Approximate Reasoning using Fuzzy Set Theory
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Lecture - 31
Fuzzy Relational Inference - MISO Case

Hello and welcome to the 4th of the lectures, in this week 6 of the course titled Approximate Reasoning using Fuzzy Set Theory. A course offered over the NPTEL platform.

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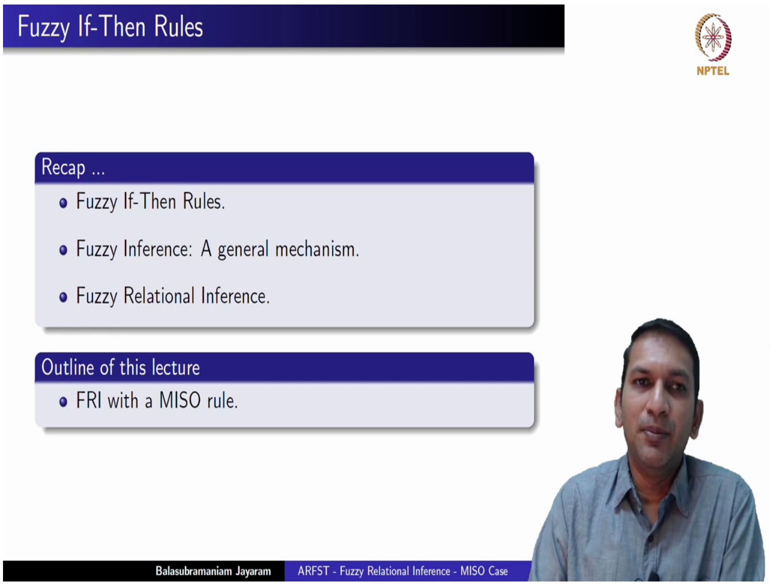

So far this week we have looked at two of the important components on fuzzy inference system that of fuzzy if-then rules in the rule base itself. We have looked at general schematic diagram of a fuzzy inference mechanism. And we moved on to the second of the important components of an FIM that of the inference engine itself. In here, we have looked at fuzzy relational reference. Of course, we have seen it only for a single rule and that too for a single input single output rule.

In this lecture, we will look at how to infer when we are given a multi-dimensional input output that means, multi input single output rule.

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A quick recap of the mechanism of fuzzy relational inference.

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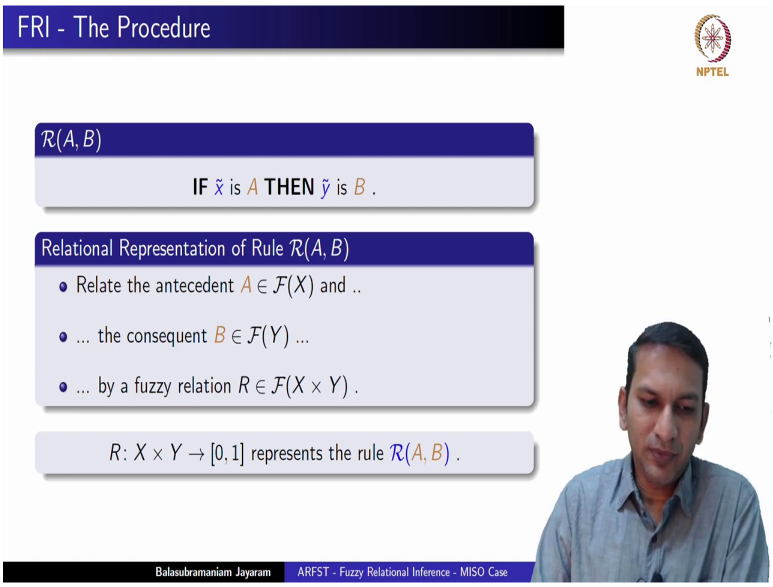

So, we are given a single input single output rule. Firstly, we would like to get a relational representation of this rule. That means, we would like to relate the antecedent A which is a fuzzy set on X to the consequent B which is a fuzzy set on Y and capture it as a fuzzy relation on the Cartesian product X cross Y. Note that X and Y are the underlying input and output domains. So, essentially we want to build a fuzzy relation from the given rule.

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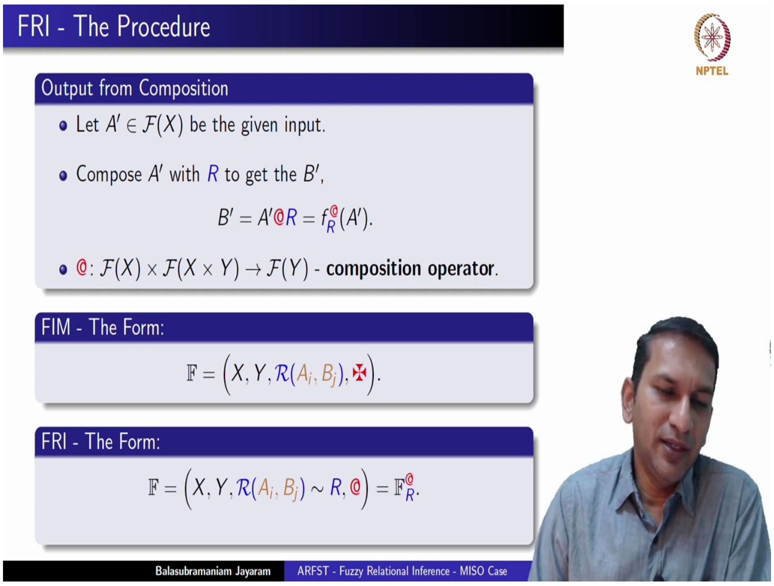

Once we have built this, given an input A dash which is again a fuzzy set on X, we obtain the output B dash which is a fuzzy set on Y by composing A dash with this relation which captures the rule. We have seen that this composition operator is essentially a function which takes fuzzy set on X and a fuzzy relation on X cross Y and outputs a fuzzy set on Y.

We have seen that so far from what we have seen a fuzzy inference mechanism can be looked at as a quadruple, wherein X and Y are the input and output domains. R of A i, B j it represents the rule base. And the final operation symbol mark is it shows that there is some inference being applied. It consists of all the operations that require, that are required for the inference itself.

Specifically, in the case of fuzzy relation inference it turns out like this. So, we have the input output domains and the rule base. And the inference is captured by these two things. One, the relation which is obtained out of the rule base, rule so far that we have seen and the composition itself. Often, it is convenient to write it in the short form.

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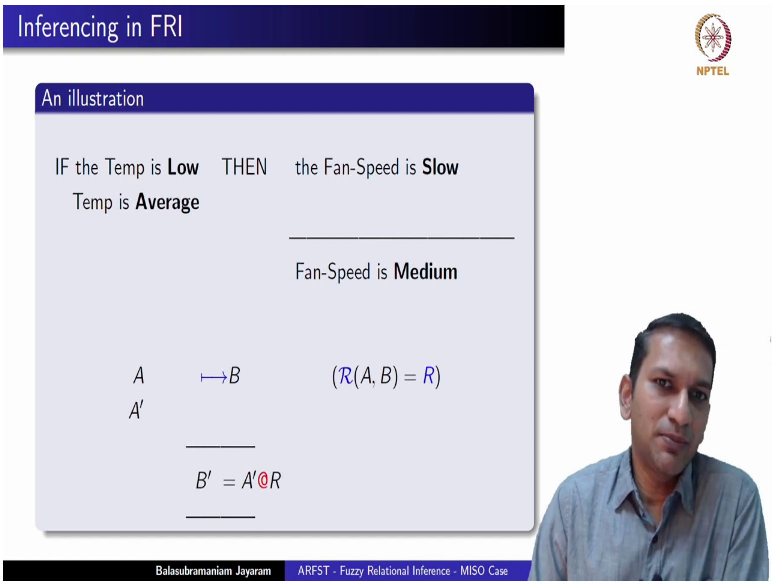

Well, let us look at an new illustration. We are given this rule. Temperature is low then the fan-speed is slow. And given the input temperature is average, we would like a reasonable inference something like fan-speed is medium. So, when you translate it into notational symbols, then we see that we have a rule of the form A implies B and given A dash we want a B dash. We know that this can be obtained by composing A dash with the relation R. And what is R? It is that fuzzy relation which captures a rule.

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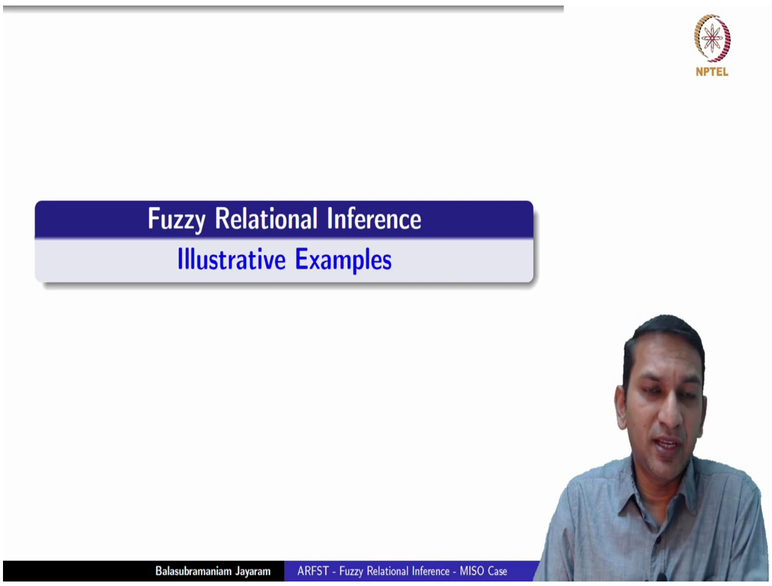

In the previous lecture, we have seen a few illustrative examples. Let us revisit a couple of them with a specific purpose in mind.

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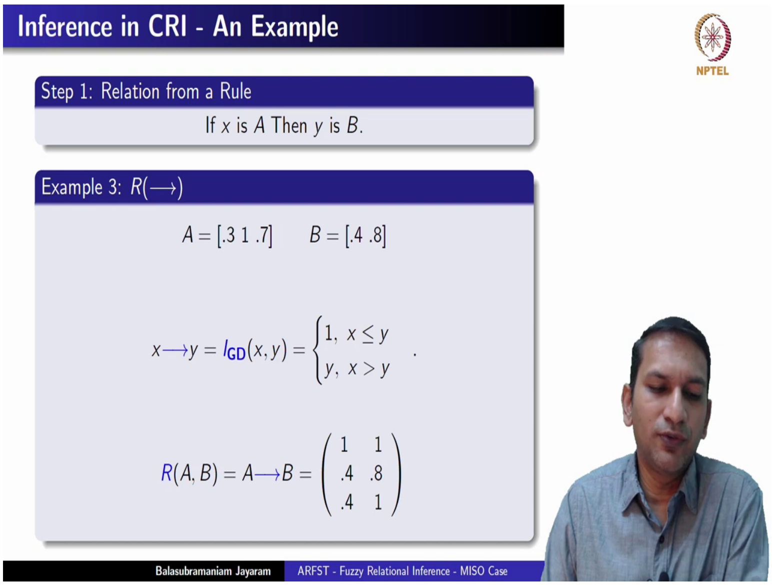

Note that we have a rule, a single input single output rule. And in this case, we would like to use the implication operator to capture the rule into a relation. A is given as this and B is given as this; that means on x and y, we have 3 points for discretization on x and, 2 points in as for discretization in the domain of y. We are using the Goguen implication. And we have seen that this is the relation that we will extract out of this rule.

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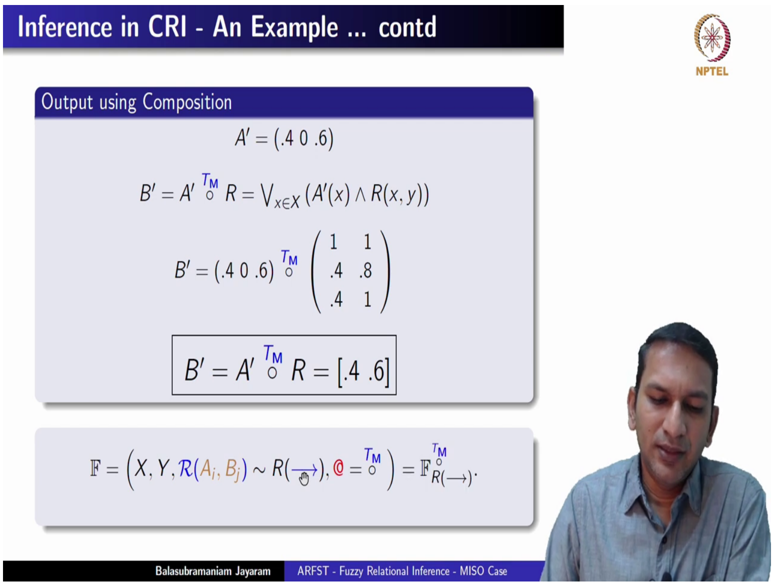

Now, given the input A dash as follows as 0.4, 0 and 0.6, we apply the sup-T composition because we are in the regime of CRI, composition rule of inference. Sup-T composition with the minimum t-norm, and we saw that we obtain the output to be 0.4 , 0.6. So, this is the output fuzzy set. Now, first thing if you look at it there is an FRI, specifically it is a CRI; that means, the composition is a sup-T composition. In this case, we are using a sup-min composition. But note that R is obtained by using the implication as the function F. This is an shorter way of indicating.

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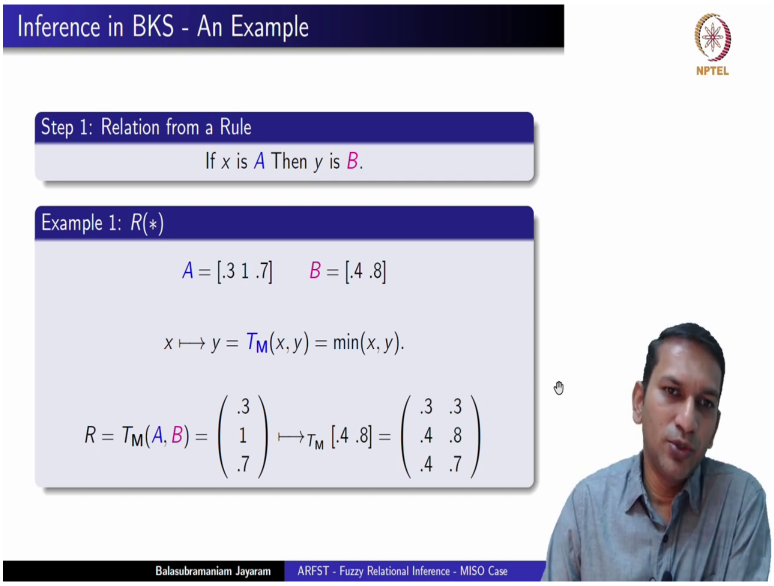

Now, in yet another example, we used t-norm to generate the relation, once again A and B remain the same. To generate the relation we have used minimum t-norm, and taking A and B in some sense doing some operation that kind of that is kind of an outer product, but using the minimum t-norm, we have come up with this relation. We have seen this in the little detail in the last lecture. Now, the relation itself was obtained using the t-norm.

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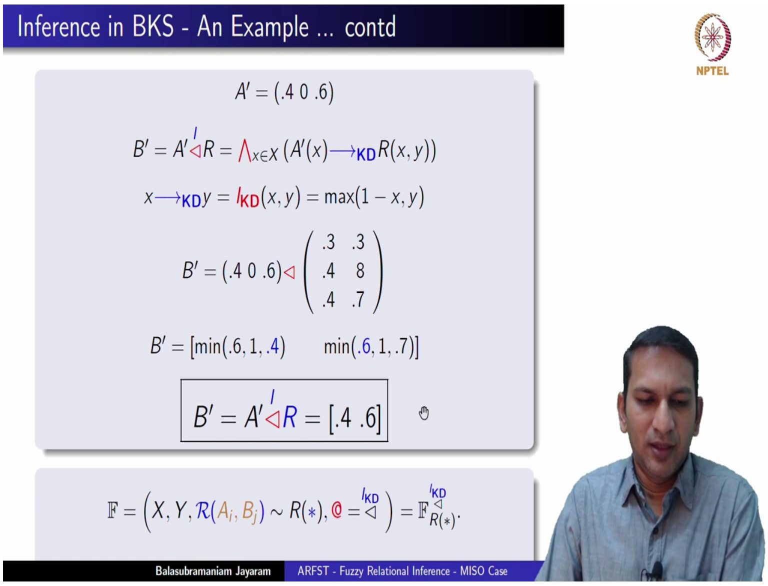

But the composition is in phi composition because we are in the regime of BKS inference, Bandler-Kohout Subproduct inference. And for the implication here, we have used the Kleene-Dienes implication which is given as max of 1 minus x , y. And we have seen that when we give this input A dash to this relation, this is the output that we obtained. So, when you take this row into this column using Kleene-Dienes implication, these are the values you get and then it is actually infimum of this.

So, minimum of them will be 0.4, minimum here is 0.6, and that is why we get 0.4, 0.6. This is an example that we have discussed in the last lecture. And if you were to write this inference, you see that X, Y in the rule base are fixed. And we know the inference is captured by the relation and the composition. In the BKS, it has an infinite composition that is clear. In this case, we have used the Kleene-Dienes implication. Note that the relation itself has been obtained by the t-norm.

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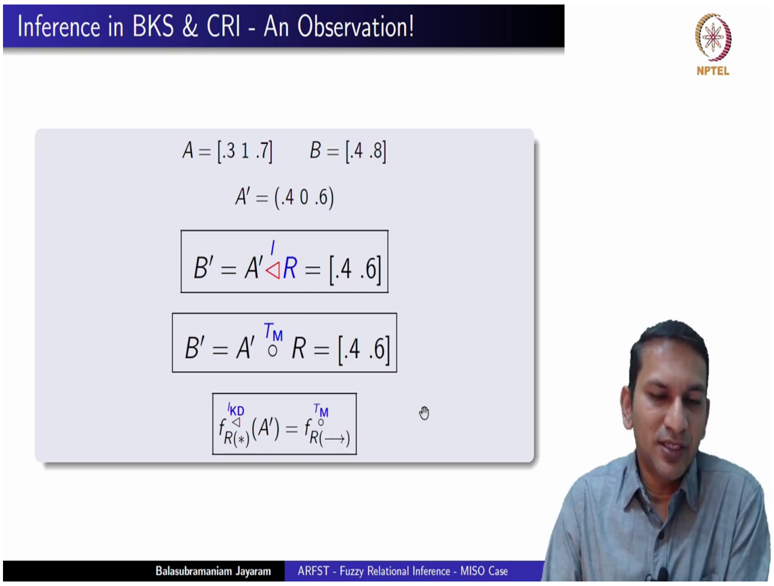

Well, what is interesting is this, we are given A, B and A dash, and now when we used BKS where the Kleene-Dienes, and obtained the relation from t-norm especially the min, we will saw that this was the output we got. And when you use CRI which sup-min composition and obtain the relation from the rule using the Goguen implication, we saw that this is essentially the same output that we have got.

So, it appears that for this A dash from two different inferences, we are actually obtaining the same output. Let us take the same inputs and try to change some operations.

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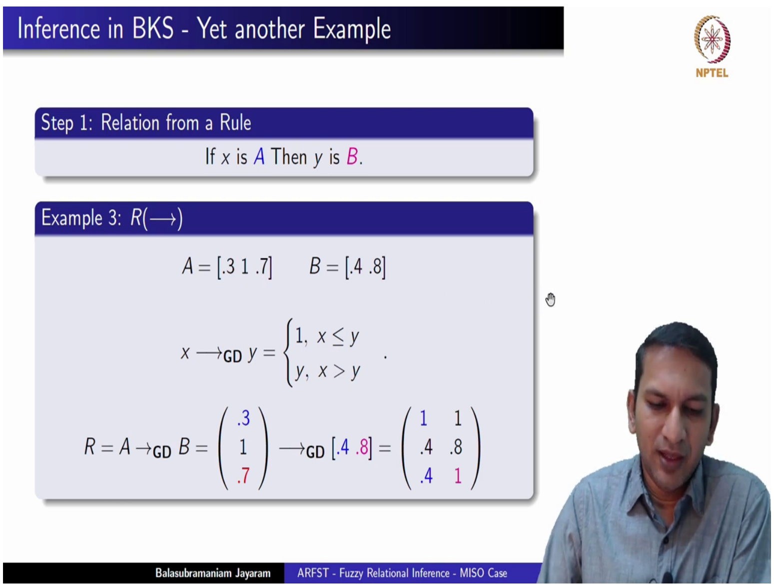

The same rule we will use an implication to capture the relation. We have used the Goguen implication itself. So, this is the relation that you will get to capture this rule, when A and B are given as this.

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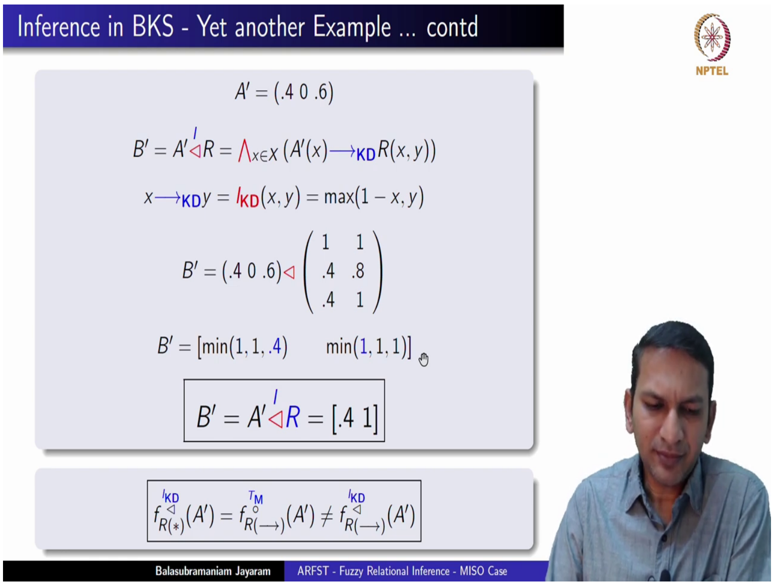

Now, let us use the BKS inference. Earlier with when we captured the rule using an implication, we use the CRI inference; that means, sup TM, sup-min composition. In this case, we are capturing the relation also with an implication and we are applying the B case which essentially uses an implication. So, to capture the rule we have used Goguen implication. And in the inference we are using the Kleene-Dienes implication. Let us look at what happens here.

Now, you look at this. Note that we have to take this row into this column using the Kleene-Dienes operation. A quick check will show you that if we are using the Kleene-Dienes implication 0.4, 1 will be 1; 0.4 fuzzy it is an implication if x is 0 it is 1, if y is 1 it is 1 and 0.6, 0.4 is 1 minus 0.6 , 0.4 max of these two which is again 0.4. So, that is how you get this. Now, 0.4, 1 means 1, 0.8 of a Kleene-Dienes applied the Kleene-Dienes would give you 1; 0.6, 1 would also give you 1. Now, minimum of all of this is 1 is 0.4, minimum of these three 1's is 1.

So, what we see is while earlier the inference that we got from two different inference schemes A dash were same. In this case, the same exactly the same rule and inputs, but with the slightly modified different combination of operations for obtaining the rule, the relation from the rule, and the composition, we see that they are no more true. This forms many questions; a few of them definitely we will be addressing as we go forward.

Well, this was just to illustrate that for a given A dash, the B dash is not unique. It depends on so many operation operators that you use. So, the degree of freedom is quite a lot and it depends on the choice of operators that you have pick.

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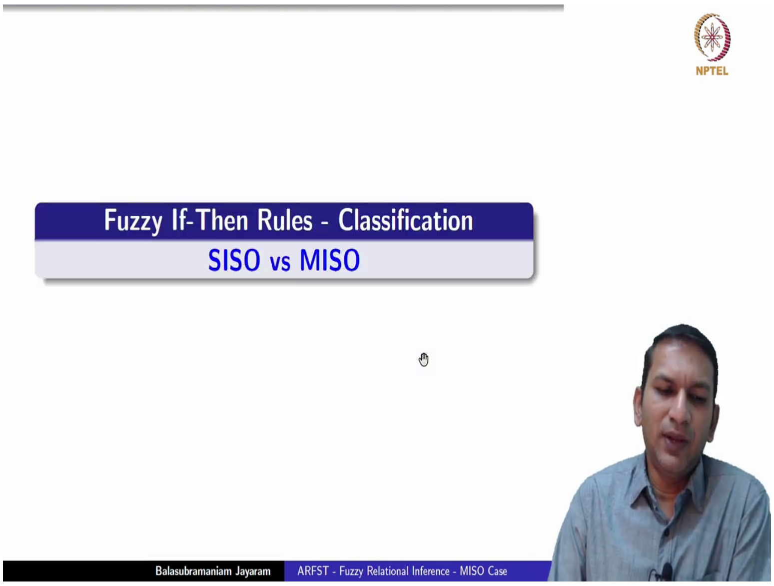

Well, now let us go into looking at how fuzzy relations inference can be applied when you are given a MISO rule.

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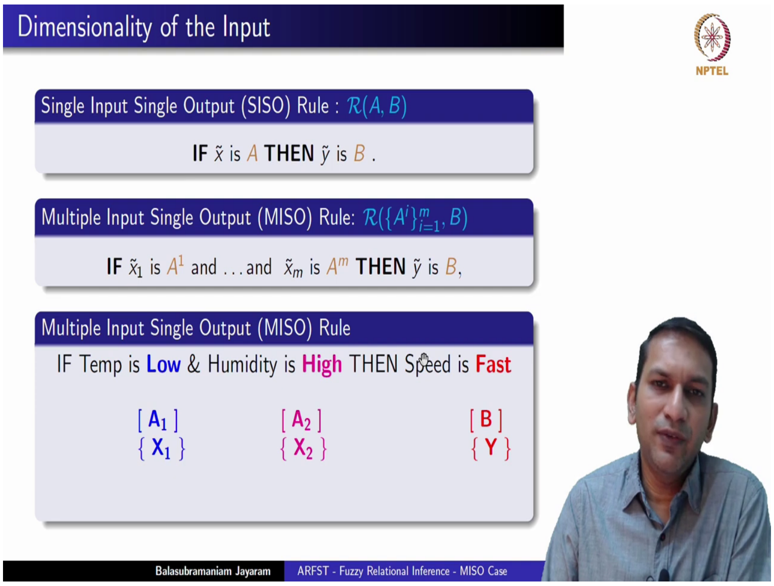

Now, what is the SISO rule? Input dimension is only 1. What is the MISO rule? There are multiple input dimension. So, we say if x tilde; x 1 tilde is A 1 and x 2 tilde is A 2, so once, if x m tilde is A m then y tilde is B. What are these x 1 tilde, x 2 tilde? They are again linguistic variables which can assume fuzzy sets linguistic values, defined over the domains x 1, x 2, x 3, so on till x m. So, the dimensionality of this rule is (Refer Time: 11:34).

We have seen an example of the rule, if temperature is low and humidity is high then speed of fan-speed is fast. Low is the linguistic value A 1, high is A 2 and fast is B. Note that A 1 is defined over X 1 its fuzzy set from X 1, A 2 is fuzzy set from X 2, and B is fuzzy set from Y. Now, the question is how do we capture this rule into a relation. Let us look at that.

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How do we represent a MISO rule by a relation?

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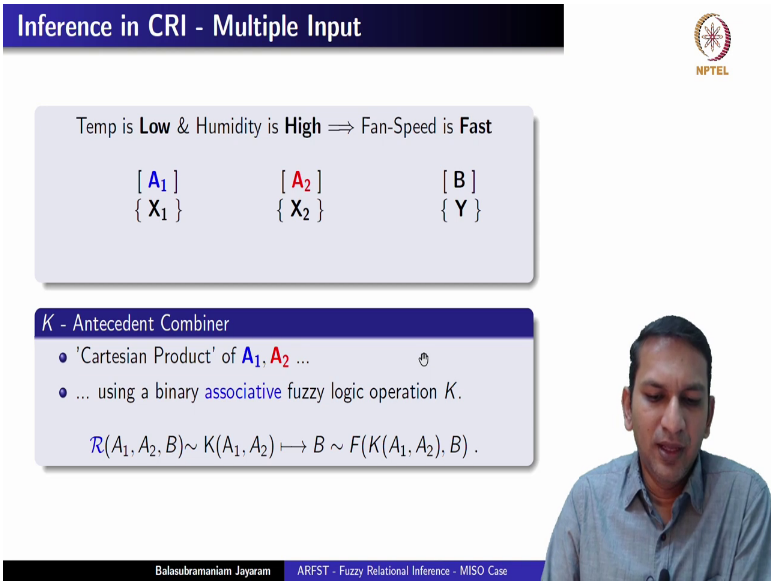

This call is for yet another operation which we call the antecedent combiner. What essentially we are doing is taking the Cartesian product of A 1 and A 2, much like how we took Cartesian product of A and B to obtain the rule, but here it is with the difference. So, we are going to consider A 1 and A 2 as both of them as antecedents. So, the semantics here could be quite different from relating an antecedent to consequent. For the moment, let us stick with just these two dimensions A 1 and A 2.

So, we can combine them, these antecedents, using a binary fuzzy logic connective, but a binary associative fuzzy logic connective. Why associative? Because we might have more than two antecedents here. So, we have seen that you do not have an m-dimensional input, so we need to be able to combine m of them. So, in that sense, the rule now becomes R of A 1, A 2 , B. It is being captured as follows. First, we use an antecedent component to combine A 1, A 2, then we apply the operation F, which relates this combined antecedent to the consequent.

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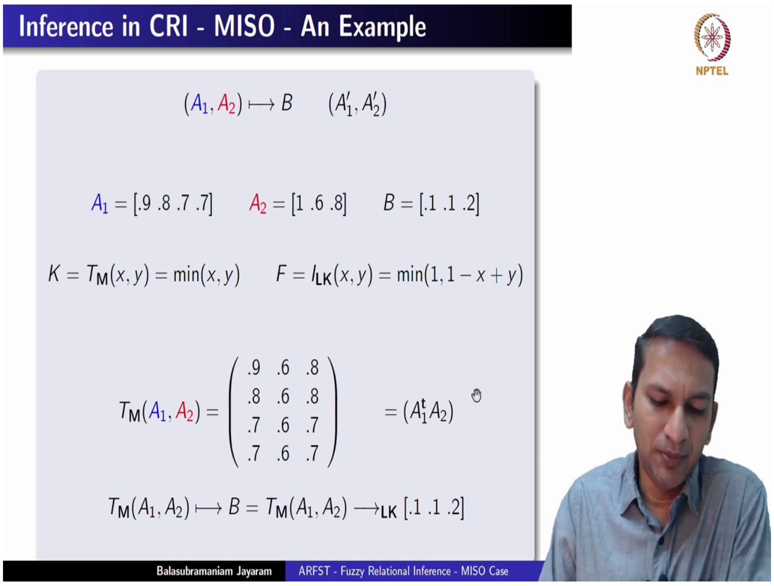

Let us look at an example. So, let us represent this multi-dimensional group as A 1, A 2 going to B and the input now will not be a single A dash, it will also be multidimensional. In the case that we are considering, it will be A 1 dash, A 2 dash. Let us give some values to these labels that A 1 look like this; that means, on the domain of X 1 we are discretising with 4 points and this is the fuzzy set A 1. Similarly, A 2 is discretized with 3 points on X 2 and B again we are obtaining as a fuzzy set from Y which is discretized into 3 points.

And for the antecedent combiner, let us use the operation min just to keep the working example simple. And to relate the combined antecedent to the consequent let us use the Lukasiewicz implication. Now, combining A 1 and A 2 means what? Essentially, what we have done earlier to obtain the relation from the rule. It, you can think of it once more as an outer product for instance.

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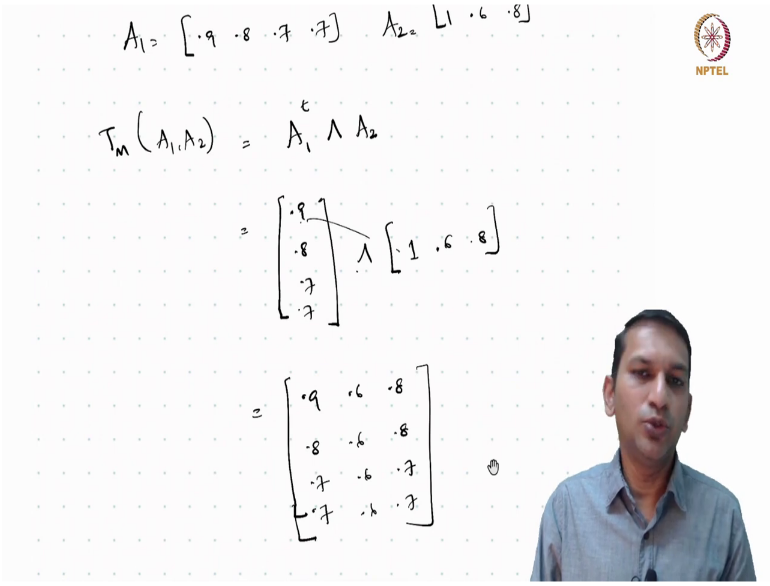

So, call A 1 was this 0.9, 0.8, 0.7 and B A 2 is 1, 0.6, 0.8. And we want to talk about T M A 1 , A 2 is nothing but A 1 transpose min A 2. What do you mean by this? 0.9, 0.7, 0.7 and apply the minimum operator 1, 0.6, 0.8. So, once again it is we are actually taking 0.9 and multiplying with this row vector that the multiplication operation is given by min. So, clearly what you would get is 0.9 and 1 minimum of from this, 0.9, 0.9 is 6 is 0.6, 0.8; 0.8 is known as 0.8, 0.8 and 0.6 is 0.6, 0.7 and 1 is 0.7, 0.7 and 0.6 is 0.6, 0.7, 7 (Refer Time: 16:20).

So, this is the matrix that you get when you combine A 1 and A 2 using the minimum and minimum as the antecedent combiner. That is exactly what we are here. Now, this is combining the antecedent. We still need to use the consequent of the rule to build the relation. So, that means, this operation is spending, and note that this is the F operation and it is given by to Lukasiewicz implication.

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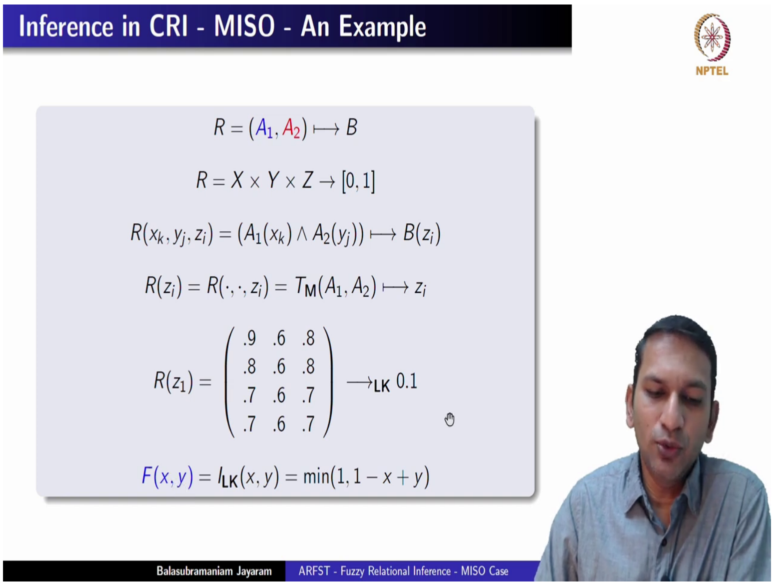

Now, how do we do this? Look at this. We want to now build a relation for this rule. Now, there are two antecedents and the consequent coming from X 1 cross X 2, X 1 cross X 2 cross Y 3 dimensions. And so the relation R is over these 3 dimensions. So, if you look at them like this R of x k, y j, z i, then we want to build this from the membership value of x k in A 1 and membership value of y j in A 2 and using F operation the membership value of z i in B.

Now, clearly this R is a three-dimensional relation. So, now, it is difficult to represent it in two dimensions. So, a better way to look at it is the fixing one of the complex. Let us fix z i and let the other two vary. So, which means essentially we are looking at the combined antecedent acting upon just one component of B. What do you mean by this?

Look at this R of z 1 is this is the matrix that we have got and using the F operation where z i is actually 0.1. Note that earlier we would have had only one column, if it was just one antecedent I mean consequent. Now, because we have combined these two antecedent in B and then combine these two antecedents we have a matrix (Refer Time: 18:35). However, it is the same outer product that we are going to take.

So, how do we do this outer product? The simple this is the combined antecedent, we take the outer product with one of the components first and repeat the process over each of the components z 1, z 2, z 3. And to do this we are using the operation F which is the Lukasiewicz implication.

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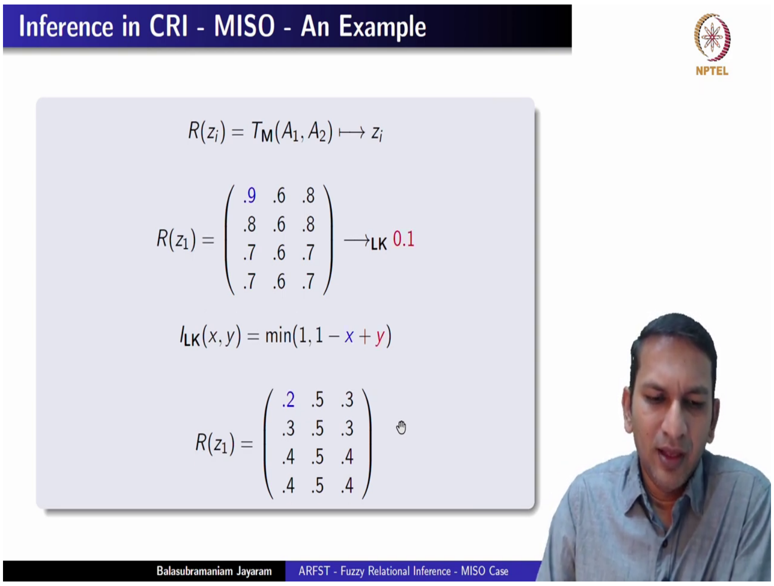

Well, let us look at doing this. R of z 1 is combined antecedent and using the implication Lukasiewicz implication and this z 1, we need to obtain one part of the relation. Remember, this relation itself is a creative relation. If you apply this, this is what we will get. Now, let us try to understand this from the calculation point of view.

How did we get this component? We are taking the outer product, so with 0.9 and 0.1, we need to apply the Lukasiewicz implication. When you apply this x is 0.9 and y is 0.1, this minimum of 1 , 1 minus 0.9 plus 0.1 which is essentially 0.2.

If you take 0.8 here and 0.1 here, then by fixing x to be 0.8 and y to be 0.1, you will get a second component. So, if you go through like this, essentially you are only doing some kind of an outer product operation and you will be able to fill this matrix. And this is R of z 1, then we need to know R of z 2, R of z 3 also with this T M of A 1 and A 2.

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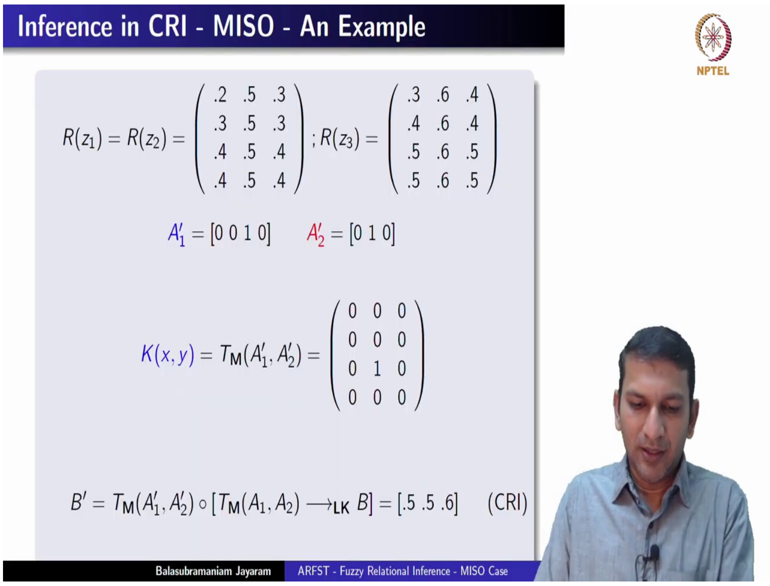

Since, z 1 and z 2 are same if you look at it, z 1 and z 2 are same what we would get is the same for R of z 1 and z 2 and R of z 3 is different. So, now, this is the three-dimensional relation that we have obtained. You could think of it as three 2D matrices perhaps on 3 sheets of paper.

Now, this is building the relation representing the rule, single multi input single output rule. Now, we need to be able to infer with this. Let us for the moment take a very simple case wherein the given inputs A 1 dash and A 2 dash are in fact, single things. That means, what? They take the value 1 at only one of those 4 discretized points in the underlying discrete as (Refer Time: 21:26) and 0 (Refer Time: 21:27).

Now, what we need to do is remember R is a 4 cross it is a 4 cross 3 cross 3 matrix. So, when you fix one of those components z 1, you get a 4 cross 3 matrix, in the 4 cross 3 matrix, but we have 3 such matrices. So, from dimensionality point of view also given these two, we need to be able to obtain a 4 cross 3 matrix to be able to compose. Now, how do you do this?

Once again, we use the antecedent component K which is the minimum in this case. Now, it is easy to see that if you take the outer product of A 1 dash and A 2 dash with respect to minimum, this is the matrix that you get. Now, this is what we need to compose with R of z 1, R of z 2, and R of z 3, especially it essentially with R.

Now, since we are dealing with CRI, it makes R calculations much simpler. You could think of performing such an operation, taking this page on which this matrix is written and in super imposing it on this matrix on the page where this matrix is written. And component wise you take the min and take the max we need. So, if you do that 0 and 0.2, 0; 0 and 0.3, 0; everything will be 0 except where this is 1, which is 0.5.

And similarly everywhere else here it will be 0 except at 0.6, thus when you compose this with the original relation R, this obtained combined input, this is what you would get as the output. Let us look at one, but one other example only from the calculation point of view.

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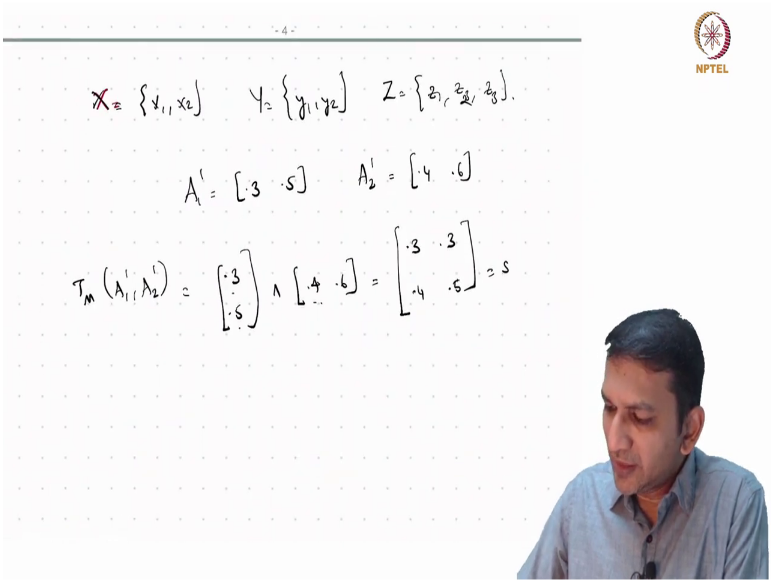

Let us look at another example. For this example, let us take X to be just x 1, x 2; Y again to be y 1, y 2; that means, both these domains are discretized by just two points and Z by 3 points. Now, let us since we are only trying to get the calculations right, let us start with the A 1 inputs. A 1 dash, let us assume it is 0.3, 0.5 and A 2 dash which is 0.4, 0.6. Now, when we want to do the inferencing, assuming we have the rule represented in terms of this relation which in this case as you can see will be the three-dimensional matrix.

So, let us start with obtaining the combined input which has to be used for the inference. Now, we need to use the minimum operator here. Once again is essentially taking the outer product, it is 0.3, 0.5; min 0.5, 0.6. What we obtain is the minimum between 0.3 and 0.4 is 0.3; 0.3 and 0.6 is 0.3; 0.5 and 0.4 is 0.4; 0.6 and 0.5 is 0.5. So, let us take this.

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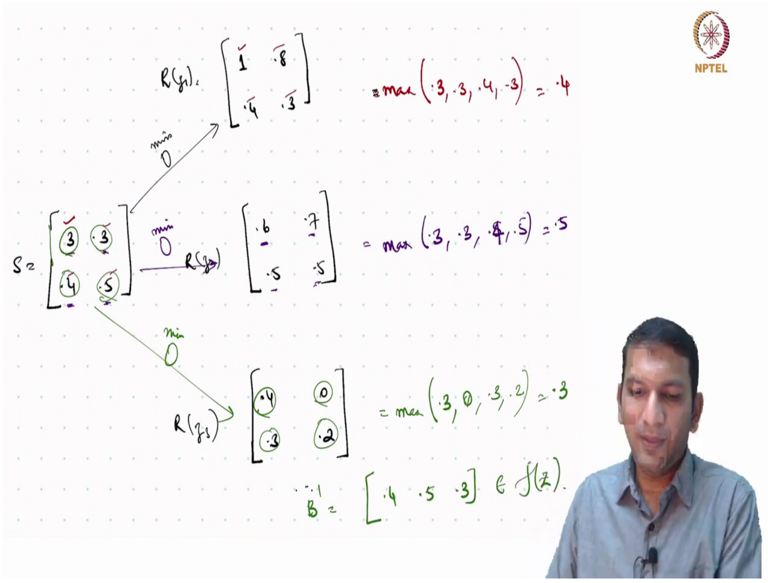

And now what we want is, let us call this S instead of (Refer Time: 25:17) here 0.3, 0.3, 0.4, 0.5. Since, R has R is a relation from X cross Y cross Z to [0,1], we saw that we can fix up z 1, z 2, z 3 and then vary the first two components in which case we get a three-dimensional matrix of course, but its A 2 cross 2 cross 3. So, we will get three 2 cross 2 matrices. So, allow me to write them directly, so this R of z 1, R of z 2 and R of z 3.

So, let us R (Refer Time: 26:05) fill the values here. Assume that these are the ways in which we have actually obtain the relation and the relation will looks like this. Now, what you are doing is this is the combined input that we have, all we are going to do is we are going to take component wise minimum and finally, the maximum. So, we need to do between these two sigma min. Let us look at it. So, we start with 0.3, 0.3, 0.3 and 1, the minimum of that we take the maximum of the minimum between the component based elements.

Minimum of 0.3, 1 is 0.3; minimum of 0.3 and 0.8 is 0.8, minimum of 0.4, 0.4 is 0.4; 0.5, 0.3 is 0.3 and the maximum of that is actually 0.4. Similarly, if I take these two matrices and once again use this sup-min composition, so we have 0.3 and 0.6, the minimum of that will be 0.3; 0.3 and 0.7 minimum is once again is 0.3; 0.4, 0.5 which is 0.5; 0.5, 0.5 is 0.4; 0.5, 0.5 is 0.5 and we take a max of this which is 0.5

Finally, we need to take sup-min composition between these two matrices. So, if we take 0.3 here, so perhaps circle it 0.3 and 0.4 here, the minimum is 0.3, 0.3 and 0, 0; 0.4 and 0.3 is 0.3; 0.5 and 0.2 is 0.2 and here get the maximum outcome. And what we get is max of the such

0.3. So, essentially finally, the overall output B dash that we get is 0.4, 0.5 and 0.3 which is an element of F of Z.

Well, you would have noticed the cumbersome tedious way to compute the output when we have a multiple input single output rule, and this is just one multiple input single output rule. So, if you have many multiple rules, then it becomes computationally even more cumbersome. Is there a way to reduce the computational effort? Yes, we can propose many. But the question is would it still give the same answer, we would have obtained by following this procedure.

Now, these are questions we will get definitely take up as we go along and answering these questions will once again show us the theoretical structures that we have discussed and developed, the crucial role they play in answering this questions.

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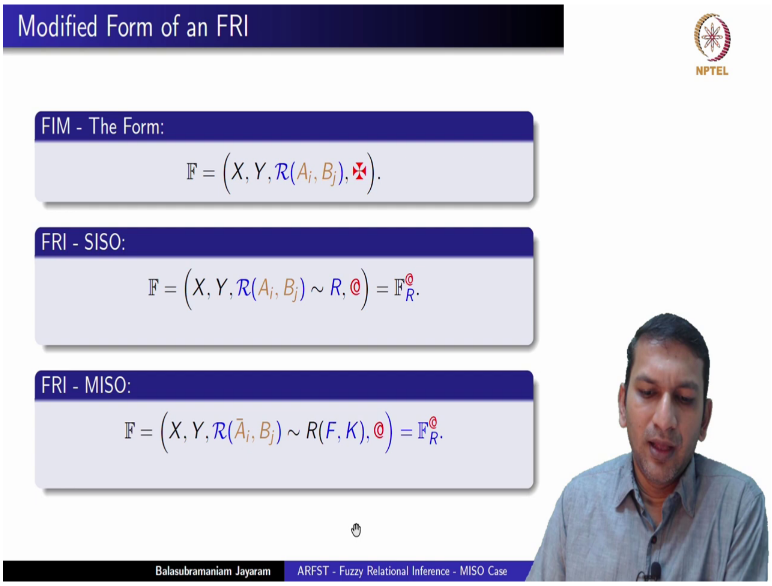

We know that as general fuzzy inference mechanism can be represented by this quadruple. In the case of single input single output rule, we saw that we could obtain the relation from just an operation F which related the antecedent to the consequent and we needed a composition operator. In the case of multi input single output rule, we not only need an F, but also an antecedent combiner K and of course, the composition of K.

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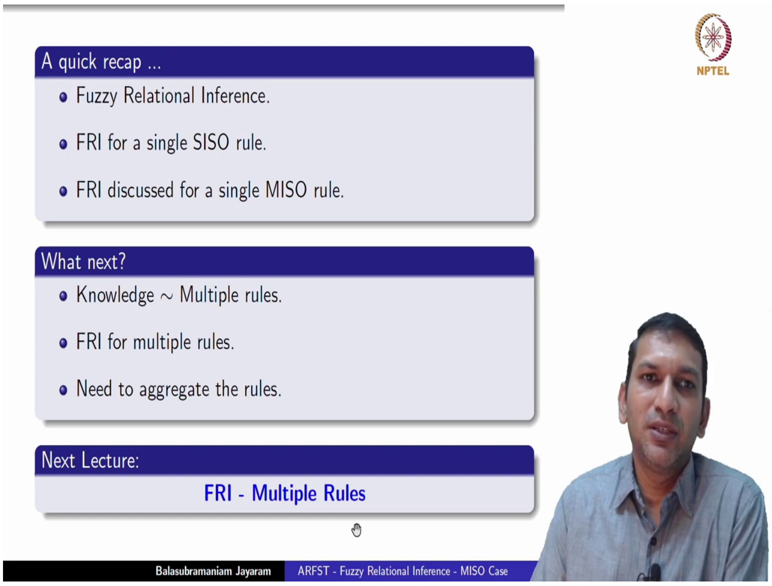

A quick recap. So, we have been discussing fuzzy relation inference. In the previous lecture, we looked at fuzzy relation inference for a single SISO rule. In this lecture, we have looked at how to perform FRI in the presence of a single multiple input single output rule. Of course, it is clear that the knowledge about any domain in which you are trying to capture and reason in cannot be captured in one single rule.

We need multiple rules. We will look at fuzzy relational inference for multiple rules in the next lecture of this week. We will also see that it necessitates aggregation of pieces of information, either the representation of the rules themselves or the local outputs. Next lecture, we will deal with how to handle multiple rules in we are inferring with fuzzy relation inference.

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Once again, a good resource for the topics that we have covered today is the book of Driankov, Hellendoorn and Reinfrank.

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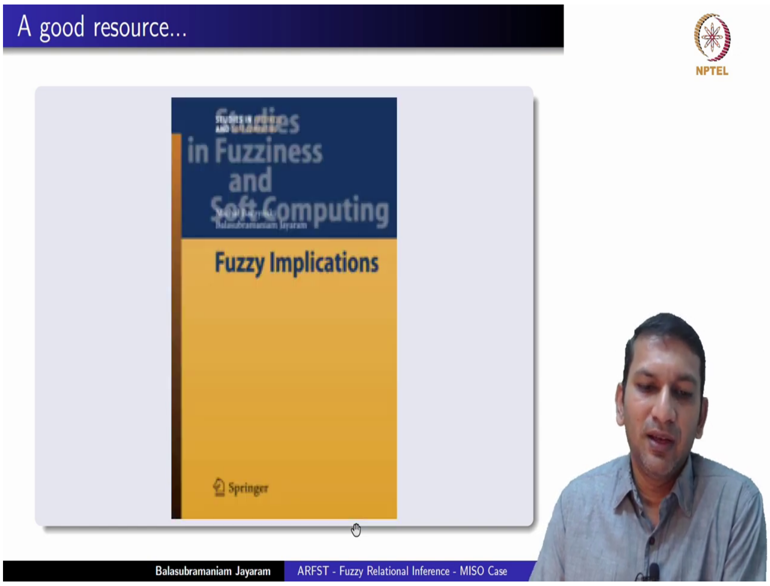

And also, you will see the case of MISO rule being handled in this book on Fuzzy Implication. Glad, you could join us for this lecture. Hope to see you soon in the next lecture.

Thank you once again.