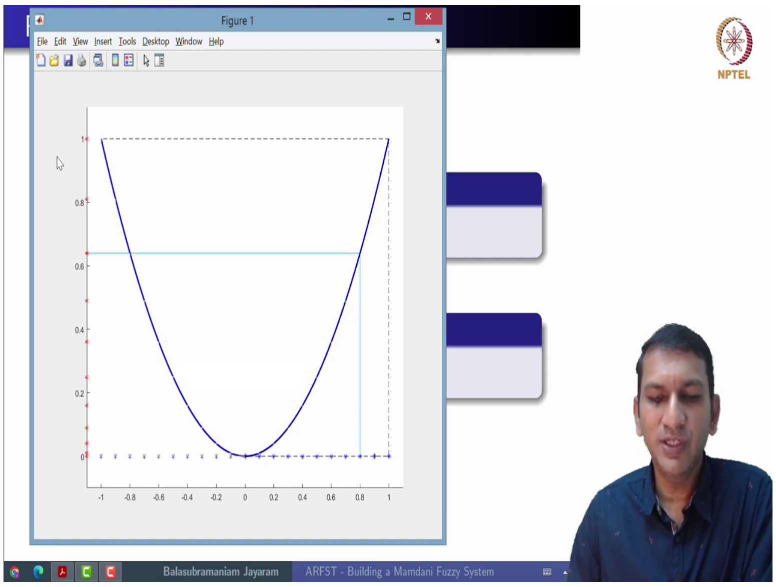
Figure 1
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NPTEL
Balasubramaniam Jayaram
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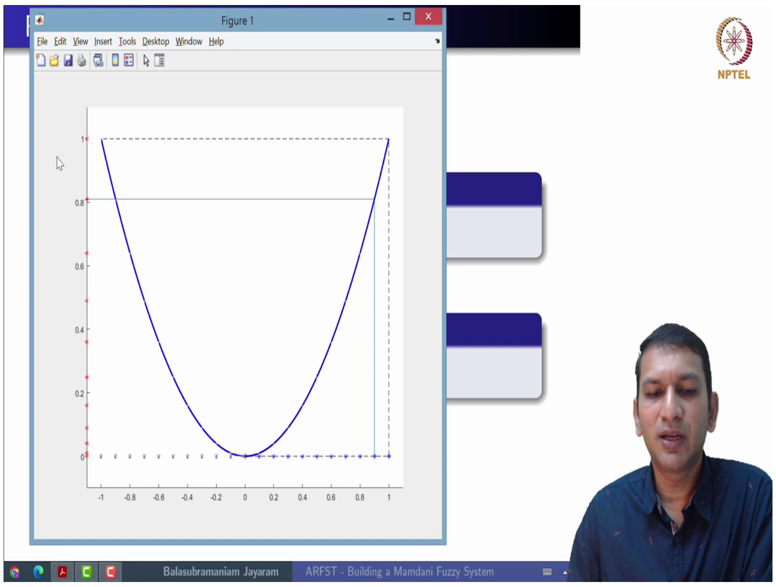
Figure 1
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NPTEL
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(Refer Slide Time: 26:48)

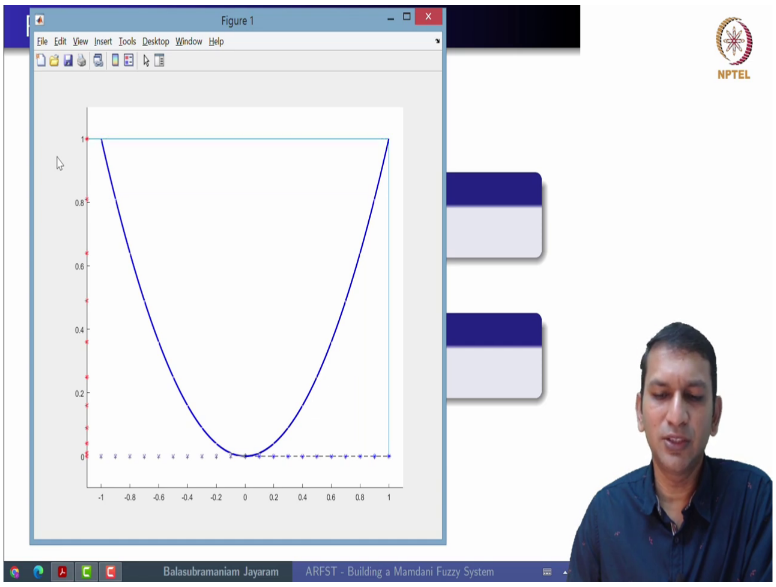
Figure 1
File Edit View Insert Tools Desktop Window Help
NPTEL
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And this gives us an idea clearly that perhaps we should be looking at these points as a prototypical points, around which we could construct the fuzzy sets to form a fuzzy curve.

(Refer Slide Time: 26:49)

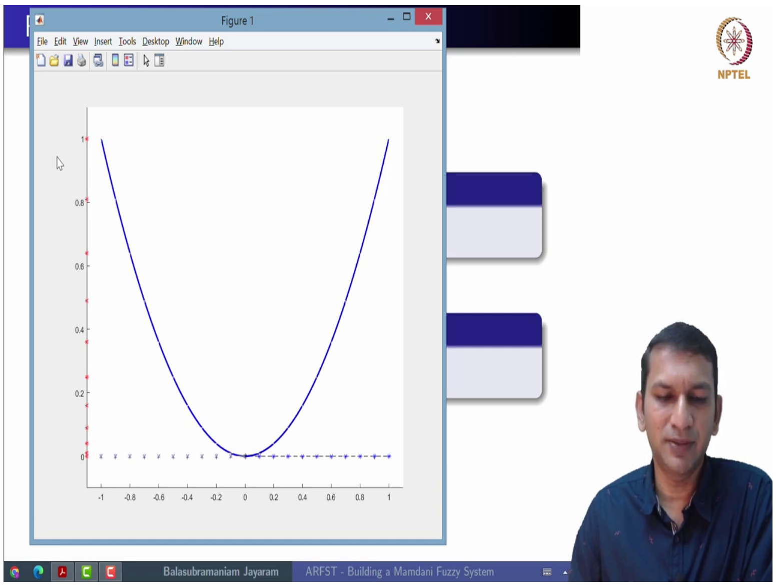
Figure 1
File Edit View Insert Tools Desktop Window Help
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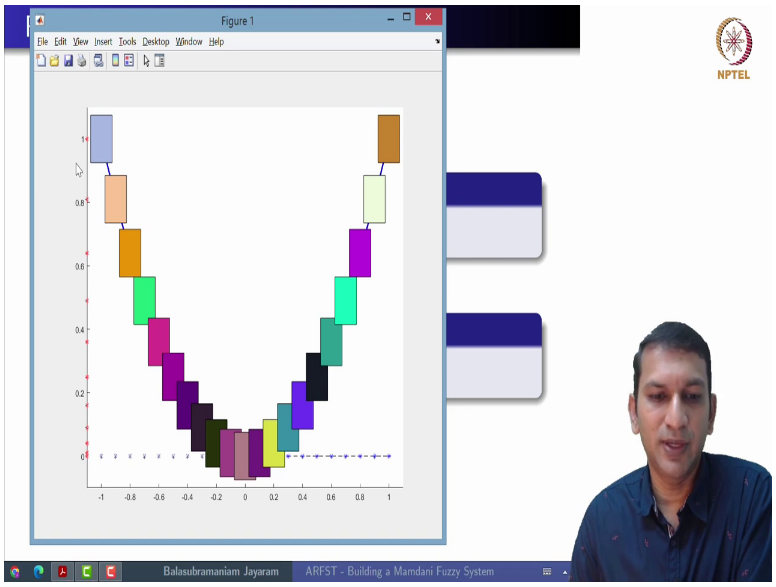
So, now it is clear that once we form the fuzzy sets around these points, then we need to intelligently relate them so as to form rules. It is clear that if you are taking minus 0.8 then we should relate it to 0.64. And similarly, if you are taking 0.7 we have to relate it to 0.49. Is the same thing with when x is if we are considering the fuzzy set which is centered around minus

0.7, because its x square we would look at relating it to the fuzzy set which is centered around 0.49.

Now, let us go back to our idea of patching up this function with rule patches. How would they look like if you if you took the supports on either side to be a constant; that means, on X we have constant support fuzzy sets and similarly having constant support fuzzy sets on Y also.

(Refer Slide Time: 27:51)



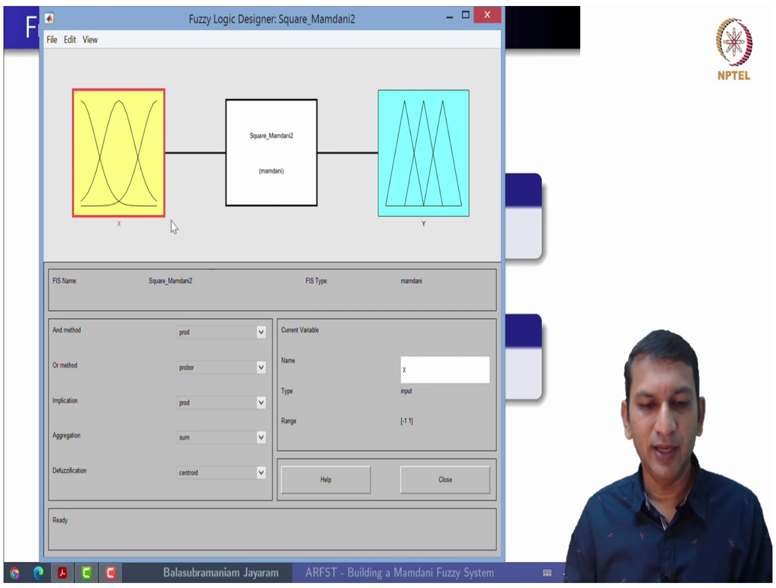
So, as was explained, this is how we should map the fuzzy sets on X to fuzzy sets on Y, these are the rule patches that we would get.

If we fix the support, the width of the supports assuming that we are considering triangular fuzzy sets the widths of widths of the support is to be equi equal on the corresponding domain. We see here that while it is actually nicely patching up overlapping with overlapping patches, here we see that the patches are not overlapping and hence these values are lost. The function is not approximated there.

It clearly shows two things. Firstly, that it is not always possible to have nice we may get symmetric fuzzy sets, but often we need to compromise on many things to be able to approximate the function. Of course, the overall goal is to approximate the function. And if you are using equi width support fuzzy sets on Y, it is not going to cut the deal for us. That is the first thing. Secondly, you see here the support of this fuzzy set centered around 0.81 is only from this point to this point and you see that if you consider such fuzzy sets they will not form a fuzzy covering on Y.

We have seen this that we have insisted only on a fuzzy cover on fuzzy covering on x to call it a complete rule base. However, to be able to approximate the function smoothly and completely, we would need often a fuzzy covering on Y also we will make use of this approach again in constructing a Mamdani fuzzy inference system to approximate this function x square.

(Refer Slide Time: 29:47)



FIS Name: Square_Mamdani2 FIS Type: mamdani

And method	prod
Or method	probor
Implication	prod
Aggregation	sum
Defuzzification	centroid

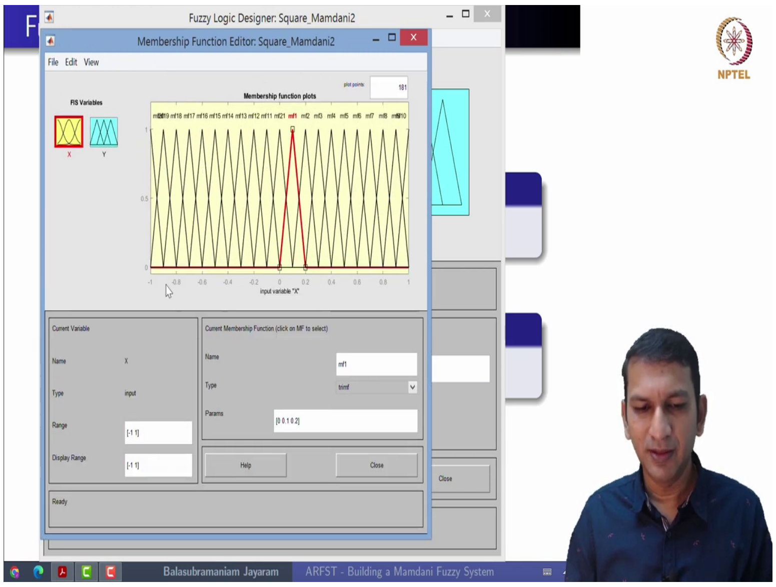
Current Variable

Name	X
Type	input
Range	[-1 1]

Ready

Well, I have taken the liberty to populate the input fuzzy sets, the way we have discussed it.

(Refer Slide Time: 29:54)


Fuzzy Logic Designer: Square_Mamdani2
Membership Function Editor: Square_Mamdani2
File Edit View
FIS Variables
Membership function plots
input variable "X"
Help
Close
Balasubramaniam Jayaram
ARFST - Building a Mamdani Fuzzy System
NPTEL

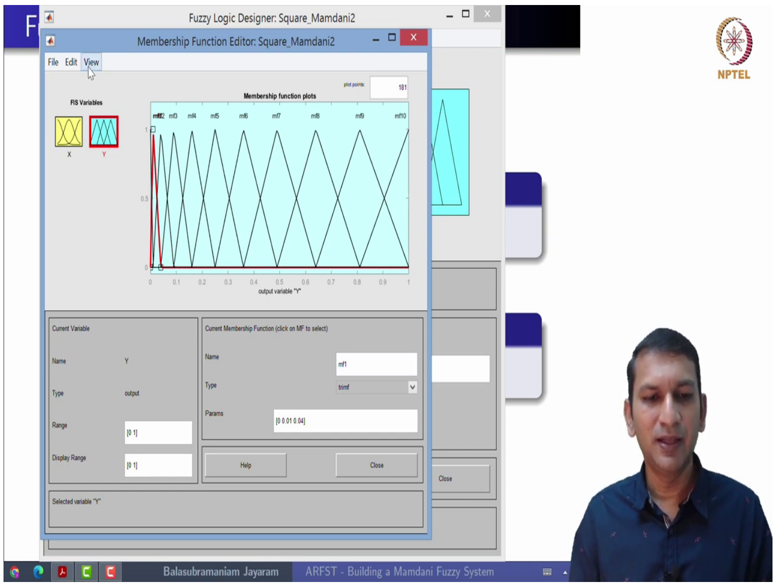
Current Variable
Name X
Type input
Range [-1 1]
Display Range [-1 1]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [0 0.1 0.2]

Ready

So, here are these 21 fuzzy sets as you can see, equi spaced clearly they form a cover, they also form a re spinning partition.

(Refer Slide Time: 30:10)


Fuzzy Logic Designer: Square_Mamdani2
Membership Function Editor: Square_Mamdani2
File Edit View
FIS Variables
Membership function plots
output variable "Y"
Help
Close
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NPTEL

Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [0 0.01 0.04]

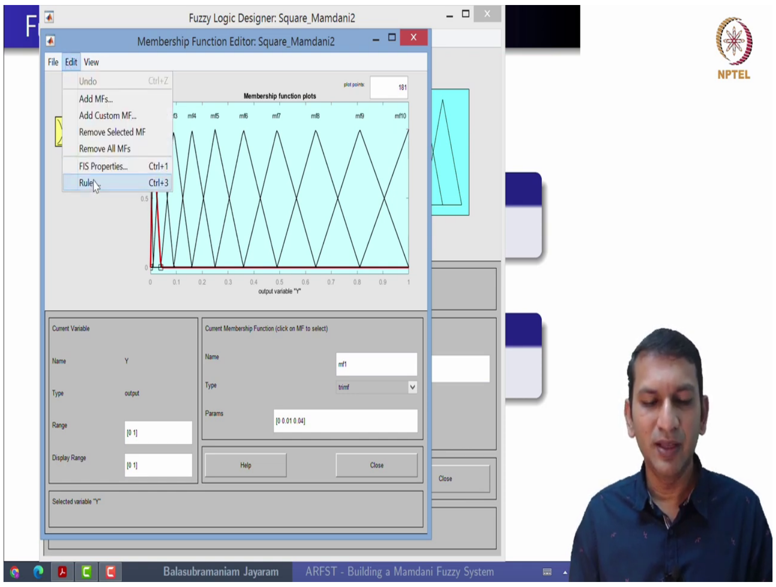
Selected variable "Y"

Similarly on the output domain between 0 and one we have only 11 of them, because it is an x square function even function. So, we have only 11 such fuzzy sets we need to relate these 21 input fuzzy sets to these 11 output fuzzy sets.

Now, at this point of time, let us also recall what we had mentioned or seen earlier, that largely when we have a rule base we have more input fuzzy sets than output fuzzy sets. This is typical and we have seen here now that there are 21 such input fuzzy sets and there are only 11 output fuzzy sets.

Now, how do we relate them? Once again it is clear that we relate them as follows.

(Refer Slide Time: 30:52)

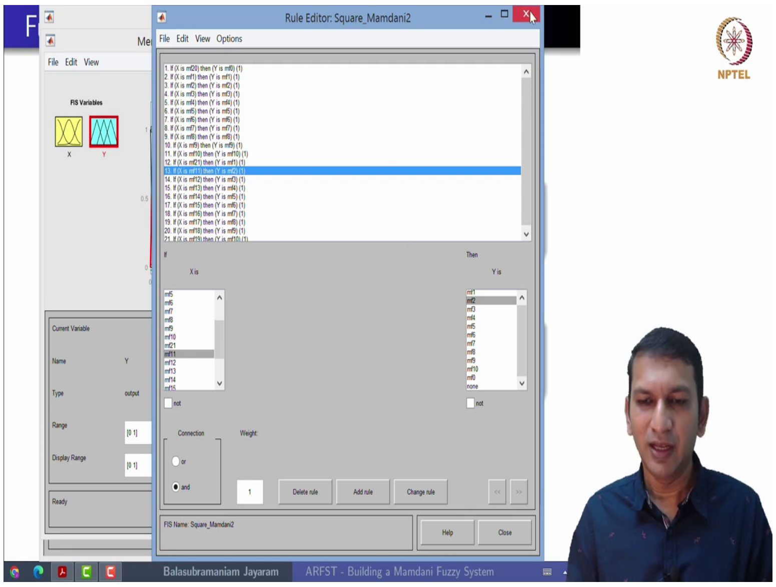


Current Variable
Name Y
Type output
Range [0 1]
Display Range [0 1]

Current Membership Function (click on MF to select)
Name mf1
Type trimf
Params [0 0.01 0.04]

Selected variable "Y"

(Refer Slide Time: 30:54)



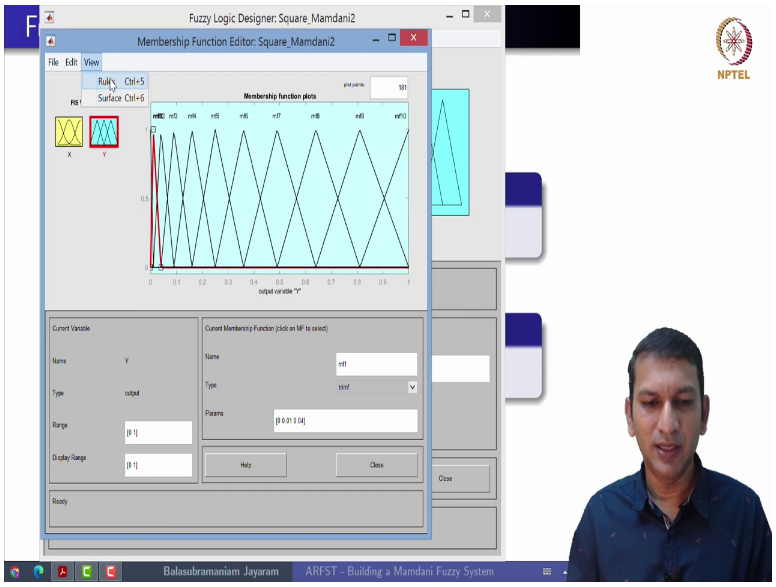
1. If (X is mf20) then (Y is mf0) (1)
2. If (X is mf1) then (Y is mf1) (1)
3. If (X is mf2) then (Y is mf2) (1)
4. If (X is mf3) then (Y is mf3) (1)
5. If (X is mf4) then (Y is mf4) (1)
6. If (X is mf5) then (Y is mf5) (1)
7. If (X is mf6) then (Y is mf6) (1)
8. If (X is mf7) then (Y is mf7) (1)
9. If (X is mf8) then (Y is mf8) (1)
10. If (X is mf9) then (Y is mf9) (1)
11. If (X is mf10) then (Y is mf10) (1)
12. If (X is mf21) then (Y is mf1) (1)
13. If (X is mf11) then (Y is mf2) (1)
14. If (X is mf12) then (Y is mf3) (1)
15. If (X is mf13) then (Y is mf4) (1)
16. If (X is mf14) then (Y is mf5) (1)
17. If (X is mf15) then (Y is mf6) (1)
18. If (X is mf16) then (Y is mf7) (1)
19. If (X is mf17) then (Y is mf8) (1)
20. If (X is mf18) then (Y is mf9) (1)
21. If (X is mf19) then (Y is mf10) (1)

Weight: 1

FIS Name: Square_Mamdani2

That we relate if 1 relates to 0.1, then we are relating it to the square of 0.1 and if 11 relates to minus 0.1, we are again relating it to the corresponding membership function of 0.1. So, this is how we have done this.

(Refer Slide Time: 31:24)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Membership Function Editor: Square_Mamdani2
File Edit View
Rules Ctrl+5
Surface Ctrl+6
Membership function plots
plot points
181
mf1 mf2 mf3 mf4 mf5 mf6 mf7 mf8 mf9 mf10
output variable "Y"
Help
Close
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NPTEL

Current Variable

Name Y

Type output

Range [0 1]

Display Range [0 1]

Current Membership Function (click on MF to select)

Name mf1

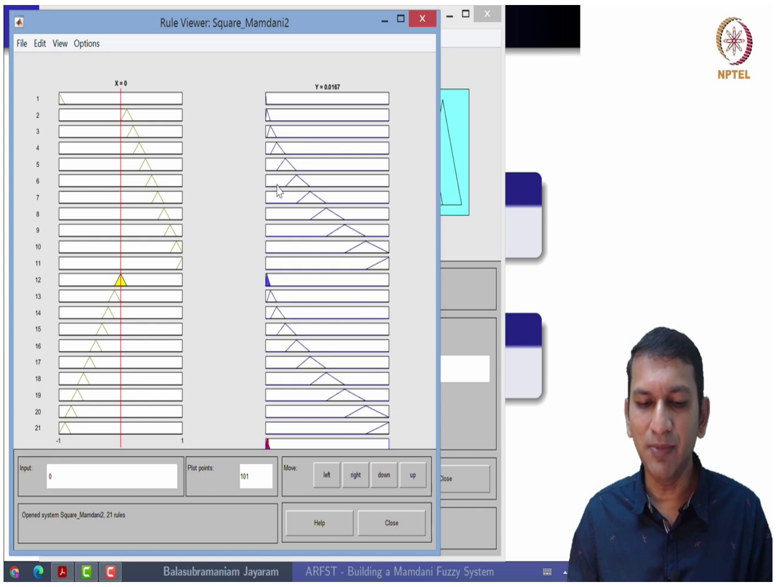
Type trimf

Params [0 0.01 0.04]

Ready

If you would like to see the rules, it is perhaps much easier to see that.

(Refer Slide Time: 31:26)



Input: 0

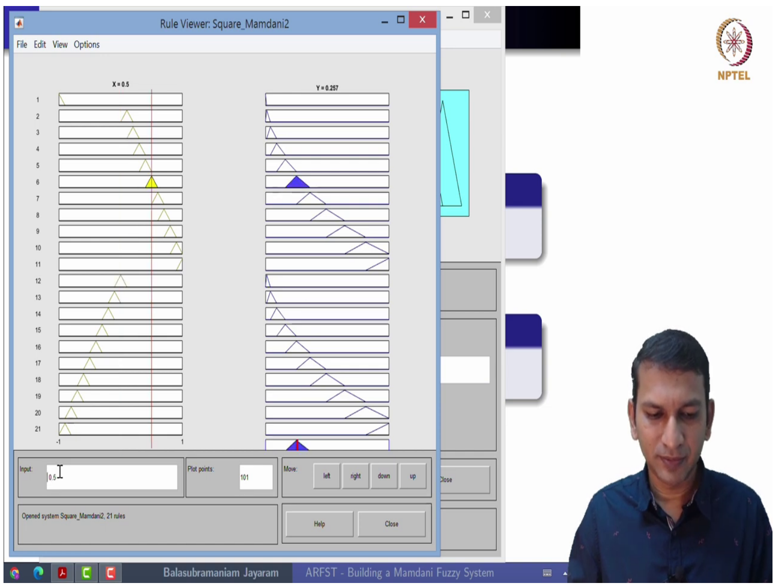
Plot points: 101

Opened system Square_Mamdani2, 21 rules

You will see here that 0 is mapped to 0, 1 is mapped to or 0.1 is mapped to 0.01, 0.2 is mapped to 0.04 and you could see almost it is tapering in the way x square tapers. And you should also notice that these are not constant with fuzzy sets and you will see from here that for every input that you have here; we do not give the input on Y, but we would see that every any element that you take from Y, every element of Y will find some b i here which to which it will belong to a membership value greater than 0. There are far too many things for us to notice here.

But let us give the value 0 and what you want is 0 square and you see here it is 0167.

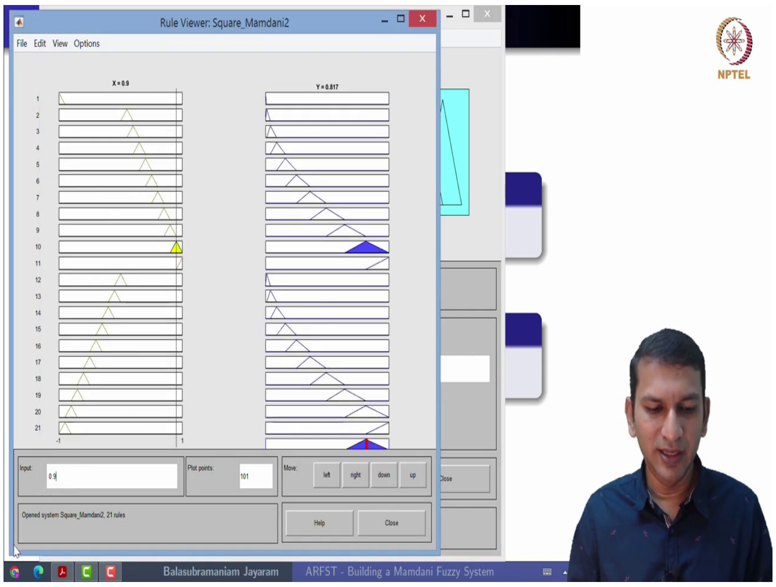
(Refer Slide Time: 32:30)



Opened system Square_Mamdani2, 21 rules

Let us give the value of 0.5 and see, you see here it is 0.257. So, its approximating it quite well.

(Refer Slide Time: 32:38)



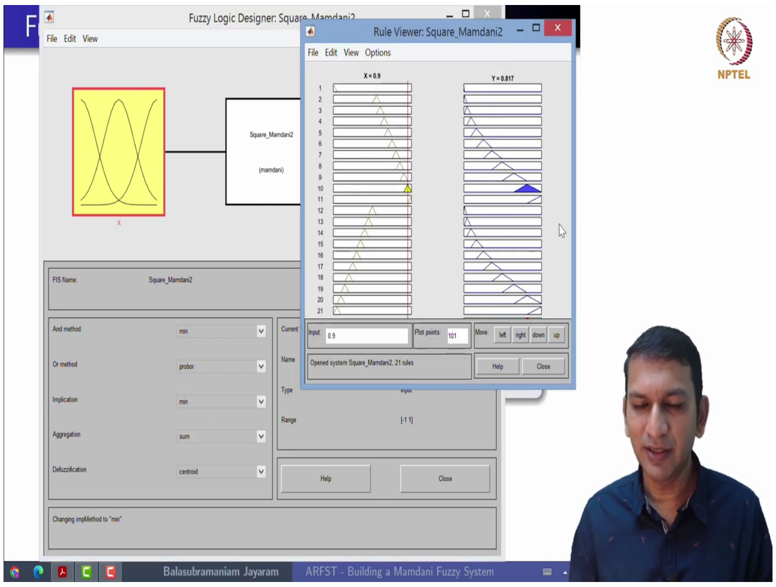
Input: 0.9

Plot points: 101

Opened system Square_Mamdani2, 21 rules

If it is 0.9 you will see it is 0.817 this is what happens. Now, let us do one thing like in the earlier case, let us try to change some of the operations and see what is it that we get, for 0.9 here we have got 0.817.

(Refer Slide Time: 33:03)



FIS Name: Square_Mamdani2

And method: min

Or method: probor

Implication: min

Aggregation: sum

Defuzzification: centroid

Range: [-1 1]

Input: 0.9

Plot points: 101

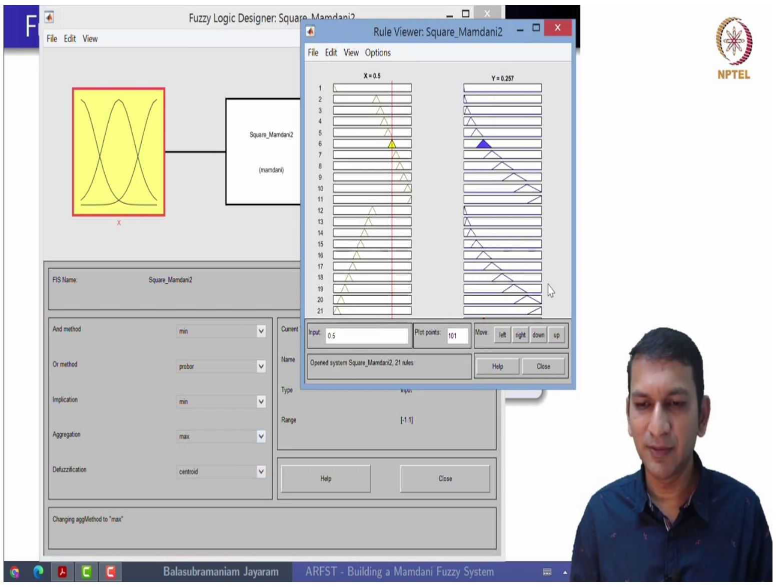
Opened system Square_Mamdani2, 21 rules

Changing impMethod to "min"

Let us change this product to minimum that will not make any difference, because we are talking about singleton inference. So, all that will come out when we are using a different t-norm or any t-norm at all for that matter when you are using the Zadeh’s matching function is the membership value of this input point x naught to the corresponding antecedent.

Instead, what we will do is we will change the modification function and see whether it has any effect.

(Refer Slide Time: 33:39)



FIS Name: Square_Mamdani2

And method: min
Or method: probor
Implication: min
Aggregation: max
Defuzzification: centroid

Range: [-1 1]

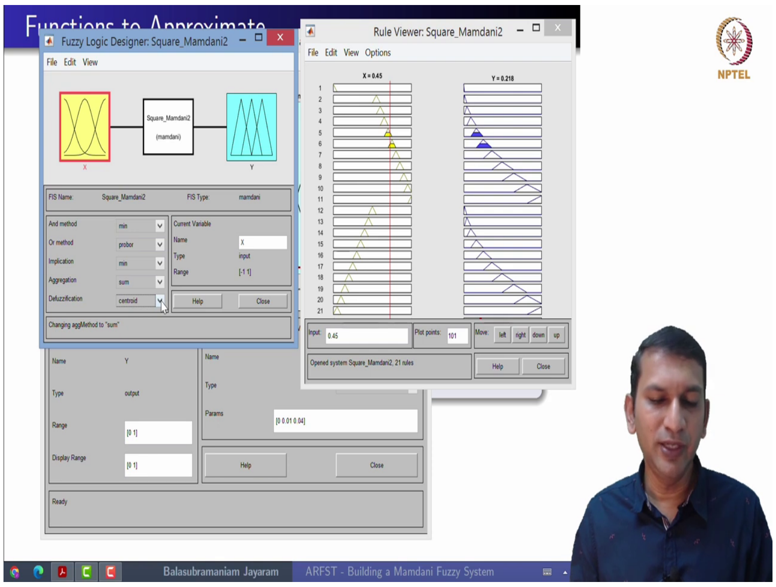
Changing aggMethod to "max"

Input: 0.5
Plot points: 101

Opened system Square_Mamdani2, 21 rules

In this case it does not seem to, that 0.5 again you get this. Now, instead of aggregating using some, let us do it with max of course, we have taken a point which is no overlap in two things.

(Refer Slide Time: 33:53)



FIS Name: Square_Mamdani2
FIS Type: mamdani

And method: min
Or method: probor
Implication: min
Aggregation: sum
Defuzzification: centroid

Current Variable
Name: X
Type: input
Range: [-1 1]

Changing aggMethod to "sum"

Name: Y
Type: output
Range: [0 1]
Display Range: [0 1]

Params: [0 0.01 0.04]

Ready

Input: 0.45
Plot points: 101

Opened system Square_Mamdani2, 21 rules

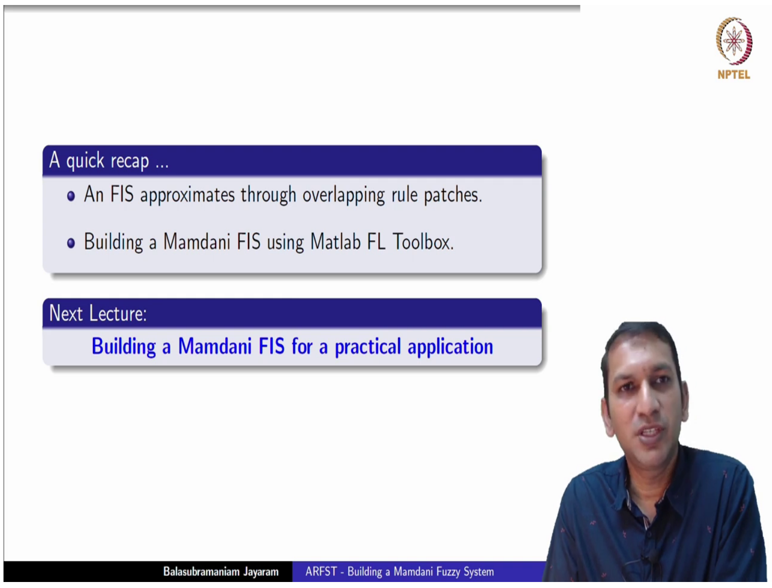
Now, you see here when it is 0.4, it is 0.016, when it is 0.45 you get 0.22, when we are actually using maximum aggregation I want to change it here. So, if you use. So, if you use sum you see that automatically it goes to 0.218.

Instead of centroid, if you use some other de fuzzification since it is not clear in this diagram here, we will change it. We will not change it now, we will look into de fuzzification the different de fuzzification methods how they change. Perhaps when we take up building a tsk fuzzy system to approximate a function. Well, so this in a nutshell is how you build a fuzzy inference system, a Mamdani fuzzy inference system to approximate a given function.

Of course, the immediate question is in this case we knew the function that we need to approximate, there was no need to approximate this function. However, as was mentioned earlier too we are only learning the tricks of the trade given it is only a some kind of proof of evidence that any function can actually, any continuous function can actually be approximated using Mamdani fuzzy inference system. And also a way of doing it, how to actually pick the fuzzy covering, how to pick the rules and what are the operations or options that you choose for the different inference operations.

In general, we do not know what is the function that we are going to approximate, the system function is not known, but we will have some data which is coming from the system and using the data we build the inference system. We will see an example of it in one of these lectures this week.

(Refer Slide Time: 35:55)



A quick recap ...

- An FIS approximates through overlapping rule patches.
- Building a Mamdani FIS using Matlab FL Toolbox.

Next Lecture:

Building a Mamdani FIS for a practical application

Balasubramaniam Jayaram ARFST - Building a Mamdani Fuzzy System

A quick recap of what we have seen so far in these lectures. We know that a fuzzy inference system approximates any given function through overlapping rule patches. In this particular lecture we have looked at how to build a Mamdani fuzzy inference system to approximate a given function.

Of course, the function is known and we were trying to approximate it. So, essentially we took the information from the given function itself; however, in practical applications it is not often that we know the exact system function; however, we will know some information about the function itself, some qualitative information about the function, the system function. Perhaps about its monotonicity or about its oscillatory properties or some qualitative properties about how it behaves over different subdomains or perhaps essentially that the graph of the function is band limited between some region.

Making use of this we may be able to build a fuzzy inference system and often we do using this heuristics. We will see in the next lecture one particular case, where we will take a practical application and show how to build a Mamdani fuzzy inference system for them. For this we will take the case of enhancing the contrast in a given grayscale image.

And we will see how we obtain the qualitative nature of the system function that we are trying to approximate. And using that how we build a Mamdani fuzzy inference system. This is what we will take up in the very next lecture itself. Glad you could join us for this lecture and hope to see you in the next lecture.

Thank you again.