Mathematical Methods 1
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Fourier Series
Lecture – 41
SHM and waves

So, starting with this lecture, we begin a new topic. So, we are going to study Fourier Series, which is basically a way of representing apparently complicated functions in terms of special kinds of sums. So, before we get there, we will recall some familiar concepts related to simple harmonic motion, and you know very basic wave phenomena right, so that is the subject of this lecture right.

So, given our you know another you know comprehensive understanding of linear algebra, now we will also try and you know bring in some of these concepts as we go along right in our discussion of Fourier series. It will turn out to be profitable to bring in some of these concepts of linear algebra right, but all of that is coming a bit later. In this lecture, we are going to describe you know it is more mostly in the nature of recall we are going to talk about Simple Harmonic Motion and Waves ok.

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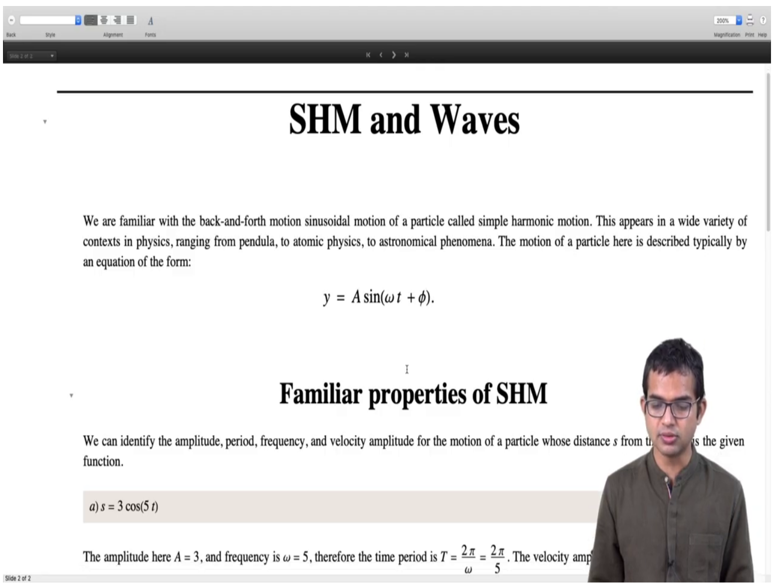

So, we are familiar with simple harmonic motion right. So, it is you know back-and-forth motion which is sinusoidal in nature right. So, I guess the most familiar object which performs simple harmonic motion is a pendulum right, I mean within the small angle approximation right. So, you have oscillatory phenomena of all kinds, but you know it becomes simple harmonic motion if it is you know if we keep terms only it is the lowest order right.

So, in fact, the solution is what I am going to write down. So, it appears in a wide variety of contexts right. So, harmonic phenomena are ubiquitous, and you know and it is often useful to start with the lowest order effect which is simple harmonic motion right, where we consider that the you know potential energy if you wishes has a purely quadratic term or you can think of it as in terms of forces it is going to involve a you know Hooke's law kind of forces involved right, all very familiar stuff right.

And so the reason why you know simple harmonic motion is particularly important is because you know all you know functions with a local minimum right. So, you know the simplest kind of minimum will involve a quadratic type of function right. And therefore, simple harmonic motion is very important to study because this is as simple as it gets.

Of course you can look at you know more complicated scenarios where higher order effects are present sometimes the lowest order is not presented. So, but for our purposes, we will start with you know the simple harmonic motion. And here we will directly look at the solution itself right, I mean the differential equation aspect of this will come later right, so that is also part of our study right.

We are going to study differential equations, but for now let's say we know intuitively or from our earlier studies that the solution for a simple harmonic motion is this y equal to A times sin omega t plus phi right. So, there is this phase factor, there is a; there is a natural frequency associated with this motion. And therefore, there is a time period which one can equivalently define, and there is an amplitude of this motion right all of these are properties.

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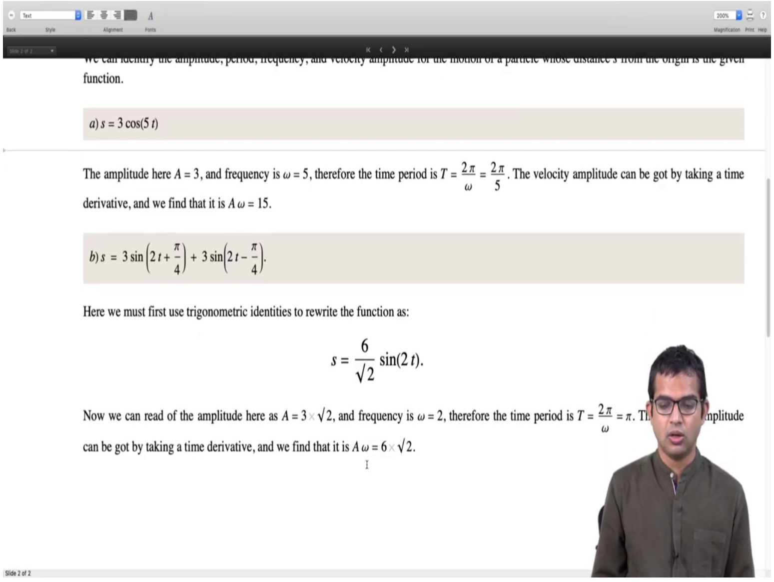

So, since we are all familiar with this we will directly look at a couple of examples and identify these various quantities associated with simple harmonic motion. So, given you know ah the distance of some particle, and if we know s as a function of time let say you are measuring s from the origin.

And if it is given to be s is equal to 3 times cosine of 5 t right. So, it does not matter whether it is cosine or sine right, I mean this is a comprehensive way of thinking about this because you have this phase right.

So, this phase allows you to treat cosines or sines, it does not matter even if there is a way to bring it back to this form. If you think of this as a standard form, of course, it is possible to look at this function 3 times cosine of 5 can bring it to this form, but we can directly read off from here the amplitude here is going to be just 3 right.

This is the amplitude. Then the frequency is so it is the angular frequency which is phi. And therefore, the time period of this motion is 2 pi by omega which is 2 pi by 5. You can also work out the velocity amplitude for this particle right. You take the time derivative ds by dt which will give you know minus 3 times 5.

You know in general it is going to give you if you take a derivative of sin. It is going to bring out this omega, and this is going to become cosine right. And so there is going to be a shift in the phase, but it does not matter so much because we are looking at if you are interested in the amplitude. So, that is velocity amplitude and that is going to be just A times omega which in this case is 15 right. So, the amplitude is always taken to be a positive number that is the magnitude.

And sometimes you know blindly looking at the functional form, you know one may be deceived into concluding that it may not be SHM, but if you are you know carefully you know arrange the terms in a suitable way.

So, like in this example right using trigonometric identities, we can recast this functional form in the following manner. And in fact, we will be able to check that this is 6 by root 2 times sin of 2 t right. So, it looks like there are two terms, but in fact, actually this is just you can pull out the 3, and then add these right.

So, you, you use the identity sign of c plus d plus sign of c minus d, and carefully collect terms, you will get only a simple harmonic motion whose amplitude is 3 times root 2, and whose velocity amplitude you can compute to be 6 times root 2, and of frequency is omega equal to 2, time period is pi right. So, this is all familiar stuff which is you know from high school we have been looking at these kinds of phenomena ok.

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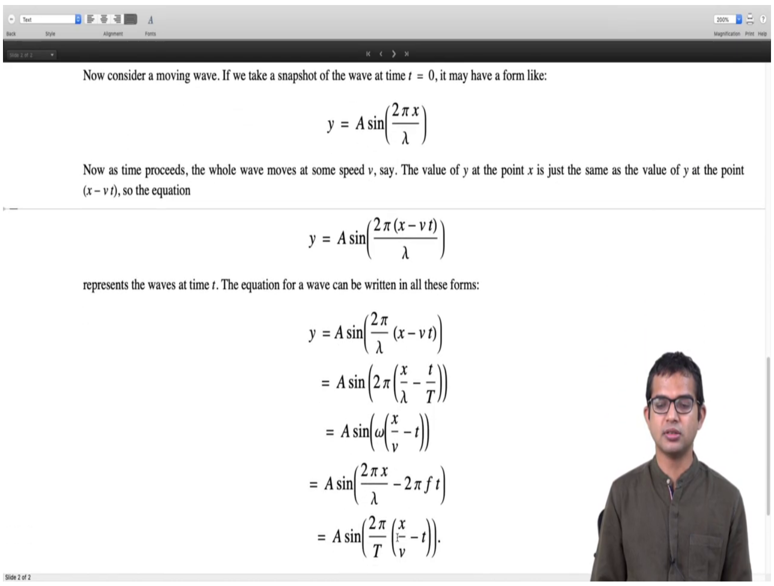

So, we are also familiar with a moving wave right in 1 d. If you take a snapshot of a wave at some time, it may look something like this. It is a sinusoidal curve right. So, to represent a wave, you need to provide an infinite amount of information in some sense.

So, you need a whole function right if you are taking a snapshot at some instant. And then if you wait for some time and take another snapshot after some time t, it is going to look sinusoidal again, but the whole sinusoidal curve would have moved right.

So, if there is some speed that you can associate with this wave, then we can argue that the value of y at the point x is just the same as the value of y at the point x minus v t, because the whole wave has moved.

So, you know using this argument you can actually write down the functional form for this wave you know in terms of x and time. So, there is so in this case I am you know not just there is a wavelength associated with it. So, it is a pure sinusoidal wave that I am thinking of here right.

So, there is a wavelength associated with the wave which is information that you can get by looking at the wave at any snapshot at any instant. And then there is also a dynamical component to this wave. The wave itself is moving as a function of time, so that you know brings in another parameter which is the speed of the wave. So, you know we can think of you know this entire function, it represents a moving wave.

Another way of thinking about a moving wave is to look at what a particular point is doing. If you fix x and look at what that particular point is doing as a function of time, you will see that it is really you know the same as this equation that we had earlier, that is the simple harmonic motion. So, basically what is happening is that you have you know infinitely many different particles if you wish of position x, each of them is performing this simple harmonic motion right.

And another is there is a very precise phase relationship. And therefore, all of them conspire together to create this wave. So, you can either think of this as every point performing a simple harmonic motion in a very you know precise manner or you can as you might see if you plot this as a function of x and t, it is the whole wave which is propagating as a function of time right.

So, there are many different ways in which the same way function can be written as right. So, the and you know all each of these forms is instructive in the sense that it you know it carries information about a different quantity involved right, but they are all connected together as you are familiar right.

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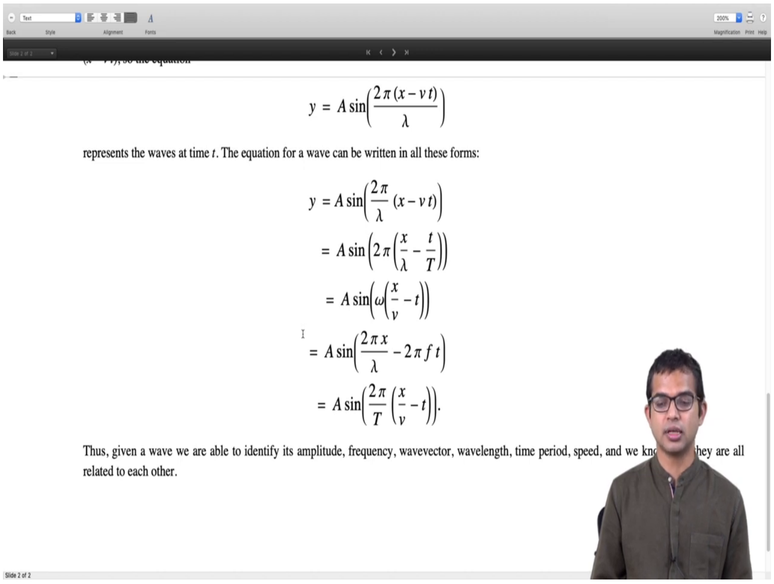

So, you can check that indeed all of these forms are completely equivalent. So, in built in this are notions of you know amplitude, amplitude is of course, the same in all of these explicitly. And then there is a wavelength which is used here sometimes.

The wavelength appears you know in this form in place of the speed it is more convenient to you know put in this kind of a time period, and or you know there is an omega that you can work with or you can work with you know a wave vector and so on right, period, speed, you know the standard relationship between these quantities. And I am just collecting together information that we all already have right.

So, this is like a warm up lecture to Fourier series right. So, we will describe in a step by step manner you know the context for coming up with a Fourier series, and then we will see how to get a Fourier series for a periodic function and so on right. That is all for this lecture.

Thank you.