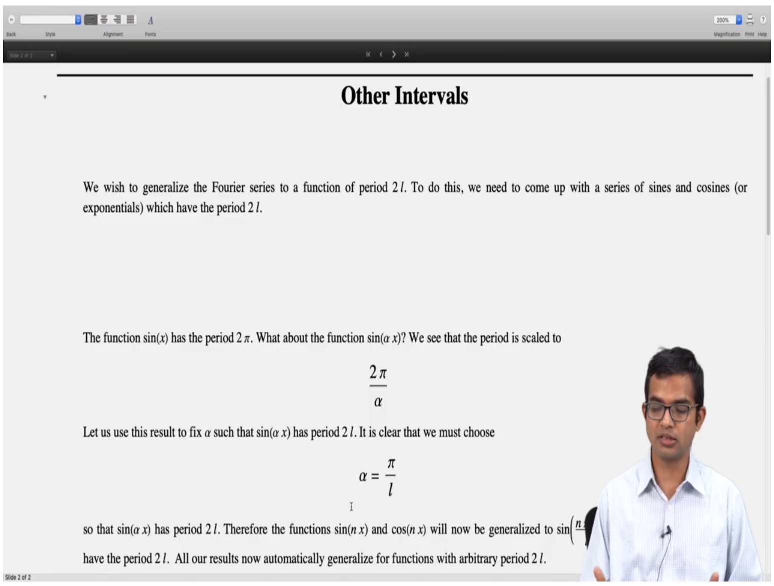
Mathematical Methods 1
Prof. Auditya Sharma
Department of Physics
Indian Institute of Science Education and Research, Bhopal

Fourier Series
Lecture – 49

Other intervals: arbitrary period

So, we have seen how a function which is periodic with period 2 pi can be written as a Fourier series. So far, we have restricted our attention to functions with period 2 pi, but there is nothing sacrosanct about this period 2 pi, right. Any function with which is periodic with an arbitrary period can be expanded as a Fourier series right. So, we will look at how this series expansion can be generalized to an arbitrary period in this lecture ok.

(Refer Slide Time: 00:53)

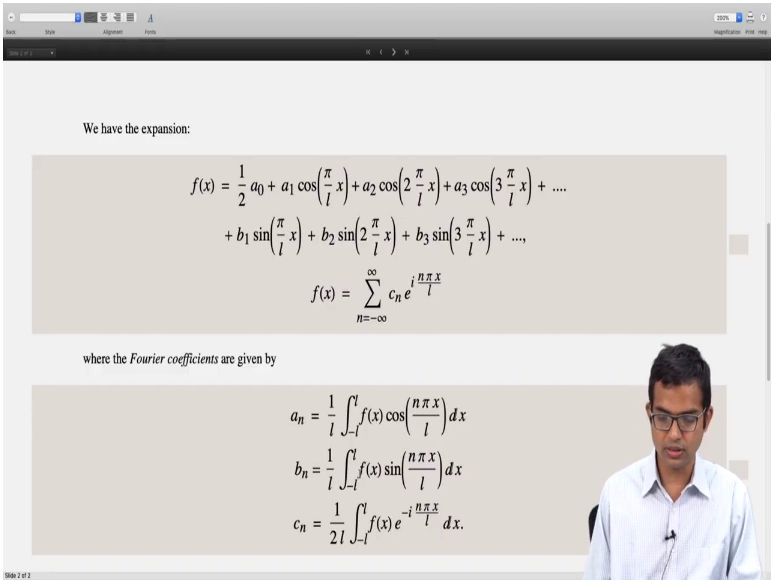


So, sine x and cosine x sine 2 x cosine 2 x so on is not going to be appropriate right. If you have some function with some other periods right because clearly sines and cosines you know as they are have period 2 pi right. So, suppose, you want to take sine x and what can you do to tweak this to get a different period?

And so, the simple answer is to just include some factor like alpha right. So, instead of sine of x, you consider sine of alpha times x and now, you see that the period of this function will become 2 pi by alpha right. So, if we want this period to be 2 l instead of 2 pi; suppose, we want the period to be 2 l, then you have to fix alpha in such a way that 2 pi by alpha is equal to 2 l.

So, if 2 pi by alpha is equal to 2 l, then alpha is equal pi by l right. So, if you choose alpha is equal to pi by l, then you know sines and cosines with this factor included are going to have the right period of interest. And so, therefore, the functions sine x and cosine n x will now be generalized to sine of n pi x by l and cosine of n pi x by l right. If we make this adjustment, you know all are results concerning Fourier series which generalize to an arbitrary period with period 2 l alright.

(Refer Slide Time: 02:34)

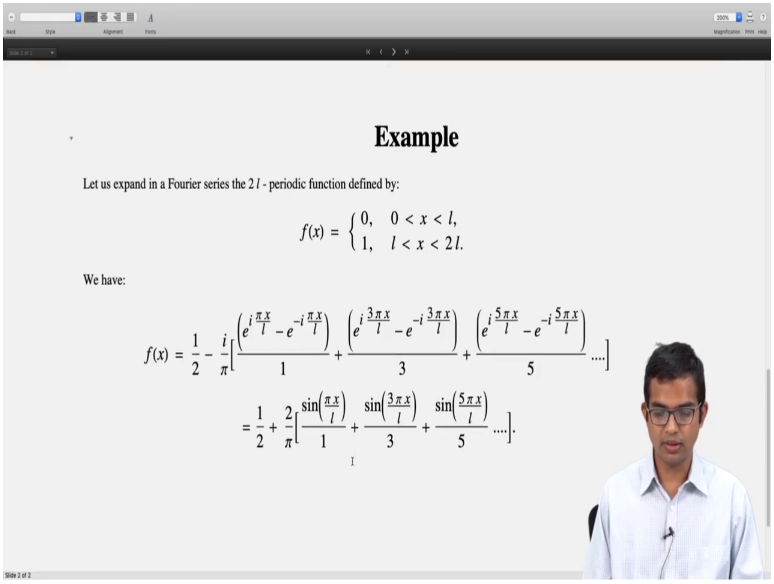


So, now we have the expansion f of x is equal to half a naught plus a 1 cosine of pi x by l plus a 2 cosine of 2 pi x by l plus a 3 cosine of 3 pi x by l plus so on and then, you know the sine part will come in; b 1 sine pi x plus by l b 2 sine 2 pi x by l so on or equivalently, you could have also written this in terms of the exponential series, summation n going from minus infinity to plus infinity c n e to the i n pi x by l right.

So, if you make this adjustment, then of course, the coefficient is also going to be given by a slightly different formula. Now, the integrals will run from minus l to plus l right. After all you have information in an entire period now and the period is like 2 l. So, you and this factor outside also changes you have 1 over l integral minus l to plus l f of x cosine of n pi x by l dx right.

So, you can quickly see that in place of l, if you just put l equal to pi, then you will recover all the equations that we have already seen and b n is 1 over l integral minus l to plus l f of x sine of n pi x by l d x and c n likewise right. So, you can quickly check that indeed these expressions are all consistent. You can directly work this out from first principles, if you wish and extract these coefficients for arbitrary period 2 l.

(Refer Slide Time: 04:04)



Let us look at an example; so, the same example but now with you know the interval going from 0 to l and l to 2 l f of x equal to 0 and f of x equal to 1 in these two intervals. If we plug in, then we will see that you know again we will get a either you know you can think of it in terms of complex exponentials, the series or you can think of it in terms of sines right; both are completely equivalent ok.

So, this is just a short lecture to show you that in fact we can make a small tweak and then, we have a general expression for an arbitrary period.

Thank you.