**Mathematical Methods 1**
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**Fourier Series**
**Lecture – 46**

# Fourier coefficients

So, we have seen that if you have a function f of x with period 2 pi, it can be expanded in a Fourier series in terms of cosines and sines. And we have also seen how this can be thought of as an expansion of a vector in an orthogonal basis right. So, where cosines and sines can be thought of as you know unit vectors which form or vectors of a basis of an orthonormal basis.

And then working out the Fourier coefficient is really you know an exercise in working out the expansion of a vector in terms of its basis vectors right. So, in this lecture, we will look at an example of you know how to carry out this expansion in practice ok.

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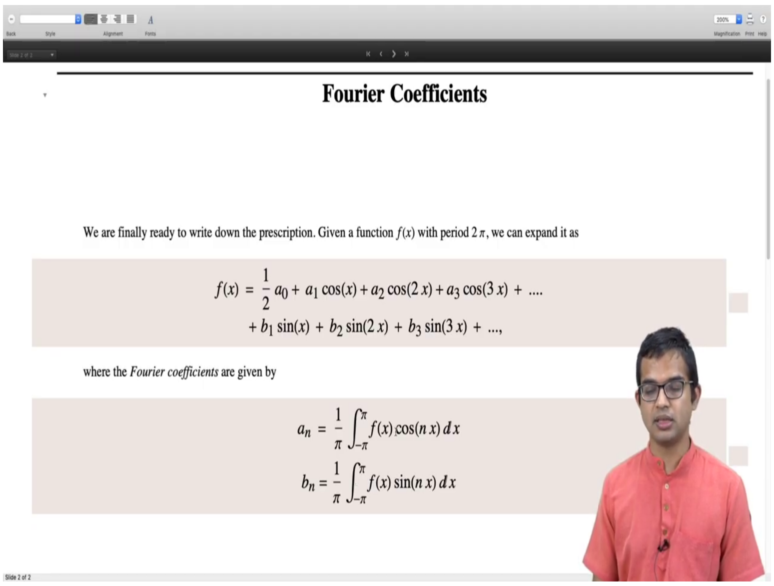


So, well, we are finally ready to write down the prescription. We have already written down the prescription. So, if you think of this as a function with period 2 pi, we can expand it as you know in this manner and where the coefficients are given by this right. So, this is you know to be viewed purely as a prescription.

You have a function of f x period 2 pi right. And so, we want to find a bunch of coefficients a naught, a 1, a 2, a 3 so on and b 1, b 2, b 3 and so on such that you know we will be able to say f of x is equal to this expansion, this infinite series which is called a Fourier series right.

So, the prescription is simply given by this. You have to do 1 over pi integral minus pi to pi f of x times cosine of n of x d x and b n is 1 over pi integral minus pi to pi f of x sine of n x d x right. So, this is the prescription. If you are interested in just purely the Fourier as the operational aspect of it, this is what you will have to keep in your head right.

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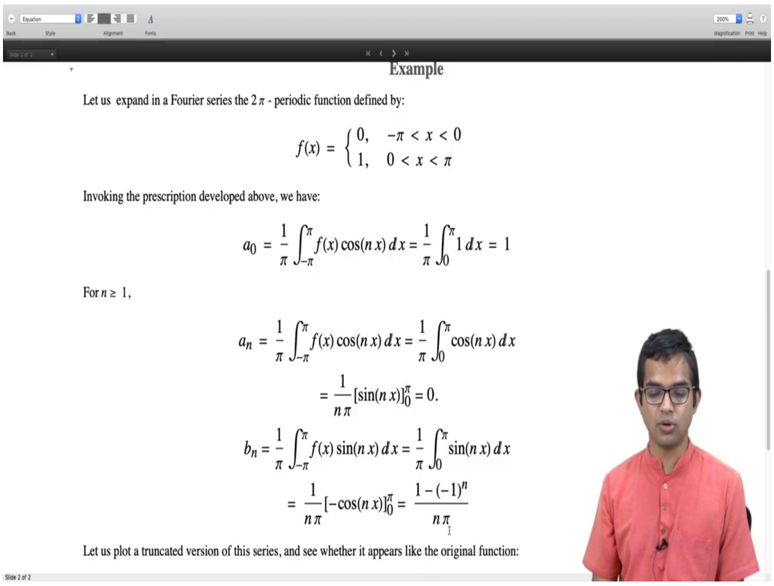


So, you are given some function f of x like in the following example. Suppose, you are given this function f of x which is 0 from minus pi to 0 and its 1 from 0 to pi and it repeats right. So, if we invoke this above prescription right, you have to be careful with this right. If you blindly do this, you might think that all the a n's are 0; but in fact, a naught is to be treated slightly differently.

So, the integral minus pi to pi reduces to an integral from just 0 to pi because f of x is 0 in the you know interval from minus pi to 0. So, it is 1 over pi integral 0 to pi 1 d x, right. So, this will just give you pi and divided by pi is just 1; so a naught is 1.

But if n is greater than or equal to 1, a n is 1 over pi integral minus pi to plus pi f of x cosine of n x d x which is 1 over pi 0 to pi cosine of n x d x. And now you see you get a 1 over n pi, sine of n x is 0 to pi; but sine is going to be 0 both at x equal to pi and x equal to 0 right unless n is 0, which we have already seen right.

That is something, that is a separate case; you will have to you cannot blindly put it into this. So, a n is 0, if n is greater than or equal to 1 and b n likewise is an integral which needs to be violated only in the limit in the interval 0 to pi right f because of the function that we have f of x is 0, in the intervals minus pi to 0.

So, we have 1 over pi integral 0 to pi sine of n x d x which gives us 1 over n pi minus cosine of n x from 0 to pi. So, now we see that unless n is odd right, this is going to vanish right. If n is even, like n equal to 2, for example, then you will see you get 1 minus 1 divided by n pi 2 pi, it does not matter, it is going to go be 0. So, if n is odd, you will get 2 in the; so, you get 2 divided by n pi, if n is odd; otherwise, b n is 0 right.

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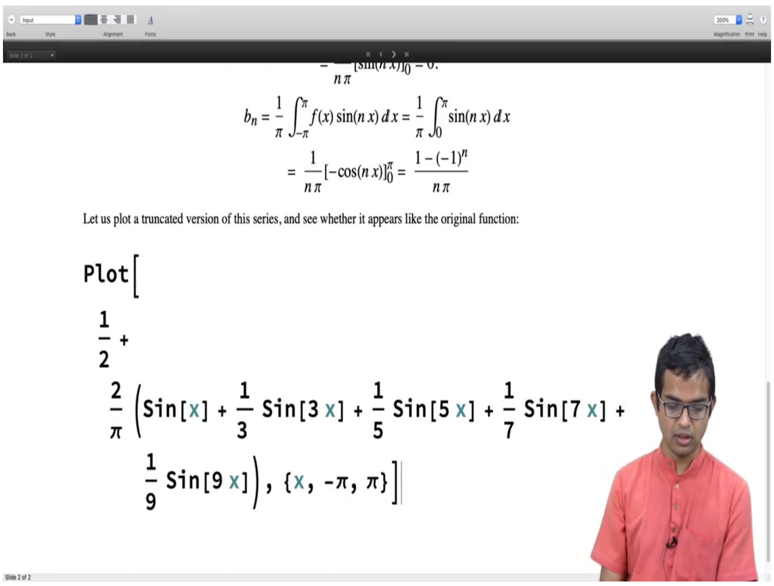

If you put all these together and we can plot this right, the function looks something like this. I have plotted I have kept terms up to you know sine of 9 x right. So, it is going to have only sines in it. There is a constant term half, then you 2 by pi, then you have sine x plus 1 by 3 sine 3 x plus 1 by 5 sine 5 x so on right.

It is 1 over n times sine of n x, where n you know takes only odd values and then, there is this factor 2 by pi outside right. So, if I let us look at what this function looks like if I plot you know truncating it up to this term.

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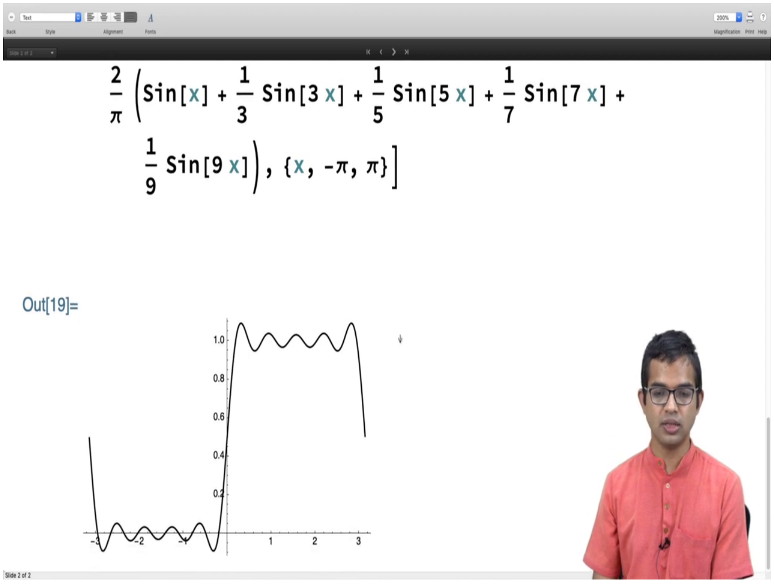


So, you see that indeed, it works out very well right. So, the actual function is something like this. It is 0, in this region and then, it is going to be 1 here. So, let us keep on reducing the number of terms.

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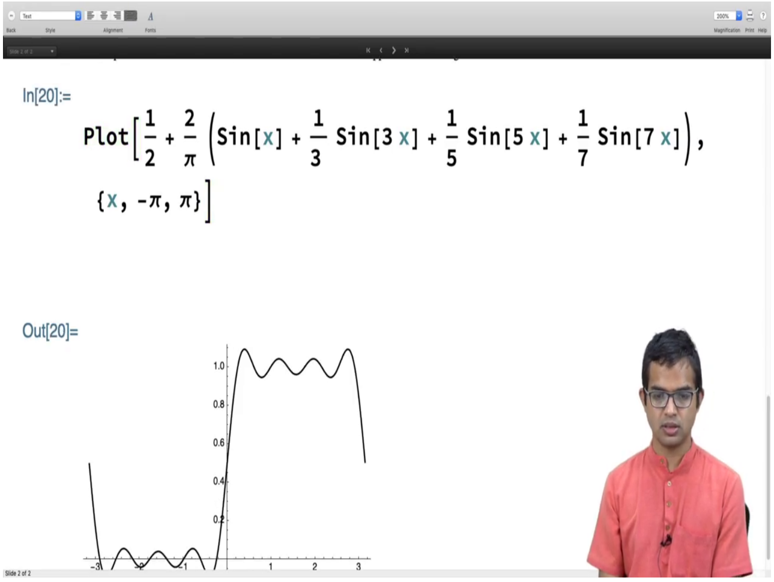


So, if I take this only up to four terms, then it looks like this. Yeah, so, it is not too bad.

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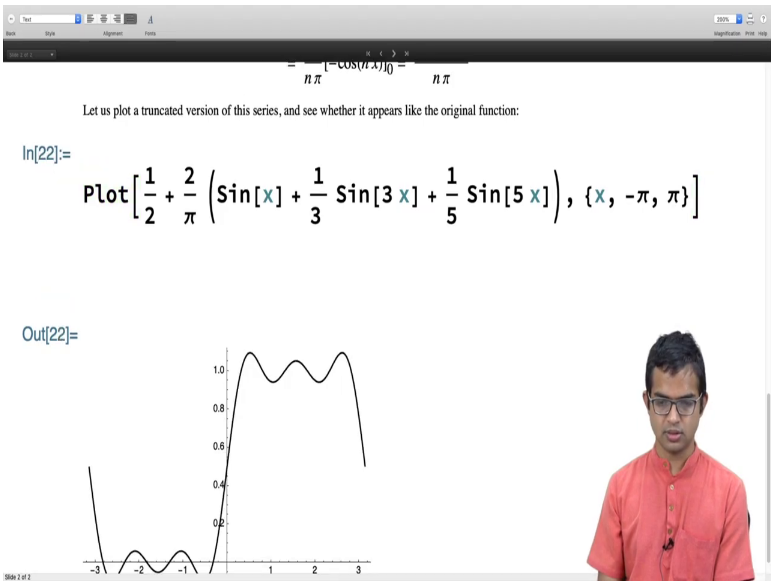


It is already looking like the original function. Let us remove one more term.

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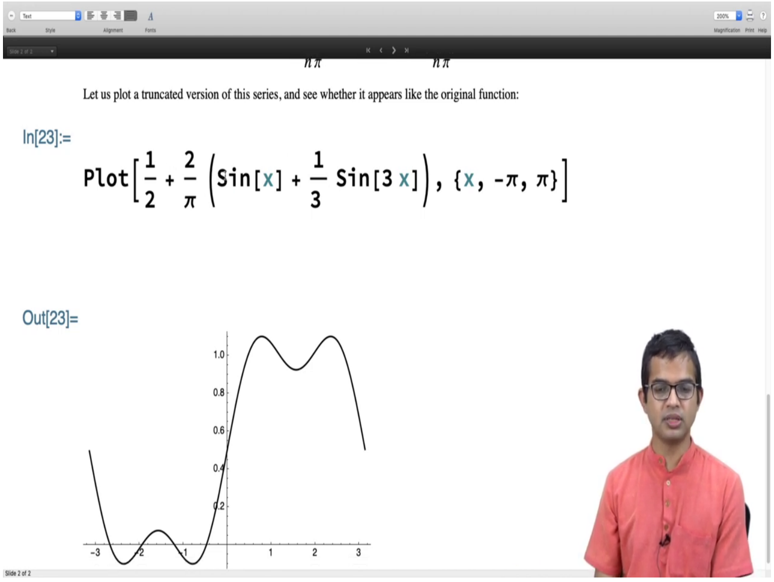


Yeah, it is already you know it is decent considering that you have just kept three terms in this infinite series and it is able to mimic your overall function. Let us see what happens if I remove one more term.

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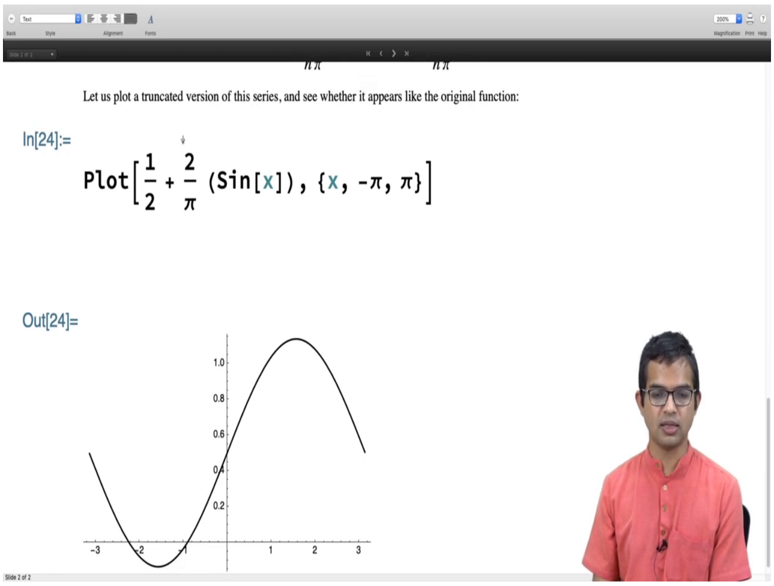

That is just this sine right. So, it is not great; but it has you know the broad properties are in there. Of course, if I remove this also, then that will be just you know that is like the BC term it is called right.

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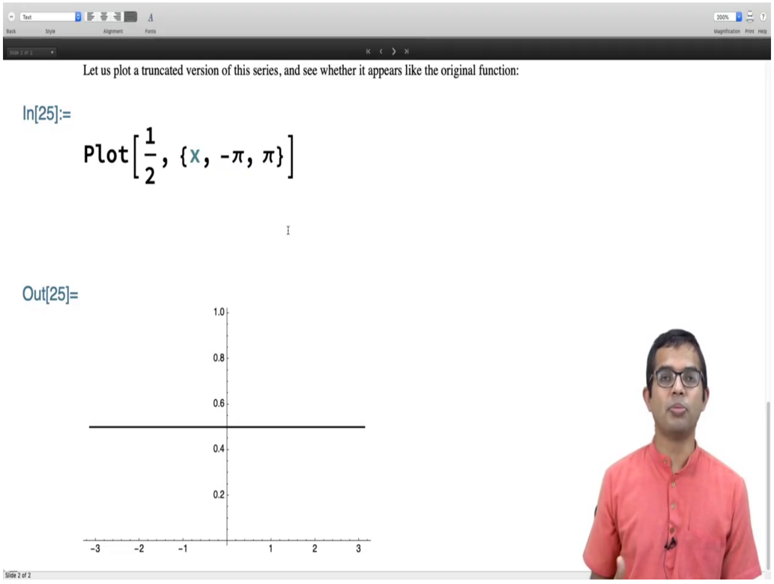


So, it has no; so, it just gives you the average value. Of course, this is bad, but it is what you would expect if you try to take a whole function and put one number to it ok. So, this lecture was an exercise in giving you an operational example of how if you are given a function f of x which is periodic with period 2 pi, you know this prescription, it works right.

So, I have considered a simple example and then, it would be part of the homework to work out many such examples and become very familiar with the you know just the operational aspect itself right. So, that is all for this lecture.

Thank you.