Mathematical Methods 1
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Linear Algebra
Lecture - 18
Homogeneous equations

So we have seen how given a system of equations, row reduction is a very general and powerful method. But if you have a system of equations which is square in nature right; if the number of unknowns is equal to the number of equations, then you can use the determinant method.

And if the determinant of the matrix of coefficients is non-zero; then you have Cramer's rule, which can be used, right. So, we had a discussion about this.

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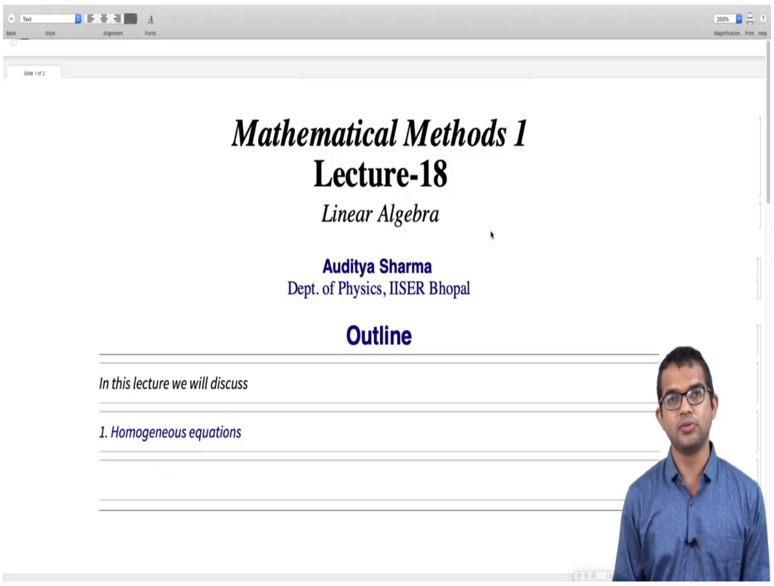

But in this lecture, we will look at a class of linear equations, which are called homogeneous linear equations, ok.

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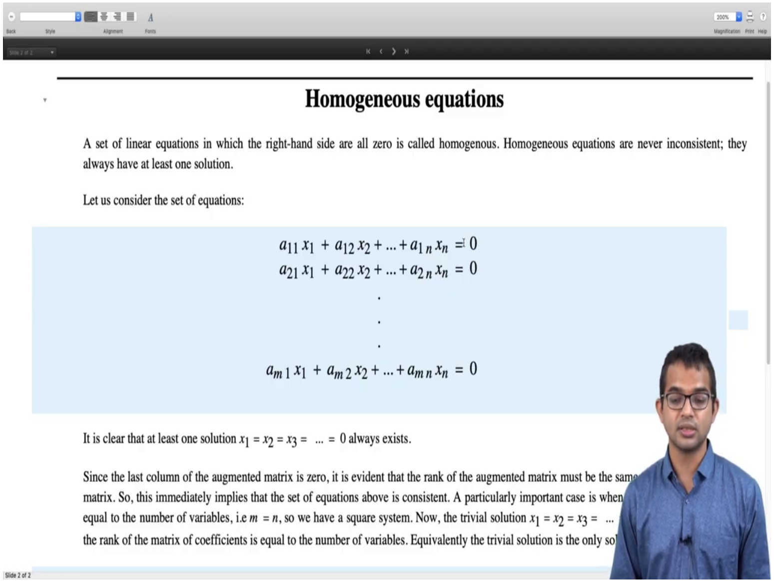

So, a system of equations becomes homogeneous, if all the constant terms on the right hand side are 0, right. So, I am looking at a system of equations of this kind. So, I have a 1 1 x 1 (Refer Time: 01:18) so on is equal to 0; then again the second equation a 2 1 x 1 so on all the variables appear on the left hand side as way usual our convention is and is equal to the constant on the right hand side is 0.

So, every single equation must have a constant which is 0 on the right hand side; if this happens, then you say that such a system of equations is homogeneous. So, an important you know aspect of homogeneous equations is that, they are never inconsistent; they always have at least one solution, which you may think of as a trivial solution right, which is simply put all the x's x 1, x 2, x 3 all of them to be 0.

It is clear that this solution will always exist. So, the question is, is this the it is; is this solution the unique solution or not, right. Sometimes this solution becomes the unique solution for a homogeneous set of equations, so that we will look at now in this discussion.

Now, since the last column of this augmented matrix is zero, it is evident that the rank of the augmented matrix must be the same as that of the coefficient matrix right; because the last column basically contributes nothing, right. So, if you think of this way of computing the rank of a matrix using determinants; so you can for all practical purposes, you can just ignore that the last column has any information at all, it does not have right. So, if therefore the rank of your augmented matrix necessarily is equal to the rank of your matrix of coefficients; so which immediately implies from our earlier discussion that this is a system of equations which is always consistent, right.

So, now a particularly important case is when it is also a square system, not only is it homogeneous the system of equations which is homogeneous right; every homogeneous set of equations is always consistent. Now it does not matter whether it is square or not. But now if this homogeneous set of equations is also square; which means the number of equations is equal to the number of variables then we can find it is determinant, right.

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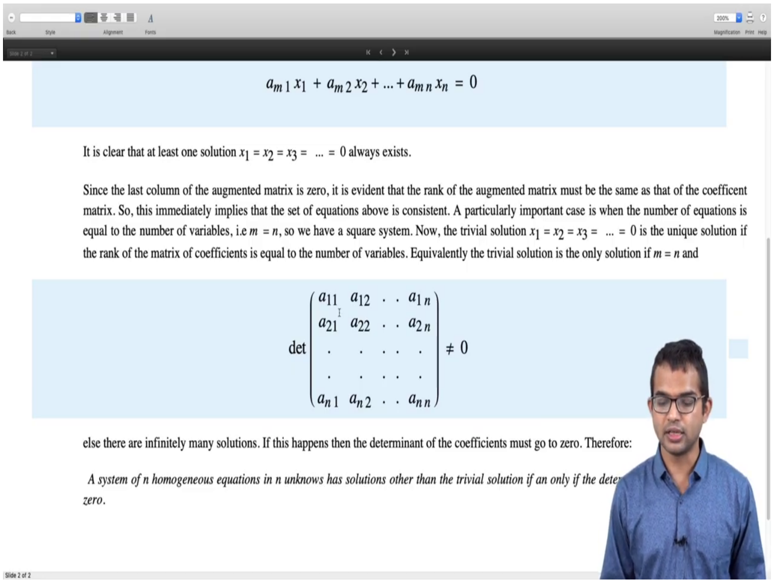

So, we have this condition. So, the trivial solution becomes the only solution; if you are able to find this determinant and find that it is non-zero, right. So, the trivial solution is always a solution. Now, whether, so either this is you know a unique solution or you have infinitely many solutions; you have another in addition to this there are many other solutions, right.

I am thinking that it is a square system, if the determinant is non-zero, then this is a unique solution; the trivial solution is the solution, it is a unique solution. So, if this happens when the determinant of the coefficients must go to zero. So, a system of n homogeneous equations in n unknowns has solutions other than the trivial solution, if and only if the determinant of the coefficients is zero, right. So, this is an important result, right. We will make use of this result, you know in later discussions on you know linear dependence and so on.

So, the whole point is that, we had this b 2 into matrices, determinant, rank and all those, so that we will use some of these ideas in our discussion of abstract linear vector spaces, crucially in the context of linear vector spaces. But before I sign off on this lecture, let me make one more point about new systems of linear equations and how one can give a geometric interpretation to these problems, right.

So, it is best understood in terms of it is a two by two system, right. So, you have a 1 x plus a 2 x equal to c 1 and b 1 x plus b 2 y equal to c 2 let us say, right. So, you can think of this as you know trying to find the intersection point between two lines, right.

You know if there are two lines in a plane, either they intersect or they do not; if they intersect, they intersect at you know a point which is unique, unless you know not only at, if they are parallel and apart, then they will never intersect, that is the case of no solution.

But if they are parallel and coincident right, they in fact intersect everywhere, right. It is a weird kind of intersection, because they are basically the same; there are two equations which lie on top, two lines which lie on top of each other, right. So, something similar happens when you have a larger dimension right.

You can think of some hyper plane and how many of these hyper planes are intersecting and whether they interconnect at the same point or if there are some of these which are coincident; whether which is when you have infinitely many solutions with, you know the number of degrees of freedom depends on how many of them are overlapping and so on, right.

You can imagine this up to three dimensions it is easy to imagine, but beyond three dimensions it is not something that you can imagine geometrically; but you know it is really the same idea which extends. And then we have seen that algebraically using you know these are the idea of the rank or the idea of the determinant, if it is a square system and so on; we have a you know prescription for telling what is the fate of a system of linear equations, right.

So, now, we will conclude this discussion on systems of linear equations; but we will use some of these properties and connect back to linear dependence, linear independence and then go back to vector spaces; that is the path ahead.

Thank you, that is all for this lecture.