Mathematical Methods 1
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Linear Algebra
Lecture – 22
Vector Subspaces

So, in this lecture, I am going to describe for you the notion of a Subspace and we will look at some simple consequences of the definition of a subspace ok.

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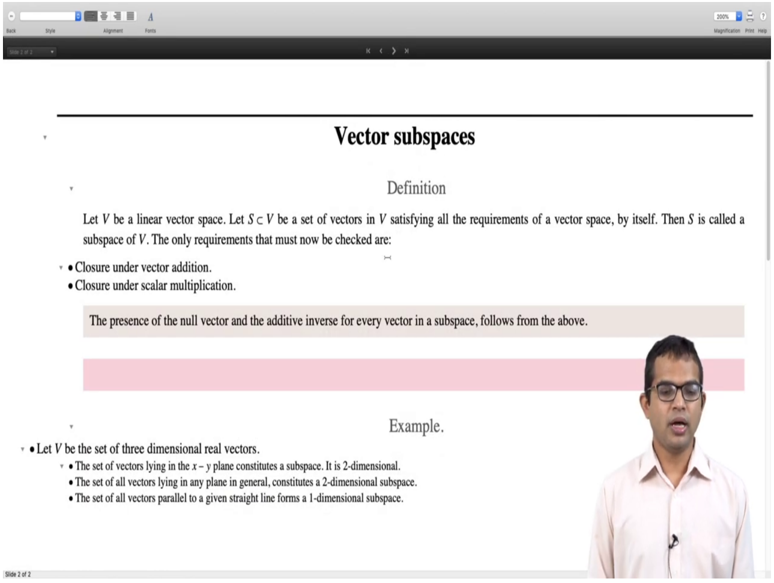

So, you have a vector space V and you know you consider some subset of this vector space. So, as you know V is a set of vectors, if you consider some subset of these vectors if it so happens that the subset itself is a vector space, then you say that that subset is a subspace of V right. So, you have a subset S of vectors which is a subset of V and which satisfies all the requirements of a vector space by itself and then that would be a subspace.

So, it turns out that you need to check only for two requirements right. If there is closure under vector addition and closure under scalar multiplication, then automatically this subset is going to be a subspace right.

So, the presence of the null vector and additive inverse for every vector space in a subspace actually follows from the above definition itself right. How does this happen right? So, if you have; if you have a vector you know suppose there is a vector V which is an element of your subspace, then closure under scalar multiplication means minus V also must be part of this and closure under scale vector addition means V minus V must also be part of this space right.

So, if V and minus and minus V are both part of this space so, the additive inverse exists in this part of the same space, the sum of these two V, minus V must also be part of this which is the null vector. So, the presence of the null vector and additive inverse are guaranteed simply because S is a subset of V and these properties hold for the bigger space, it automatically implies that you know just you have to just check for vector closure under vector addition and closure under scalar multiplication and you are done.

And let us look at a few examples. So, if you look at the set of three-dimensional real vectors as your vector space. Now, the set of vectors lying in the x-y plane, it will constitute a subspace and it is a 2-dimensional space right. You can check that the set of vectors in the x-y plane itself is a vector space as we have seen before right and so, and clearly the x-y plane is a subset of the three-dimensional space and therefore, it is a subspace.

The set of all vectors lying in any plane in general, constitutes a 2-dimensional subspace. You know it does not have to be an x-y plane; it could be any plane right. Again, you can see that you know you can think of any two vectors in that space and that will form a basis and that is going to be a vector space by itself and it is a subspace of the set of three-dimensional vectors.

Also the set of all vectors parallel to a given straight line, maybe for example, x axis, they all form a 1-dimensional space and that too is a subspace ok.

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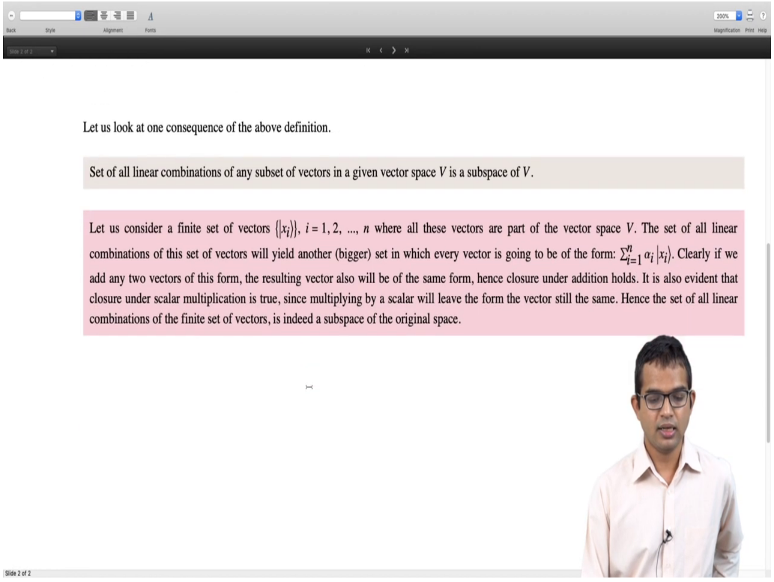

So, let us look at one consequence of the above definition. The set of all linear combinations of any subset of vectors. So, you have an overall space, vector space, you consider some subset of vectors and you make a set which is you know the span of this subset basically the set of all linear combinations of any subset of vectors is a subspace right.

So, let us work this out for a finite set of vectors right, you can also have an infinite set, a subset of which is made up of an infinite set of vectors. We can extend this to this case right after the result using a finite set of vectors.

Let us say you have a finite set of vectors x i, i equal to 1, 2 all the way up to n. Now, if you take a linear combination of all these vectors right that is the set, that is clearly an infinite set right. You are working with a you know the span of this set of vectors will be a space. Now, the question is this a subspace? Right. So, the argument is the following. So, any vector in this space is going to be of this form right by definition which is alpha i times x i linear combination of these vectors x i.

Now, clearly, if you add any two vectors of this form, the resulting vector will also be of the same form. You will have some in place of alpha i, you will have some alpha i plus beta i. If you have taken another vector of the same kind, then alpha i plus beta i, the set of coefficients is also another set of coefficients. So, therefore, that is also a vector of the same form so, it must lie in the same space.

So, closure under addition holds and also closure under scalar multiplication also holds. If you multiply throughout with some scalar, the resulting vector also has the same form so it must also be part of the space. Therefore, it is evident that we have verified closure under vector addition and scalar multiplication and this is evidently a subset of the overall space therefore, this itself is subspace right.

Now, I mean if you had a an infinite you know subset of vectors, then you know you can construct you know you can come up with the idea of a basis, there will be a basis which will have a finite number of vectors right we are working with you know finite dimensional spaces.

So, if you have, if you consider a subset of your overall space that the span of you know the vectors you have considered in this subset will also be representable in terms of a finite number of basis vectors and when you work with that and use the same type of argument, you can show that in the set of all linear combinations of any subset of vectors is a subspace of V right.

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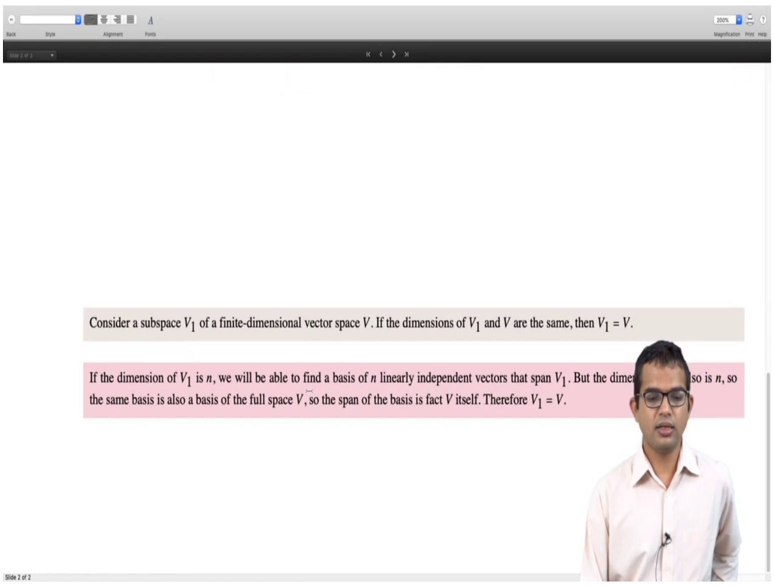

And so, finally, we will see that if there is a subspace V 1 of a finite-dimensional vector space V. If the dimension of V 1 and V 2 are the same, then V 1 is equal to V right. So, if the dimension of V 1 is n right, it is a finite dimensional space so n, we will be able to find a basis of n linearly independent vectors that span V 1.

But the dimension of V is also n and we have the result that any n vectors which are linearly independent will form a basis for V. So, the basis we have form for V 1 it is in fact, a basis for V itself. So, the span of since the span of this so, the span of this basis that we had is in fact, the whole space V itself.

Therefore, the span of you know the V 1 is equal to V because we have seen that the span of this basis will give you V 1, but the span of this basis also going to give you V and the span of its the sense the span of the same basis so, you have V 1 is equal to V right.

It is not something very surprising I guess, but you know if you look at the chain of arguments, it shows you how using the basic properties in a very clean and systematic way, we are able to obtain all these results right in a more or less rigorous way right.

So, although that is not the emphasis of this course, but by and large many of the results that we have covered, we have also tried to be as rigorous as possible without making it you know without letting it go out of hand in some sense right that is not the you know the emphasis of course, of this course is not you know rigor, but when possible why not also be as rigorous as possible ok. So, that is all for this lecture.

Thank you.