

**Computational Mathematics with SageMath**  
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**Lecture - 58**  
**Review and What next in SageMath?**

Hi, welcome to this last session on Computational Mathematics with SageMath course. I hope all of you enjoyed learning this course. I particularly learnt several things during this course. In this particular section, I will tell you what are the topics, that you can explore after going through this course using SageMath.

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What we have covered?

- Basics of Python Programming
- Introduction to SageMath
- Basics of calculus of one and multivariable
- Computational Aspect of Linear Algebra with Applications
- Basic Numerical Methods and Applications

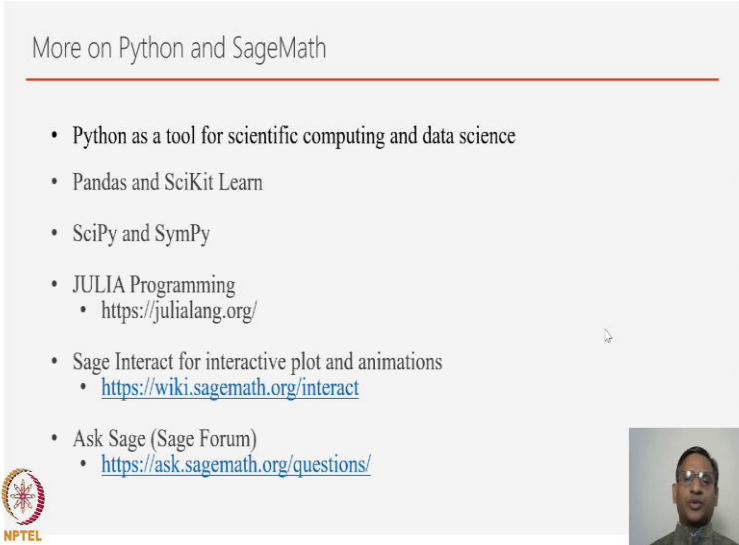
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So, let us first look at what are the things that we covered? First of all we started with Python programming language and this enabled us to use Sage more effectively. In fact, at many places we wrote our own program to solve mathematical problems. And this was possible, because we learnt basics of Python programming language.

After 2 weeks of Python programming language, we started with introduction of SageMath. We looked at SageMath as a tool to explore concepts in calculus of one variable and also multi variables. We further looked at computational aspect of linear algebra along with several applications and visualizations, and we saw that SageMath is extremely good, when it comes to concepts in linear algebra.

We then looked at basic techniques of numerical analysis, along with several applications. So, these are the things we went through in this last 8 weeks. Though we did not look at lot of concepts, one of my aim was to introduce this SageMath and use this as a tool to explore various concepts in calculus linear algebra and numerical methods.


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More on Python and SageMath

- Python as a tool for scientific computing and data science
- Pandas and SciKit Learn
- SciPy and SymPy
- JULIA Programming
  - <https://julialang.org/>
- Sage Interact for interactive plot and animations
  - <https://wiki.sagemath.org/interact>
- Ask Sage (Sage Forum)
  - <https://ask.sagemath.org/questions/>

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Now, what more in Python and SageMath? As you know Python is very useful tool for scientific computing and data science. So, you can explore Python further, for scientific computing and in case you want to use it as a tool in data science then you would try to learn pandas a package using which data can be explored.

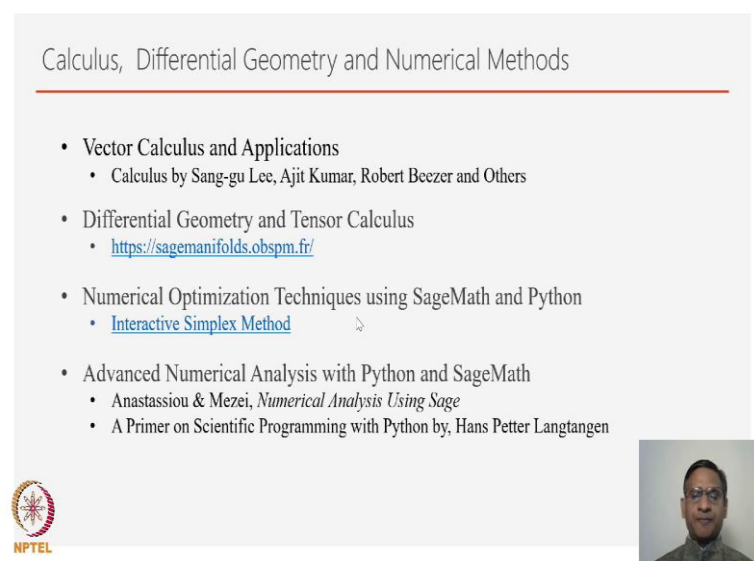
And there is another package call scikit learn, if you go to scikit learn website, it has several machine learning algorithms, examples and some of these are basically application of linear algebra. You can further explore SciPy and SymPy. We did introduce SciPy and also SymPy little bit, but as you must have seen Python. For example, SageMath also uses SymPy in its background.

You can also use a programming language called JULIA, this is a latest programming language, and actually it is a combination of Python, R and MATLAB. It has power of all these languages and it is extremely fast. In fact many people believes that, this is going to be future programming language when it comes to scientific computing and data science. So, try to explore this, you can look at its official website: [julialang.org](https://julialang.org).

We did not explore much on Sage Interact, which allows you to create interactive modules and also animations. We did few examples, but not much. So, I request you to further explore this and you can look at wiki page: [wiki.sagemath.org/interact](http://wiki.sagemath.org/interact), where you can find several examples of Sage Interact.

In case you have some doubts in SageMath, you can go to SageMath forum. This is Ask Sage: [ask.sagemath.org](http://ask.sagemath.org) and you can ask your doubts, questions, most often you will get answer immediately.


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Calculus, Differential Geometry and Numerical Methods

- Vector Calculus and Applications
  - Calculus by Sang-gu Lee, Ajit Kumar, Robert Beezer and Others
- Differential Geometry and Tensor Calculus
  - <https://sagemanifolds.obspm.fr/>
- Numerical Optimization Techniques using SageMath and Python
  - [Interactive Simplex Method](#)
- Advanced Numerical Analysis with Python and SageMath
  - Anastassiou & Mezei, *Numerical Analysis Using Sage*
  - A Primer on Scientific Programming with Python by, Hans Petter Langtangen

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Now, in calculus actually we explored very basic concepts in one variable and also differential calculus mainly for multivariable calculus. But in case you want to learn let us say vector calculus, and its applications Sage can be used as a tool. For example, you can go through our book on calculus along with Sang-gu Lee, Robert Beecher Beezer and few others on calculus, this has chapters on vector calculus as well.

In case you want to expose some concepts in differential geometry and tensor calculus, Sage has a package known as SageManifold. So, you can visit this particular website and explore this one. It is very nice, specially when you want to visualize surfaces etcetera and various concepts related to surface theory, you can use this.

You can further do numerical optimization techniques using SageMath and also Python programming language, because Python has facilities to deal with lot of numerical

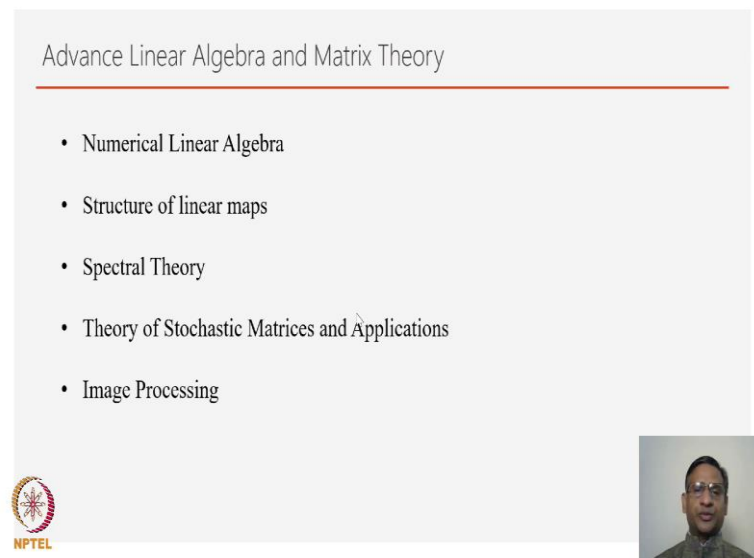
computations. For example SageMath has an interactive simplex method, along with other simplex methods, which is known as mixed integer programming problem.

You have a tool which is known as interactive simplex method using which you can solve problems using simplex method and it will give you step by step solutions along with visualizations.

You can look at advanced numerical analysis with Python and SageMath. What we covered was very basic ones, but if you want to explore more on this, especially the going to error analysis, exploring several concepts in differential equation. Solving these differential equations numerical, especially for example, boundary value problems, etcetera, we did not look at several examples. So, you can use it further.

For example, you can look at a book which is on Numerical Analysis Using Sage by Mezei, but this is very basic book. But you can also look at, A book on a Premier on Scientific Programming with Python. This is by Hans Petter langt Langtangen. This is also a very nice book and it is available, actually freely. You can also look at the Python codes on its website.

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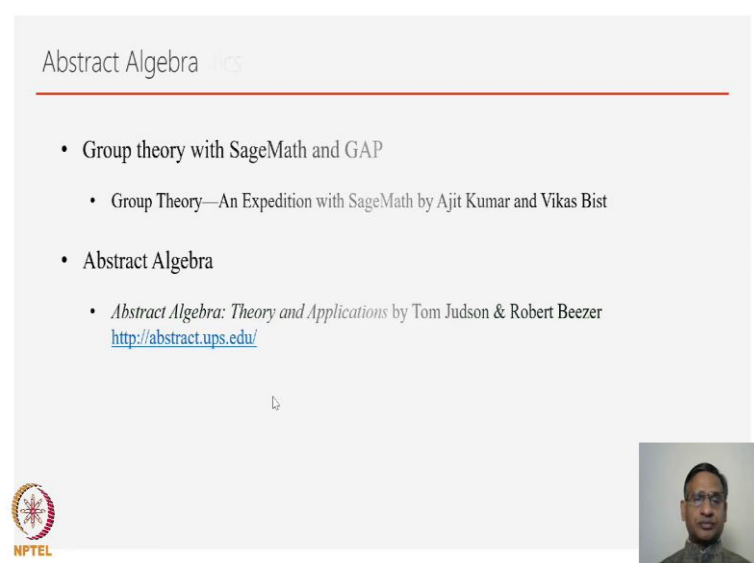
In case you want to explore more concepts in linear algebra, for example, a concepts in numerical linear algebra, you can make use of SageMath. For example, you can also explore structure of linear maps. We made use of Jordan canonical form as a tool, when

we wanted to solve differential equations, 1st order linear differential system of linear differential equations, when the coefficient matrix is not diagonalizable.

But, if you want to further explore this topic, you can use SageMath. You can also explore the spectral theory, you can look at theory of stochastic matrices and its applications. We saw stochastic matrix appeared in Google search algorithm. So, you can dwell more on these concepts of stochastic matrices its convergence and other things.

Image processing is also very nice application of linear algebra. So, you can explore image processing using SageMath. We for example, looked at image compression as an application to singular value decomposition, but further aspect of image processing can be also explored very nicely.

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The slide is titled "Abstract Algebra" and contains a bulleted list of topics. The first bullet is "Group theory with SageMath and GAP", which has a sub-bullet "Group Theory—An Expedition with SageMath by Ajit Kumar and Vikas Bist". The second bullet is "Abstract Algebra", which has a sub-bullet "Abstract Algebra: Theory and Applications by Tom Judson & Robert Beezer" with a link to <http://abstract.ups.edu/>. In the bottom left corner is the NPTEL logo, and in the bottom right corner is a small video inset showing a man speaking.

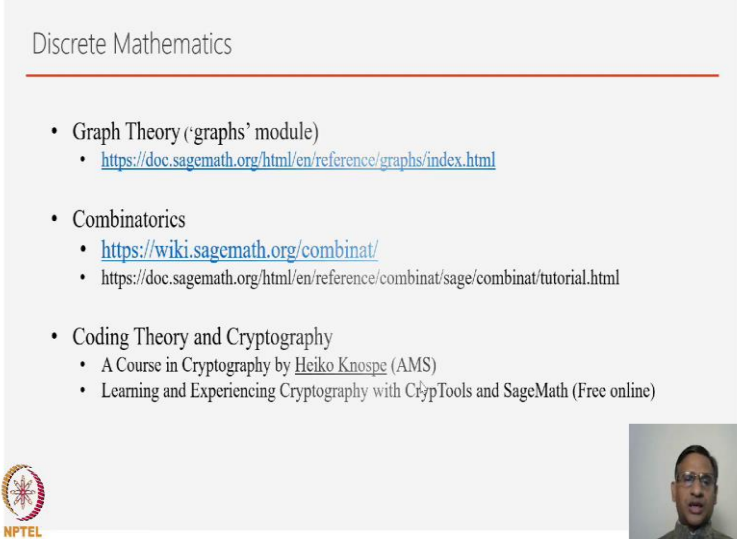
- Group theory with SageMath and GAP
  - Group Theory—An Expedition with SageMath by Ajit Kumar and Vikas Bist
- Abstract Algebra
  - *Abstract Algebra: Theory and Applications* by Tom Judson & Robert Beezer  
<http://abstract.ups.edu/>

You can even explore concepts in abstract algebra. For example, if you want to do group theory with SageMath, it also has a package called GAP. There is a package called GAP, which is used mainly for exploring concepts in group theory. In fact, SageMath uses GAP in the background. You can look at there is a forthcoming book on

Group theory--An Expedition with SageMath, which I have co authored with Vikas Bist and it should be available anytime now. It is published by Narosa publication.

You can also explore concepts in abstract algebra. For example, there is a nice book which is freely available, free and online book on Abstract Algebra; Theory and Applications, it deals with basic number theory, groups, rings, fields including finite fields along with this Sage codes.


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Discrete Mathematics

- Graph Theory ('graphs' module)
  - <https://doc.sagemath.org/html/en/reference/graphs/index.html>
- Combinatorics
  - <https://wiki.sagemath.org/combinat/>
  - <https://doc.sagemath.org/html/en/reference/combinat/sage/combinat/tutorial.html>
- Coding Theory and Cryptography
  - A Course in Cryptography by Heiko Knospe (AMS)
  - Learning and Experiencing Cryptography with CrypTools and SageMath (Free online)

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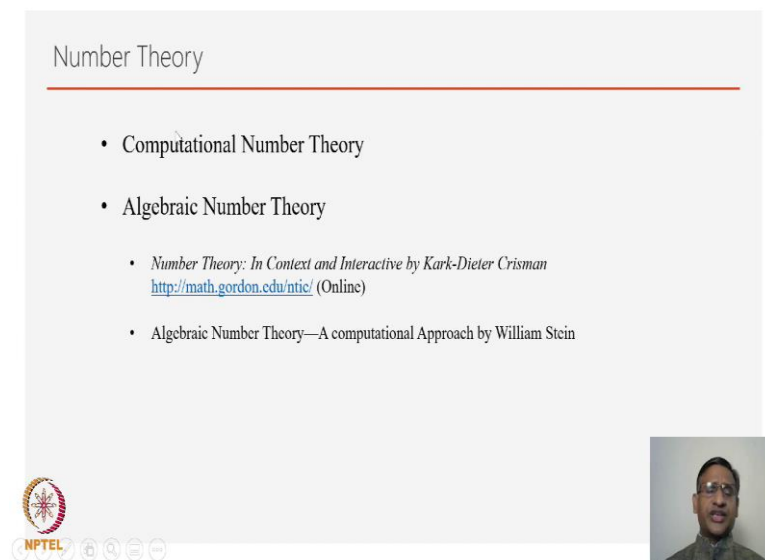


Discrete mathematics is another branch, which is very useful and it has lot of applications. For example SageMath has a package, which is called graphs. It has a module graphs, module and using that you can explore most of the concepts in graph theory. So, in case you are interested in learning graph theory, Sage can be used as a tool. For example, we used this module to generate random network, when we were looking at Google search page rank algorithm right.

So, you can look at inbuilt reference manual on SageMath, you can also explore combinatorics using SageMath, there is a package called combinat, it has a wiki site. You can go through this website and of course, Sage itself has a reference manual on combinatorics, basically in the form of tutorial. So, that also you can go through.

You can explore concepts in coding theory and cryptography. For example, there is a book on A course in Cryptography, this is a AMS book. This uses SageMath, also there is a book on Learning and Experiencing Cryptography with CrypTools and SageMath, this again is freely available. So, if you are interested to explore concepts in coding theory and cryptography you can make use of this.

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Number Theory

- Computational Number Theory
- Algebraic Number Theory
  - *Number Theory: In Context and Interactive* by Kark-Dieter Crisman  
<http://math.gordon.edu/ntic/> (Online)
  - Algebraic Number Theory—A computational Approach by William Stein

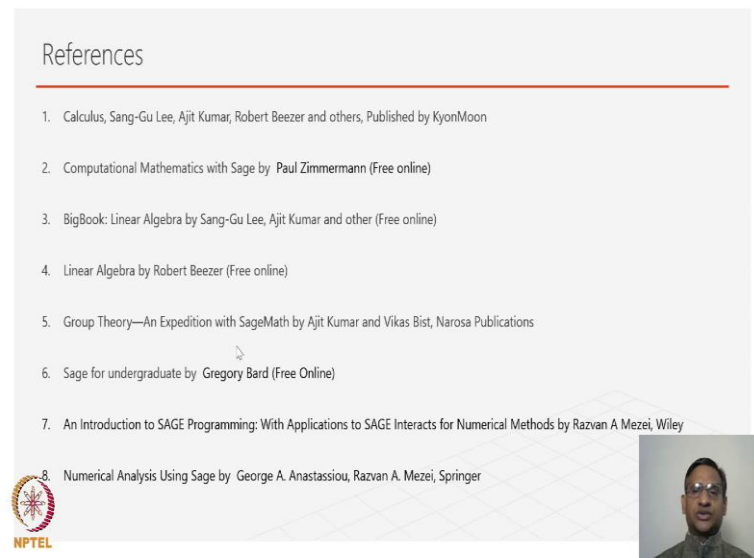
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You can also look at concepts in computational number theory and algebraic number theory using SageMath, as I mentioned this SageMath is developed by Professor William Stein, who himself is a number theorist. And you can expect this SageMath to be very powerful, when it comes to exploring concepts in number theory and it is.

For example, you can look at, these are some few references.

For example, there is a book on Number Theory: In context and interactive by Crisman and this has several SageMath module. This is again freely available. You can also look at Algebraic Number Theory, short book A Computational Approach by William Stein and many people actually uses this SageMath for their research in number theory.

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### References

1. Calculus, Sang-Gu Lee, Ajit Kumar, Robert Beezer and others, Published by KyonMoon
2. Computational Mathematics with Sage by Paul Zimmermann (Free online)
3. BigBook: Linear Algebra by Sang-Gu Lee, Ajit Kumar and other (Free online)
4. Linear Algebra by Robert Beezer (Free online)
5. Group Theory—An Expedition with SageMath by Ajit Kumar and Vikas Bist, Narosa Publications
6. Sage for undergraduate by Gregory Bard (Free Online)
7. An Introduction to SAGE Programming: With Applications to SAGE Interacts for Numerical Methods by Razvan A Mezei, Wiley
8. Numerical Analysis Using Sage by George A. Anastassiou, Razvan A. Mezei, Springer

These are some of the references, which could be useful. Many of these things I have already told you during the course, but I thought of just putting all these together. So, whatever we have done during these 8 weeks of lectures you can find most of it in all these references.

So, let me thank, all of you for attending this course. I hope all of you enjoyed this course and learnt SageMath and used SageMath to explore various concepts in basic calculus, linear algebra and numerical methods.

I really enjoyed this bringing for you and I am sure, all of you would have become now confident of using Python and SageMath. I am sure you would explore this further. You can also visit my personal website, on which you can also find a series of video lectures, which I gave during May-June this year. That it is a series of 12 videos.

So, thank you very much and I wish you all the best.

Thank you.