Mathematical Methods 1
Prof. Auditya Sharma
Department of Physics
Indian Institute of Science Education and Research, Bhopal

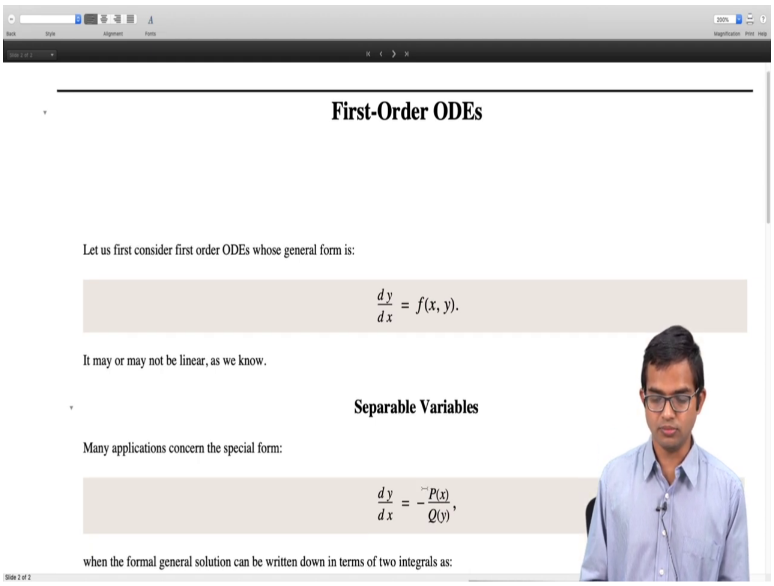
Ordinary Differential Equations
Lecture - 57
First order ODEs

Ok so, in this lecture, we will discuss First Order Ordinary Differential Equations, right. So, we have seen how we can have linear and nonlinear differential equations, and we have seen how the order of a differential equation can be defined. So, we will see that first order differential equations are special, right.

So, in general although nonlinear differential equations are extremely hard problems, if they are first order it turns out that there are general methods available you know.

We will look at a few examples, right, ok.

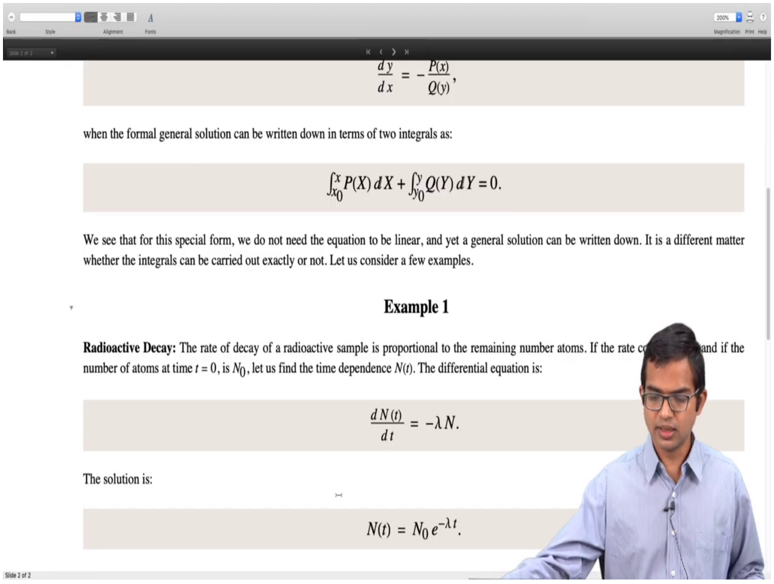
(Refer Slide Time: 00:56)



So, a completely general form for a first order ODE is dy by dx is equal to f of x comma y, right. It may be linear or it may be non-linear, right. So, because f of x comma y can be as complicated as you want, right say, right; as far as you know the general development is concerned, right.

So, there is a special form which is particularly convenient if dy by dx is of this form equal to minus P of x divided by Q of y, right. So, then it is in a form which is called a separable form, right.

(Refer Slide Time: 01:35)



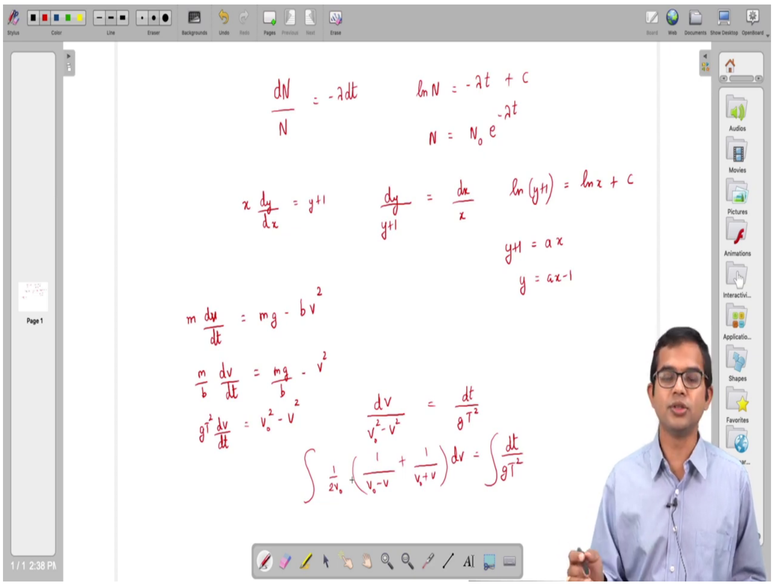
So, the formal solution can be written down, right. So, all you do is you bring this Q of y d y to the left side then you can write plus P of x d x and then integrate both of them, right. So, since it is a you know function purely in x and another function purely in y, you can formally integrate both of these and that is the solution, right, right. So, it is another matter whether this integration can be carried out and if you can write down a solution in closed form or not, right.

But if it is a separable differential equation then at least there is a formal solution available, right. So, let us look at a few examples. So, one is that of radioactive decay, this is a simple problem which most of us must have seen something of this kind. So, the rate of decay of a radioactive sample is proportional to the remaining number of atoms, right.

So, it keeps on reducing in number and so, the rate also drops as a function of time, right. If the rate constant is lambda and if the number of atoms at time t equal to 0 is n naught let us find the dependence of n of t. So, the differential equation is just given by the rate d N by dt is equal to minus some constant times N, right.

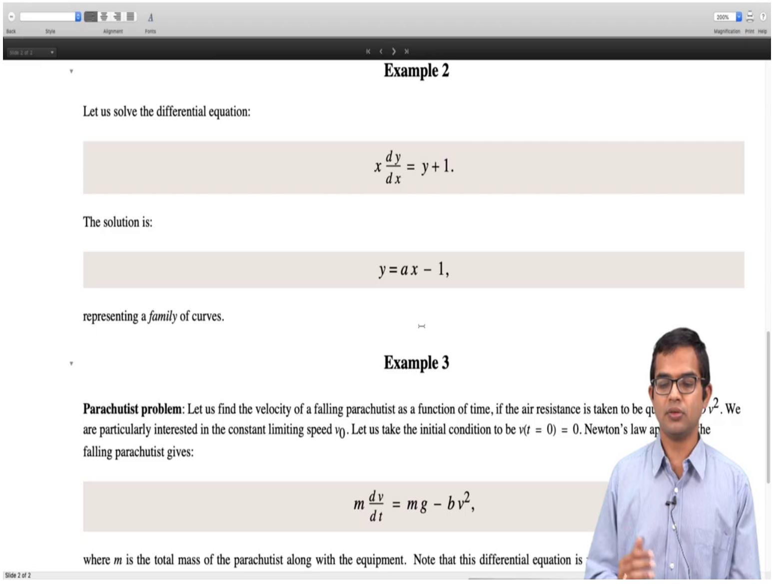
So, the rate constant is also given. So, the negative sign is important, lambda is a positive number. So, it is a decaying function N of t, right. So, now the solution is just this and it is straightforward to see this. So, it is in a separable form, right.

(Refer Slide Time: 03:14)



So, we can write this as d N by N, d N by N is equal to minus lambda dt is equal to minus lambda dt. So, integrate both sides, so you get log N is equal to minus lambda t plus some constant. So, therefore, N is equal to you can absorb this constant into this and it is just N naught times e to the power minus lambda t, right. So, a very familiar differential equation. I think most of us must have seen this, ok.

(Refer Slide Time: 03:48)

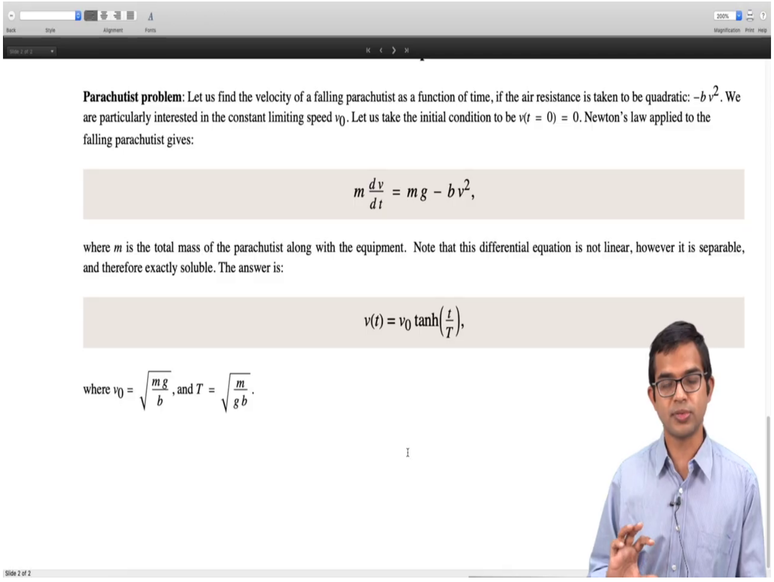


Example 2. So, if you solve the differential equation x times dy by dx. So, once again this is the separable differential equation. So, let us look at this x dx, x dy by and dx is equal to y plus 1, is equal to y plus 1. So, you have dy divided by y plus 1 is equal to dx by x. So, log of y plus 1 is equal to log x plus some constant. So, y plus 1 is equal to some is equal to e to the c times some constant, some other constant which I have called a, right.

So, its y is equal to a x plus very straightforward, right. So, just an exercising integration, right, ok. So, we are starting simple, very straightforward stuff. And so, what it gives you is a family of curves, right. So, there is this free constant. So, notice that it is a first order equation and then you have a one free constant which comes in, right and this will turn out to be a general rule for linear differential equations. So, it is a family of curves that you get.

So, let us look at one more example. So, the parachute problem. So, you have the velocity of a falling parachutist as a function of time; the air resistance is taken to be quadratic. So, you have a minus b v square and we are interested in the limiting speed of this parachutist. So, you have m dv by dt is equal to mg minus bv square, right.

(Refer Slide Time: 05:38)



And the answer here turns out to be, so let us very quickly look at this. So, we have m dv by dt is equal to mg minus, so we have m dv by dt is equal to mg minus bv squared. So, let us divide throughout by b. So, we have m by b, dv by dt is equal to mg by b minus v squared. So, we have defined mg by b, mg by b as v naught. So, we have the equation and m by b is g T square.

So, we have g T square bv by dt is equal to v naught square minus v square. So, then we bring in; this is slightly harder than the earlier too, but it is still probably familiar. So, we have v naught squared minus v squared is equal to dt divided by g T squared. So, then you write this as a sum of partial fractions. So, you have v naught minus v, and then you have v naught plus v.

So, if I add these two then I will get v plus v naught, so v will cancel. So, I will get 2 v naught. So, I will have to divide by 2 v naught. So, I can write this as 1 plus this divided by 1 over 2 v naught dv is equal to dt by g T square.

So, now we can go ahead and integrate both sides. So, you will get log of v naught minus v with some coefficient minus sign, then you have another log of v naught plus v on the left-hand side you can collect these two log terms. So, and then on the right-hand side you just get an exponential of t. So, this is divided by 2 v naught, not v naught v squared, but 2 v naught, right.

So, you have to carefully work out the algebra. It is not difficult. So, it is just 1 over 2, 1 over 2 v naught, right 1 over 2 v naught. And so, this is a straightforward integral to do, right. You have to collect the terms carefully and then you have just a t which comes out on the right hand side and then you have to solve for v in terms of t and so, you can see that because of the presence of these two logs, there is a few more steps involved, but I will not go into it you will complete it as homework.

You can show that you get a tanh function, alright. So, you get this sinh over cosh t by T is involved, and all the details you will complete. If you plot this also it is a very instructive graph, it is going to saturate for long times. It starts at time t equal to 0, it is given to be just 0 and then as a function of time it is going to saturate to v naught, right. So, in fact, we have managed to find v naught as well, that was one of the questions.

So, this lecture is in the nature of warm up. Most of these examples we have already seen, but we have seen how if you have a first order differential equation of a special form, which is the separable form, then you can formally write down the solution even if it's non-linear, right. That is all for this lecture.

Thank you.