Mathematical Methods 1
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Linear Algebra
Lecture - 16
Cramer's rule

So, we have seen how with the aid of row reduction and back substitution, we can find the solution for a set of equations if it exists, right. Whether it exists and when it exists what the solution is, all of these can be obtained with the aid of row reduction, right.

If when you have a set of equations, which is you know which gives you a square matrix right; when you have a you know the number of equations is equal to the number of unknowns; in that case there is a an elegant you know way of obtaining the solution using something called the Cramer's rule, using determinants which uses determinants, right. So, that is the subject matter of this lecture, ok.

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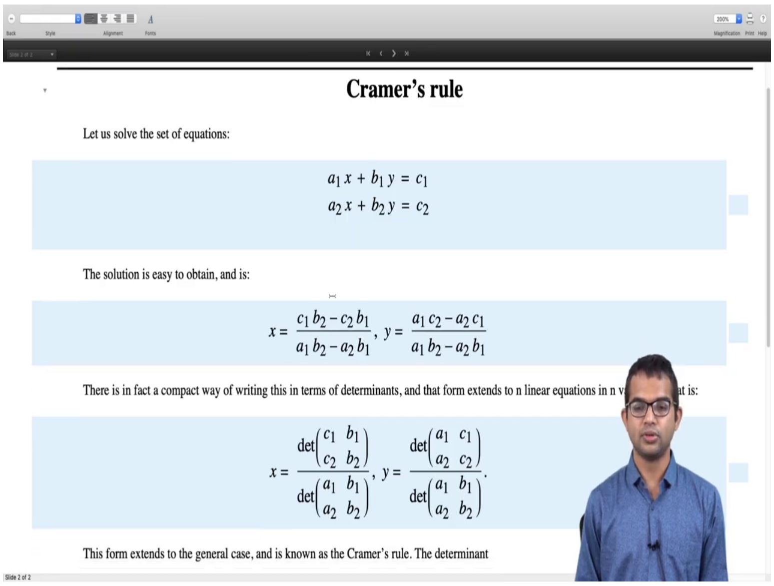

Let us start with a 2 by 2 system of equations, right. So, I have a 1 x plus b 1 y is equal to c 1 and then a 2 x plus b 2 y equal to c 2, right. The solution is of course, easy to obtain; x is equal to you know the c 1 b 2 minus c 2 b 1 divided by a 1 b 2 minus a 2 b 1 and y is a 1 c 2 minus a 2 c 1 divided by a 1 b 2 minus a 2 b 1, right.

So, notice that the denominator of both you know the expression for x and y involves a 1 b 2 minus a 2 b 1, right. So, this is actually nothing, but the determinant of your coefficient matrix, right. So, in fact you can rewrite this solution in a compact way, which lends itself to generalization to n linear equations in n variables. So, x is determinant of, right. So, you form this coefficient matrix a 1 a 2, b 1 b 2 and you put that in the denominator and find its determinant, right.

But if you are interested in finding x; so in place of you know a 1 and a 2 which are the coefficients corresponding to x. So, whichever variable you are interested in, you implant the right hand side the coefficient c 1 and c 2 in place of the coefficients for you know, c 1 and c 2 in place of a 1 and a 2 comes in here, right.

So, this is what we have here, right. You have to compare between the matrices in the numerator and the denominator. So, the only thing that is different between the numerator and the denominator is, in place of a 1 a 2, you have c 1 c 2. And likewise in place of a 1 a 2 here, I have. So, in place of b 1 b 2 here, I have c 1 c 2, right.

So, wherever you have you know coefficients corresponding to a certain variable that you are interested in, those coefficients have to be omitted and in place you bring in the constants from the right hand side here, right; this is the prescription and this is going to, this works for you know n linear equations in n variables. I have shown you for 2 by 2 explicitly, but this form extends, right. So, I am not going to write down the general form explicitly, you can work this out.

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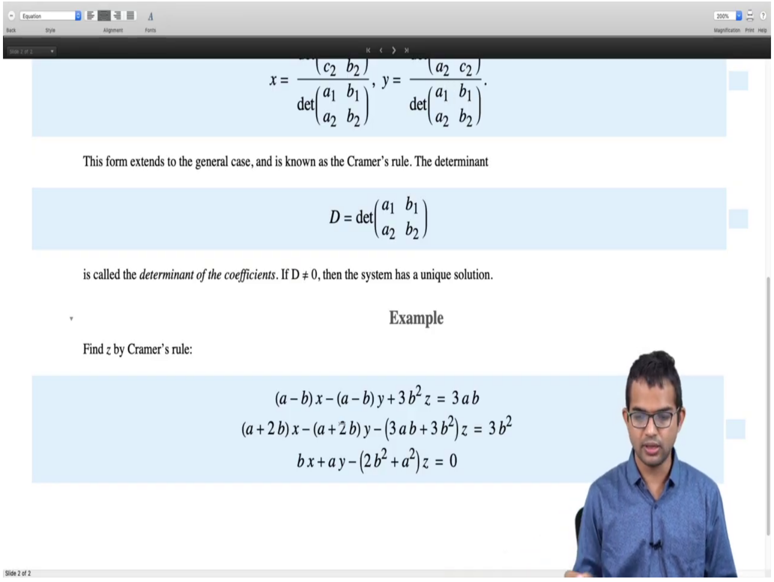

So, let me just state this again, right. So, this is what is called the Cramer's rule. So, the key point is that, you must look at the determinant of the coefficients right; the determinant of the coefficient matrix is also called the determinant of the coefficient. So, only if this determinant D is not equal to 0, Cramer's rule holds, right.

So, that is when you have a unique solution, right. So, this is actually a criterion for whether a system of n linear equations in n variables has a unique solution, right. If there is a unique solution, Cramer's rule can give it to you, right.

Now, if the determinant is 0, then you know you have to be careful and go about studying it you know separately. So, that can be the subject of a later lecture, which is which we will follow; but let us look at what happens when D is not equal to 0 here, then you have the Cramer's rule.

So, to end this lecture, I am going to give you one example right; how of all these can be exploited and then in conjunction with the properties of determinants that you have already seen. So, let us look at this, you know apparently complicated looking set of equations; but it is, so it gives us a 3 by 3 matrix. So, let me just pull out you know these coefficients. So, let me first write down. So, here the goal is to find z. So, I have a minus b.

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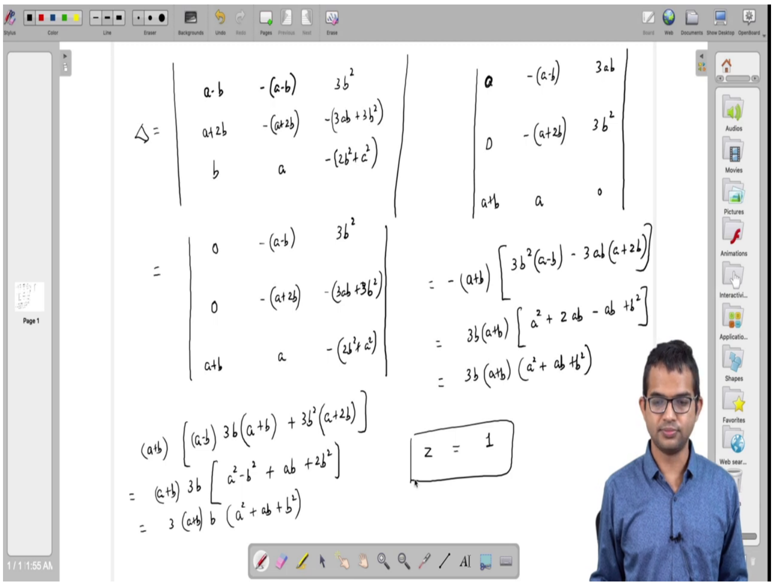

Let me write this down, I have a minus b minus a minus b; then I have 3 b squared 3 b squared. And likewise I have a plus 2 b, a plus 2 b minus a plus 2 b and then I have minus 3 a b plus 3 b squared, minus 3 a b plus 3 b squared with the brackets covering both terms.

And then I have b a and b a and minus 2 b squared plus a squared, minus 2 b squared. So, I need to find this determinant and then I also need to find another determinant. So, this I can call it delta. I also need to find this other determinant, ok. Let me try, let me compute this.

So, the key observation that I make here is, you know I have a minus b, I have minus a minus b; I have a plus 2 b and minus a plus 2 b. So, whenever I see something like this, the immediate suggestion that I have is to subtract the second column from the first column or to add the two; if I add the two, then I am going to get 0's, right. So, 0's are good things to have when you are evaluating determinants.

So, if I add the second column to the first column. So, I have 0, I have 0, then I have a plus b, then I have minus a minus b, minus a plus 2 b, then I have 3 b squared, minus 3 a b plus 3 b squared, then I have a, then I have minus 2 b squared plus a squared. So, actually these last two quantities do not even matter.

So, I see that if I go along the first column and I just have this one term, it is just a plus b times the product of these two minus the product of these two. So, which will give me, I can pull out. So, let me write this as a minus b, the minus sign cancels; a minus b times 3 b times a 3 b times a plus b.

Then I have a minus, there is a minus sign, so that becomes plus 3 b squared times a plus 2 b. So, which is a plus b times 3 b times a squared minus b squared plus a b plus 2 b squared. So, my answer is 3 times a plus b times b into a squared plus a b plus b squared. So, the other determinant that I need to evaluate is. So, to get the answer for z, it is going to be, so this determinant; I will have to find, so a minus b.

So, in fact the first two columns are the same. So, I will do exactly the same transformation, which is to add the first column into the second column. So, I might as well write this as 0, 0, a plus b and then this will be minus a minus b, this will be minus of a plus 2 b and I have a. And only the third column changes and the third column is 3 a b, 3 b squared and 0; 3 a b, 3 b squared and 0. I need to evaluate this determinant.

So, once again this is just equal to a plus b. So, I have to find the determinant of this 2 by 2 matrix; there are these two minus signs which are sticking out, so which I can actually pull out. So, I have a plus b, overall minus sign I will pull out. So, then I have 3 b squared into a minus b minus 3 a b into a plus 2 b, right. So, which is equal to, so again I can pull out 3 b; and so, in fact I also have a minus sign. So, I have 3 b into a plus b into a squared plus 2 a b.

So, the second term becomes positive and the first term is negative. So, minus 3 I have pulled out a 3 b. So, I have a minus a b plus b squared; so, which is equal to 3 b into a plus b into a squared plus a b plus b squared. So, we have all these nice simplifications. And so, the answer is z is equal to 1, very simple final answer right; it looks like a mess, but there is a way to get to the final answer using determinants using the Cramer's rule.

So, that was just an example to illustrate how this may play out in a real setup, that is all for this lecture.

Thank you.