Mathematical Methods 1
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Linear Algebra
Lecture - 19
The rank of a matrix and linear dependence

So, we have looked at the concept of a rank and we have seen two different ways in which you know we could think of the rank right. So, one was of course using row reduction, which is perhaps the most foolproof way of getting to the rank of a matrix. You know you do a certain series of transformations and recast your matrix into a form, where you know the bottom few rows.

If there are any 0 rows in the matrix they are all pushed systematically to the bottom few rows and then you count the number of rows which are nonzero and that is the rank of the matrix right. And then we saw that there is another way of you know computing the rank of a matrix which was to locate all you know determinants of all square sub matrices right of a given matrix and find out the largest sized matrix which has a nonzero determinant and that would be the rank of the matrix.

Now, there is a third way of thinking of the rank of a matrix and this connects to the notion of linear dependence and linear independence which we introduced many lectures ago. So, I am going to talk about this third approach in this lecture.

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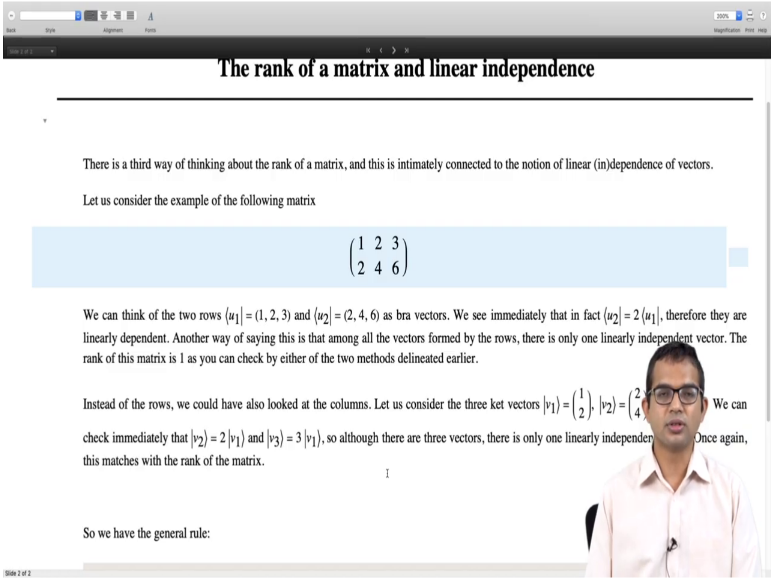

So, the idea is as follows. Just look at any matrix - I have taken a 2 by 3; 2 rows and 3 columns matrix as an example. So, the rank of this matrix is really the number of independent rows right. So, you have. So, there are 2 rows u 1 and u 2 right. We can use our bra notation to represent these rows right. So, there are two row vectors u 1 and u 2.

And we immediately see that in fact u 2 is 2 times u 1, so in fact u 2 and u 1 are not linearly independent, there is only one although you have 2 vectors there is only one linearly independent vector. So, the number of linearly independent rows of this matrix is the rank of this matrix right.

So, the rank of this matrix is 1, because there is only 1 linearly independent row and you can check this right; using some other method like row reduction will essentially come down to this, the fact that there is only one linearly independent vector right. When you do these row operations really you know underlying these row operations is also the property of the linear independence is hidden there, but it was not explicitly stated there right.

So, that is it is just another perspective that we are giving here. And so instead of so one very interesting and you know somewhat non trivial aspect of matrices is the fact that you could also have thought of this in terms of the columns right. You can ask yourself how many columns you know there are these three vectors you know, it is a three column matrix.

So, there are three vectors v 1, v 2 and v 3; 1 2, 2 4 and 3 6. So, you can ask how many linearly independent column vectors are there in this matrix and it turns out that the answer is going to be 1, because we have already seen that there is only 1 linearly independent row in this matrix.

And so it is a theorem really that for any matrix the number of linearly independent rows has got to be equal to the number of linearly independent columns. And so you know in some discussions you might see these defined as the row rank and the column rank and then there is a theorem which shows that the row rank is equal to the column rank.

And therefore there is no need to call it a row rank or column rank; you might as well just call it the rank of the matrix right. So, this is a non-trivial result. But it is a you know for our purposes we will use it like a rule right, I mean we are not going into some you know rigorous you know statement and proof you know type of approach is not the philosophy of this course.

So, it is important to be aware of this result, that the rank of a matrix is not just something that you obtain from row reduction. But it is also the number of linearly independent rows or equivalently the number of linearly independent columns right. So, this is the general rule.

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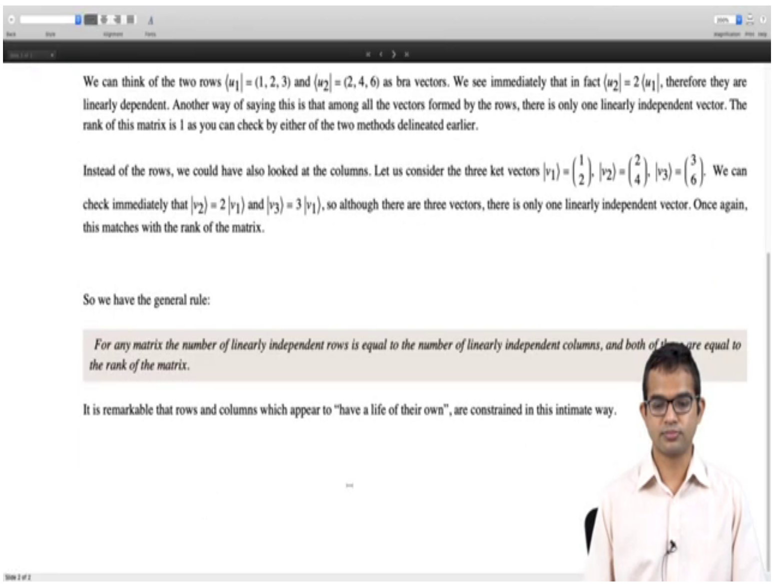

For any matrix the number of linearly independent rows is equal to the number of linearly independent columns and both of these are equal to the rank of the matrix right. So, this is the rule which I would like you to verify right; look at all the, you know different examples which you have already considered.

Use your or make up your own matrices 2 by 3s; 3 by 4 you know a simple matrices not too large and check for yourself that indeed you know the number of linearly independent rows is going to be equal to the number of linearly independent columns and each of them is equal to it is going to be equal to the rank of the matrix, as you would obtain either you know from the determinant approach or from the row reduction approach right. That is all for this lecture.

Thank you.