Mathematical Methods 1
Prof. Auditya Sharma
Department of Physics
Indian Institute of Science Education and Research, Bhopal

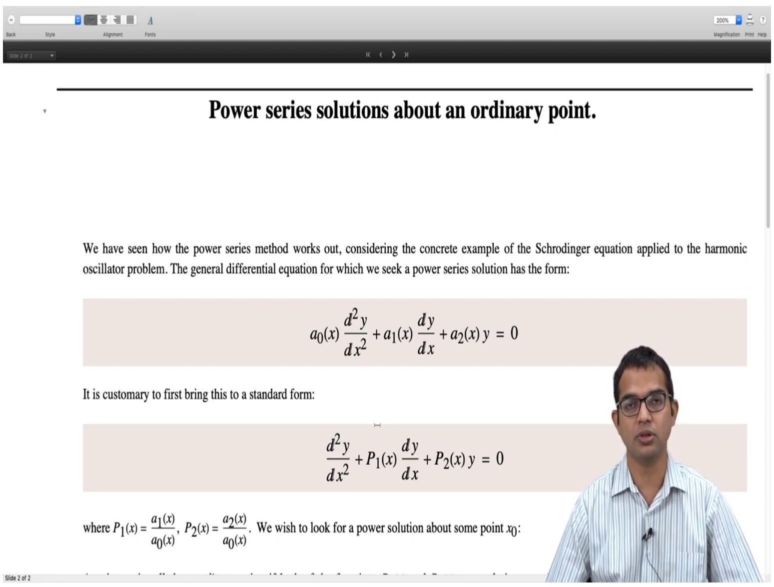
Ordinary Differential Equations
Lecture - 88
Power series solutions about an ordinary point

So, we saw how the Power series solution works out with a concrete example of the Schrodinger equation for the harmonic oscillator potential. So, there we did not worry too much about you know convergence issues and things like that.

So, it was one of those problems where it all worked out well and in fact, we found the solution that we obtained. You know based on you know physical conditions and all the final solution was in fact a polynomial. So, which would be you know convergent everywhere right.

So, but in general if you are going to come up with a power series, you know questions of convergence become important. So, but so in this lecture we will only sort of touch upon, you know this aspect and you know talk about how if you have something called an ordinary point, about which you are writing a power series solution. There is a theorem which guarantees that it is going to be a convergent series within some region of convergence right. So, we will only barely touch upon this topic in this lecture ok.

(Refer Slide Time: 01:29)



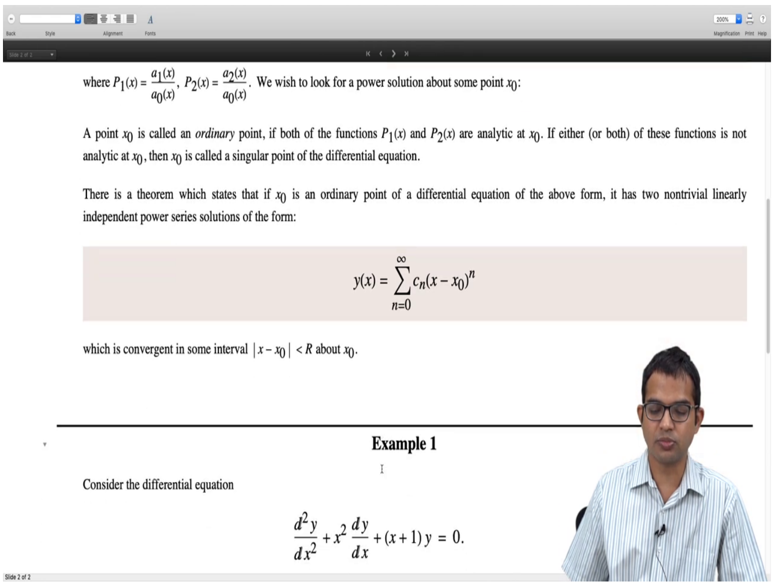
So, we are interested in power series solutions, when you have a differential equation of this specific form right. So, it is a homogeneous differential equation and it is not trivial what appears on the left hand side is not trivial. And we are focusing on second order differential equations, because lots of you know physics problems are second order in nature right.

So, some of these ideas can be extended to higher orders as well, but anyway let us look at the second order differential equation. So, a standard form to bring this in is you know to divide throughout by a naught right. So, we look at a revised form of the same differential equation which is d square y by dx square plus P 1 of x dy by dx plus P 2 of x times y is equal to 0 right.

So, you may like to make the coefficient corresponding to the highest derivative which is the second order derivative in this case to be 1 right. So, P 1 is a1 of x divided by a naught of x and P 2 is a 2 of x divided by a naught of x right. So, there can be trouble, if a naught of x you know goes to 0 at some point right and you know it is not always going to be trouble right. If there is a you know suitable term in a 1 or a 2 which cancels with this then perhaps there is no issue.

But, anyway so in general one has to be careful when you are dividing by a function of this kind.

(Refer Slide Time: 03:06)



And so, here is a you know classification of the type of points about which you can do a power series solution right. So, when we did the problem for the harmonic oscillator problem, it was natural to just write down a power series expansion about the origin. But in general actually you can come up with a power series solution about some point x naught right. So, a power series solution about a point x naught, it does not have to be the origin.

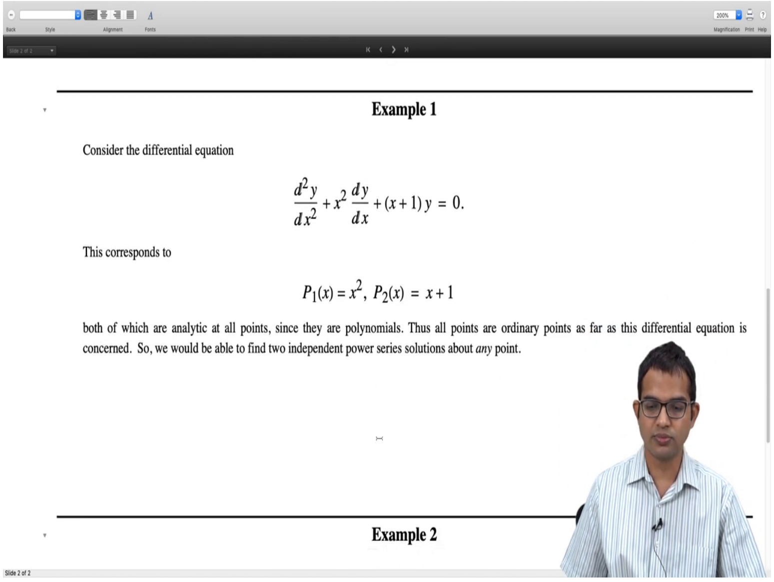
Now, there is a classification of you know points about which you are doing a power series expansion right. The point x naught is said to be an ordinary point if both these functions P1 of x and P 2 of x are analytic at that point, which means that there is no singularity there. So, nothing you know awkward is happening with respect to a naught.

So, if a naught is 0 and which does not cancel with a 0 with of a 1 or a 2 you know if either of these has a singularity either P1 or P 2 has a singularity at that point, then it is a problem, then it is not an ordinary point it is called a singular point. So, there is this theorem which says that if you are doing an expansion about an ordinary point, then you are guaranteed to have 2 non trivial linearly independent power series solutions of this form right.

So, you know the kind which we saw for the Schrodinger equation right. So, y of x is equal to this and this is going to be convergent in some region of convergence, mod of x minus x naught less than r. It is a region of convergence around x naught where this is going to be convergent.

So, this is guaranteed by a theorem the details of which we will definitely not be able to go into, but the fact that it exists is useful for right. So, we can go ahead and crank up the machinery. So, we are interested in some assurance that it would work and then we want to be able to actually use the method.

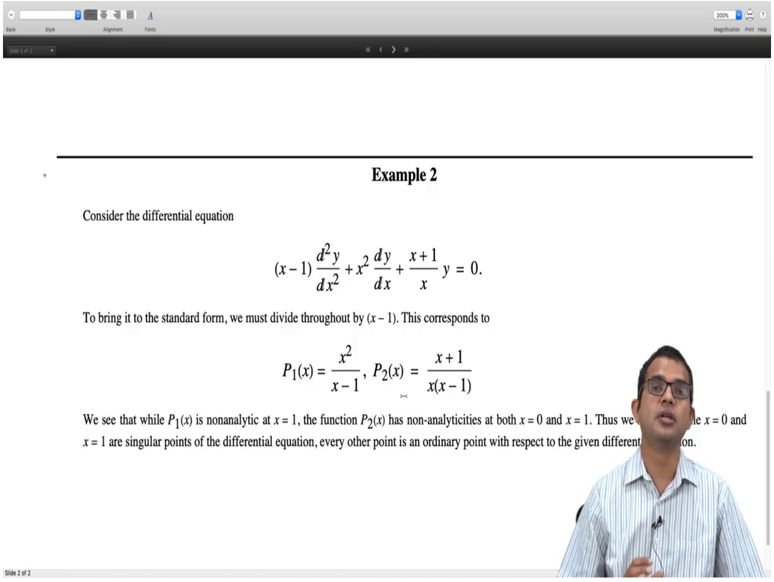
(Refer Slide Time: 05:08)



So, quickly let us look at a few examples of you know this classification of this point. So, let us look at example 1, so if you have this, consider a differential equation like this d squared y by dx squared plus x squared dy by dx plus x plus 1 times y equal to 0. So, this corresponds to P1 of x is equal to x squared because there is already one sitting you know is the coefficient of d squared y by dx squared, so P 2 of x is just x plus 1.

So, both of which are analytic at all points. So, since they are polynomials, there is no problem at all. So, in fact, all points are ordinary points as far as this differential equation is concerned. So, you should be able to write down a power series expansion about any point you wish and you do not have to do it about the origin, you can do it about any point right. So, that is very convenient.

(Refer Slide Time: 05:57)



Let us look at another example where it is not so trivial if you wish. So, suppose you have a differential equation like this you have x minus 1 d squared y by dx squared plus x squared times dy by dx plus x plus 1 divided by x times y equal to 0 right. So, I mean it is similar to the first differential equation, but I have x minus 1 as a coefficient for the first term here and I also have this division by x in the last term here.

So, if I want to bring it to the standard form then I must divide throughout by x minus 1. So, I will get P 1 of x is equal to x squared divided by x minus 1 and P 2 of x is going to become x plus 1 time divided by x times x minus 1. So, what we see is that this first function P1 of x has a non analyticity or a singularity at a point x equal to 1. So, at this point I think we intuitively see what a singularity means right.

So, you know when we discuss complex analysis in the next part of this course, we will talk about these ideas you know a little more in depth. So, but intuitively we see what singularity here means. So, if x equal to 1 you know there is a singularity as far as P 1 of x is concerned P 2 of x has you know non analyticity or singularity at 2 points 1 is at x equal to 0 and x equal to 1.

So, as far as the differential equation is concerned both the points x equal to 0 and x equal to 1 are singular points, they are not regular points. So, the key idea is that you know, if even one of these 2 functions P 1 or P 2 has a singularity it is not a regular point, it is not an ordinary point or it is not an ordinary point right. So, it is a singular point.

So, every other point is an ordinary point with respect to the given differential equation and you will be able to use this theorem and you know write down a power series expansion about that point, except x equal to 0 and x equal to 1. So, although we observe that the point x equal to 0 is a singularity only for P 2 of x, P 1 of x is not singular here and yet as far as the differential equation is concerned x equal to 0 is also a singular point it is not an ordinary point.

So, we will look at how you know how you know the power series solution would work out for an ordinary point, and also we will see that when there are certain very special kinds of singularities will still allow us to write, you know power series solutions. But that is coming up later, in this lecture we just wanted to quickly point out what an ordinary point is that is all for this lecture.

Thank you.